

Measurement of railroad track irregularities using an automated recording vehicle

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Abstract

This paper presents the design, construction and experimental validation of a scaled railway vehicle for the automated and quick survey of the geometry of an experimental scaled track. The method, that can be extended to full scale tracks, validates the measurements of the automated recording vehicle (ARV) by comparison with respect to a precise but slow manual measuring method (MMM). The two-axle vehicle, which is powered by a brushless DC motor, has its leading axle instrumented with a LVDT, an inclinometer, a reflector and a precision encoder. These sensors, together with the help of a total station, allow the measurement of the scaled track geometry and its irregularities following an optimization procedure that first determines the ideal track centreline. The ARV has been tested on a scaled track and its measurements are validated with the MMM results showing a good agreement between both approaches.

Keywords: Track surveying, instrumented railway vehicle, scaled track, track irregularities, experimental validation, automated recording vehicle.

1. Introduction

An accurate knowledge of the track geometry is essential to guarantee the safety of the rolling stock. This is because the dynamic response of a railway vehicle [1] is highly influenced by track irregularities [2, 3]. Thus, operator companies are continuously investing human and material resources on the survey and maintenance of their railway infrastructures. Large track irregularities result on larger wheel-rail contact forces, that end up compromising the integrity of the track and, the most important, the safety of the passengers. These irregularities can also produce disturbing sounds for the nearby towns to the track [4] and reduce the comfort of the vehicle [5, 6].

Rail irregularities can be understood as geometrical deviations of the rail cross-sections from an ideal track geometry. They can be divided into two general groups: (1) distributed track irregularities and (2) isolated track defects. The first group appears as regular patterns along the track with multiple wavelengths that depend on the damage mechanism and they are quantified by the four well-known variables used in the industry: track gauge, cross level, vertical profile, and alignment. The isolated track defects, which are the responsible of many of the unsafe vehicle responses, appear more rarely but may

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also have regular patterns. They account for changes in the rail cross section at specific locations, such as in poor drain areas, bridges, or turnouts [7]

Engineers have been working for decades on the development of new methodologies and measuring apparatus for the precise measurement of track irregularities and simulation models and procedures that help to determine and improve the comfort of the ride [8, 9, 10]. This is evidenced by the numerous patented ideas and scientific studies found in the literature. Some of the oldest inventions are based on simple mechanical measuring methods [11] or instrumented wheelsets [12] and others more recent use sensorized trailed vehicles [13] or computer vision techniques [14]. Nowadays, there are also laboratory vehicles such as the Taiwanese EM-50 for the inspection of high-speed lines, the Bulgarian EM-120 for the inspection of the catenaries [15] or the Spanish Seneca track and catenary inspection vehicle. These are just a small sample of the numerous measuring systems that have been and are currently being used in the railway industry for surveying the track.

In general, track measuring methods can be classified into manual surveys and automated or dynamic surveys. On one hand, manual methods are normally based on the use of an instrumented trolley that is manually pushed by a human operator along the track under analysis. On the other hand, automated methods are based on the use of a laboratory vehicle. This one can be instrumented with inertial and navigational sensors for a measurement of the track based on the dynamic response of the vehicle or distance measuring devices. The latter are used in the so-called chord methods which are applied in the horizontal and vertical versine measurement of rails [16]. An extended survey of track irregularity measuring methods can be found in [17]. The European standard EN-13848-3:2009 [18] regulates the measuring features and requirements for track measuring systems.

For their simplicity, precision and robustness, manual trolleys are the most widely-used apparatus for the inspection of the track in the railways industry. Their use can be considered to provide a direct measurement since the trolley is equipped with sensors such as a distance transducer and an inclinometer for the measurement of track gauge and cross-level, respectively, both also known as relative track irregularities. It also includes a total station and a GPS receiver for the measurement of the absolute position of the track centre line. An example of one of the commercial track measuring trolleys that can be found in the market is the Amber Survey GRP 1000 by Amberg Technologies [19]. There are also other versions of manual trolleys focus on the measurement of rail corrugation. The latter are based on the use of several high-precision laser sensors for the survey of the rail heads [20]. It can be also found in the literature some prototype trolleys that, on the basis of the conventional models, try to overcome their slow performance. An example is the work presented by Chen et al. in [21, 22]. The authors propose a sensors fusion algorithm between the GPS and an Inertial Measuring Unit (IMU) that allows a faster survey of the track using this trolley without leaving behind the accuracy requirements for a high speed line. A similar experimental trolley called REGEOS is presented in the work of Akpınar et al. [23, 24]. In this case, a Kalman filter algorithm is proposed to determine track centre line geometry, track gauge and super-elevation, that shows an excellent performance according to the International Union of Railway (UIC) standards. Jiang et al. present in [25] an algorithm that tries to overcome the low acquisition frequency of a total station combining its measurement with a high precision inertial sensor installed on

the trolley and the measurement of some known landmarks along the track under analysis. Despite of their precision and good performance, the use of these devices have the handicap of being very slow when measuring the track, that used to be around one kilometre per hour. They are a valid solution for the inspection of short track segments or occasional measurement but not for a full track measurement or continuous track monitoring.

As an alternative to the manual survey of a track, automated or dynamic methods are presented. In this case, the track irregularities are obtained in an indirect manner using the measurement of several sensors of different types that are installed on a laboratory vehicle. The key of these methods lies in a computational model of the vehicle and the track [26] that uses as inputs the measurement of the installed sensors and it is able to determine the track irregularities based on the dynamic response of the vehicle. As can be expected, the sophistication of the model affects the final results of the estimation model. In the work of Sagadeghi et al. [27] the authors demonstrate that in a slab track a 3D model of the wheel-rail interaction is required for a correct estimation of the contact forces. An example of an automated method is the work of Tsunashima et al. [28] where the vertical irregularity of the track is estimated from the car-body vibration using a Kalman filter and a simplified model of the vehicle. Westeon et al. in [29] present an alternative way to measure the vertical track irregularity using the pitch angle measured by a gyro sensor and accelerometers installed on the bearing boxes from an in-service railway vehicle. The estimation of track irregularities from the dynamic response of a vehicle is not an easy task as shown in the work of Karis et al. [30]. In this work, the authors try to find a correlation between the dynamic response of the vehicle and the excitation generated by the track irregularities using a simple model of the vehicle. They conclude that they do not always coincide in absolute values and standard deviations. The work of A De Rosa et al. [31] is focused on the indirect measurement of lateral track irregularities. In this work the authors use a modern machine learning algorithm to identify the lateral defects of the track. Another interesting work is the one presented by Wei et al. [32] where the alignment of the track is determined through a double integration of the acceleration measured by several accelerometers placed on the vehicle. Introducing the measurement of the sensors in a computational simulation model of the vehicle they are able to determine different range of alignment on the measured track. In the work of Escalona et al. [33], a track measurement system that can be installed on in-service vehicles and combines a kinematic model, a computational vision system and inertial measurement is presented. The mentioned system demonstrate a strong potential to the automated inspection of tracks. In conclusion, dynamic methods have the clear advantage of being much faster compared with the manual trolleys due to the fact that they are based on the use of in-service vehicles. However, the track measurement obtained from an in-service vehicle is not as accurate than using a manual trolley since the dynamics of the vehicle is included in the measurement. This is explained in [34], where a very interesting analysis of the measuring precisions obtained with manual trolleys and laboratory vehicles is presented. As a summary, it can be said that the railway industry is tending toward automated track survey systems installed on in-service vehicle combining conventional sensorization (inertial or distance sensors) with modern computational vision techniques [35] and artificial intelligence algorithms [36]. This could allow a daily monitoring of the track resulting on a reduction of the maintenance cost of the infrastructure

thanks to an anticipated intervention. There are also intermediate solutions that are neither a manual method nor an automated one. An example is the track inspection wagon presented by Chang et al. in [37]. An innovative self-propelled measuring trolley instrumented with a wide variety of sensors such as accelerometers and computational vision cameras, that allows a fast and accurate measurement of the track. The mentioned vehicle is tested on a laboratory track which irregularities has been artificially introduced. Finally it must be noted that computational vision techniques are gaining ground on the track surveying business. An examples is the ballast track inspection system presented by Sadegui et al. in [38]. This system provides an index that is a function of deficits and excess of ballast in the track.

As a response to the needs posed by the railways industry in terms of track measurement based on the dynamic response of the vehicle, the Mechanical Engineering research group of the University of Seville have been working for over two decades on the development of new computational multibody model formulations [39, 40] for the state observation from in-service railway vehicles. To guarantee the accuracy and good performance of any computational model, the scientific method requires that it must be experimentally validated. However, getting the access to a real railway vehicle and track is not always inexpensive neither possible. As an alternative, this research group has been recently using scaled vehicles and tracks for its investigations on railway dynamics [41, 42]. Although the dynamic response of an scaled vehicle cannot be directly extended to full-scaled vehicles, the utilisation of scaled systems represent an easy and relatively inexpensive way to experimentally validate new computational multibody models and new theoretical approaches without compromising the safety of the passengers or the integrity of the track. In this sense, the proposed method applied to a scaled vehicle and track, is not affected by the vehicle dynamics, and can serve as a design strategy to the application of the measurement of track irregularities at real tracks. In these tracks, geometry is continuously changing due to temperature effects, movements of the foundation and the most important, due to the effect of the wheel-rail contact forces exerted by the vehicles. Considering the fact that the vehicle dynamics is highly influenced by the track irregularities, it is fundamental to know the geometry of the track under analysis with the greatest possible accuracy, reducing this way uncertainties during the validation process of new computational models or a running vehicle safety analysis. Indeed, in an ideal scenario, the track should be measured just immediately before an experiment and in that case a fast an reliable measuring method is required. In the work of Aceituno et al. [43] a possible measuring method is presented but despite its precision, its slowness does not make it very operational. That is reason why the goal of this manuscript is to present a novel scaled railway vehicle for the automated track survey and calculation of track irregularities. The vehicle is not based on the traditional "chord-method", that would requires a large system to account for high wavelengths. The vehicle is powered by a DC motor and equipped with a LVDT, an inclinometer, a precision encoder and a reflector (target prism for the total station). Using the vehicle with the support of the total station, the scaled track geometry can be easily obtained within minutes. The track measurement and irregularities obtained with the proposed automated inspection method has been compared with a very accurate but slow manual measuring method, resulting on an great agreement between both approaches.

The paper is organised as follows: Section 2 is devoted to the design and instrumentation of the ARV and the measuring procedure. In Section 3 the MMM is explained in detail. Section 4 deals with

the track centre line optimization process and the calculation of track irregularities. The experimental validation of the ARV and the results comparison with the MMM are described in Section 5. Summary and conclusions are presented in Section 6.

2. Automated recording vehicle (ARV) and measuring procedure

Track irregularities have a great importance for railway operation. Depending on their magnitude, they might be only a matter of ride comfort or even compromise the ride safety leading to a derailment scenario. Thus, a correct identification and characterization of track irregularities results fundamental in railway simulation. In the railways industry, track irregularities are divided in two groups, relative and absolute track irregularities. Relative irregularities are the gauge variation and the cross-level. While absolute track irregularities are the alignment and the vertical profile. An extended description of track irregularities can be found in [44].

The original idea of this work was the development of an scaled Automated Recording Vehicle (ARV) such that, in a short time period, the track geometry could be measured and later on, track irregularities calculated. In the case of the scaled track subject of study in this research, having a fast procedure for measuring the track is even more important since the geometry can be manually modified if needed. Figure 1 (a) shows the mechanisms that are used as the sleepers of the scaled track. Each of them allows the variation of track gauge, cant angle and the relative height between both rails. The full track includes 900 mechanism distributed along its 90 meters of length (see Fig. 1 (b)). With the automated recording vehicle presented in this paper, the scaled track can be easily measured before any experimental campaign with an scaled railway vehicle [45, 42], minimizing that way possible uncertainties in the computational simulation due to errors in the definition of track irregularities.



Figure 1: (a) Track sleepers. (b) Track overall view

2.1. Mechanical design of the ARV

The automated recording vehicle CAD design is illustrated in Fig. 2 (a). It has two different parts: the drive system, consisting on the vehicle's body and traction wheels, and the measuring axle (see Fig.

2 (b)). Both parts are connected through a spherical joint as sketched in the figure. This mechanical joint isolates the rotations of the vehicle's body and the measuring axle.

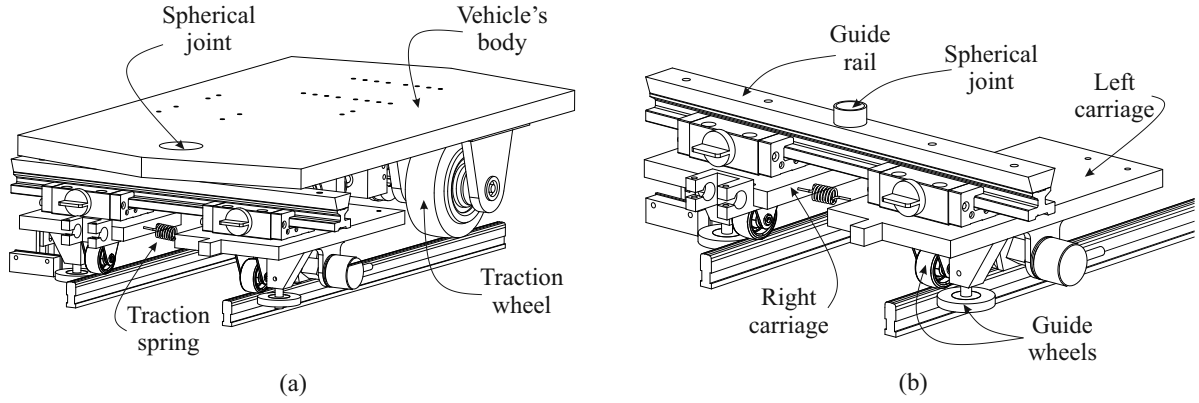


Figure 2: (a) Automated recording vehicle CAD. (b) Isolated measuring axis CAD

The fundamental element of the inspection vehicle is its measuring axle represented on Fig. 2 (b). It includes two carriages that slide along a guide rail. Two pairs of guide wheels keep the carriages attached to the rails attached to the rails, which ensures that vertical wheels lie in the same relative location with respect to the rail cross-section thus avoiding lateral wheel-rail displacement that may occur in other measuring vehicles. A traction spring guarantees the contact between the lateral guide wheels and the outer side of both rails. The stiffness of the traction spring has been calculated according with the maximum vehicle's velocity in order to guarantee a permanent constant between both lateral wheels and the rails. A third vertical wheel installed on the left carriage (see Fig. 3 (b)) avoids the measuring axis to pitch-over when moving forward. This mechanical assembly allows the measuring axis to follow the track geometry perfectly while it is pushed by the vehicle's drive system.

The drive system is shown with further details on Fig. 3. As it can be observed, the belt drive is actuated by a brushless DC motor. The output shaft is connected to a differential that drives the traction wheels, which are cylindrical, not conical, to minimize the influence that the kinematics with respect to the track of the relative traction axle may have in the measuring axle. In addition, because the guidance of the vehicle is achieved due to the lateral wheels and traction spring of the measuring axle, the traction wheels are wide enough to negotiate the curves properly. The differential guaranties the correct performance of the vehicle while it negotiates the curve sections of the track.

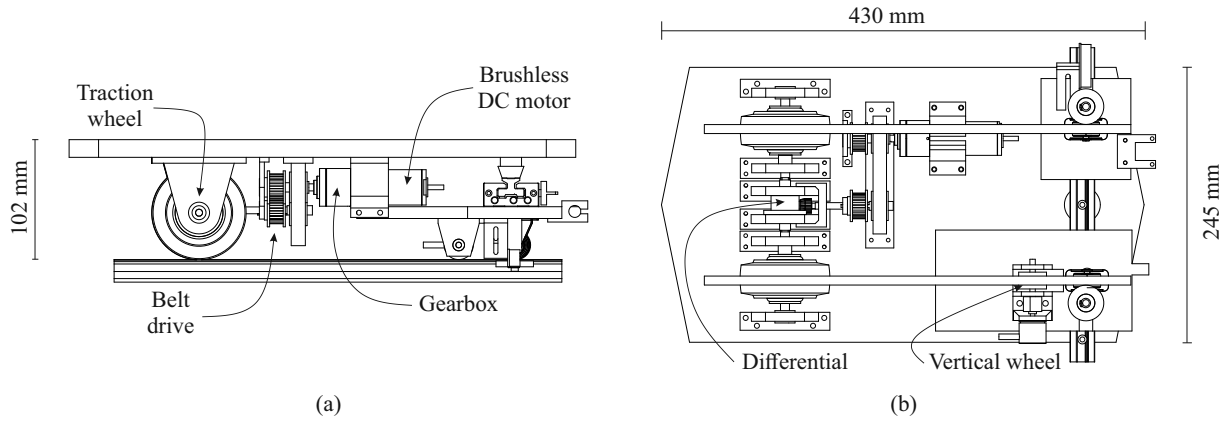


Figure 3: (a) Side view of the recording vehicle. (b) Bottom view of the recording vehicle

The vehicle has been manufactured in aluminium and 3D printed PolyLactic Acid (PLA). It is 430 mm long and 245 mm wide with a total mass of 6.5 kg. The measurable track gauge range is 107 mm to 157 mm and the maximum yaw rotation of the measuring axis is $\pm 35^\circ$. The vehicle is powered with a industrial quality brushless electric motor Maxon-ECi30-GP32C-ENC16EASY-MK2. The drive wheels can reach a maximum angular velocity of 20π rad/s that correspond to a forward velocity of 5 m/s. It must be noted that, such velocity does not correspond with the maximum velocity recommended for the measurement of the track as it will be explained later in this manuscript

2.2. Instrumentation of the ARV and data acquisition system

Figures 4 and 5 show the final assembly of the automated recording vehicle. It has been instrumented with an LVDT, an inclinometer, a precision encoder and a reflector. These sensors allow the measurement of track geometry and irregularities. As explained before in this section, relative track irregularities are the gauge variation and cross-level. They are the easiest to determine since they are directly obtained from the measurement of the LVDT and the inclinometer. In this case, the vehicle uses a 5mm-range high precision LVDT SCHREIBER SM347.10.1.S with a maximum accuracy of ± 12.5 microns. The sensor body is attached to the right carriage while the rod is in permanent contact with the left carriage thanks to an internal spring. The inclinometer has a range of $\pm 10^\circ$ and it is installed on the left carriage. The traction spring, which is expanded when the vehicle is on the track, tends both carriages towards a minimum separation between both guide wheels, being in that way the measuring axle perpendicular to both rails. Thus, it can be assumed that the vehicle follows the track irregularity. While measuring, the movement of the left carriage is locked to the guide rail and it is the right carriage the one that slides along the guide rail (see Fig. 5 (a)).

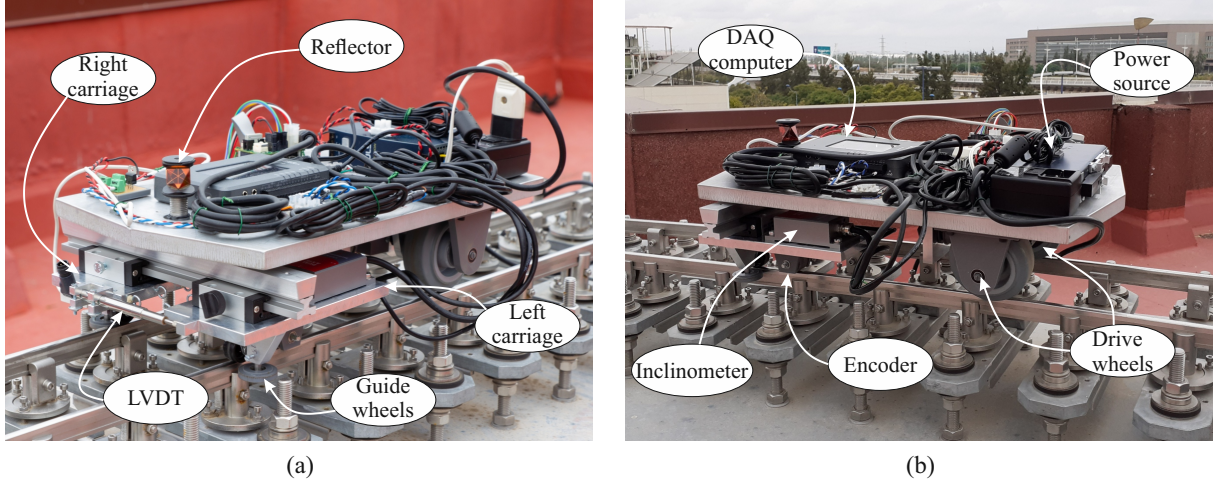


Figure 4: (a) Front view of the recording vehicle. (b) Lateral view of the recording vehicle

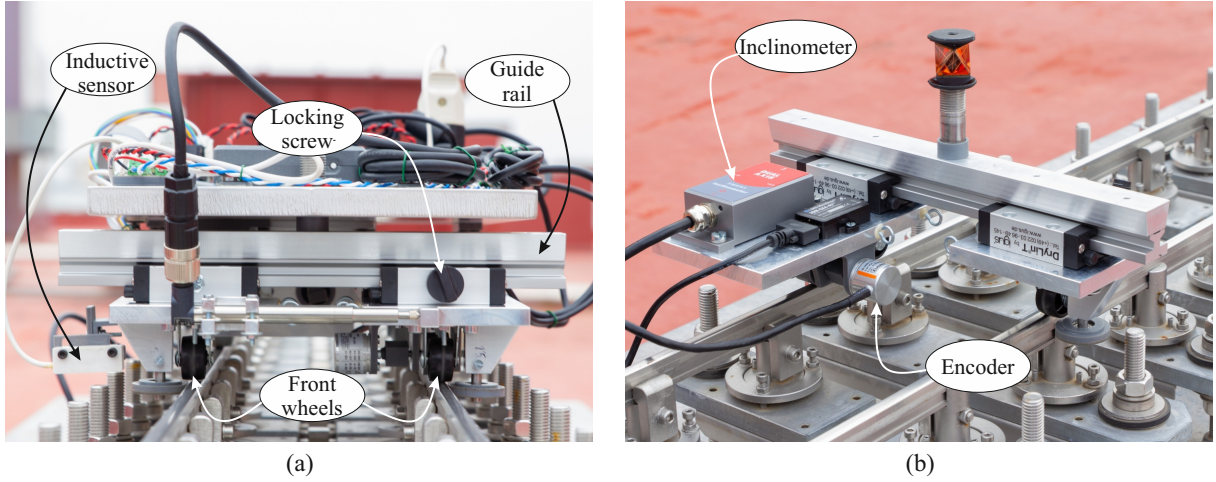


Figure 5: (a) Front view of the instrumented axle. (b) Top view of instrumented axle

For the measurement of absolute track irregularities (alignment and vertical profile), the sensors installed on the vehicle are complemented with the measurement of a robotic high precision total station (see Fig. 6) that follows the trajectory of a reflector rigidly installed on the vehicle's measuring axis (see Fig. 4 (a)). This method allows to capture the trajectory of a fixed point on the measuring axis that will be used to determine the absolute track irregularities, as will be shown in the next section. The speed operation of the vehicle depends on the total station capabilities and required frequency content of the measured irregularities. In this context, according to the limits of track irregularities in full scaled tracks given by EN-13848 [46], the smallest wavelength at D1 band (3 m) results in 0.3 m in the scaled track. Note that the scaled track is ten times smaller than a full scaled one. This minimum wavelength combined with the capabilities of the total station results in a maximum ARV operation speed of 0.9 m/s which guarantees the desired frequency content of the measured irregularities. The process to calculate the absolute track irregularities is explained later in this manuscript. Finally, the vehicle includes a precision encoder that measures the distance travelled by the instrumented axis. The encoder is rigidly attached to the rotation axis of one of the wheels of the measuring axis. In order to avoid possible errors in the measurement of the distance travelled by the vehicle, due to eventual slides of the measuring wheel,

the measurement of the encoder is corrected with an inductive sensor (see Fig. 5 (a)) that detects the position of several magnetic beacons (see Fig. 1 (a)) distributed along the track and whose positions have been previously measured. The odometry algorithm merges the measurement of the encoder and the inductive sensor to precisely locate the vehicle on the track. A Real-Time (RT) computer NI-myRIO installed on the vehicle's body is in charge of the control of the vehicle's DC motor and the acquisition of the sensors measurements using LabView. Due to the computational power and storage capacity of the acquisition computer, the measured data is oversampled for post processing purposes in this research. On one hand, all the sensors installed on the vehicle are sampled at 250 Hz, that based on the 0.9 m/s maximum forward velocity of the vehicle allows a measurement every 3.6 mm. On the other hand, the measurements of the total station are sampled at 20 Hz, that based on the same forward velocity allows a measurement every 45 mm guaranteeing in that way the required smallest wavelength $D1$ of 300 mm. Note that both sets of data, the measurement of the total station and the measurement of the TRV, are acquired with two different computers, thus the synchronization between both measurements must be guaranteed. The process is accomplished as follows: Using the absolute position of the reflector measured by the total station, the longitudinal distance s travelled by the TRV can be determined. This longitudinal distance is also measured by the precision encoder installed on the instrumented axle of the vehicle, whose measurement is time-synchronized with the rest of sensors of the TRV. In that way, both measuring systems can be space-synchronized.

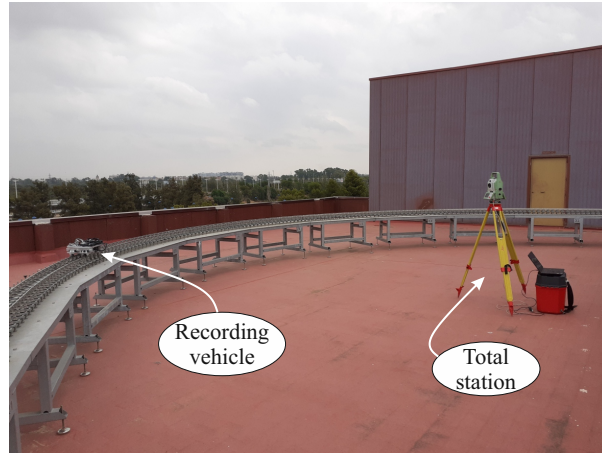


Figure 6: Robotic total station following the trajectory of the vehicle

3. Manual measuring method (MMM)

The accuracy of the track geometry measuring process using the automated recording vehicle could be jeopardized by the dynamics of the vehicle itself or possible clearances between mechanical parts due to errors in the manufacturing and assembly process. In order to guarantee the correct performance of the vehicle, its measurement has been compared with a manual measuring method of the track for validation purposes. Although the manual measuring method represents a very accurate and reliable way for measuring the track geometry, its execution is slow and tedious. A human operator requires approximately 20 hours to measure the 90 meter long scaled track presented in this paper. This is a huge difference with respect to the automated recording vehicle which is able to accomplish such task within

minutes. The manual approach used in this work, which is described next, is an upgraded version of the method previously presented by Aceituno et al. in [43].

3.1. Description of the MMM

The execution of the manual measurement of the track is accomplished in two stages. The first one entails the measurement of the track centre line, while the track gauge and cant angle are obtained in the second phase. The track centre line is determined using the total station and the reflector which is installed on an special measuring plate. Figure 7 (a) shows the mentioned plate on the track. An ellipsoidal profile with long and short axis of 129.5 and 126 mm, respectively (values which are out of the limits of admissible gauges of the scaled track), is machined on its bottom side (see Fig. 8). The ellipse has the property that if it is placed in between two parallel lines and it contacts both, the centre of the ellipse stays in the mid point of both lines. Thus, when the ellipsoidal plate is placed on the scaled track being tangent to both rails, its centre point where the reflector is located, is exactly over the track centre line. The plate is manually moved at regular intervals along the track while the total station registers the position of the track centre line. This assumption is also applicable in curved sections since the radius of curvature is large enough to consider both rails always parallel. Figure 7 (b) shows one of the authors when measuring the track centre line using the ellipsoidal plate.



Figure 7: (a) Ellipsoidal measuring plate on the track. (b) Manual measurement of the track centre line

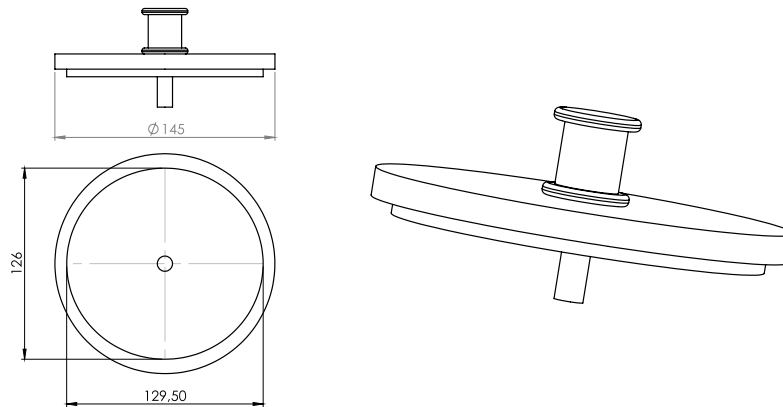


Figure 8: CAD of the ellipsoidal measuring plate

The total station used in the measurement of the scaled track with the automated recording vehicle and the measuring plate is a Leica Nova MS50, which according to its technical specifications could eventually reach a maximum accuracy of 0.2 mm. However, it has been proved by the authors in previous tests that, such precision is only achievable if the machine and the target are separated less than 20 meters. For this reason, due to the large dimension of the scaled track surveyed in this work, and in order to guarantee the maximum precision in the measurement with the total station, the track has been measured in three sections. This is a common practice also when measuring a full scaled track. In the railways industry when a section of the track needs to be accurately surveyed, either because it has been modified or simply because its condition needs to be known, the measuring trolley must be used together with a total station. The latter must be located in a place such that it can observe the reflector placed on the measuring trolley while moving on the section of the track under analysis. Depending on the features of the machine, the required accuracy and the length of the section, the machine is placed in one or several spots, which represents a tedious task. Nowadays measuring trolleys are also equipped with GNSS, that helps to locate the absolute position of the track. However, these systems are not as precise yet as the total station. Figure 9 shows the three sections the scaled track has been divided, coloured in orange, blue and purple, respectively, and the three positions (O_1 , O_2 and O_3) where the total station has been located during the measurement process. Two sets of fixed landmarks, P_i and Q_i , measured from the three locations of the total station allow the alignment of the three sets of data points in a common reference frame.

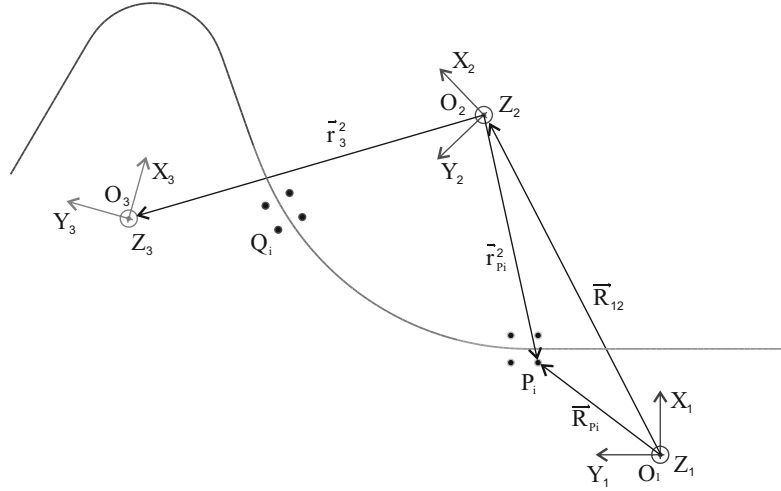


Figure 9: Scaled track measurement

In the second phase of the manual measuring process, the track gauge and cant angle are obtained using the gadget shown in Fig. 10. It is instrumented with the same LVDT and inclinometer than the measuring axis of the automated recording vehicle. Its performance is simple. Distance and tilt sensors are rigidly attached to the central guide which lies over the rails. An angle bracket under the bar keeps the central plate perpendicular to one of the rails while the edge of the slider, which is pushed by the internal spring of the LVDT, remains in contact with the opposite rail. Thus, the track gauge and cant angle are simultaneously measured when the gadget is moved along the track by a human operator. The two stages of the MMM have been accomplished independently. Note that the measured discrete points

with the total station and the gadget are not exactly the same. To solve that, both measurements are interpolated in a common trajectory coordinate s along the track centre line in a post-processing stage.

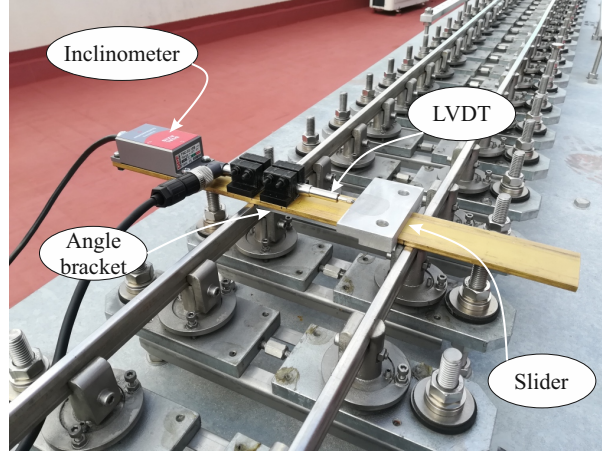


Figure 10: Gadget for the manual measurement of the relative track irregularities

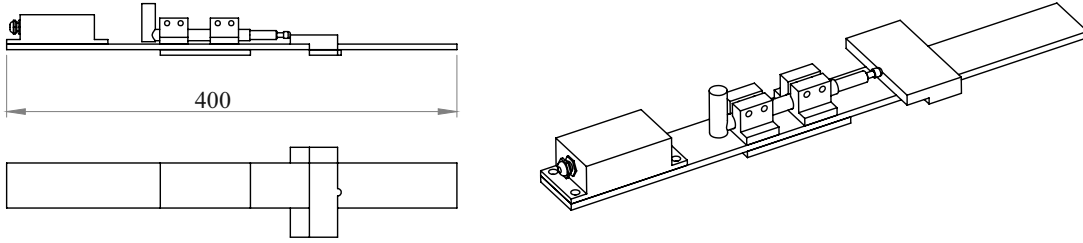


Figure 11: Gadget for the manual measurement of the relative track irregularities

4. Track centre line optimization and calculation of irregularities

This section describes a general procedure to calculate track irregularities based on experimental measurements. Although in this particular case it has been applied to a 1:10 scaled track, this procedure would be equivalent in a full-scaled one. Track irregularities can be obtained based on the difference between the design track geometry and the measured geometry. In railway engineering the design track geometry is defined by an horizontal profile and a vertical profile. The horizontal profile projection is composed of a combination of tangent (straight), constant curvature (circular) and transition segments. The projection of the vertical profile includes constant slope and transition sections. Due to errors made during the construction of the track, natural movements on the foundation and the continuous passage of vehicles make the design geometry to vary resulting on an irregular track.

4.1. Track centre line optimization

As previously mentioned, track irregularities can be quantified based on the distance difference between the design geometry and the measured geometry. In this context, it might be considered that the design track geometry could be adopted from construction drawings. However, as it will be shown next, the differences between the design geometry and the real track could lead to considerable high absolute deviations of track alignment ξ_a and vertical profile ξ_v due to the fact that the ideal geometry

does not properly fit the measured one. For this reason, this section describes an optimization procedure to determine the horizontal and vertical projections of the ideal track centreline that better fits the measured one where track irregularities can be determined. Figure 12 shows a comparison between the design geometry of the horizontal and vertical profiles of the scaled track centre line and the measured geometry using the automated recording vehicle. As it can be observed, there is a notorious difference between both lines caused by errors made during the construction process of the scaled track. In view of that, the original designed geometry cannot be considered as the ideal geometry that better adjusts the measured track. Thus, the optimization algorithm, starting from the original designed geometry of the scaled track, summarized in Tables 1 and 2, must modify the lengths of the segments and radii of curvature in order to find a new design geometry. The algorithm minimizes the mean squared error of the distances between both curves given by the following equation:

$$MSE = \frac{1}{n} \sum_{i=1}^n (\mathbf{R}_i^{ideal} - \mathbf{R}_i^{measured})^T (\mathbf{R}_i^{ideal} - \mathbf{R}_i^{measured}) \quad (1)$$

where $\mathbf{R}_i^{ideal}$ and $\mathbf{R}_i^{measured}$ represent the absolute components (x, y, z) of the ideal and measured track points, respectively, and n is the total number of points the track centre lines have been measured. Note that the real track has been measured as a set of discrete points. Using a track pre-processor [47] each measured point can be associated with a point of the design geometry. The track pre-processor is a subroutine that provides any geometrical data associated to the ideal track (position vectors, tangent, curvatures) as a function of a trajectory coordinate based on analytically defined curve segments. It uses a text file that parametrically describes the different segments that define the design geometry. These parameters are modified at each iteration of the optimisation process until the MSE reaches an admissible tolerance. Figure 13 shows the obtained horizontal and vertical projection of the optimized track centre line after the optimization process which numerical values are summarised in Tables 3 and 4. As it is now observed, the optimized geometry fits quite well with the measurement of the recording vehicle.

Note that, in view of the results presented in Fig. 13, the proposed minimization procedure does not account for any regularization term [48, 49], although it could be augmented in case the measurements suffer from high noise levels.

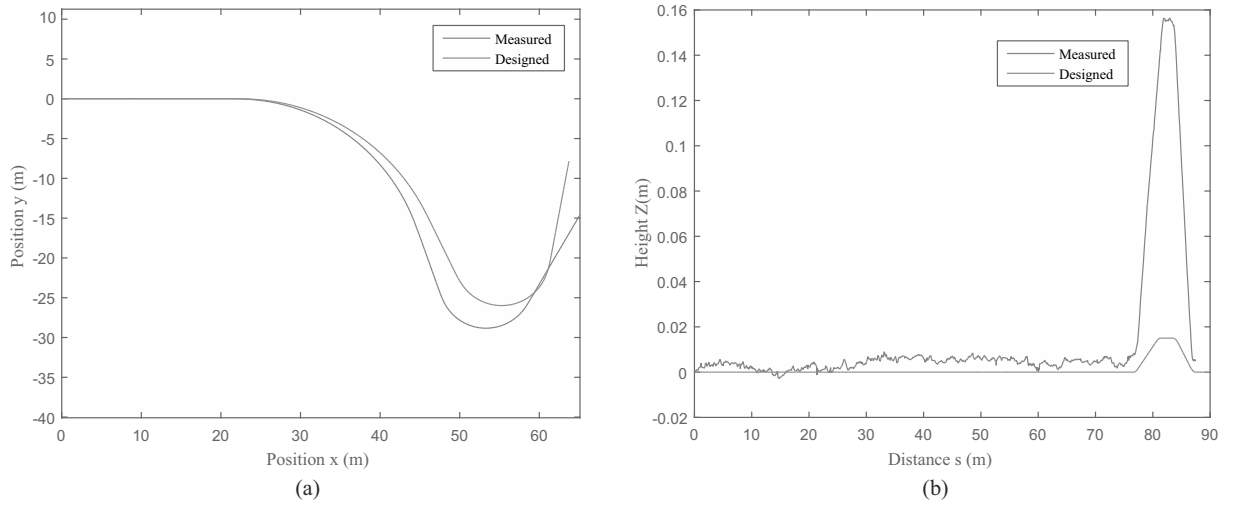


Figure 12: (a) Horizontal projection, measured vs designed geometry. (b) Vertical projection, measured vs designed geometry

Table 1: Horizontal projection of the ideal track geometry, initial guess

Section	Type	Length (m)
A	Tangent	20
B	Transition	3
C	Constant radius (24m)	26
D	Transition	3
E	Tangent	6
F	Transition	3
G	Constant radius (6m)	12
H	Transition	3
I	Tangent	12

Table 2: Vertical projection of the ideal track geometry, initial guess

Section	Type	Length (m)
A	Tangent	76
B	Constant slope (+3.75%)	4
C	Tangent	2
D	Constant slope (-5%)	3
E	Tangent	2

Table 3: Horizontal projection of the ideal track geometry, optimized

Section	Type	Length (m)
A	Tangent	21.235
B	Transition	1.358
C	Constant radius (24m)	25.139
D	Transition	7.258
E	Tangent	4.483
F	Transition	2.766
G	Constant radius (6m)	10.989
H	Transition	2.169
I	Tangent	12.603

Table 4: Vertical projection of the ideal track geometry, optimized

Section	Type	Length (m)
A	Tangent	20.9
B	Constant slope (+0.035%)	15.2
C	Tangent	40.9
D	Constant slope (+3.2%)	4.7
E	Tangent	2
F	Constant slope (-4.7%)	3.2
G	Tangent	0.6

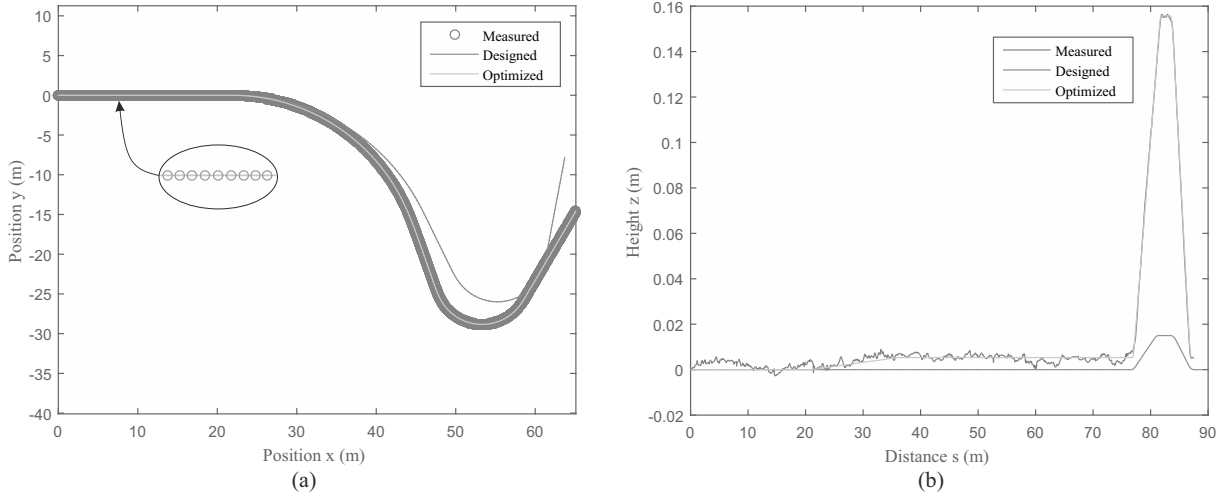


Figure 13: (a) Horizontal projection, measured vs designed geometry vs optimized. (b) Vertical projection, measured vs designed geometry vs optimized

4.2. Calculation of the irregularity

Once the ideal track geometry is optimized, track irregularities can be calculated as the difference between the ideal position of the rails with respect to their actual positions. Figure 14 sketches the track irregularity definition. In the figure, rail sections coloured in pale blue represent the ideal position while the dark green ones are the measured positions of the rails. Irregularity vectors are defined as $\vec{r}^{lir} = \begin{bmatrix} 0 & y^{lir} & z^{lir} \end{bmatrix}^T$ and $\vec{r}^{rir} = \begin{bmatrix} 0 & y^{rir} & z^{rir} \end{bmatrix}^T$ respectively. These vectors are expressed in a frame

called track frame (TF), that is attached to the track centre line being its X^t axis tangent to it, and the Y^t axis contained in the line that joins the left and right railheads, as shown in Fig. 14. The alignment ξ_a , gauge ξ_g , cross-level ξ_c and vertical profile ξ_v are defined as a function of the irregularity vectors components using the following expressions:

$$\begin{aligned}\xi_a &= (y^{lir} + y^{rir})/2 \\ \xi_g &= (y^{lir} - y^{rir}) \\ \xi_c &= (z^{lir} - z^{rir}) \\ \xi_v &= (z^{lir} - z^{rir})/2\end{aligned}\tag{2}$$

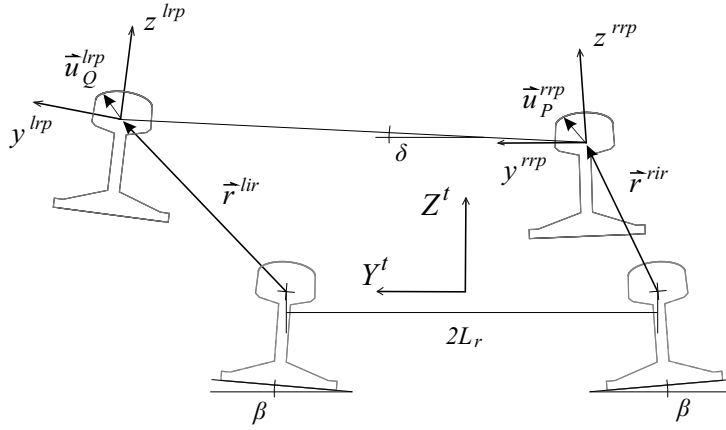


Figure 14: Definition of the track irregularities

Experimentally, gauge and cross-level are obtained directly from the measurement of the LVDT and the inclinometer. The gauge irregularity is defined as the difference between the measured gauge and the nominal value of the track. The cross-level is the difference of height between both rails and it can be easily obtained as:

$$\xi_c = \sin(\alpha_{inc}) \cdot d_{LVDT}\tag{3}$$

where α_{inc} is the angle measured by the inclinometer and d_{LVDT} is the track gauge measured by the LVDT.

Alignment and vertical profile cannot be obtained in such a direct way. It is first required to obtain the distance between the optimized track centre line points and the measured ones. To do that, a system of non-linear algebraic equations must be solved, which calculates the distance between every single measured point and the closest point at the optimised track centre line. Point P_i in Fig. 15 represents a measured point on the track while point Q_i is an arbitrary point at the optimized track centre line. The distance between both points is the modulus of vector $\vec{d}_i$ that can be calculated using the expression:

$$\vec{d}_i = \vec{R}_{Q_i} - \vec{R}_{P_i}\tag{4}$$

where $\vec{R}_{Q_i}$ is the absolute position of one point of the optimized track centre line and $\vec{R}_{P_i}$ is the absolute position of the measured point.

In Fig. 15, $\vec{t}_{Q_i}$ denote the tangent vector to the scaled track ideal centre line at point Q_i . Vector $\vec{d}_i$ has to be perpendicular to $\vec{t}_{Q_i}$. Thus, the following equation has to be fulfilled:

$$\vec{d}_i \cdot \vec{t}_{Q_i} = 0 \quad (5)$$

The process concludes after projecting vector $\vec{d}_i$ on the track reference frame according to the next expression:

$$\vec{r}^{irr} = (\mathbf{A}^t)^T \mathbf{d}_i \quad (6)$$

being $\mathbf{A}^t$ the transformation matrix of the Track Frame (TF) to the Global Frame (GF). As usual in railways kinematics, the GF is a fixed reference frame, meanwhile the TF follows the vehicle on its movement along the track. In this research, the GF is assumed to be located in the first location where the total station was installed. Due to the orientation of the TF with respect to the track centre line, the longitudinal coordinate X is equal to zero. Thus, second and third components of vector $\vec{r}^{irr}$ are the irregularities of alignment and vertical profile, respectively. Once the alignment, cross-level, gauge and vertical profile have been determined, system of equations (2) can now be solved to obtain the irregularity vector components y^{lir} , z^{lir} , y^{rir} and z^{rir} .

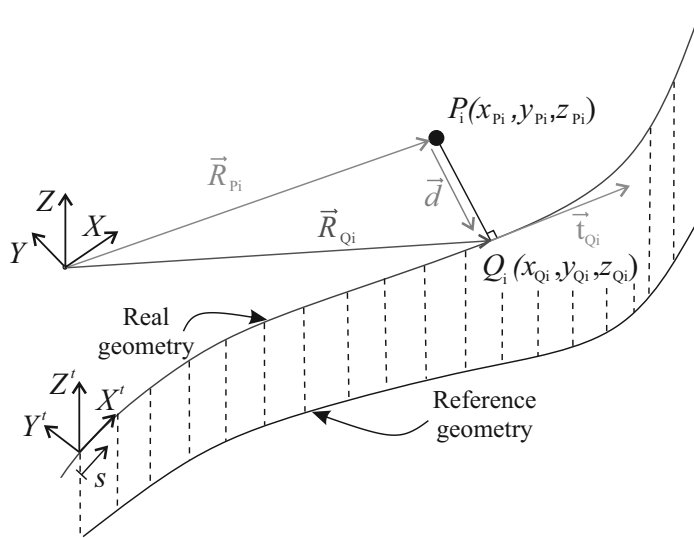


Figure 15: Alignment and vertical profile calculation procedure

4.3. Determination of the rails positions using the ARV

For a correct performance and accurate measurement of the total station, the reflector is located on the ARV in a position such that, it is always visible to the total station during the measurement of the track. This requires the reflector to be installed on the top of the ARV as shown in Fig. 5 (b). That means, the total station is not directly measuring the track centre line, but a line parallel and close to the latter. Considering that, the left carriage (where the sensors are installed) is locked to the guide rail and the relative position of the reflector with respect to the left rail is known. As it is explained next, using the position of the reflector measured by the total station, the measurement of the inclinometer, the

368 LVDT, the encoder and the final help of the track pre-processor, the absolute coordinates of the actual
 369 track centre line can be determined.

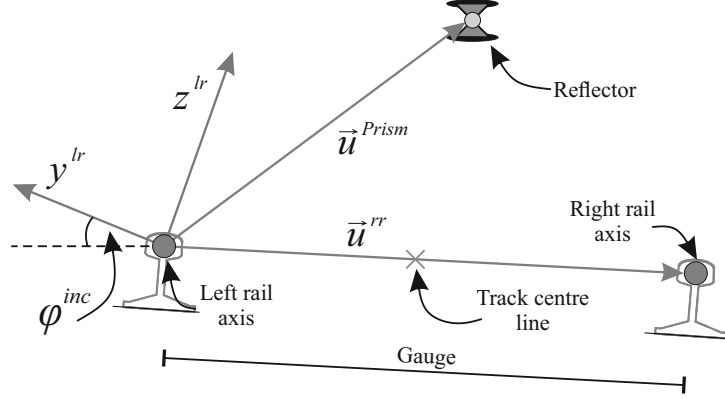


Figure 16: Kinematics of the reflector and the rails

370 Figure 16 shows a kinematics sketch of the rails and the reflector, in witch the latter is assumed to
 371 be rigidly attached to a reference frame $\langle y^{lr}, z^{lr} \rangle$ that moves together with the left-front wheel that lies
 372 over the left rail. Let define A^{lr} as the semi-experimental rotation matrix that transform a vector from
 373 the left rail reference frame to the global frame,

$$\mathbf{A}^{lr} = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & \varphi^{inc}\sin(\psi) + \theta^{inc}\cos(\psi) \\ \sin(\psi) & \cos(\psi) & -\varphi^{inc}\cos(\psi) + \theta^{inc}\sin(\psi) \\ -\theta^{inc} & \varphi^{inc} & 1 \end{bmatrix} \quad (7)$$

374 where φ^{inc} and θ^{inc} are the angles experimentally measured by the dual-axis inclinometer and ψ is the
 375 yaw angle provided by the track pre-processor. Matrix $\mathbf{A}^{lr}$ has been linearised assuming that the angles
 376 of the inclinometer are small values. Taking into account that the local position of the reflector with
 377 respect to the left rail reference frame $\bar{\mathbf{u}}^{Prism}$ is known from the mechanical design of the vehicle, the
 378 following expression can be established:

$$\mathbf{R}^{lr} = \mathbf{R}^{Prism} - \mathbf{A}^{lr} \bar{\mathbf{u}}^{Prism} \quad (8)$$

379 where $\mathbf{R}^{lr}$ denotes the position of the left rail axis in the global reference frame and $\mathbf{R}^{Prism}$ is the absolute
 380 position of the reflector measured by the total station, also expressed in the global reference frame. Then,
 381 the absolute position of the right rail $\mathbf{R}^{rr}$ and the track centre line $\mathbf{R}^{TCL}$ can be easily obtained using
 382 the following expressions:

$$\mathbf{R}^{rr} = \mathbf{R}^{lr} + \mathbf{A}^{lr} \bar{\mathbf{u}}^{rr} \quad (9)$$

$$\mathbf{R}^{TCL} = \mathbf{R}^{lr} + \mathbf{A}^{lr} \bar{\mathbf{u}}^{TCL} \quad (10)$$

383 being $\bar{\mathbf{u}}^{rr} = [0 \quad -gauge \quad 0]^T$ and $\bar{\mathbf{u}}^{TCL} = [0 \quad -gauge/2 \quad 0]^T$, respectively. Note that, the *gauge* is
 384 obtained throughout the measurement of the LVDT.

5. Experimental validation and comparison of results

In order to validate the performance and accuracy on the measurement using the automated recording vehicle, this section presents an experimental comparison between the track irregularities obtained with the vehicle and the irregularities drawn from the manual measurement of the track. The manual measurement is considered as the reference since it has been accomplished in totally controlled conditions. However, the measurements of the vehicle, taking into account that they are obtained while the vehicle moves at a certain speed along the track, might be affected by the vehicle-track dynamics.

Figures 17, 18 and 19 show the comparison between the relative track irregularities obtained using the automated recording vehicle and the manual method. In this experiment the vehicle moves at a constant velocity of 0.5 m/s along the track. It can be observed in Fig. 17 how the gauge variation drawn from the measurement of the LVDT almost coincide in both experiments. These results have been obtained considering a nominal track gauge of 127.8 mm. In view of the results, it can be said that the measurement of the LVDT is not affected by the dynamics of the vehicle at the speed of 0.5 m/s. In Fig. 19 it is observed that the inclinometer installed in the ARV follows the trend of the manual measurement but experiencing larger oscillations. This is something expected due to the fact that, the cross-level is obtained using the combined measurement of the LVDT and the inclinometer, and the latter is a type of sensor that normally presents a bad dynamic performance. However, the observed differences at the speed of 0.5 m/s can be assumed to be sufficiently accurate because the trend of the manual measurement is well identified. A possible alternative to the utilisation of an inclinometer to measure the cant angle of the track is the use of an Inertial Measurement Unit (IMU) and a sensor fusion algorithm. Two examples are the algorithms proposed by Madwick et al. in [50] or Sabatini in [51, 52]. These algorithms are a priori suitable for dynamic measurements. They combine the measurement of the acceleration and the angular velocities of the IMU to estimate the absolute orientation of the sensor.

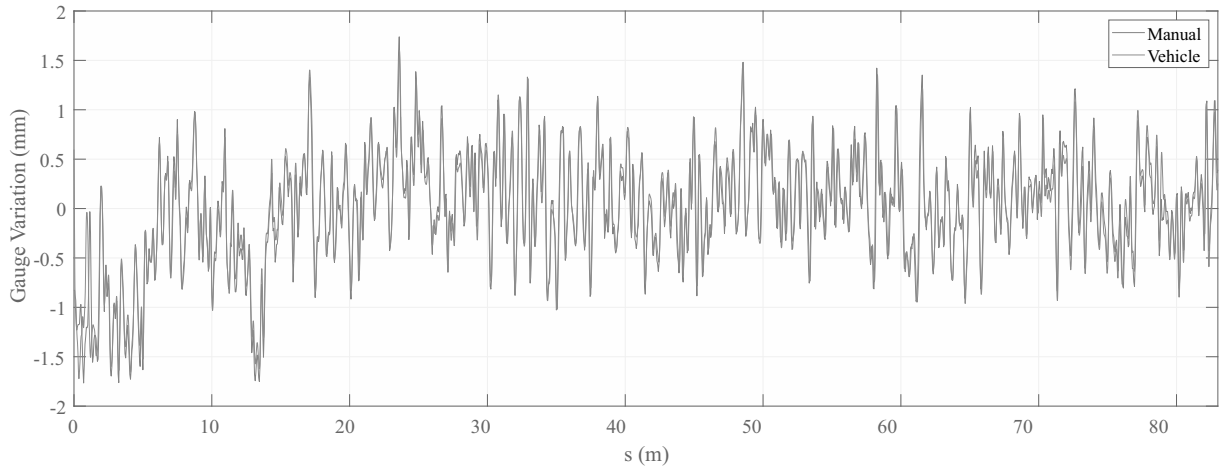


Figure 17: Gauge variation methods comparison

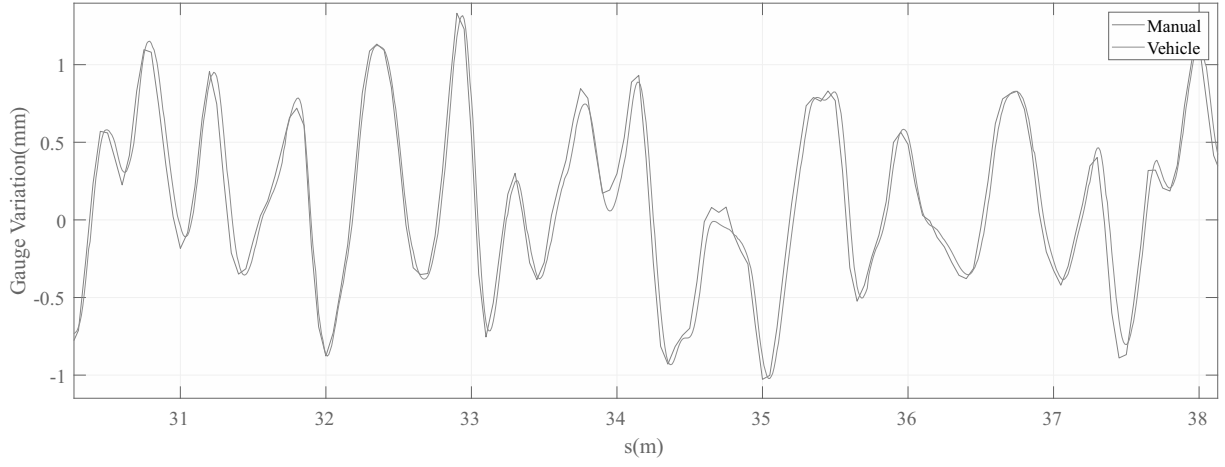


Figure 18: Magnification of gauge variation methods comparison

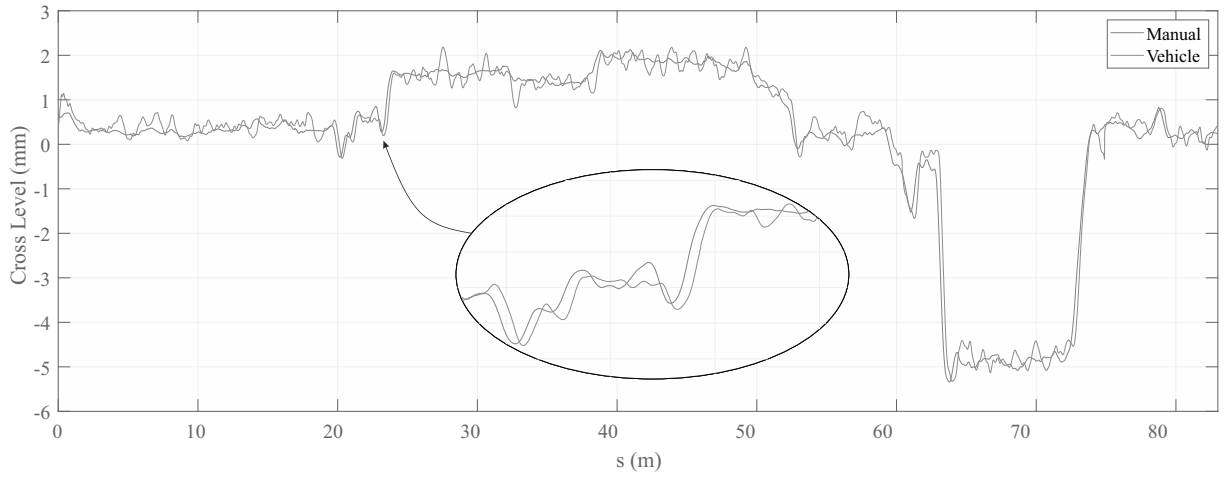


Figure 19: Cross-level methods comparison

The repeatability of the sensors measurement when the vehicles moves at different forward velocities is also analysed. Figure 20 shows the measurement of the track gauge obtained with the LVDT in three different experiments accomplished at 0.5, 0.7 and 0.9 m/s. It is clearly observed how the measurement of the LVDT remains stable in the three scenarios. However, the inclinometer measurement shown in Fig. 21 varies significantly at different velocities. This is observed when the vehicle negotiates the second curve of the scaled track located between $s = 60$ m and $s = 75$ m, approximately. In this case, the centrifugal force experienced by the sensor installed in the vehicle strongly affects its measurement when it moves faster than 0.5 m/s. This can be explained given the fact that the inclinometer bases its measurement on the capacitive micro pendulum principle and the Earth gravity principle sketches in Fig. 22. Since the inclinometer is an inertial sensor, the angle α represented in the figure is highly affected by the vehicle's dynamics. In view of these results, it is concluded that the measurement of the track using the vehicle must be done at a maximum speed of 0.5 m/s (which is in range of admissible operational speeds described in Section 2) in order to guarantee the correct performance of the tilt sensor.

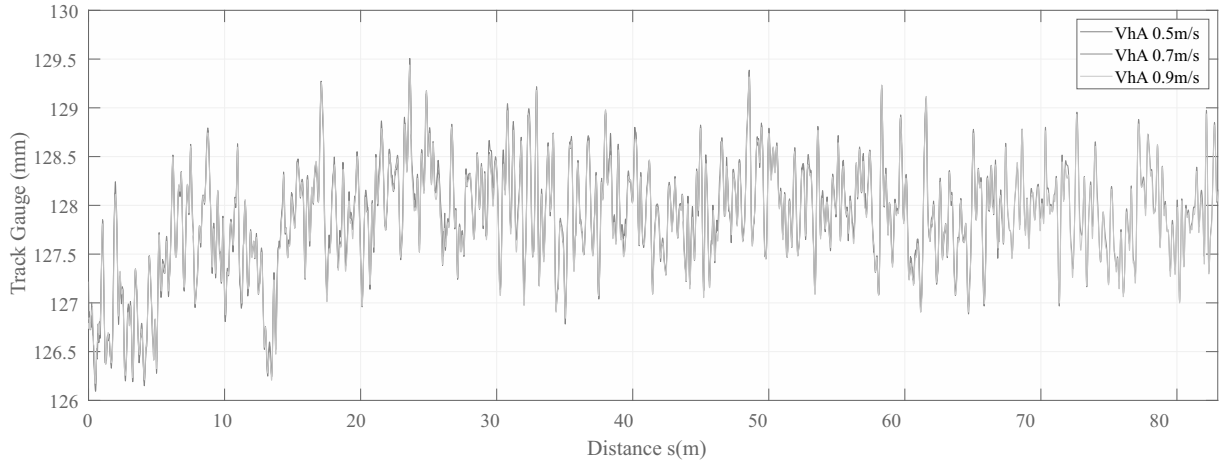


Figure 20: Repeatability of LVDT measurement

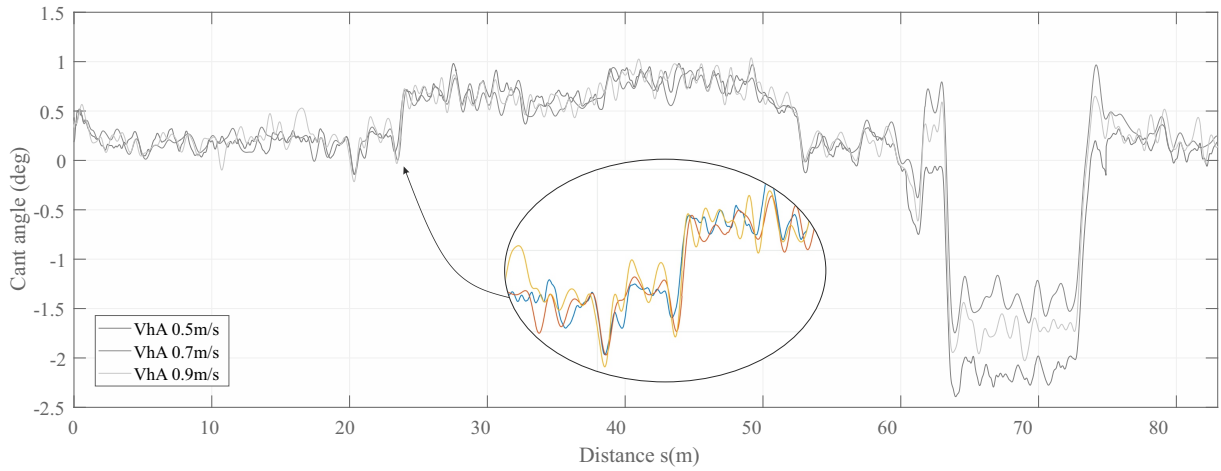


Figure 21: Repeatability of inclinometer measurement

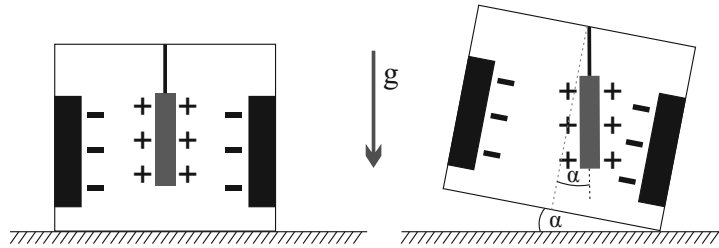


Figure 22: Functioning principle of the inclinometer sensor

Moreover, the absolute track irregularities have been calculated according the procedure explained in the previous section. Figures 23 and 24 show the obtained alignment and vertical profile of the scaled track using the measurement of the ARV and the MMM. The signals are filtered according to the European standard [46] using a band-pass filter applied in the length domain with a cut-off frequencies between $1/7 \text{ m}^{-1}$ and $1/0.3 \text{ m}^{-1}$, which correspond with $1/70 \text{ m}^{-1}$ and $1/3 \text{ m}^{-1}$ in a full scaled track. These frequencies are determined taking into account the scale reduction of 1:10, and they correspond with the D1 to D2 range given in the European standard. In what follows, all figures concerning measurements both with the manual and vehicle methods, are filtered equally. In the Fig. 23 it is observed a good agreement in the

429 magnitude of the alignment between both signals. The resultant Root Mean Squared (RMS) level of the
430 difference between both signals is summarized in Table 5. In this analysis the track has been divided in
431 three sections which correspond with the tangent section located at the beginning of the track, the large
432 radius curved section ($R = 24$ m) and the sharp radius curved section ($R = 6$ m).

Table 5: RMS analysis in different track sections

Irregularity	Tangent ($s=0-21\text{m}$)	Curve ($s=21-47\text{m}$)	Curve ($s=60-73\text{m}$)
Gauge	$2.02 \cdot 10^{-4}$ m	$1.26 \cdot 10^{-4}$ m	$1.24 \cdot 10^{-4}$ m
Cross-level	$1.79 \cdot 10^{-4}$ m	$2.08 \cdot 10^{-4}$ m	$4.71 \cdot 10^{-4}$ m
Alignment	$4.21 \cdot 10^{-4}$ m	$4.99 \cdot 10^{-4}$ m	$6.31 \cdot 10^{-4}$ m
Vertical profile	$4.58 \cdot 10^{-4}$ m	$4.29 \cdot 10^{-4}$ m	$4.31 \cdot 10^{-4}$ m

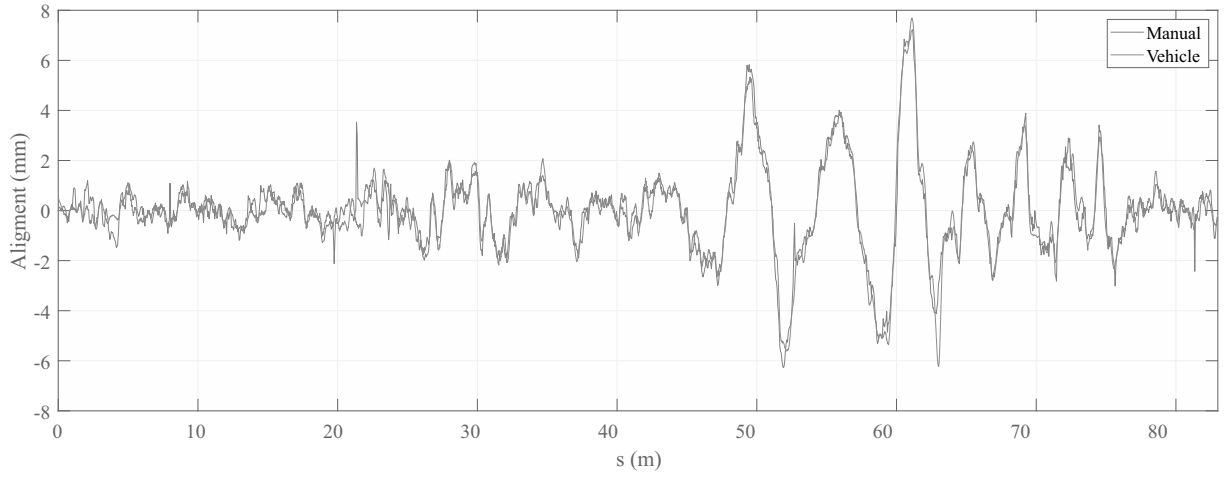


Figure 23: Alignment of the scaled track

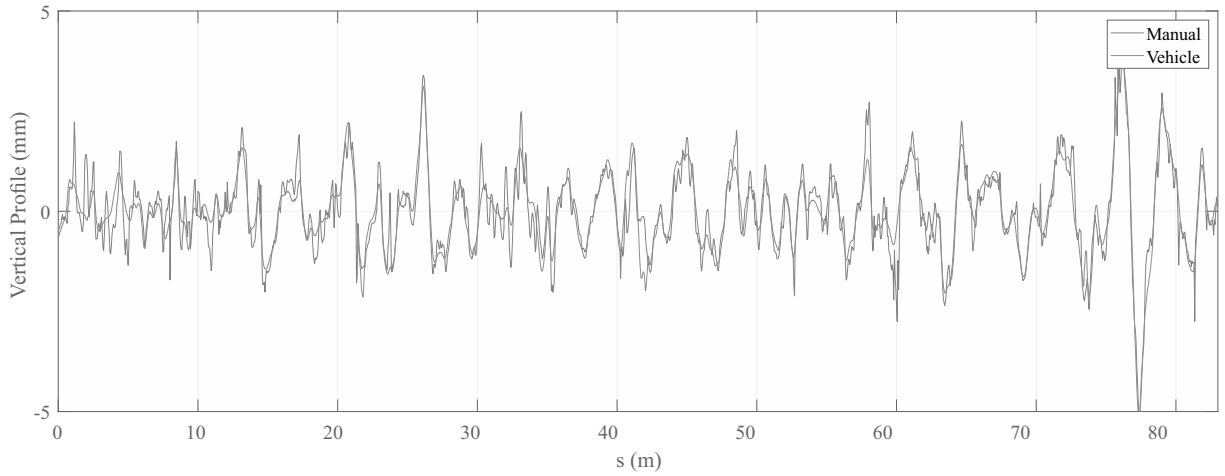
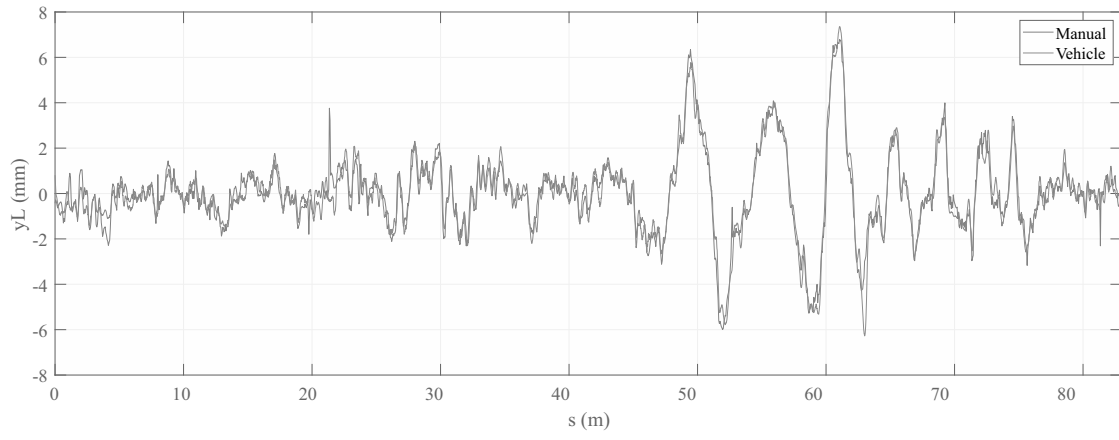


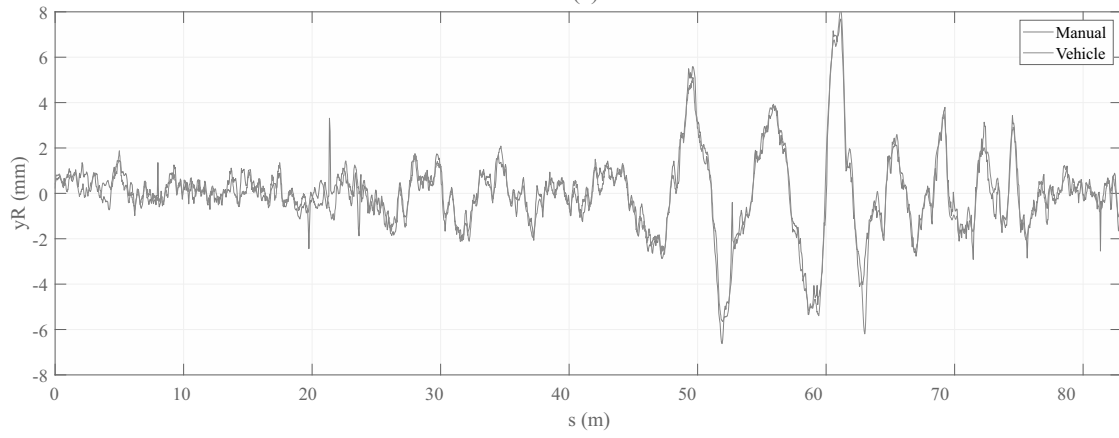
Figure 24: Vertical profile of the scaled track

433 Finally once the four irregularities of track gauge, cross-level, alignment and vertical profile are calcu-
434 lated, the real position of the rail's head (see Fig. 9) given by y^{lir} , y^{rir} , z^{lir} and z^{rir} can be determined
435 using Eq. 2. Figure 25 shows the obtained results. Looking at the Figs. 25 (c) and 25 (d) it can be
436 observed the difference of height between both rails in the curved sections located $s = [21 - 47]$ m and $s =$

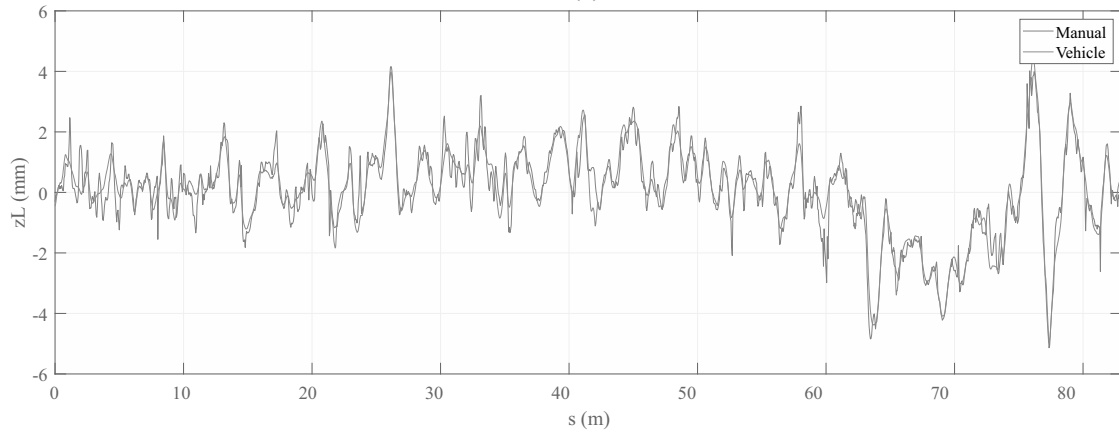
437 [60 - 73] m. This difference of height corresponds with the cant angle manually introduced in the scaled
438 track. In view of these results it can be concluded that the ARV demonstrates an excellent performance
439 when measuring the scaled track geometry, being the obtained irregularity results analogous to the ones
440 drawn from the MMM. In addition, the use of the ARV reduces the measurement time twenty times
441 compared with the MMM. The ARV allows a fast and accurate measurement of the track just before an
442 experimental campaign with a scaled vehicle.



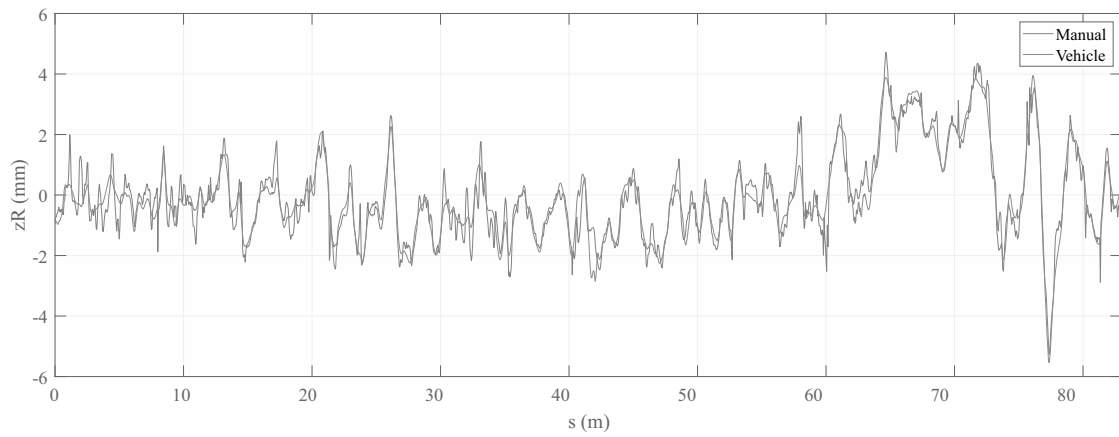
(a)



(b)



(c)



(d)

Figure 25: (a) y_L , (b) y_R , (c) z_L , (d) z_R displacements

6. Summary and conclusions

In this work, a novel Automated Recording Vehicle (ARV) and a track irregularity measuring process are proposed for the measurement of a scaled railroad track. The vehicle is intended to be used for the regular measurement of a 1:10 scaled track built at the School of Engineering of the University of Seville. The mechanical design of the aforementioned track allows the manual introduction of track irregularities, which implies that, the geometry of the track can be easily modified. The ARV includes a mechanical front axle which adjusts its geometry to the track and records the irregular lines. This axle is instrumented with several sensors that allow the measurement of the track geometry and its irregularities. The vehicle is used together with a high precision total station that register its position along the track. Using this new vehicle the scaled track can be measured within minutes, that represents a huge advantage compared with a long and tedious Manual Measuring Method (MMM) accomplished by a human operator.

The track geometry and irregularities measured using the ARV is compared with the results obtained using a manual approach. Both the vehicle and the manual process use identical sensorization. The track geometry obtained using the manual approach is considered as reference geometry since it is not affected by the vehicle's dynamics. The vehicle has been tested at different velocities in order to analyse the response of the sensors' measurements with the velocity. It has been determined that the inclinometer sensor used for the measurement of the track cant angle limits the vehicle's maximum speed. The inclinometer measurement misrepresents the reality when the vehicle moves forward at 0.7 m/s due to the inertial forces. The final results shown in this manuscript with the vehicle moving at 0.5 m/s demonstrate that the track irregularities obtained using the automated recording vehicle are equivalent to the ones obtained using the manual method.

As a conclusion, it can be said that the proposed ARV is able to satisfactorily measured the geometry of the scaled track in a fast, precise and reliable way. This is something fundamental in railway research since track geometry, which is the main source of excitation, is measured just before experimental campaigns take place. This allows computational models to be adjusted in detail for validation purposes. This updated and accurate measurement of the irregularities reduces the uncertainties in the posterior dynamic analysis of the vehicle since the track geometry is well defined. The technology and methodology used in this particular work, that is applied to an scaled system can be extended to a full scaled scenario, although in that case, the utilization of the total station would reduce the functionality of the system. A possible improvement is to develop a sensor fusion algorithm that combines the low frequency measurements of the inclinometer and the high frequency measurements of an IMU to capture the cross-level more accurately using the TRV. In that case the total station would be unnecessary and the survey velocity higher. The authors are currently working on the development of such algorithms.

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