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Risk-aware Strategies for Optimal Participation of Parking Lots in Day-ahead Electricity Markets

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Abstract. The decarbonization of the mobility industry rules the massive deployment of charging infrastructures worldwide. Frequently, charging points are installed by private companies and entities, which pursue a monetary profit through providing charging services. It is therefore interesting looking for business opportunities that maximize the monetary profit of such infrastructures. In this regard, large-scale parking lots can partake in wholesale electricity markets, where they can buy or sell energy, thus actuating as a high-capacity virtual battery storage system. In this paper, a novel methodology for optimal participation of parking lots in day-ahead electricity markets is developed. The new proposal contributes with two main advantages compared to other similar research. On the one hand, we properly consider both charging and discharging modes of electric vehicles, which enable full participation in electricity markets as load or generator. On the other hand, an adaptive uncertainty-aware model is proposed and accommodated into the developed tool, thus casting as a unified framework that allows adopting both risk-averse and risk-seeker operational strategies. To this end, a tailored mathematical model is proposed, wherein uncertainties and binary variables related to operational statuses of batteries are properly accommodated via an original Benders' decomposition algorithm. Moreover, different improvement strategies are proposed, thus resulting in a practical tool with potential fields of application in industry. A number of numerical results are provided to validate the new tool, as well as analyse how the adoption of risk-averse or risk-seeker strategies affects the strategic participation of parking lots in electricity markets. Furthermore, the developed methodology is compared with other conventional approaches proposed in the literature. The new proposal is further validated through a sensitivity analysis regarding the parking size and charging/discharging prices, whereas results obtained in 50 different market instances confirm that the developed tool performs well in a number market environments.

Keywords. Electricity markets; Parking lots; Strategic bidding; Risk-aware optimization

Nomenclature

Indices (sets)	
$l(\mathcal{L})$	Loads (demand)
$g(\mathcal{G})$	Generators
$t(\mathcal{T})$	Time
Parameters and functions	
$\alpha_{l,t}$, $\alpha_{g,t}$	Utility/offer of the l^{th}/g^{th} load/generator at time t (€/MWh)
$\underline{(\cdot)}$, $\overline{(\cdot)}$	It denotes the minimum/maximum value of a variable or parameter
η	Roundtrip efficiency (pu)
$\mathbb{E}(\cdot)$	Expected value
Γ	Uncertainty budget (pu)
$\Delta^{\downarrow}r$, $\Delta^{\uparrow}r$	Lower/upper variation bands for the r^{th} uncertain parameter
θ, κ	Iteration counters
$C^{P,c}$, $C^{P,d}$	Cost of charging/discharging electric vehicles in the parking lot (€/MWh)
M	Positive number (normally a large number)
tol	Convergence threshold ($tol \in \mathbb{R}_+$)
ρ	Penalty term ($\rho \in \mathbb{R}_+$)
Primal decision variables	
$p_t^{P,c}$, $p_t^{P,d}$	Charging/discharging power of the parking lot at time t (MW)
$p_{l,t}$, $p_{g,t}$	Power demanded/generated by the l^{th}/g^{th} load/generator at time t (MW)
α_t^P	Bidding strategy of the parking lot at time t (€/MWh)
$y_t^{P,c}$, $y_t^{P,d}$	Charging/discharging status of the parking lot at time t (binary)
SOC_t^P	Energy available in the parking lot at time t (MWh)
u	Auxiliary binary variable (binary)
β	Auxiliary variable for approaching costs (-)
$\mathrm{on}_t^{P,c}$, $\mathrm{on}_t^{P,d}$	It indicates ON transition to charging/discharging modes (binary)
$\mathrm{off}_t^{P,c}$, $\mathrm{off}_t^{P,d}$	It indicates OFF transition to charging/discharging modes (binary)
Dual decision variables	
λ_t^{spot}	Market price at time t (€/MWh)
$\underline{\mu}_{l,t}$, $\overline{\mu}_{l,t}$	Linked to lower/upper bounds of the l^{th} demand at time t (€/MWh)
$\underline{\mu}_{g,t}$, $\overline{\mu}_{g,t}$	Linked to lower/upper bounds of the g^{th} generator at time t (€/MWh)
$\underline{\mu}_t^{P,c}$, $\overline{\mu}_t^{P,c}$	Linked to lower/upper bounds of the parking lot (charging mode) at time t (€/MWh)
$\underline{\mu}_t^{P,d}$, $\overline{\mu}_t^{P,d}$	Linked to lower/upper bounds of the parking lot (charging mode) at time t (€/MWh)

1 - Introduction

1.1 - Background & motivation

Sales of electric vehicles (EVs) exceeded 10 million in 2022, and expects to continue through 2023 [1]. This data sets forth a clear tendency towards massive integration of EVs worldwide. The integration of EVs must be accompanied by a proper deployment of charging infrastructures, especially fast and upcoming super-fast charging points [2]. The installation of charging points requires a high investment from private companies which pursues a monetary profit by offering charging services. Typically, EV owners pay for charging their vehicles, thus supposing the main (and typically unique) monetary income. However, secondary monetary activities can be leveraged in order to boost up the profit of charging infrastructures. Note that increasing monetary incomes for private-owned charging points brings up two principal advantages:

- Encourages private investment in charging infrastructures, thus increasing the number of charging points available, thus easing the penetration of EVs.
- May allow reducing the charging cost, thus encouraging users to renew their vehicles opting for electric technologies.

It is therefore of great interest making charging infrastructures economically viable. Indeed, it encourages the use of EVs, increases the profit of charging owners and reduces the cost for users.

Large-scale parking lots (PLs) can trade energy in electricity markets thus accessing to market-driven economic opportunities [3]. Indeed, EVs provide large-scale storage capacity which may be harnessed for energy arbitrage in electricity markets. Thereby, it results interesting studying bidding strategies so as to maximize the economic profit obtained by participating in electricity markets.

Bidding strategies for PLs differ from conventional strategies already developed for demands and generators. Certainly, a PL may partake as a load (charging mode) or as a generator (discharging mode), thus being viewed as a virtual battery at market level. Nevertheless, unlike to conventional stationary battery systems, available storage capacity within PLs varies dynamically due to the number of vehicles plugged any time instant as well as their particular features. Owing to these reasons, bidding strategies for PLs require developing novel mathematical models as a response to the particularities of these installations. This paper focuses on this issue.

1.2 - Bidding strategies for PLs

Market players can partake as price-takers or price-makers [4]. In the former, a generator or load assumes that spot market price is fixed onwards and it decides on its own strategy assuming this fixed price. This way, price-taker strategies typically treat the market price as a parameter rather than a decision variable. In contrast, price-maker strategies (also known as strategic bidding/offering) aim at impacting on the spot market price. This way, both the offering strategy and market price are actually decision variables in price-maker strategies.

Most of the works related to the participation of PLs in electricity markets cast as price-taker strategies. In [5], a stochastic-based price-taker strategy is described for participation of EV aggregators in day-ahead electricity markets. In this case, the spot price is modelled via scenarios whereas features of EVs are modelled using confidence bounds (robust optimization). Similarly, the authors in [3] consider real-time markets, whose price is modelled via robust optimization while the day-ahead spot price is treated using scenarios, as in [5]. On the other hand, ref. [6] incorporates the conditional value at risk (CVaR) into the objective function in order to orient the strategy to a risk-averse perspective. Likewise, Sadati et al. [7] describe a bi-level operational model for PLs considering charging-discharging routines. This methodology considers uncertainties via scenarios and CVaR. Moreover, this methodology does not constitute a strategic participation model as PLs are taken as followers within a Stackelberg game framework. Ref. [8]

includes a penalty term to consider user preferences of users regarding charging and idle periods. In [9], the role of stationary batteries to support the operation of PLs is investigated. To this end, a stochastic quadratic model is raised and further linearized to be tractable in practise.

Focusing on the mathematical formulation, the references above render the optimal bidding strategy as a Mixed Integer Linear Programming (MILP) model, being easily solvable using off-the-shelf solvers. Nevertheless, other references develop linear programming approaches in which binary variables are omitted. Han et al. [10] investigate a grouping strategy for clusters of EVs. In particular, EVs are grouped according to their charging time window and then treated as stationary batteries in electricity markets. In [11], a scenario-based bidding strategy for EV aggregators is proposed, in which regression models are adapted to predict the realization of uncertain day-ahead and real-time prices. More specifically, this reference establishes an auxiliary optimization problem to determine the parameters of the forecasting technique, which casts as a quadratic optimization model. The strategy developed in [12] incorporates a real-time adjustment module responsible of correcting possible deviations of uncertain parameters. In [13], a risk-averse stochastic bidding strategy is developed, which raises as a nonlinear programming model further linearized using different tricks.

Enabling discharging mode of EVs (vehicle-to-grid mode) supposes a key point when describing an optimal biding strategy for PLs. Note that this is mostly important in the mathematical modelling as well as to disclose opportunities in electricity markets. Indeed, if the discharging mode is disabled, the PL operates as a pure load and incomes from exporting energy are inaccessible. In this sense, albeit most of the studied references consider both operational modes, some exceptions should be remarked [3, 11-13]. This way, charging-oriented bidding strategies are close to conventional ones already developed for elastic demands.

While the references above focus on price-taker strategies, only a few references deal with price-maker strategies for PLs. Existing literature devote on risk-averse strategies, for which different uncertainty models are considered. In most cases, risk aversion is modelled via scenarios and the inclusion of the CVaR term in the objective function. Thus, [14] considers market-level uncertainties such as wind production, total demand or failures of generators. On the other hand, [15] and [16] focus on the cooperation between wind producers and EVs. These references highlight the benefits of engaging in bilateral energy transactions for both wind producers and EVs. To this end, a bi-level optimization framework is developed which is solved using genetic algorithms. In contrast to these stochastic-based approaches, a distributionally robust model is proposed in [17], in which uncertainties arise from the behaviour of EVs, and are in turn treated as an uncertain demand parameter.

1.3 – Research gaps

The references studied in the previous section focus on risk-averse strategies, for which either stochastic or robust approaches are considered for modelling uncertain parameters. Typically, market prices and EV-related parameters are treated as uncertainties in price-taker strategies, while price-maker focus on the behaviour of EV users (e.g. arrival and departure times) or other exogenous parameters (e.g. wind production or outages). In this regard, there is a clear lack of risk-seeker strategies as a way to compare both risk-aware perspectives. Indeed, risk-seeker strategies may provide access to higher profits if the operator accepts the risk inherent to assuming optimistic realization of uncertainties. In this sense, instead of seeking for a robust scheduling strategy, the charging owner may assume a risk and plan a charging-discharging strategy under favourable conditions.

It is remarkably that existing price-maker strategies only consider charging modes of EVs. In this sense, the impact of vehicle-to-grid is neither modelled nor studied. This simplification does not allow evaluating the benefits of enabling discharging capability in electricity markets. Note that enabling the discharging mode of EVs requires the installation of bidirectional chargers with the consequent investment. In this sense, existing strategies do not allow evaluating the profitability of installing bidirectional chargers, limiting the decision capability of charging owners.

Neglecting the discharging capability of EVs in price-maker strategies have, in turn, some advantages from a mathematical point of view. Indeed, properly modelling charging and discharging states require the use of binary variables [18]. It is worth noting that price-maker strategies typically render as bi-level Stackelberg games (see [19]), in which the PL acts as the leader and the market as the follower. In most of cases, the follower-related problem is reduced to its Karush-Kuhn-Tucker (KKT) conditions, for which the model needs to be continuous and convex and therefore the inclusion of binary variables supposes a problem. It hinders the consideration of discharging modes in price-maker strategies.

On the basis of the above, there is a clear lack of robust models for strategic participation of PLs in wholesale electricity markets, including a proper modelling of their charging-discharging statuses via binary variables in order to avoid physically unrealizable results.

1.4 - Contributions & paper organization

This paper aims at solving the limitations discussed above. To this end, a novel price-maker bidding strategy for PLs in day-ahead electricity markets is developed, in which risk-averse and risk-seeker strategies are modelled in a unified framework. This idea allows PL owners to fairly compare both risk-aware strategies and take decisions in consequence, unlike to the existing approaches in which only risk-averse strategies are contemplated. In this sense, we consider typical EV-related uncertainties via confidence bounds. The new approach allows to evaluate different solutions according to the risk level, resulting in an adaptive approach according to the level of risk assumed by the manager. Note that this parametrization principle results valuable in decision-making strategies under uncertainties, as pointed out in [20].

On the other hand, the developed approach properly models both charging and discharging modes of EVs. To this end, binary variables are incorporated into the bi-level strategic bidding structure using an original Benders' decomposition algorithm [21]. In this sense, the new proposal supposes (to the best of our knowledge) the first attempt to considering the role of PLs as full-capable virtual battery storage in price-maker strategies.

With the aim of validating the new bidding strategy and shows its potential and practical implications, it is profusely tested in a large number of cases and scenarios, analysing the impact of the risk level in both monetary and energy aspects. For the sake of simplicity, Table 1 compares the main features of the methodology proposed in this paper with its closest related literature analysed above.

Table 1 – Comparison of the proposed methodology with others in the literature

Ref.	**Vehicle-to-grid**	**Bidding strategy**	**Uncert. Model.**	**Risk strategy**
[3, 11, 13]	No	Price-taker	Scenarios	Risk-averse
[5-7]	Yes	Price-taker	Scenarios	Risk-averse
[14-16]	No	Price-maker	Scenarios	Risk-averse
[17]	No	Price-maker	Robust	Risk-averse
Present	**Yes**	**Price-maker**	**Robust**	**Risk-averse/seeker**

In the rest of this paper, Section 2 describes the market structure and states the strategic bidding problem. Section 3 develops the proposed solution strategy via decomposition. A variety of accelerating techniques and other improvements are described in Section 4, with the objective of reducing the computational burden of the developed approach. Different experiments with results are reported and discussed in Section 5. Finally, the paper is concluded with Section 6.

2 - Preliminaries

2.1 - Market structure

This paper focuses on day-ahead electricity markets, in which generators and loads submit their offers/bids to trade energy throughout the following day. In particular, we consider that PLs can partake in the market selling or buying energy (discharging and charging modes, respectively) under day-ahead market spot prices revealed by the market operator (namely λ_t^{spot}).

It is assumed that vehicles plugged at the parking pay for charging and are paid for discharging, both at fixed costs $C^{P,c}$ and $C^{P,d}$, respectively, thus following the same market logic that loads and generators (see Fig. 1). In this sense, the profit obtained by the PL will be the balance between the incomes and expenditures for offering charging services as well as the monetary result of trading energy in the market.

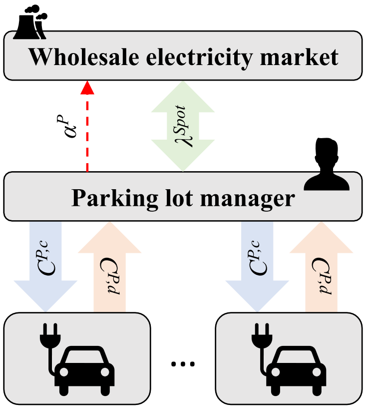

Fig. 1 - Market structure considered in this paper

2.2 - Basic notations and assumptions

A set $\mathcal{L}$ of elastic demands partake in the wholesale electricity market by submitting their hourly utility price $\alpha_{l,t}$, looking for buying energy up to a limit $\overline{p}_{l,t}$. Likewise, a set of generators $\mathcal{G}$ (encompassing both dispatchable and renewable units) partake in the market by selling energy at offering prices $\alpha_{g,t}$. Generators cannot offer more than a cap given by $\overline{p}_{g,t}$. In case of dispatchable generators, this bound corresponds to their corresponding rated power, while in case of renewable generators, they are limited to forecasted potential generator strongly affected by weather parameters such as wind speed or solar irradiance [22]. This way, the total number of players in the market is $|\mathcal{L}| + |\mathcal{G}| + 1$, thus counting with one PL which is the scope of this paper.

A number of wholesale markets $|\mathcal{T}|$ is cleared daily. In the case of PLs, hourly markets are coupled by inter-temporal energy stored constraints. However, we assume that the rest of players cannot store energy and therefore perform decoupled market strategies.

We do not consider battery degradation in our formulation since EV owners are actually compensated by discharging at a price $C^{P,d}$. In this sense, we assume that EV owners agree with enabling vehicle-to-grid services and therefore the parking manager can discharge batteries at her own benefit. Note that this simplification is feasible in real cases, where the EV owner is the unique responsible of maintaining her own batteries and the responsibility of the parking manager is limited to provide charging services.

In order to catch real-life day-ahead wholesale market environments, the grid is not included in the model. This way, we preserve the privacy of the network operator, whose role is typically assumed by a stakeholder different to the market regulator. In such a case, parameters of the network are not revealed to market players, who need to decide on their bidding strategies without network parameters knowledge. Note that this assumption is common in other references related to wholesale electricity markets (e.g. see [23]).

2.3 - Deterministic problem statement

When a PL partakes in wholesale electricity markets, it pays/is paid by buying/selling energy. As such, the PL is paid by offering charging services while has to pay to EV users for enabling vehicle-to-grid capability. Thereby, the daily profit obtained by the PL is given by:

$$Profit = \sum_{t\in\mathcal{T}}\{p_t^{P,c}(C^{P,c} - \lambda_t^{spot}) + p_t^{P,d}(\lambda_t^{spot} - C^{P,d})\} \tag{1}$$

The strategic market participation of generators or loads in wholesale electricity markets can be stated as a bi-level Stackelberg game [19], wherein the strategic player acts as the leader while the market is cleared within the follower problem. In the follower problem, the market operator seeks for clearing the market with the aim of maximizing the collective welfare, given by [24]:

$$WF = \sum_{t\in\mathcal{T}}\{\sum_{l\in\mathcal{L}}\alpha_{l,t}\,p_{l,t} - \sum_{g\in\mathcal{G}}\alpha_{g,t}\,p_{g,t} + \alpha_t^P(p_t^{P,d} - p_t^{P,c})\} \tag{2}$$

As seen, (2) includes both charging and discharging modes of the PL. Keeping the above in mind, the strategic participation of a PL in day-ahead electricity markets reads as a bi-level optimization model, as follows:

$$\max_{\alpha_t^P} Profit \tag{3a}$$

Subject to:

$$\alpha_t^P \geq 0; \forall t \in \mathcal{T} \tag{3b}$$

$$\lambda_t^{spot} \in \underset{\boldsymbol{p}^P,\mathrm{SOC}_t^P,\boldsymbol{y}^P,\boldsymbol{p}^G,\boldsymbol{p}^L}{\operatorname{argmax}} \; WF \tag{3c}$$

Subject to:

$$\sum_{l\in\mathcal{L}}p_{l,t} - \sum_{g\in\mathcal{G}}p_{g,t} + p_t^{P,c} - p_t^{P,d} = 0 : \lambda_t^{spot}; \forall t \in \mathcal{T} \tag{3d}$$

$$0 \leq p_{l,t} \leq \overline{p}_{l,t} : \underline{\mu}_{l,t}, \overline{\mu}_{l,t}; \forall l \in \mathcal{L} \wedge t \in \mathcal{T} \tag{3e}$$

$$0 \leq p_{g,t} \leq \overline{p}_{g,t} : \underline{\mu}_{g,t}, \overline{\mu}_{g,t}; \forall g \in \mathcal{G} \wedge t \in \mathcal{T} \tag{3f}$$

$$0 \leq p_t^{P,c} \leq y_t^{P,c}\mathbb{E}(\overline{p}_t^P) : \underline{\mu}_t^{P,c}, \overline{\mu}_t^{P,c}; \forall t \in \mathcal{T} \tag{3g}$$

$$0 \leq p_t^{P,d} \leq y_t^{P,d}\mathbb{E}(\overline{p}_t^P) : \underline{\mu}_t^{P,d}, \overline{\mu}_t^{P,d}; \forall t \in \mathcal{T} \tag{3h}$$

$$\mathrm{SOC}_t^P - \mathrm{SOC}_{t-1}^P - p_t^{P,c}\sqrt{\eta} + p_t^{P,d}/\sqrt{\eta} = 0;\; \forall t \in \mathcal{T}\backslash\{1\} \tag{3i}$$

$$\mathbb{E}(\underline{\mathrm{SOC}}_t^P) \leq \mathrm{SOC}_t^P \leq \mathbb{E}\left(\overline{\mathrm{SOC}}_t^P\right); \forall t \in \mathcal{T} \tag{3j}$$

$$y_t^{P,c} + y_t^{P,d} \leq 1; \forall t \in \mathcal{T} \tag{3k}$$

$$y_t^P \in \{0,1\}; \forall t \in \mathcal{T} \tag{3l}$$

where the dual variables are shown at the right-hand side of each constraint, $\boldsymbol{p}^P = [p_t^{P,c}, p_t^{P,d}]$, $\boldsymbol{y}^P = [y_t^{P,c}, y_t^{P,d}]$, $\boldsymbol{p}^G = [p_{1,1}, \dots, p_{1,|\mathcal{T}|}, \dots, p_{|\mathcal{G}|,|\mathcal{T}|}]$ and $\boldsymbol{p}^L = [p_{1,1}, \dots, p_{1,|\mathcal{T}|}, \dots, p_{|\mathcal{L}|,|\mathcal{T}|}]$.

In (3), the upper problem (3a) and (3b) seeks for maximizing the PL profit by strategically deciding its offering strategy α_t^P. In the lower problem, the market is cleared by maximizing the social welfare subjected to some well-known market-related constraints. (3d) draws the power balance in the market including charging and discharging modes of the PL. (3e) and (3f) limit the power of loads and generators, respectively. (3g) and (3h) limit the power traded by the PL. As seen, power bounds of PL are considered uncertain parameters, as discussed later, taking expected values in this problem (deterministic assumptions). (3i) expresses the instantaneous energy stored in the PL as a function of the power exchanged in the market, roundtrip efficiency and the amount of energy at the previous time instant [25]. (3j) limits the energy available in the PL to uncertain bounds determined by the number of vehicles plugged and their features. (3k) makes charging and discharging modes complementary, thus discarding the realization of physically unfeasible solutions, whereas binary variables are defined in (3l).

At its present form, (3) is hardly solvable using conventional analytic solvers due to the presence of binary variables in the lower problem. Indeed, bi-level optimization problems are customarily solved by reducing them to a single-level structure. To this end, the lower-level is reduced to its KKT conditions, which are sufficient optimality conditions for convex problems [26]. However, the presence of binary variables in (3) makes the problem non-convex. Note that binary variables are still necessary to properly model charging and discharging modes of storage units, as said [18], and therefore we consider that some relaxing techniques (e.g. see [27]) should not be applied in our problem. Moreover, it is worth noting that the follower problem (market clearing) in (3) includes energy-related constraints (3h) and (3i), which are not frequently dealt by market operators and therefore the problem (3) may result unrealistic. In Section 3, these issues are solved.

2.4 - Uncertainty modelling

When operational models are subjected to uncertain parameters (i.e. those parameters whose exact value is not known in advance), both the uncertainties and risk preferences of users need to be modelled. In the former, various models have been proposed in the literature, encompassing stochastic [11] or robust programming [17]. While the former models different realization of uncertainties via scenarios, the latter seeks for the worst or best-case realization of uncertainties depending on the risk strategy adopted. Risk-averse (conservative) or risk-seeker (optimistic) operational strategies allow for accounting and managing the risk associated to uncertainties. This way, unlike to deterministic approaches, risk-aware models allow measuring the risk associated with decisions and enable accessing to further information brought by uncertain parameters, thus refining the decision-making process.

In particular, we consider that PLs operate in day-ahead markets under the following sources of uncertainty:

- The total energy stored in the parking must be limited to the minimum and maximum storage capacity available. Hence, these bounds are actually determined by the number of vehicles plugged at the parking any time instant as well as their particular features. Specifically, we consider that the number of vehicles plugged any time instant cannot be predicted with exactitude and therefore the energy bounds are assumed to be uncertain parameters in our problem.
- By the same reasons, the maximum power deliverable for the PL is considered uncertain, as it depends on the number of vehicles plugged.

For simplicity, the uncertain parameters are gathered in the vector $\boldsymbol{r}$ given by:

$$\boldsymbol{r} = \left\{ \underline{\mathrm{SOC}}_t^P, \overline{\mathrm{SOC}}_t^P, \overline{p}_t^P \right\} \tag{4}$$

In this paper, uncertainties are modelled using confidence bounds (robust optimization). Thereby, given an uncertain parameter $r \in \boldsymbol{r}$, its value is restricted by the following box constraint (see Fig. 2) [28]:

$$r \in \left[\mathbb{E}(r) - \Gamma \Delta^{\downarrow} r, \mathbb{E}(r) + \Gamma \Delta^{\uparrow} r \right]; \forall r \tag{5}$$

where $0 \leq \Gamma \leq 1$ is the so-called risk level, which makes the proposed framework adaptive. Indeed, a higher value of Γ makes wider the feasible space of r, thus implying a higher tendency to risk. Note that the risk level is tuned by the manager, which allows her to compare different risk-aware adoptions and their implications.

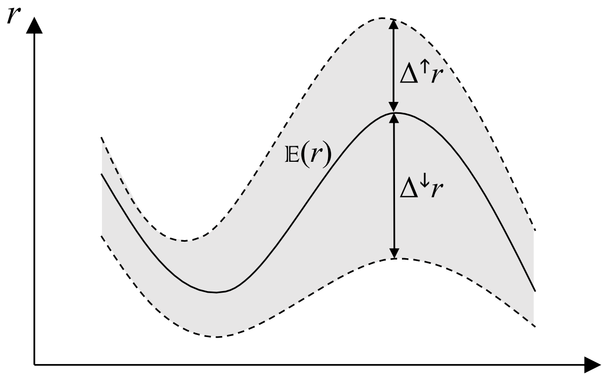

Fig. 2 - Uncertainty modelling via confidence bounds used in this paper

3 - The proposed solution strategy

3.1 - Foundations

As commented previously, the solution of (3) is not a trivial due to the presence of binary variables and energy-related constraints in the market clearing problem. To overcome this issue, we propose a solution strategy based on the well-known Benders' decomposition [21]. This technique transforms a single-level optimization problem into a master-slave structure, by which variables are divided into two groups. On the one hand, the so-called complicating variables are considered uniquely in the master problem, which is a relaxed version of the original one. In contrast, the subproblem deals with the rest of variables and supposes a more restricted version of the master problem (typically it shares structure with the original optimization framework). The basic foundation of the Benders' decomposition is assuming that, when the complicating variables are relaxed (e.g. fixing their values), the entire problem becomes simpler to solve.

This paper proposes to apply the same idea for solving (3). This way, we separate the set of variables into two groups:

- Complicating variables: in our case, the complicating variables will be the binary variables $\boldsymbol{y}^P$, the energy stored in the parking SOC_t^P and the set of uncertain parameters $\boldsymbol{r}$. By this adoption, the binary and energy-related variables vanish from the follower problem in (3), which becomes linear and therefore reducible to its KKT conditions. In addition, including the vector $\boldsymbol{r}$ into the decision space, the entire optimization framework turns into risk-aware.
- The rest of variables, i.e. power dispatch of loads and generators ($\boldsymbol{p}^P$, $\boldsymbol{p}^G$ and $\boldsymbol{p}^L$) as well as the bidding strategy of the PL, α_t^P, will be treated as easy variables.

As explained below, we actually consider $\boldsymbol{p}^P$ as complicating variables as well in order to derive valid sensitivities. Under this variable-splitting strategy, the framework (3) is decomposed into a master-slave solution paradigm by which the final solution is iteratively approximated by exchanging information between both levels. Each level of the proposed solution strategy is explained in detail in subsequent subsections.

3.2 - Slave problem

The slave problem corresponds to the PL strategic bidding, which is derived from (3) but reducing the lower-level by its KKT conditions. To this end, we assume that binary variables are given coming from the master problem, which will be described later, and thus treated as parameters. This way, the slave problem for the θ^{th} iteration within the developed solution algorithm reads as:

$$\boldsymbol{p}^{G,(\theta)}, \boldsymbol{p}^{P,(\theta)}, \lambda_t^{spot,(\theta)}, \alpha_t^{P,(\theta)} \in \underset{\substack{\boldsymbol{p}^P, \boldsymbol{p}^{G,(\theta)}, \boldsymbol{p}^{L,(\theta)}, \\ \alpha_t^{P,(\theta)}, \lambda_t^{spot,(\theta)}, \boldsymbol{\mu}}}{\text{argmax}} \; Profit^{(\theta)} \tag{6a}$$

Subject to:

(3b) (6b)

(3d) (6c)

$$-\alpha_t^{P,(\theta)} - \underline{\mu}_t^{P,c} + \overline{\mu}_t^{P,c} + \lambda_t^{spot,(\theta)} = 0; \forall t \in \mathcal{T} \quad (6d)$$

$$\alpha_t^{P,(\theta)} - \underline{\mu}_t^{P,d} + \overline{\mu}_t^{P,d} - \lambda_t^{spot,(\theta)} = 0; \forall t \in \mathcal{T} \quad (6e)$$

$$-\alpha_{l,t} - \underline{\mu}_{l,t} + \overline{\mu}_{l,t} + \lambda_t^{spot,(\theta)} = 0; \forall l \in \mathcal{L} \wedge t \in \mathcal{T} \quad (6f)$$

$$\alpha_{g,t} - \underline{\mu}_{g,t} + \overline{\mu}_{g,t} - \lambda_t^{spot,(\theta)} = 0; \forall g \in \mathcal{G} \wedge t \in \mathcal{T} \quad (6g)$$

$$0 \leq p_t^{P,c} \perp \underline{\mu}_t^{P,ch} \geq 0; \forall t \in \mathcal{T} \quad (6h)$$

$$0 \leq y_t^{P,c,(\theta)} \overline{p}_t^{P,(\theta)} - p_t^{P,c} \perp \overline{\mu}_t^{P,ch} \geq 0; \forall t \in \mathcal{T} \quad (6i)$$

$$0 \leq p_t^{P,d} \perp \underline{\mu}_t^{P,d} \geq 0; \forall t \in \mathcal{T} \quad (6j)$$

$$0 \leq y_t^{P,d,(\theta)} \overline{p}_t^{P,(\theta)} - p_t^{P,d} \perp \overline{\mu}_t^{P,d} \geq 0; \forall t \in \mathcal{T} \quad (6k)$$

$$0 \leq p_{l,t} \perp \underline{\mu}_{l,t} \geq 0; \forall l \in \mathcal{L} \wedge t \in \mathcal{T} \quad (6l)$$

$$0 \leq \overline{p}_{l,t} - p_{l,t} \perp \overline{\mu}_{l,t} \geq 0; \forall l \in \mathcal{L} \wedge t \in \mathcal{T} \quad (6m)$$

$$0 \leq p_{g,t} \perp \underline{\mu}_{g,t} \geq 0; \forall g \in \mathcal{G} \wedge t \in \mathcal{T} \quad (6n)$$

$$0 \leq \overline{p}_{g,t} - p_{g,t} \perp \overline{\mu}_{g,t} \geq 0; \forall g \in \mathcal{G} \wedge t \in \mathcal{T} \quad (6o)$$

$$\lambda_t^{spot,(\theta)}: \text{free} \quad (6p)$$

$$\boldsymbol{\mu} \geq \mathbf{0} \quad (6q)$$

$$\boldsymbol{p}^P = \boldsymbol{p}^{P,(\theta)}: \boldsymbol{\omega}^{P,(\theta)} \quad (6r)$$

where $\perp$ stands for complementarity and $\boldsymbol{\mu}$ collect the μ's.

The strategic bidding problem (6) aims at maximizing the profit of the PL in (6a) on the decision space formed by easy variables as well as the set of dual variables. (6b) and (6c) ensure the feasibility of the reduced single-level problem. (6d)-(6g) are the set of stationary conditions, which are derived from equaling zero the first partial derivatives of the Lagrangian w.r.t. the set of easy variables (see [29] for further information). On the other hand, the complementarity conditions are given in (6h)-(6o), which are linked to inequality constraints in (3). Dual feasibility constraints are given in (6p) and (6q), while (6r) is imposed to derive sensitivities (the ω's).

Note that (6) considers $\boldsymbol{p}^P$ as complicating variables fixing them equal to known values coming from the master problem (i.e. $\boldsymbol{p}^{P,(\theta)}$). This is due to sensitivities need to describe the impact of complicating variables in the objective function. However, the complicating variables described in 3.1 do not appear in (6a) and therefore valid sensitivities cannot be derived on the basis of them. Thus, we need to consider $\boldsymbol{p}^P$ instead, which does appear in (6a) and therefore (6r) produces valid sensitivities that can be used in the master problem, as explained in the following subsection.

It is worth noting that energy-related variables vanished in (6) due to the energy available in the parking has been considered a complicating variable. It is important to see that by fixing $\boldsymbol{p}^P$, the feasibility of the problem with respect to (3i) and (3j) is ensured, as commented later.

The problem (6) is nonlinear because the presence of bilinear terms in the objective function and complementarity constraints. To linearize the objective function, we apply the Strong Duality Theorem [30], which establishes that at the optimum point the primal and dual objectives have the same value if the Slater's conditions hold. Thus, the profit (1) can be replaced by its linear dual counterpart given by:

$$\widetilde{Profit} = -\sum_{t\in\mathcal{T}} \left\{ \begin{matrix} C^{P,d} p_t^{P,d} - C^{P,c} p_t^{P,c} - \sum_{l\in\mathcal{L}} \alpha_{l,t}\, p_{l,t} + \sum_{g\in\mathcal{G}} \alpha_{g,t}\, p_{g,t} + \\ \sum_{l\in\mathcal{L}} \overline{\mu}_{l,t}\, \overline{p}_{l,t} + \sum_{g\in\mathcal{G}} \overline{\mu}_{g,t}\, \overline{p}_{g,t} \end{matrix} \right\} \quad (7)$$

On the other hand, complementarity constraints can be linearized using the big-M method [31], which replaces the complementarity term $0 \leq a \perp b \geq 0$ by the following set of disjunctive constraints:

$$a \geq 0, b \geq 0 \tag{8a}$$
$$a \leq uM^P \tag{8b}$$
$$b \leq (1-u)M^D \tag{8c}$$
$$u \in \{0,1\} \tag{8d}$$

The relaxation above is exact if the value of M's is properly tuned. Unfortunately, tuning up the M's is challenging and there no exist a unified tuning strategy [32]. However, in market-related problems, owing to dual variables have a clear economic meaning, the value of the M's can be set easily following the principles and ideas described in [33].

Note that the inclusion of binary variables in (8) converts (6) into a MILP, from which sensitivities cannot be derived [34]. To solve this issue, we reformulate (6) as the following equivalent primal-dual linear problem

$$\boldsymbol{\omega}^{P,(\theta)} \in \underset{\boldsymbol{p}^P,\boldsymbol{p}^G,\boldsymbol{p}^L,\boldsymbol{\mu}}{\operatorname{argmax}} \widetilde{Profit} \tag{9a}$$

Subject to:

$$\text{(6b)-(6h)} \tag{9b}$$
$$\widetilde{Profit} = Profit^{(\theta)} \tag{9c}$$
$$\text{(6q)} \tag{9d}$$
$$\boldsymbol{p}^G = \boldsymbol{p}^{G,(\theta)} \tag{9e}$$
$$\boldsymbol{p}^L = \boldsymbol{p}^{L,(\theta)} \tag{9f}$$
$$\text{(6r)} \tag{9g}$$

One can easily see that (6) and (9) will give rise the same result by enforcing the value of the objective function in (9c). Nevertheless, (9) is fully linear as complementary constraints vanish in this problem and therefore sensitivities can be derived from (9g).

3.3 - Master problem

The master problem is a relaxed version of (3) involving complicating variables and their related constraints. This way, the market clearing problem is irrelevant at this step, thus resulting in the following optimization problem:

$$\boldsymbol{p}^{P,(\theta)}, \boldsymbol{y}^{P,(\theta)}, \boldsymbol{r}^{P,(\theta)} \in \underset{\substack{\boldsymbol{p}^{P,(\theta)},\boldsymbol{y}^{P,(\theta)},\\ \boldsymbol{r}^{P,(\theta)},\beta^{(\theta)}}}{\operatorname{argmax}} \beta^{(\theta)} \tag{10a}$$

Subject to:

$$\text{(3g)-(3l)} \tag{10b}$$
$$\text{(5)} \tag{10c}$$
$$\beta^{(\theta)} \geq M \tag{10d}$$
$$\beta^{(\theta)} \geq Profit^{(\kappa)} - \boldsymbol{\omega}^{P,(\kappa)} \left(\boldsymbol{p}^{P,(\theta)} - \boldsymbol{p}^{P,(\kappa)}\right)^{\intercal}; \forall \kappa \in \{1,2,\ldots,\theta-1\} \tag{10e}$$

where $\intercal: \mathbb{R}^{m\times n} \mapsto \mathbb{R}^{n\times m}$ is the transpose operator. In (10a), the profit function is replaced by the auxiliary variable β, which iteratively approaches the value of the original objective function using the information provided by sensitivities, which build up the optimality cuts in (10e). In (10b), those constraints related to complicating variables are included. Note that the inclusion of (10b) and taking a fixing value of $\boldsymbol{p}^P$ ensure the feasibility (6) with respect to the energy-related constraints. (10c) models the uncertain variables according to the box modelling given in (5). Finally, (10d) ensures feasibility of the algorithm at first iteration.

Normally, feasibility cuts need to be included to ensure feasibility of solutions obtained at the master problem [35]. However, we replace these cuts by valid inequalities, as shown in Section 4.

3.4 - Risk-aware strategies

The inclusion of (5) in (10) entails two important issues:

- It expands notably the decision space of the master problem leading to a slow convergence.
- The algorithm may arbitrarily evolve towards risk-averse or risk-seeker strategies.

To solve these two issues at once, we propose a forcing bound strategy. According to [36], the extreme case realization of uncertainties corresponds to extreme or vertex of the polyhedron representing the uncertainty set (5), whereby this box constraint can be directly replaced by its limits. Moreover, one can choose the risk orientation adopted by simply selecting the activated bound (upper or lower) properly. This way, risk-averse conditions imply tight energy limits, as in (3j), while power limits in (3g) and (3h) hit at the minimum. Keeping this in mind, (5) can be equivalently replaced by risk-averse conditions as:

$$\overline{p}_t^P = \overline{p}_t^P - \Gamma\Delta^{\downarrow}\overline{p}_t^P; \forall t \in \mathcal{T} \tag{11a}$$
$$\mathrm{SOC}_t^P \le \overline{\mathrm{SOC}}_t^P - \Gamma\Delta^{\downarrow}\overline{\mathrm{SOC}}_t^P; \forall t \in \mathcal{T} \tag{11b}$$
$$\mathrm{SOC}_t^P \ge \underline{\mathrm{SOC}}_t^P + \Gamma\Delta^{\uparrow}\underline{\mathrm{SOC}}_t^P; \forall t \in \mathcal{T} \tag{11c}$$

Intuitively, assuming the contrary leads to risk-seeker strategies, given by:

$$\overline{p}_t^P = \overline{p}_t^P + \Gamma\Delta^{\uparrow}\overline{p}_t^P; \forall t \in \mathcal{T} \tag{12a}$$
$$\mathrm{SOC}_t^P \le \overline{\mathrm{SOC}}_t^P + \Gamma\Delta^{\uparrow}\overline{\mathrm{SOC}}_t^P; \forall t \in \mathcal{T} \tag{12b}$$
$$\mathrm{SOC}_t^P \ge \underline{\mathrm{SOC}}_t^P - \Gamma\Delta^{\downarrow}\underline{\mathrm{SOC}}_t^P; \forall t \in \mathcal{T} \tag{12c}$$

3.5 - Convergence checking

The algorithm evolves exchanging information between the master and slave problems until the objectives of both problems remain similar, which is checked by the following condition:

$$res^{(\theta)} = \frac{\beta^{(\theta)} + Profit^{(\theta)}}{\beta^{(\theta)}} \le tol \tag{13}$$

where $tol = 10^{-7}$ in this paper.

3.6 - The algorithm

The following steps describe the designed algorithm for solving the bi-level problem (3) using the decomposition strategy described throughout Section 3:

1. Define $tol \in \mathbb{R}_+$, set $\theta = 1$ and provide input parameters. Decide on the risk-aware strategy adopted.
2. Solve the master problem (10) replacing (10c) by the set of constraints (11) or (12) depending on the risk-aware strategy adopted. If $\theta = 1$, omit the constraint (10e). Obtain the tupple $\{\boldsymbol{p}^{P,(\theta)}, \boldsymbol{y}^{P,(\theta)}, \boldsymbol{r}^{P,(\theta)}\}$ and store the value of $\beta^{(\theta)}$.
3. Solve the slave problem (6). Obtain the tupple $\left\{\boldsymbol{p}^{G,(\theta)}, \boldsymbol{p}^{P,(\theta)}, \lambda_t^{spot,(\theta)}, \alpha_t^{P,(\theta)}\right\}$ and store the value of $Profit^{(\theta)}$.
4. Solve (9) and derive sensitivities $\boldsymbol{\omega}^{P,(\theta)}$.
5. Check convergence by (13). If the algorithm converges, then stop. Else, go to Step 2.

4 - Algorithm improvements

The Benders' decomposition is a powerful technique but not free of serious shortcomings that need to be solved. In this Section, we discuss several improvements that are incorporated to the methodology developed in Section 3 in order to improve its performance.

4.1 - Genuine master problem costs

In our particular problem, the unique objective in the master problem (10) is the auxiliary variable β. In other words, the master and slave problems share objective function. However, the inclusion of genuine master problem costs helps to obtain stronger optimality cuts that may eventually lead to a faster convergence [33]. In order to improve the convergence features of the developed methodology, the objective (10a) is modified by including extra costs, as follows:

$$\min_{\substack{\boldsymbol{p}^{P,(\theta)},\boldsymbol{y}^{P,(\theta)},\\ \boldsymbol{r}^{P,(\theta)},\beta^{(\theta)},\mathrm{on}_t^{P,c},\\ \mathrm{on}_t^{P,d},\mathrm{off}_t^{P,c},\mathrm{off}_t^{P,d}}} \beta^{(\theta)} + \rho \sum_{t\in\mathcal{T}} \left\{ \mathrm{on}_t^{P,c} + \mathrm{on}_t^{P,d} \right\} \tag{14}$$

where $\rho \in \mathbb{R}_+$. The extra variables flagged the transition between commitment statuses. For example, $\mathrm{on}_t^c = 1$ indicates that the PL has transitioned from the discharging or idle mode to the charging status at time t, while off_t^c has the opposite meaning (the same is applied to on_t^d and off_t^d for the discharging mode). In order to capture these transitions, the following constraints must be included in (10).

$$y_t^{P,c} - y_{t-1}^{P,c} = \mathrm{on}_t^{P,c} - \mathrm{off}_t^{P,c};\ \forall t \in \mathcal{T}\backslash\{1\} \tag{15a}$$

$$y_t^{P,d} - y_{t-1}^{P,d} = \mathrm{on}_t^{P,d} - \mathrm{off}_t^{P,d};\ \forall t \in \mathcal{T}\backslash\{1\} \tag{15b}$$

$$\mathrm{on}_t^{P,c}, \mathrm{on}_t^{P,d}, \mathrm{off}_t^{P,c}, \mathrm{off}_t^{P,d} \in \{0,1\};\ \forall t \in \mathcal{T} \tag{15c}$$

The extra costs in (14) have a twofold objective: on the one hand, it improves the convergence features of the algorithm, as commented. On the other hand, it naturally filters out potential solutions involving many transitions between commitment statuses, which might be infeasible in practise or lead to rapid battery degradation.

4.2 - Integer cuts

The master problem is basically a combinatorial problem on the values of $\boldsymbol{y}^P$. It is known that forcing the change the value of $\boldsymbol{y}^P$ improves the convergence when the algorithm is stuck at a local optimum [37]. Thus, when the number of iterations is large ($\theta \geq 50$), the following integer cut is included:

$$\sum_{t\in\mathcal{Y}^{c,(\theta-1)}} \left(1 - y_t^{P,c,(\theta)}\right) + \sum_{t\notin\mathcal{Y}^{c,(\theta-1)}} y_t^{P,c,(\theta)} + \sum_{t\in\mathcal{Y}^{d,(\theta-1)}} \left(1 - y_t^{P,d,(\theta)}\right) + \sum_{t\notin\mathcal{Y}^{d,(\theta-1)}} y_t^{P,d,(\theta)} \geq 1 \tag{16}$$

where the set $\mathcal{Y}^{c,(\theta-1)} = \left\{t | y_t^{P,c,(\theta-1)} = 1\right\}$ and $\mathcal{Y}^{d,(\theta-1)} = \left\{t | y_t^{P,d,(\theta-1)} = 1\right\}$. Indeed, one can easily check that activation of (16) implies $\boldsymbol{y}^{P,(\theta)} \neq \boldsymbol{y}^{P,(\theta-1)}$ in at least one term. It is worth commenting that (16) may lead to instability or even infeasibility, and therefore it should be added in (10) as fewest times as possible and only when the algorithm is stuck.

4.3 - Price-guided cuts

Additional feedback from the slave problem might help to better estimate the costs in the master problem [23]. In our case, the slave problem provides market spot prices, which can accelerate convergence by the inclusion of the following price-guided cut:

$$\beta^{(\theta)} \geq \sum_t \left\{ p_t^{P,c,(\theta)} \left(C^{P,c} - \lambda_t^{spot,(\theta-1)} \right) + p_t^{P,d,(\theta)} \left(\lambda_t^{spot,(\theta-1)} - C^{P,d} \right) \right\} \tag{17}$$

Cuts (17) coherently guide the solution of the master problem according to market prices. Nevertheless, the inclusion of (17) may lead to convergence at sub-optimal solutions and therefore it is uniquely included when the solution is approached sufficiently. In particular, we activate (17) if the residual (13) lies below 10^{-2}.

4.4 - Refined upper bound

In the master problem, the value of the objective function is approached by means of the auxiliary variable β. In order to better approximate its value, information coming from the subproblem can be used to better refine the bounds of β, as follows:

$$\beta^{(\theta)} \leq Profit^{(\theta-1)} \tag{18}$$

Indeed, (18) uses a better estimation of the upper bound of β to cap its value and thus orienting the master problem to a more refined estimation of the objective function.

4.5 - Valid inequalities

The master problem (9) does not include feasibility cuts and therefore its solution might be infeasible for the slave problem. To solve this issue, the following valid inequalities are included:

$$p_t^{P,c,(\theta)} \leq \sum_{g\in\mathcal{G}} \overline{p}_{g,t} \,; \forall t \in \mathcal{T} \tag{19a}$$
$$p_t^{P,d,(\theta)} \leq \sum_{l\in\mathcal{L}} \overline{p}_{l,t} \,; \forall t \in \mathcal{T} \tag{19b}$$

Indeed, (19) ensure that power dispatching is physically realizable in the market clearing problem.

4.6 - Forcing upper bound

When the follower problem is a bi-level problem solved by reduction to KKT conditions, the follower problem may yield an invalid upper bound and therefore lead to incoherence in (13) and bad convergence (see the discussion in [38]). To prevent this issue, the following constraint is enforced in (6).

$$\widetilde{Profit}^{(\theta)} \geq \beta^{(\theta)} \tag{20}$$

As seen, (20) is equivalent to (18) but in the follower problem, ensuring the coherence of the whole algorithm.

5 - Numerical results

Throughout this Section, we present a variety of numerical results with three main objectives. Firstly, validate the developed methodology through a simple toy example. Secondly, further validate the new proposal considering a variety of instances of different sizes. Thirdly, evaluate the proposed tool from a computational point of view (time consumption), validating it for industrial tools.

5.1 - Illustrative example

We consider a large-scale PL with 1000 expected charging events. Fig. 3 (top) shows the energy and power limits for the case study, which were generated using the methodology described in [28]. This illustrative example considers a small-scale market environment involving a wind farm, a dispatchable generator and an elastic demand. The expected generation/load for the renewable generator and demand is plotted in Fig. 3 (bottom) whereas the rest of necessary parameters are reported in Table 2. For the PL, we consider conventional charging prices in fast-charging stations [39], whilst the discharging price is estimated to be competitive in the market (note that the marginal cost of the dispatchable generator is slightly higher). With these settings, the discharging mode becomes profitable in the market.

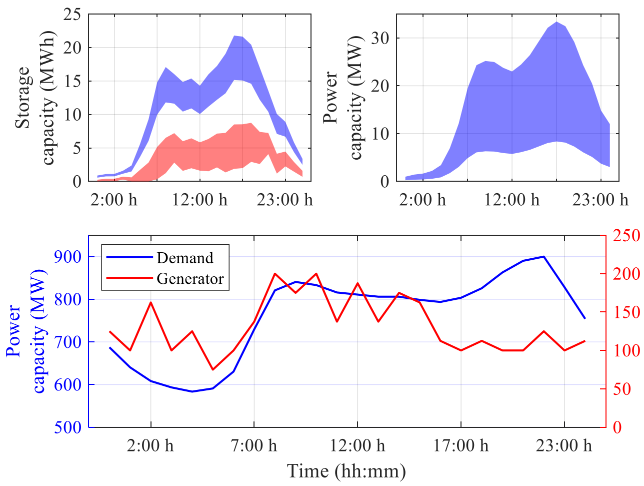

Fig. 3 - Expected energy and power limits for the PL (top) and generation/load of the renewable generator and elastic demand (bottom)

Table 2 - Parameters of the illustrative example

Parameter	Value
$\overline{p}_{g,t}$ (dispatchable)	1000 MW
$\alpha_{g,t}$ (dispatchable)	180 €/MWh
$\alpha_{g,t}$ (renewable)	200 MW
$\alpha_{l,t}$ (elastic)	750 €/MWh
$C^{P,c}$, $C^{P,d}$	500 €/MWh, 150 €/MWh
η	0.80 pu
ρ	10^4 €

Fig. 4 plots the scheduling result for two extreme risk-aware cases (i.e. adopting risk-averse and risk-seeker strategies with $\Gamma = 0.5$). As seen, when adopting a risk-seeker strategy, the PL assumes favourable realization of uncertainties. In this sense, both the storage and power capacity increases, enabling a more active participation in the market. It is worth observing that, under a risk-seeker strategy, peak charging is observed at morning and evening. At these hours, the parking reaches its maximum storage capacity. In contrast, power demand under a risk-averse strategy is softer and frequently the energy stored does not hit its upper limit. On the other hand, it is remarkable that the parking is able to sell a considerable amount of energy when adopting a risk-seeker strategy, while tighter bounds limit the capability for discharging energy when adopting a risk-averse strategy.

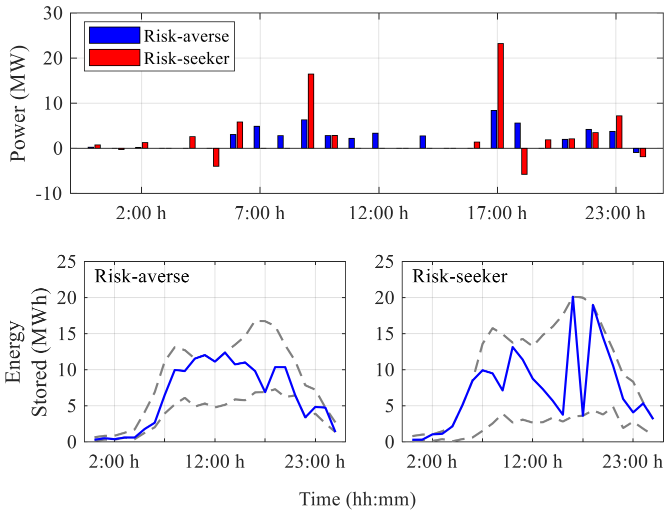

Fig. 4 - Results for the illustrative example. Power scheduling (top) and energy stored in the parking (bottom). In this figure, $\Gamma = 0.5$ was considered for risk-aware strategies. Negative values in the top plot stands for discharged power

Table 3 reports a summary of the results in the illustrative example. As observed, adopting a risk-seeker strategy leads to a higher expected profit (+ 25.2 %). It is worth noting that this increment is only accessible under optimistic realization of uncertainties and assuming a high risk level ($\Gamma = 0.5$). Under these conditions, the energy charged by the parking notably increments (+ 24.12 %), which supposes the main economic activity of the parking thus ruling the profit of the installation. Indeed, marginal costs of generators are notably lower than the cost for charging. It supposes an attractive economic environment for the parking, which can buy energy in the market at low cost and thus obtaining a considerable profit for providing charging services. In contrast, selling energy is not as profitable due to the considered cost for discharging is closer to the marginal cost of generators. Even so, the parking recurs to this activity to complement the monetary incomes but only when adopting a risk-seeker strategy, as pointed out previously.

Table 3 - A summary of the results in the illustrative example

Result	Risk-averse	Risk-seeker
Profit (€)	16,741	22,382 (+25.2 %)
Total energy charged (MWh)	52.22	68,82 (+24.12 %)
Total energy discharged (MWh)	0.96	11,94 (+92 %)
Profit for charging (€)	16,712	22,024 (+24 %)
Profit for discharging (€)	29	358 (+92 %)

5.2 - Sensitivity analysis

We perform a sensitivity analysis regarding two parameters. On the one hand, we study how the number of vehicles in the parking impacts on final results. On the other hand, we scale the charging and discharging prices by a real factor. Not surprisingly, incrementing both the number of vehicles and charging/discharging costs leads to increment the parking profit notably. Indeed, the more vehicles plugged, the higher payments received, but also the market-sharing of the parking increases, being capable of partaking more actively in the market and thus accessing to economy opportunities. On the other hand, incrementing the charging price leads to receive more money for providing charging services. In this case, incrementing the discharging price does not retract the parking profit, due to payments for discharging are much lower than incomes from charging, as discussed above. It is worth noting that the number of vehicles impacts more notably on the profit. This is important to the time of deciding whether increasing the number of charging points or the charging price.

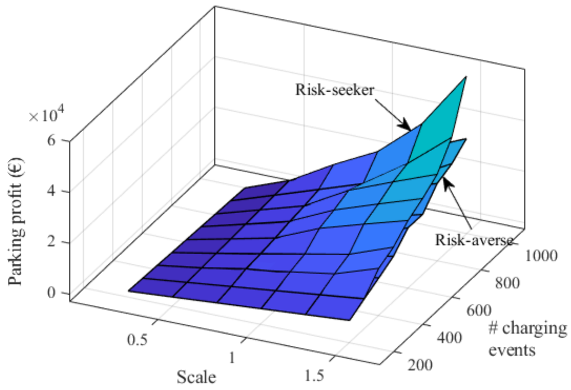

Fig. 5 – Parking profit for different number of charging events and charging/discharging prices

5.3 - Comparison with stochastic programming

In this section, we provide a comparison with stochastic programming, which supposes one of the benchmark uncertainty models. To provide a fair comparison, 1000 scenarios were created following a normal distribution with mean at expected values of uncertainties. To avoid a computational burden explosion due to the high number of variables involved, we reduce the original number of scenarios to 7 following the clustering methodology proposed in [24]. Table 4 provides various key results of this comparison with $\Gamma = 0.5$. As seen, our risk-aware strategies provided more extreme results than stochastic programming. This is due to stochastic models optimize over a number of possible realization of uncertainties, including optimistic and pessimistic scenarios. Thus, the results obtained with stochastic approaches are coherently less conservative than those obtained with the proposed risk-averse methodology, which is based on extreme values of uncertain parameters. As expected, the risk-seeker strategy yielded more optimistic results than the risk-averse one and the stochastic approach.

Table 4 – Comparison of the proposed risk-aware strategies and stochastic programming with ($\Gamma = 0.5$)

Method	**Profit (€)**	**Total charged (MWh)**	**Total discharged (MWh)**
Risk-averse	16,741	52.22	0.96
Risk-seeker	22,382	68.82	11.94
Stochastic	16,938	52.93	1.22

5.4 - Further validation

Next, we further validate the developed tool in a number of different market environments. To this end, 50 instances are randomly generated considering the parameters given in the Appendix, whereas the rest of parameters are as in Table 1. With these settings, we aim at checking the performance of the developed methodology under different market sizes and conditions. Hereinafter, we show and discuss different relevant results obtained after solving the generated market instances.

In this regard, Fig. 6 compares the profit in both risk-aware strategies. As observed, the profit expected under optimistic conditions (i.e. risk-seeker strategy) improves until reach ~30 % in most of cases with $\Gamma = 0.5$. As expected, the difference between both strategies increases with the value of Γ. This result further validates the developed tool as the results obtained follow an expected pattern. Indeed, as pointed out in the previous section, risk-seeker conditions lead to overestimate the available storage capacity and power, which allows the PL to partake more actively in the market obtaining a higher profit.

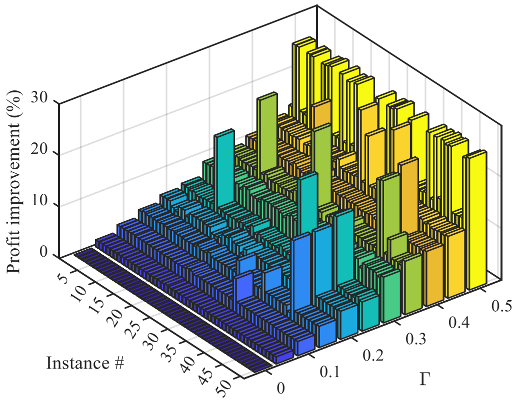

Fig. 6 - Profit improvement when considering a risk-seeker strategy in comparison with a risk-averse strategy

To further validate the developed approach, Fig. 7 and 8 compare the total energy charged and discharged by the PL, respectively. As seen, energy charged increases with the value of Γ, thus confirming our previous assumptions (i.e. optimistic conditions motivate the PC to partake more actively in the market). It is worth noting that Fig. 7 is rather similar to Fig. 6, which confirms that the profit obtained by the PL is mainly ruled by the total energy charged. Fig. 8 shows similar results but more extreme. Indeed, the value of the uncertainty budget notably impact on the total energy discharged, to the point of being unable to sell energy to the market for $\Gamma = 0.5$. This way, the PL sees hindered its access to higher profits due to its own limited exportable and importable power capacity.

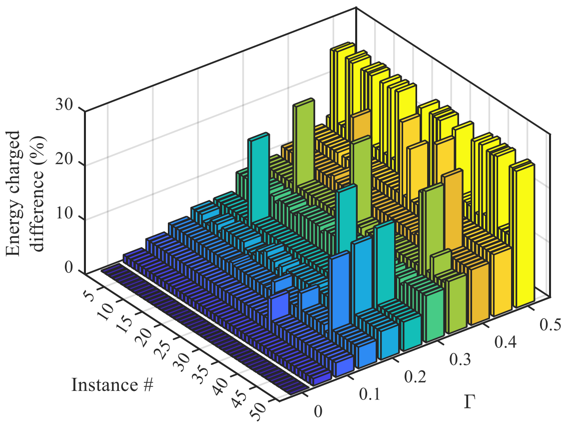

Fig. 7 - Total energy charged difference when assuming a risk-seeker strategy in comparison with a risk-averse strategy

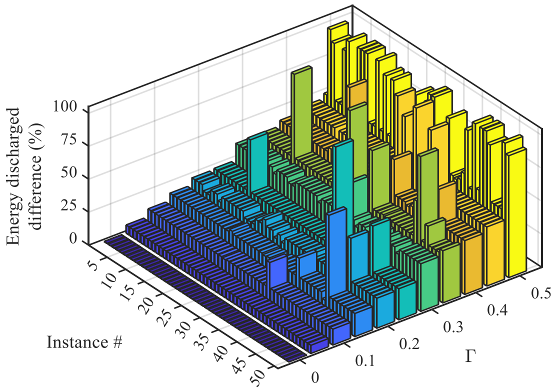

Fig. 8 - Total energy discharged difference when assuming a risk-seeker strategy in comparison with a risk-averse strategy

5.5 - Algorithm performance

Lastly, we focus on the computational performance of the developed solution strategy. To this end, the proposed Benders' algorithm has been coded under Matlab R2021b and solved using Gurobi [40], for which free license agreements for research and academic purposes are provided currently. In order to obtain an accurate solution and ensure the global optimum reachability, the optimum and integer gaps were set to their minimum possible values. All the simulations were run on an Intel Core i7-10700K CPU 3.80GHz 3.79 GHz with 32 GB RAM.

Fig. 9 shows the total computational time for solving the instances considered in previous analysis. When assuming a risk-averse strategy, the computational time typically increases with the value of the uncertainty budget. This is due to, under pessimistic realization of uncertainties, the feasible solution space is reduced and the solution is harder to reach, as pointed out in [20]. For the same reasons, the behaviour is opposite when adopting a risk-seeker strategy. Indeed, in this case the total computational time reduces with the value of Γ due to the feasible set expands. Nevertheless, in both cases the computational time was kept below 125 seconds, which is quite reasonable for day-ahead market-oriented tool. These results validate the new proposal for practical tools, showing a reasonable performance even under large-scale market environments.

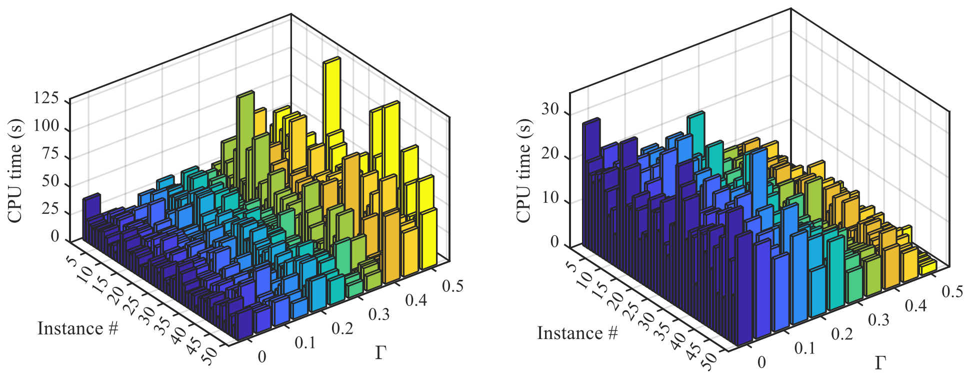

Fig. 9 - Total computational time when assuming a risk-averse (left) or risk-seeker (right) strategy

5.6 – Limitations & countermeasures

Results above have validated the new bidding model and the computational analysis in Section 5.5 proved its applicability in real-life tools. However, the proposed methodology requires advanced computational models in optimization environments. For instance, the implementation of the Benders' decomposition algorithm may be challenging if a high-level code instance is not employed. Nevertheless, there is a variety of commercial software that implement friendly coding environments for optimization models. For example, Matlab and GAMS enable model-oriented codes that allow replicating the original formulation of optimization problems in a simple way, thus allowing to implement optimization codes without an in-depth knowledge of the solution process. Currently, Pyomo [41] is being developed as a library for high-level optimization coding in Python. Therefore, although its codification might be difficult, we honestly believe that the use of open-source coding languages and in-cloud computational machines allow implementing the developed methodology without any problem in real-life tools.

6 - Conclusions

A novel risk-aware model for optimal participation of PLs in day-ahead wholesale electricity markets has been developed. The new proposal casted as a bi-level optimization framework in which the charging/discharging strategy of the PL is decided at the upper level, whereas the lower-level clears the market and reveals prices. Binary variables appearing at the upper level as well as risk-aware operational strategies have been accommodated through an original Benders' decomposition algorithm, thus resulting in a tractable paradigm solvable by off-the-shelf solvers and average machines. Moreover, the modelling of risk-averse and risk-seeker operational strategies has been illustrated and discussed.

A number of results have been provided to illustrate the performance of the developed tool and compare how the adoption of risk-aware strategies affects the market strategy of the PL. Remarkably, the available power is the most important uncertain parameter in PLs when partaking in energy markets, strongly determining the capacity of the parking to participate in energy markets and thus accessing to economic profits. In such a case, the uncertainty budget notably impacted on the economy of the parking, leading to increment the expected profit by 30 % when assuming optimistic realization of uncertainties. These results were confirmed under a number of different instances, showing that enabling risk-aware strategies strongly determine the market-sharing of the PL, which may be limited its participation in the market under conservative assumptions. A sensitivity analysis reveals that higher monetary opportunities are accessible when more vehicles charge in the parking. Moreover, increasing charging and discharging prices led to increment the profit for the PL notably under both risk-aware strategies. In addition, it was shown that the capability of selling energy in markets is notably limited when adopting a risk-averse strategy, but rather promoted under optimistic realization of uncertainties. Moreover, an in-depth computational analysis showed that the new proposal is efficient and can be solved below 125 seconds in a number of different scenarios, further confirming the practicability of the developed tool.

Future works should be devoted on investigating different monetary activities that PLs can put on practise with the aim of increasing their profit. Moreover, the developed mathematical solution strategy could be applied in other similar problems involving binary variables or uncertainty modelling.

CRediT authorship contribution statement

Marcos Tostado-Véliz: Conceptualization, Methodology, Software, Investigation, Writing – Original Draft, Writing – Review & Editing, Visualization, Funding acquisition. **Hany M. Hasanien:** Formal analysis, Investigation, Writing – Original Draft, Writing – Review & Editing, Visualization. **José Carpio:** Conceptualization, Validation, Formal analysis, Data curation, Supervision. **Francisco Jurado:** Formal analysis, Resources, Supervision, Project administration, Funding acquisition.

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Appendix - Data for market instances

In sections 5.4 and 5.5, different market instances were considered to validate the new proposal. These instances were generated taking the different parameters randomly within the ranges reported below:

- Total number of rival players: [5, 150]
- Capacity of dispatchable generators: [500, 1000] MW
- Marginal cost of dispatchable generators: [50, 180] €/MWh
- Capacity of renewable generators: [50, 200] MW
- Marginal cost of renewable generators: [0, 50] €/MWh
- Utility of demands: [600, 1000] €/MWh

The data regarding the PL (i.e. efficiency, ρ, charging and discharging prices), were considered the same as in the illustrative example in Section 5.1. On the other hand, the demands were constructed taking the profile in Fig. 3 as benchmark. Two types of renewable generators were considered, namely wind and photovoltaic. The wind potential follows the profile in Fig. 3 as benchmark, while the profile in Fig. 10 was assumed for photovoltaic generators.

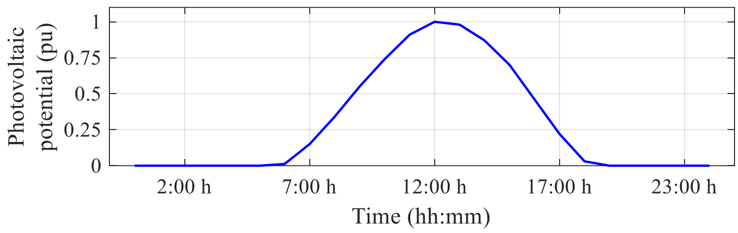

Fig. 10 - Photovoltaic potential considered in market instances

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