A Local Electricity Market Mechanism for Flexibility Provision in Industrial Parks involving Heterogenous Flexible Loads

Balgynbek Turdybek[1], Marcos Tostado-Véliz[1], Seyed Amir Mansouri[2], Ahmad Rezaee Jordehi[3], Francisco Jurado[1,*]

1. Department of Electrical Engineering, University of Jaén, 23700, Linares, Spain (e-mail: bt000011@red.ujaen.es (B.T.), mtostado@ujaen.es (M.T.-V.) and fjurado@ujaen.es (F.J.)).
2. Institute for Research in Technology (IIT), ICAI School of Engineering, Comillas Pontifical University, 28015 Madrid, Spain (e-mail: smansouri@comillas.edu).
3. Department of Electrical Engineering, Rasht Branch, Islamic Azad University, Rasht, Iran (e-mail: ahmadrezaeejordehi@gmail.com).

* Correspondence: fjurado@ujaen.es

Abstract. Industrial parks allow industries to share infrastructure and thus saving money, finally redounding in improving the economy of many countries worldwide. Given the objectives of carbon neutrality imposed by different entities, it results mandatory promoting energy efficiency in industrial parks. Aligning with such objective, encouraging industries to provide energy flexibility becomes essential. In the electricity sector, such flexibility can be provided through optimally managing local assets such as energy storage and flexible loads. However, flexibility provision should be promoted by implanting proper pricing mechanisms. This paper focuses on this issue by developing a local market clearing mechanism for industrial parks, whose main novelty redounds in the inclusion of a fair pricing mechanism through which industries are paid by flexibility provision. Different types of flexible loads are considered and modelled (i.e. curtailable, interruptible and deferrable), so that the new proposal is suitable for leveraging fully capabilities of industrial flexible loads. The whole pricing mechanism is raised as a bi-level game-based model, by which local energy and flexibility prices are revealed in a coordinated way. Challenges brought by the inclusion of binary variables (needed for modelling some types of flexible loads) are solved by proposing a solution algorithm based on the well-known Column & Constraint Generation Algorithm. The resulting optimization framework is Mixed Integer Linear Programming, being therefore solvable by off-the-shelf solvers. A case study is presented to validate the new proposal as well as highlight some important aspects related to local markets in industrial parks and its practical implantation.

Keywords. Energy efficiency, Energy flexibility, Industrial park, Local electricity market

Nomenclature

Indices (sets)	
$i(\mathcal{I})$	Industry
$t(\mathcal{T})$	Time
$\Theta_i^D = [L_i^D, \dots, U_i^D]$	Allowable time window of deferrable loads in the i^{th} industry
Superscripts	
IP	Industrial park
b/s	It refers to power imported (bought)/exported (sold)
RES	Local renewable generation
$B, c/d$	Battery system in charging/discharging mode
L	Non-controllable (non-flexible) demand
$C/S/D$	Curtailable/interruptible/deferrable loads
DR	Flexible demand
M	Park manager
G	External grid (distribution network)
$\underline{(\cdot)}/\overline{(\cdot)}$	Minimum/maximum value of a variable or parameter
$\widehat{(\cdot)}$	Estimated value
$\widetilde{(\cdot)}$	Given value
Parameters and constants	
$\Delta\tau$	Time step [h]
E_i^{Net}	Daily net energy demand of the i^{th} industry [kWh]
ν^s	Penalty/incentive for exporting energy [pu]
η	Efficiency [pu]
Z_i^C	Minimum daily demand of curtailable loads in the i^{th} industry [kWh]
h_i^D	Duty cycle of deferrable loads in the i^{th} industry [h]
h_i^S	Minimum number of hours that interruptible loads must work over a day in the i^{th} industry [h]
Compact notation	
$\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{D}, \boldsymbol{H}$	Matrix of coefficients
$\boldsymbol{b}, \boldsymbol{d}$	Vector of coefficients
$\hat{\boldsymbol{p}}$	Vector of estimated powers
$\hat{\boldsymbol{c}}^G$	Vector of estimated grid energy prices
$\boldsymbol{h}$	Vector of time-related parameters (i.e. duty cycles or maximum interruptible hours)
Decision variables (primal)	
p	Power [kW]
λ	Energy price [€/kWh]
ϵ	Energy stored [kWh]
u	Commitment statuses [binary]
ζ	Auxiliary variable for modelling costs [€]
Decision variables (dual)	
ϕ	Dual variable linked to equality constraints [-]

$\underline{\mu}, \overline{\mu}$	Dual variables linked to inequality constraints [-]
$\psi^{\downarrow}, \psi^{\uparrow}$	Auxiliary variables to linearize complementarity terms [-]
Variables (vector notation)	
$\boldsymbol{x}_i/\boldsymbol{x}^M$	Continuous primal variables of the i^{th} industry/park manager
$\boldsymbol{u}_i$	Binary primal variables of the i^{th} industry
$\boldsymbol{\lambda}^M$	Energy prices in the park (i.e. local prices)
$\boldsymbol{\phi}$	Dual variables linked to equality constraints
$\boldsymbol{\mu}$	Dual variables linked to inequality constraints

1 – Introduction

1.1 – Motivation

Industries often come together in industrial parks (IPs) where they can reduce costs by sharing infrastructure. Currently, there are more than 20,000 IPs worldwide [1], which have contributed significantly to economy development in many countries. Nevertheless, industrial activity in such areas supposes one of the main sources of greenhouse emissions, either due to manufacturing and other industrial processes, or electricity consumption [2]. Thus, the massive use of energy compromises the carbon neutrality target by 2050 [3]. To achieve such ambitious goal, the report [4] points out different measures and initiatives that should be put on practise in IPs and urban areas: advancing towards massive use of renewable energy sources (RESs), and incentivizing a more efficiency use of energy.

1.2 – Energy Efficiency in IPs: Current Status & Research

In the 90's, a number of countries launched initiatives in order to promote a more efficiency use of energy IPs [5, 6]. In particular, [6] conducted an ex-post study about the results of such initiatives, pointing out that the promotion of energy efficiency in IPs led to reduce up to 30 % CO2 emissions in South Korea in less than 10 years. More currently, the Mo Industrial Park in Norway [7] is powered by local hydroelectricity and becomes leader in recycling energy for different uses. For instance, waste heat is used by the local manufacturer Elkem to breed smolt for aquaculture. Other examples can be found in the Pôle Metropolitain de l'Artois [8], or the Ulsan IP in South Korea [9], for which a 1.5 GW is projected. This kind of projects seek for clean local generation for IPs as well as converting industrial zones economically attractive by boosting up local energy efficiency.

IPs provide an optimal environment for sharing resources locally such as individual generation or storage. However, the optimal coordination of such assets become challenge, being necessary the use of energy management tools in order to enhance the efficiency and economy of the entire system. Applying energy management strategies in IPs devotes on actively managing local resources. Focusing on electricity, such resources include RESs through photovoltaics (PVs), as well as energy storage and flexibility provided by some kind of loads. In this regard, energy management strategies for IPs may differ from other specific tools used in other paradigms like energy communities [10].

Energy management strategies in IPs may be performed at industry level, in which each industry pursues individual objectives; or collectively, in which all the users partaking in the park cooperate on achieving common objectives. Typically, the second option is preferred as the cooperation among users is promoted and, generally, better results are achieved globally when common objectives are pursued instead of individual goals [11].

A number of researches have dealt with energy management in IPs in recent years. In particular, Zhao et al. [12] propose an energy management strategy for IPs connected to gas and power networks and encompassing heating, cooling and electricity sub-systems. In [13], a game-based coordination framework for combined heat-power units integrated in IPs is developed. The game approach seeks for establishing compensatory payments for ancillary services, while energy is directly purchased from the grid under fixed prices.

Heuristic-based rules were applied in [14] for power allocation in IPs, while [15] develops a hierarchical energy management strategy for an IP involving flexible loads. Zare Oskouei et al. [16] propose an optimization framework for participation of IPs in electricity and gas markets, for which an upscale structure called virtual energy hub is described. A model predictive control for energy management in IPs is developed in [17], with the particularity of modelling uncertainties via robust optimization.

1.3 – Justification of Local Energy Markets in IPs

As mentioned, one of the most effective ways of improving energy efficiency in IPs is promoting energy cooperation. In this sense, industries partaking in the park can share resources via peer-to-peer (P2P) transactions. Motivated by this principle, many initiatives have been launched in order to promote energy sharing in IPs. For example, the European Commission launched in 2018 a specific project call focused on energy cooperation in IPs [18].

From a mercantilist point of view, energy cooperation can be performed under cooperative or market-based rules. In the first case, participants exchange resources without receiving a monetary counterpart, establishing an ex-post sharing mechanism to distribute costs proportionally among users (see [19]). In contrast, local markets can be launched, thus establishing local energy prices that are revealed by a local market entity, so that local energy transactions are performed on the basis of these prices. This idea has been put on practise for microgrids or energy communities [20], but its application in IPs is still scarce and only treated in a few references [13].

1.4 – Contributions & Paper Organization

Local markets are recognised as an effective way to promote electricity sharing, thus improving economy of users, while reducing dependence of volatile upward markets [11]. Based on this idea, we believe that the implantation of local markets mechanisms in IPs may be positive for improving energy efficiency in industrial areas. So far, existing market structures for IPs are rather focused on electricity trading (see [13]). However, the third objective marked in [4] makes mention on promoting energy flexibility. Thereby, local markets initiatives should not be uniquely focused on energy cooperation, but also on encouraging users to modify their consumption patterns in order to improve the energy efficiency collectively.

The idea of flexible demand is not new and is commonly known as demand response (DR) [21]. Electricity retailers and service provides worldwide have focused on promoting DR during years, putting on practise incentive or price-based mechanisms in order to encourage end-users to adapt their consumption profiles. Such initiatives establish coherent monetary incentives/penalties so that consumers receive a fair pricing signal and can response consequently. In this sense, we believe that local market mechanisms for IPs should contemplate proper pricing signals focused on flexibility provision, so that industries can accommodate their consumption according to local pricing signals.

This paper focuses on this issue. More specifically, this work develops a novel local electricity market model for IPs, whose main merit is the inclusion of a specific market product focused on paying for flexibility provided by users. In contrast to other local market mechanisms rather focused on electricity trading (e.g. [13]), our proposal incorporates a fair market product devoted on incentivizing flexibility provision from end-users. This way, we expect that the developed market model supposes a useful tool for park managers in order to improve the economy and energy utilization in IPs.

One particularity of IPs compared with other energy paradigms (e.g. energy communities), is the presence of a variety of loads that can partake in flexibility provision in different ways. For example, industrial processes can provide energy flexibility by means of curtailable, deferrable and interruptible loads. The models proposed in the revised literature oversimplify the flexibility capability of industries by only considering one type of loads. Thus, while [12] includes interruptible loads, only curtailable demand was modelled in [14, 15]. In other cases, flexibility is restricted to flexible generation units such as combined heat-power devices in [13]. In the new proposal, heterogeneous flexible loads are properly modelled making use of binary variables, which are properly accommodated making use of a solution strategy based on the Column & Constraint Generation Algorithm (C&CGA), which eventually preserves the convexity of the optimization model.

The developed market model is thus raised as a bi-level Stackelberg game framework, by which local electricity and flexibility prices are cleared in a coordinated way, seeking for incentivizing flexibility provision from users. The resulting optimization model is Mixed Integer Linear programming (MILP), being so easily solvable by off-the-shelf solvers. Lastly, the developed market model is validated on a benchmark IP. For the sake of simplicity, the main contributions of this paper are numerated below:

- Developing a novel local market model suitable for IPs (but adaptable to other paradigms) with the inclusion of a fair market product to incentivize flexibility provision from end-users.
- Developing a bi-level game-based model for the proposed market mechanism, through which local electricity and flexibility prices are cleared in a coordinated way.
- Including the model of different flexible loads (i.e. curtailable, deferrable and interruptible) within the developed optimization model by proposing a solution strategy based on C&CGA.

In the rest of this paper, Section 2 introduces some concepts and states the mathematical problem. Section 3 describes in detail the mathematical models. Section 4 develops a solution strategy based on C&CGA for market clearing including flexible loads. A case study with results is analysed in Section 5, whereas Section 6 concludes the paper.

2 – Preliminaries

2.1 – Local Market Model

In this paper, we consider the IP layout described in Fig. 1. We assume that industries within the park install various assets capable to provide flexibility in different ways. On the one hand, we consider that local generation and storage are enabled through PV panels and battery energy systems (BESs), respectively. On the other hand, each industry is capable of managing different types of flexible loads. In this sense, we consider three different categories as in [22], which are described below:

- Curtailable loads can reduce their consumption below a power bound, but keeping power supply above an established lower margin. There is a variety of loads that can be classified into this category and therefore curtailed for convenience (e.g. lighting). Nevertheless, we consider that a minimum energy requirement must be satisfied over a day in order to ensure safe operation.
- Interruptible loads can be disconnected a number of hours over a day. Within this category, thermal-driven devices are typically considered interruptible within some feasible margins [23].
- Deferrable loads are a priori scheduled at a specified hour, either due to personal or working requirements. However, their operation can be deferred for convenience, but respecting their duty cycles, which must be completed within established time windows. A number of industrial processes can be included within this category (e.g. steel furnaces).

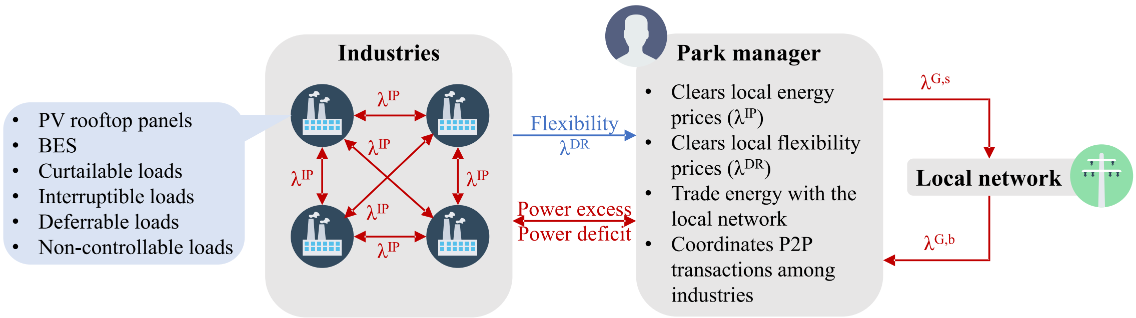

Fig. 1 – Sketch of the considered IP and agents involved

The main focus of this paper is developing a local market mechanism for the IP sketched in Fig. 1. To this end, we consider that industries are electricity end-users and engage in local electricity trading through P2P transactions under local electricity prices (i.e. λ^{IP}). Such prices are revealed daily in a market-clearing fashion by a central entity called park manager. In practise, this agent encompasses spatial arbitrageur and price-setter roles [20]. In this sense, the manager decides on electricity trading with the distribution/transmission network to which the IP is connected, as well as sets local electricity and flexibility (i.e. λ^{DR}) prices.

Thus, the main novelty of the proposed market model with respect others proposed for microgrids or energy communities [20], is the inclusion of a market product devoted on incentivizing demand flexibility from users, which will be further explained in Section 3.

2.2 – Assumptions

We assume perfect market competition, which implies that all the industries follow a price-taker strategy. This way, any of the industries partaking in the IP is keen on impacting on local prices through managing own resources. Therefore, all the players taking part in the proposed market structure follow maximizing the collective welfare.

We also assume deterministic conditions on our optimization problems. Note that volatile prices fixed by the retailer as well as non-predictable demand and renewable generation make the energy management problem uncertain in practise. However, this paper focuses on day-ahead energy markets, which are set on the basis of forecasted profiles. In this sense, real-time and intraday markets could be launched to correct possible deviations from forecasts [20]. Nevertheless, this paper is out of scope of this issue.

We assume that the developed market mechanism is cleared locally, following pricing signals from wholesale energy markets. Under this assumption, local entities, such as the park manager or industries, have no access to the local transmission or distribution network information and, therefore, it is not realistic including the transmission network model in the market framework. On the other hand, the power network within the park is assumed to be properly sized to avoid bottlenecks. Note that bottlenecks can be avoided locally by simply tuning up the power limit $\overline{p}_i^{IP}$ properly. Under this plausible assumption, including the network model does not impact on the scheduling plan of the park and therefore the results are not altered [24]. Additionally, this assumption allows us to focus on pricing results, which is the main scope of this paper.

Finally, we assume that the formed IP cannot be decomposed. In other words, any of the players partaking in the IP can leave the coalition and engage individually in energy trading with the grid. In this regard, we assume that industries agreed a long-term coalition, which is plausible in practise.

2.3 – Problem Statement

Next, we present the mathematical modelling of the agents involved in the considered IP in compact (matrix-vector) notation. Firstly, industries aim at minimizing their expected cost by trading energy and flexibility with the park manager while managing own resources, which motivates the following optimization problem for the i^{th} industry in the IP:

$$\min_{\boldsymbol{x}_i,\boldsymbol{u}_i} F_i = \boldsymbol{\lambda}^M \boldsymbol{x}_i^\intercal \tag{1a}$$

Subject to:

$$\boldsymbol{A}_i \boldsymbol{x}_i^\intercal + \left(\boldsymbol{u}_i \circ \widehat{\boldsymbol{p}}_i^{D/S}\right)^\intercal + \left(\widehat{\boldsymbol{p}}_i^{L/RES}\right)^\intercal = \boldsymbol{0} \tag{1b}$$

$$\boldsymbol{H}_i^D \boldsymbol{u}_i^\intercal + \left(\boldsymbol{h}_i^D\right)^\intercal = \boldsymbol{0} \tag{1c}$$

$$\boldsymbol{B}_i \boldsymbol{x}_i^\intercal \leq \boldsymbol{b}_i^\intercal \tag{1d}$$

$$\boldsymbol{H}_i^S \boldsymbol{u}_i^\intercal \leq \left(\boldsymbol{h}_i^S\right)^\intercal \tag{1e}$$

$$\boldsymbol{u}_i \in \{0,1\} \tag{1f}$$

where $\circ : \mathbb{R}^{m\times n} \mapsto \mathbb{R}^{m\times n}$ is the Hadamard (i.e. element-wise) product and $(\cdot)^\intercal : \mathbb{R}^{m\times n} \mapsto \mathbb{R}^{n\times m}$ stands for the transpose operator. Above, (1a) is the objective function of the i^{th} industry in the IP, which encompasses costs for energy and flexibility trading in the IP. To minimize its costs, each industry is able to manage own resources, which are modelled through continuous (i.e. $\boldsymbol{x}_i$) and binary (i.e. $\boldsymbol{u}_i$) variables.

The constraints in (1) are formed by equality, inequality and binary constraints. In particular, (1b) encompasses balance constraints, in which continuous and binary variables are implied, as well as forecasted powers (denoted by $\widehat{\boldsymbol{p}}$). More specifically, the first term in (1b) makes mention to power trading with the park, charging-discharging decisions on BES and power curtailed. The second term is related to interruptible and deferrable loads, whose consumption is forecasted and can be managed via integer variables. Lastly, the third term encompasses forecasted renewable generation and non-controllable demand. On the other hand, (1c) encompasses equality constraints uniquely involving binary variables, which model interruptible loads, as explained in the following section.

The set of inequality constraints contemplate power bounds in (1d) and also deferrable loads modelling in (1e), which require the use of binary variables. Finally, the variables in $\boldsymbol{u}_i$ are declared binary in (1f).

Thus, the park manager engages in energy trading with the local network (through a retailer), coordinates P2P trading among industries and reveals local energy and flexibility prices. In this sense, the park manager pays for power imported from the network and power exported from the park, while is paid by power exported to the network and power imported by industries. Moreover, the park manager pays industries by providing flexibility. Keeping this on mind, the park manager aims at minimizing her costs, while coordinates local power exchanges and clears local prices, which motivates the following optimization problem:

$$\min_{\boldsymbol{x}^M,\boldsymbol{\lambda}^M} F^M = \widehat{\boldsymbol{c}}^G (\boldsymbol{x}^M)^\intercal - \textstyle\sum_i F_i \tag{2a}$$

Subject to:

$$\boldsymbol{A}^M (\boldsymbol{x}^M)^\intercal = \boldsymbol{0} \tag{2b}$$

$$\boldsymbol{B}^M (\boldsymbol{x}^M)^\intercal \leq (\boldsymbol{b}^M)^\intercal \tag{2c}$$

$$\boldsymbol{D}^M (\boldsymbol{\lambda}^M)^\intercal \leq (\boldsymbol{d}^M)^\intercal \tag{2d}$$

The objective function (2a) encompasses costs for power trading with the network, which are performed under forecasted retailer's prices (i.e. $\widehat{\boldsymbol{c}}^G$). Note that such prices will be forecasted if one assumes a real-time-pricing mechanism for the park, while can be assumed deterministic if other kind of tariff is agreed (e.g. time-of-use tariffs). The second term in (2a) includes cost for trading power and flexibility with industries. It is worth noting that the manager decides on local and with-the-network power trading and local prices (i.e. $\boldsymbol{x}^M$ and $\boldsymbol{\lambda}^M$, respectively), thus being unnecessary the use of binary variables in (2). On the other hand, (2c) and (2d) express bounds for the decision variables $\boldsymbol{x}^M$ and $\boldsymbol{\lambda}^M$, respectively.

3 – Detailed Formulation

Section 2.3 introduced the problem statement using compact notation. In subsequent sections, the models (1) and (2) are explained in detail. Note that dual variables are shown at the right-hand side of their corresponding equation.

3.1 – Industries

The industries modelling (1) is fully given below:

$$\min_{\boldsymbol{x}_i,\boldsymbol{u}_i} F_i = \Delta\tau \sum_t \left\{ \underbrace{\lambda_t^{IP}\left(p_{i,t}^{IP,b} - \nu^s p_{i,t}^{IP,s}\right)}_{\substack{Cost\ for\ exchanging \\ power\ in\ the\ park}} \right\} - \underbrace{\Delta\tau\lambda_i^{DR}\left(\hat{E}_i^{Net} - E_i^{Net}\right)}_{\substack{Incomes\ for \\ providing\ flexibility}} \tag{3a}$$

Subject to:

$$E_i^{Net} = \sum_t \left\{ p_{i,t}^{IP,b} - p_{i,t}^{IP,s} \right\} \tag{3b}$$

$$p_{i,t}^{IP,b} + \hat{p}_{i,t}^{RES} + p_{i,t}^{B,d} = p_{i,t}^{IP,s} + \hat{p}_{i,t}^{L} + p_{i,t}^{B,c} + p_{i,t}^{C} + u_{i,t}^{S}\hat{p}_{i,t}^{S} + u_{i,t}^{D}\hat{p}_{i,t}^{D} : \phi_{i,t}; \forall t \in \mathcal{T} \tag{3c}$$

$$\epsilon_{i,t}^{B} = \epsilon_{i,t-1}^{B} + \Delta\tau\left(p_{i,t}^{B,c}\eta_i^B - p_{i,t}^{B,d}/\eta_i^B\right) : \phi_{i,t}^{B}; \forall t \in \mathcal{T} \backslash t = 1 \tag{3d}$$

$$\epsilon_{i,1}^{B} = \epsilon_{i,|\mathcal{T}|}^{B} = \overline{\epsilon}_i^{B} : \phi_i^{B(0)}, \phi_i^{B(end)} \tag{3e}$$

$$0 \leq p_{i,t}^{IP,x} \leq \overline{p}_i^{IP} : \underline{\mu}_{i,t}^{IP,x}, \overline{\mu}_{i,t}^{IP,x}; \forall t \in \mathcal{T} \wedge x \in \{b,s\} \tag{3f}$$

$$0 \leq p_{i,t}^{B,x} \leq \overline{p}_i^{B} : \underline{\mu}_{i,t}^{B,x}, \overline{\mu}_{i,t}^{B,x}; \forall t \in \mathcal{T} \wedge x \in \{c,d\} \tag{3g}$$

$$\underline{\epsilon}_i^{B} \leq \epsilon_{i,t}^{B} \leq \overline{\epsilon}_i^{B} : \underline{\mu}_{i,t}^{\epsilon}, \overline{\mu}_{i,t}^{\epsilon}; \forall t \in \mathcal{T} \tag{3h}$$

$$\underline{p}_{i,t}^{C} \leq p_{i,t}^{C} \leq \hat{p}_{i,t}^{C} : \underline{\mu}_{i,t}^{C}, \overline{\mu}_{i,t}^{C}; \forall t \in \mathcal{T} \tag{3i}$$

$$\Delta\tau \sum_t p_{i,t}^{C} \geq \mathrm{Z}_i^{C} : \overline{\mu}_{i,t}^{Z} \tag{3j}$$

$$\sum_t u_{i,t}^{D} = \frac{h_t^D}{\Delta\tau} \tag{3k}$$

$$u_{i,t}^{D} = 0; \forall t \notin \Theta_i^D \tag{3l}$$

$$\sum_t u_{i,t}^{S} \geq |\mathcal{T}| - \frac{h_i^S}{\Delta\tau} \tag{3m}$$

$$u_{i,t}^{S}, u_{i,t}^{D} \in \{0,1\}; \forall t \in \mathcal{T} \tag{3n}$$

The objective (3a) encompasses two terms. The first one expresses the cost for exchanging power locally through P2P mechanisms. As seen, the industries pay for importing power from other industries or the network (through the manager), and are paid by exporting power to other industries or the network (through the manager). Note that either local or grid-level power transactions, both are performed under local energy prices (i.e. λ_t^{IP}). It is worth noting that power exports are multiplied by a constant $\nu^s \geq 0$, which in turn makes different the cost of exporting power. This factor aims at motivating/demotivating local exports in order to reflect the coherent purchasing/selling prices with the local grid, from which different prices are assumed for power imported or exported, as customary [25].

The second term in (3a) reflects the monetary incomes from providing flexibility. These costs are a function of the net energy demand calculated in (3b), and an estimated energy demand (i.e. $\hat{E}_i^{Net}$). This value (whose calculation is explained in Appendix A), reflects the net energy demanded by the i^{th} industry if flexibility is disabled. Thus, flexibility provided by an industry is measured as the difference between the estimated net demand without and with flexibility. Thereby, the higher the difference between $\hat{E}_i^{Net}$ and E_i^{Net}, the higher value of the second term in (3a) and consequently higher incomes are obtained by the industry.

It is interesting to note that local energy prices are a function of time, which means that the we follow a dynamic pricing strategy. In practise, dynamic prices are considered an effective pricing strategy that reflect the generation-consumption balance of the entire system coherently [26]. In contrast, prices for flexibility are not dynamic and only different for each industry. By this assumption, industries receive a unique price signal for flexibility provision over a day, which eventually reduce the negative effects of response fatigue (see [27]).

In the set of constraints, (3c) expresses the power balance of the industry, involving batteries, renewable generation, non-flexible and flexible loads as well as power exchanging with the park. On the other hand, (3d) stands for the instantaneous state-of-charge of BES, which is expressed as a function of the previous state-of-charge and the energy exchanged by the batteries. Since (3d) is not declared for $t = 1$, (3e) fixes the initial and final energy stored in batteries.

The power that each industry can exchange is assumed to be limited in (3e) by either contractual or physical restrictions (e.g. for avoiding network congestions). On the other hand, (3f) and (3g) give power and energy limits for batteries, respectively. While power limits are considered a function of the energy-to-power ratio [28], the energy limits are given by the total capacity and depth-of-discharge settings.

In (3i), we assume that power dispatched to curtailable loads must lie within their forecasted demand and a preestablished minimum bound, considered for safe operation or other requirements. In any case, a minimum daily demand of curtailable loads must be satisfied, as said (3j). Interruptible loads can be interrupted a maximum number of hours over a day, as said (3k), while (3l) and (3m) ensure that deferrable loads complete their daily duty cycles within established time windows. Finally, binary variables for interruptible and deferrable loads are declared in (3n).

Note that the model (3) has been particularized here to IPs to be as close as possible to the real particularities of IPs (i.e. the presence of heterogeneous flexible loads is more likely in IPs than in other market structures). Nevertheless, the same market idea could be easily implemented in other networks like microgrids. Indeed, note that (3) can be adapted to microgrids easily by only introducing minor modifications. In this sense, although the new proposal is considered in this paper for IPs, its versatility allows adapting it to other paradigms, thus easing its implementation.

3.2 – Park Manager

The detailed formulation for the park manager reads as:

$$\min_{\boldsymbol{x}^M, \boldsymbol{\lambda}^M} F^M = \Delta\tau \sum_t \{ \hat{\lambda}_t^{G,b} p_t^{G,b} - \hat{\lambda}_t^{G,s} p_t^{G,s} \} - \sum_i F_i \tag{4a}$$

Subject to:

$$p_t^{G,b} + \sum_i p_{i,t}^{IP,s} = p_t^{G,s} + \sum_i p_{i,t}^{IP,b} \; ; \forall t \in \mathcal{T} \tag{4b}$$

$$F_i \leq \Delta\tau \sum_t \{ \lambda_t^{G,b} p_{i,t}^{IP,b} - \lambda_t^{G,s} p_{i,t}^{IP,s} \} ; \forall i \in \mathcal{I} \tag{4c}$$

$$0 \leq p_t^{G,x} \leq \overline{p}^G ; \forall t \in \mathcal{T} \wedge x \in \{b, s\} \tag{4d}$$

$$\underline{\lambda}_t^{IP} \leq \lambda_t^{IP} \leq \overline{\lambda}_t^{IP} \; ; \forall t \in \mathcal{T} \tag{4e}$$

$$\underline{\lambda}_i^{DR} \leq \lambda_i^{DR} \leq \overline{\lambda}_i^{DR} ; \forall i \in \mathcal{I} \tag{4f}$$

In (4a), the first term involves costs for energy trading with the network, similar to the first term in (3a), whereas the second term in (4a) encompass the cost for local energy trading as well as flexibility, as explained in Section 2.3.

The constraints in (4) expresses power balance in (4b), encompassing local P2P exchanges, while (4c) ensures that any industry would gain by engaging individually in energy trading with the local network, which in turns ensure the stability of the coalition [29]. On the other hand, (4d)-

(4f) establish limits in power exchanges with the network, as well as local energy and flexibility prices, respectively.

3.3 – On the Role of the Park Manager

In this paper, we assume that the park manager pays users for flexibility provision (and electricity exports), while is paid by electricity supplying through the local distribution network. Thus, we assume that the park manager absorbs monetary exchanges between the park and the network as in [20]. Since the proposed optimization problem falls into a Stackelberg game framework, the solution obtained is an equilibrium among players (i.e. industries and park manager). So that the resulting pricing strategy is assumed to benefit both industries and manager.

Nevertheless, this is still an open topic in the literature regarding local market environments. There are some approaches that circumvent this issue by enforcing the budget balance character of the manager (meaning neither taking a loss nor making a profit) [19]. However, one could think that this assumption may discourage private agents to partake as managers in IPs or energy communities. Other approaches simply allocate costs among users according to some preestablished rules [30]. In contrast, assuming the park manager as a profit-seeker agent may result in monetary losses in some cases (e.g. absence of local generation). Nonetheless, discussing these different adoptions is out of scope of this paper and should be studied in future works.

4 – The Proposed C&CGA Solution Strategy

4.1 – Foundations and Motivation

Our aim is developing a pricing mechanism for both local energy trading and flexibility provision. There exist different price-setting approaches in local markets or frameworks, comprising coordinated and non-coordinated strategies. Under non-coordinated mechanisms, central entity fixes prices without taking into account collective welfare and users' reaction (see e.g. [31]). In contrast, coordinated strategies bear in mind the reaction of users, so that prices are set in a coordinated way in order to maximize the collective welfare. By these reasons, coordinated pricing mechanisms are typically preferred in deregulated market structures, where users can eventually choose among different market options [32].

Coordinated pricing mechanisms are conveniently raised as bi-level optimization problems, in which one of the levels represent the users' model, while the other level establishes the price-setter strategy. This way, the price-setting problem is established as a game framework and its solution is, therefore, an equilibrium candidate in the Nash sense [33].

Based on the evidences above and according to the structure of our market model, we derive the price-setting mechanism as a Stackelberg game. As such, one agent acts as leader while other/s play as followers. In Stackelberg games, the leader takes a decision trying to anticipate the reaction of followers [34]. This concept aligns perfectly with the proposed structure, in which the park manager would act as leader, clearing local prices, while the industries play as followers.

Mathematically, Stackelberg games are raised as bi-level optimization problems in which the followers' problem (lower level) constraints the leader's problem (upper level). To do that, the lower level needs to be reduced by its Karush-Kuhn-Tucker (KKT) conditions, which are sufficient conditions for optimality in convex problems [33].

In our particular market model, the followers' problem is nonconvex due to the presence of binary variables and therefore non-reducible to its KKT conditions. To solve this issue, we propose solving the market mechanism using C&CGA. This solution strategy has been used in other related problems (e.g. [35]), and its convergence properties are well-known and proved.

Therefore, by using a C&CGA algorithm, we aim at eliminating binary variables in the lower-level problem and making it linear and therefore reducible to its KKT conditions. To do that, we decompose the solution procedure into two stages, namely master problem and sub-problem. Indeed, the master problem seeks for adequate values of binary variables, which are transferred to the sub-problem, where the market solution is pursued by solving a bi-level Stackelberg game problem where binary variables are treated as parameters. This way, the solution is achieved by iteratively solved both stages, as described in Fig. 2.

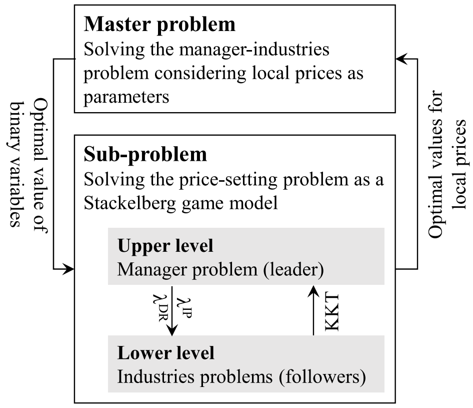

Fig. 2 – Flowchart of the proposed market solution methodology

4.2 – Master Problem

The master problem jointly solves the manager-industry problem within the C&CGA, which results in the following optimization model:

$$\widetilde{\boldsymbol{u}}_i \in \underset{\boldsymbol{x}^M,\boldsymbol{x}_i,\boldsymbol{u}_i,\zeta}{\operatorname{argmin}}\ \boldsymbol{c}^G(\boldsymbol{x}^M)^\intercal + \zeta \tag{5a}$$

Subject to:

$$\zeta \geq -\textstyle\sum_i F_i\left(\tilde{\boldsymbol{\lambda}}^M\right) \tag{5b}$$

$$\boldsymbol{A}_i\boldsymbol{x}_i^\intercal + \left(\boldsymbol{u}_i \circ \widehat{\boldsymbol{p}}_i^{D/S}\right)^\intercal + \left(\widehat{\boldsymbol{p}}_i^{L/RES}\right)^\intercal = \boldsymbol{0}; \forall i \in \mathcal{I} \tag{5c}$$

$$\boldsymbol{B}_i\boldsymbol{x}_i^\intercal \leq \boldsymbol{b}_i^\intercal; \forall i \in \mathcal{I} \tag{5d}$$

$$\boldsymbol{H}_i^D\boldsymbol{u}_i^\intercal + \left(\boldsymbol{h}_i^D\right)^\intercal = \boldsymbol{0}; \forall i \in \mathcal{I} \tag{5e}$$

$$\boldsymbol{H}_i^S\boldsymbol{u}_i^\intercal \leq \left(\boldsymbol{h}_i^S\right)^\intercal; \forall i \in \mathcal{I} \tag{5f}$$

$$\boldsymbol{u}_i \in \{0,1\}; \forall i \in \mathcal{I} \tag{5g}$$

$$\boldsymbol{A}^M(\boldsymbol{x}^M)^\intercal = \boldsymbol{0} \tag{5h}$$

$$\boldsymbol{B}^M(\boldsymbol{x}^M)^\intercal \leq (\boldsymbol{b}^M)^\intercal \tag{5i}$$

In the problem above, the optimization models (1) and (2) are solved jointly, assuming known local prices (i.e. $\tilde{\boldsymbol{\lambda}}^M$). As result, the optimal value for binary variables is obtained (i.e. $\widetilde{\boldsymbol{u}}_i$). In this sense, this stage involves decision variables related to industries but also to the park manager.

The objective (5a) is identical to (4a), but its second term is replaced by an auxiliary variable ζ, which is modelled by (5b). Note that this is an arrangement needed for C&CGA. Note that the proposed solution scheme is in fact a variant of the traditional Benders decomposition, in which only primal information is exchanged between stages.

Regarding the constraints, (5c)-(5g) are the constraints related to the industries problems, whereas (5h) and (5i) encompass the constraints of the park manager. Note that (2d) have not been included in (5) due to prices are considered parameters and therefore not included in the decision-space.

4.3 – Sub-problem

Once the optimal value of binary variables is calculated at the master problem, the sub-problem seeks for calculating the optimal value for local prices (energy and flexibility). As commented, this price-setting model is raised as a bi-level Stackelberg framework in which the lower level (industries' problem) is reduced by its KKT conditions. Note that, since the binary variables are considered parameters and equal to $\tilde{\boldsymbol{u}}_i$, the lower level results linear and therefore reducible to its KKT conditions, resulting in the following optimization framework:

$$\tilde{\boldsymbol{\lambda}}^M \in \underset{\substack{\boldsymbol{x}^M,\boldsymbol{\lambda}^M,\\ \boldsymbol{x}_i,\boldsymbol{\phi}_i,\boldsymbol{\mu}_i}}{\operatorname{argmin}} \boldsymbol{c}^G(\boldsymbol{x}^M)^\intercal - \textstyle\sum_i F_i \tag{6a}$$

Subject to:

$$\boldsymbol{A}_i\boldsymbol{x}_i^\intercal + \left(\tilde{\boldsymbol{u}}_i \circ \hat{\boldsymbol{p}}_i^{D/S}\right)^\intercal + \left(\hat{\boldsymbol{p}}_i^{L/RES}\right)^\intercal = \boldsymbol{0}; \forall i \in \mathcal{I} \tag{6b}$$
$$\boldsymbol{B}_i\boldsymbol{x}_i^\intercal \leq \boldsymbol{b}_i^\intercal; \forall i \in \mathcal{I} \tag{6c}$$
$$\boldsymbol{A}^M(\boldsymbol{x}^M)^\intercal = \boldsymbol{0} \tag{6d}$$
$$\boldsymbol{B}^M(\boldsymbol{x}^M)^\intercal \leq (\boldsymbol{b}^M)^\intercal \tag{6e}$$
$$\boldsymbol{D}^M(\boldsymbol{\lambda}^M)^\intercal \leq (\boldsymbol{d}^M)^\intercal \tag{6f}$$
$$\boldsymbol{\lambda}^M + \boldsymbol{A}_i^\intercal\boldsymbol{\phi}_i + \boldsymbol{G}_i^\intercal\boldsymbol{\mu}_i = \boldsymbol{0}; \forall i \in \mathcal{I} \tag{6g}$$
$$\boldsymbol{0} \leq \boldsymbol{G}_i\boldsymbol{x}_i^\intercal - \boldsymbol{b} \perp \boldsymbol{\mu}_i \geq \boldsymbol{0}; \forall i \in \mathcal{I} \tag{6h}$$
$$\boldsymbol{\phi}_i\text{: free}; \forall i \in \mathcal{I} \tag{6i}$$
$$\boldsymbol{\mu}_i \geq \boldsymbol{0}; \forall i \in \mathcal{I} \tag{6j}$$

where $\perp$ stands for complementarity. In (6), the objective (6a) is identical to (2a) while the rest of constraints stand for a primal-dual formulation involving primal and dual variables (namely $\boldsymbol{\phi}_i$ and $\boldsymbol{\mu}_i$). In particular, (6b)-(6f) are the primal feasibility constraints, which ensure that the solution of (6) is feasible for the primal models (1) and (2). On the other hand, (6g) and (6h) are the stationary and complementarity KKT conditions of the lower level, respectively, which are fully given in Appendix B. Lastly, (6i) and (6j) are the dual feasibility constraints declaring free variables the ϕ's and positive variables the μ's.

At its present form, (6) is hardly solvable due to the presence of nonlinear terms in (6a) and (6h). In particular, nonlinearities arise in (6a) due to the product of variables, while in (6h) the complementarity operator redounds in further nonlinearities. To solve this issue and transform the nonlinear model (6) into a solvable MILP, the objective (6a) is linearized following the methodology described in Appendix C, whereas complementarity constraints are linearized using the formulation given in [36], where it is demonstrated that a complementarity term $0 \leq y_1 \perp y_2 \geq 0$ can be effectively replaced by the following set of linear constraints.

$$y_1 \geq 0, y_2 \geq 0 \tag{7a}$$
$$\psi^\uparrow - \psi^\downarrow = (y_1 - y_2)/2 \tag{7b}$$
$$(y_1 - y_2)/2 - \left(\psi^\uparrow + \psi^\downarrow\right) = 0 \tag{7c}$$
$$\left\{\psi^\uparrow, \psi^\downarrow\right\} \in \mathbb{SOS}1 \tag{7d}$$

Note that (7) is based on the auxiliary variables $\psi^\uparrow$ and $\psi^\downarrow$, which are declared special ordered set 1 (SOS1) in (7d), which implies that at most one of them is greater than zero. The linearization trick (7) based on SOS1 variables is considered more suitable than the traditional big-M method [37], in which the use of big M's may provoke numerical instability or infeasible results [38].

By applying the linearization tricks above, the nonlinear model (6) is transformed into a tractable MILP, which is easily solvable using off-the-shelf solvers.

4.4 – The Algorithm

The 5-steps procedure below describes the proposed C&CGA for price-setting in the considered IP.

1. Initialize energy and flexibility prices. In this paper, it is suggested to initially take $\tilde{\boldsymbol{\lambda}}^M = \left(\overline{\boldsymbol{\lambda}}^M - \underline{\boldsymbol{\lambda}}^M\right)/2$

2. Solve the master problem (5) and obtain the solution tupple $\{\boldsymbol{x}^{M,*}, \boldsymbol{u}_i^*, \zeta^*\}$. Update $F^{MP} = \boldsymbol{c}^G(\boldsymbol{x}^{M,*})^\intercal + \zeta^*$
3. Take $\tilde{\boldsymbol{u}}_i = \boldsymbol{u}_i^*$ and solve the linearized sub-problem (6), obtaining the solution tupple $\{\boldsymbol{x}^{M,*}, \boldsymbol{x}_i^*, \boldsymbol{\phi}_i^*, \boldsymbol{\mu}_i^*, \boldsymbol{\lambda}^{M,*}\}$. Update $F^{SP} = \boldsymbol{c}^G(\boldsymbol{x}^{M,*})^\intercal - \sum_i F_i(\boldsymbol{x}_i^*, \boldsymbol{\phi}_i^*, \boldsymbol{\mu}_i^*, \boldsymbol{\lambda}^{M,*})$
4. Check convergence as
$$\frac{|F^{MP} - F^{SP}|}{F^{MP}} \leq \epsilon \tag{8}$$
where $\epsilon = 0.1$ in this paper.
5. If (8) is met, then stop and take $\{\boldsymbol{x}^{M,*}, \boldsymbol{x}_i^*, \boldsymbol{u}_i^*, \boldsymbol{\lambda}^{M,*}\}$ as final solution. Otherwise, update $\tilde{\boldsymbol{\lambda}}^M = \boldsymbol{\lambda}^{M,*}$, and go to step 2.

Note that the algorithm above only exchanges primal information between stages, thus avoiding the computation of sensibilities. It is worth mentioning that, typically, the considered algorithm converges within a few iterations and its convergence properties have been well-proved in several references [39].

5 – Case Study

In this section, we present a case study with results. The objective of this section is twofold: on the one hand, we aim at validating the developed market model, observing that results obtained are coherent; on the other hand, we will analyse the behaviour of the different agents partaking in the park under the proposed market mechanism. To this end, the developed mathematical models and the algorithm proposed in Section 4 were coded under Matlab R2021a and solved using Gurobi [40]. All simulations were run on an Intel Core i7-10700K CPU 3.80GHz 3.79 GHz with 32 GB RAM taking a 24-h time horizon with 1-h time resolution (i.e., $|\mathcal{T}| = 24$ and $\Delta\tau = 1$), as customary in energy market based on hourly price signals.

5.1 – Input Data

We consider a three-industry IP connected to a local distribution network, from which can exchange energy under real-time prices, which are plotted in Fig. 3 and corresponds to prices observed in PJM FE Ohio [41]. The selling price was taken 0.9 times the purchasing one, thus simulating a typical scenario under generation excess, in which surplus generation is not incentivized [25]. Bounds for local energy prices are set on the basis of grid purchasing costs, which is a plausible assumption. In particular, $\overline{\lambda}_t^{IP} = 1.2\hat{\lambda}_t^{G,b}$, $\underline{\lambda}_t^{IP} = \underline{\lambda}_i^{DR} = 0$, $\overline{\lambda}_i^{DR} = \max_t \hat{\lambda}_t^{G,b}$ and $\nu^s = 0.9$ were considered in simulations.

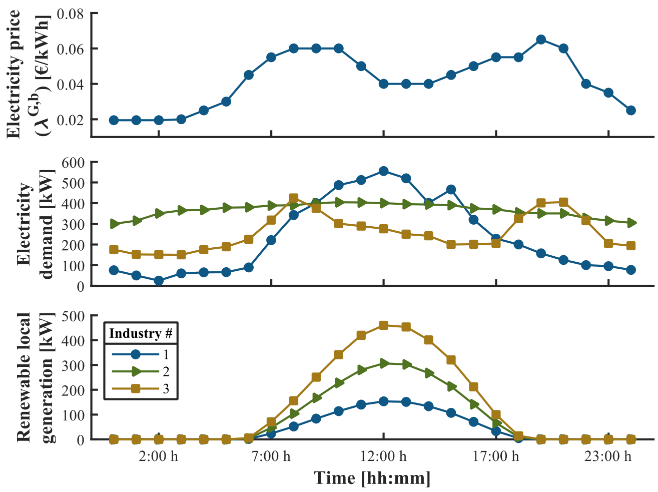

Fig. 3 – Expected grid purchasing price (top), non-flexible demand (middle) and renewable local generation (bottom)

Fig. 3 also shows expected non-flexible demand and renewable generation for each industry. The expected demand corresponds to typical profiles in industries [42], whereas renewable generation was built up assuming weather data (i.e. solar irradiance and temperature) in Madrid (Spain) in 2018 from [43], and the PV panel modelling described in [44]. Lastly, other relevant data are collected in Table 1, which are based on benchmark references [27], while features of flexible loads are shown in Table 2.

Table 1 – Some parameters used in simulations

Parameter	Value	Parameter	Value
$\overline{p}^{IP}$	2 MW	$\overline{p}_i^B$	$\overline{\epsilon}_i^B/2$
$\overline{p}^{G}$	8 MW	$\underline{\epsilon}_i^B$	$0.2\overline{\epsilon}_i^B$
$\overline{\epsilon}_i^B$	500, 300, 400 kWh	η^B	0.95 pu

Table 2 – Data of flexible loads

Parameter	Industry #		
	1	2	3
$\hat{p}_{i,t}^C$	$0.6\hat{p}_{i,t}^L$	$0.6\hat{p}_{i,t}^L$	$0.6\hat{p}_{i,t}^L$
$\underline{p}_{i,t}^C$	$0.5\hat{p}_{i,t}^C$	$0.5\hat{p}_{i,t}^C$	$0.5\hat{p}_{i,t}^C$
Z_i^C	$0.25\Delta\tau\sum_t \hat{p}_{i,t}^C$	$0.25\Delta\tau\sum_t \hat{p}_{i,t}^C$	$0.25\Delta\tau\sum_t \hat{p}_{i,t}^C$
$\hat{p}_{i,t}^D$	200 kW	200 kW	200 kW
h_t^D	5 h	6 h	3 h
Θ_i^D	10:00-20:00 h	3:00-14:00 h	16:00-20:00 h
$\hat{p}_{i,t}^S$	150 kW	150 kW	150 kW
h_i^S	5 h	5 h	5 h

5.2 – Studied Scenarios

In order to validate the developed market model, we analyse three different scenarios:

- Scenario 1 is the considered base case in which flexibility can be fully provided through batteries and flexible loads.
- In Scenario 2, local storage is disabled and therefore batteries are no longer considered.
- In Scenario 3, local renewable generation is disabled.

5.3 – Economic Analysis

Firstly, we focus on economic aspects. Fig. 4 plots the resulting local energy prices in the three studied scenarios. As expected, local prices follow grid prices. Nevertheless, it is interesting to see how peak periods are shaved when batteries are enabled. This fact remarks the energy arbitrage capability, which can be charged during valley periods to balance prices at peak hours.

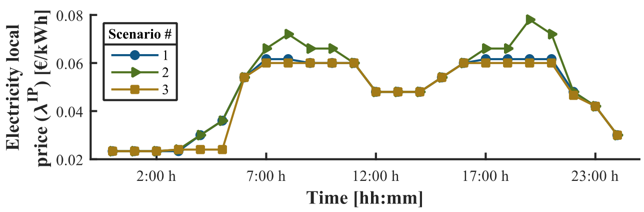

Fig. 4 – Resulting local energy prices (i.e. λ^{IP}) in the studied scenarios

It is worth commenting that local flexibility prices were always equal to the maximum allowed, thus remarking that the manager is keen on incentivizing flexibility provision through pricing signals. This fact demonstrates that flexibility provision is beneficial for industries, but rather for the park manager, who is able to improve her economy notably through this mechanism.

Table 3 shows a summary of the economic balance for industries under each studied scenario. As observed, the scenario 3, in which local generation is disabled, draws the worst scenario for industries. Total costs for industries are also increased in Scenario 2, thus demonstrating that local storage has a positive impact on the economy of users, as expected. Table 3 also reports total incomes for flexibility provision, which do not vary notably among scenarios, thus demonstrating that industries are capable to partake in flexibility provision even under pessimistic conditions.

Table 3 - Total industry costs under different scenarios

Cost [€]	**Scenario #**		
	1	**2**	**3**
Energy	1819	1921	2154
Flexibility	257	247	222
Total	1562	1674	1932

5.4 – Energy Analysis

Next, we focus on energy indicators. Fig. 5 plots the power imported/exported by industries and the park in the studied scenarios. As observed, the absence of renewable generation in Scenario 3 provokes peak demand periods beyond 2.5 MW, much higher than in the other scenarios, which demonstrates that renewable local generation is the key factor for reducing imported power. The absence of batteries in Scenario 2 draws a more flattened power profile compared to Scenario 1, when the charging/discharging routines of batteries provoke frequent power variations. Regarding exported power, it is worth distinguishing two clear periods, namely early morning and midday. In the former case, batteries are discharged during night leveraging low demand in order to draw a surplus generation in the park. In contrast, in the presence of renewable generation, exports are rather observed at midday propitiated by high PV potential.

On the other hand, Fig. 5 shows the scheduling result for deferrable loads in the park. As seen, industries 1 and 2 mainly schedule their deferrable loads at midday, driven by high PV potential. In contrast, industry 3 defers its loads to evening in scenarios 1 and 2. This is due to this industry presents the highest PV potential, which is mainly exploited at midday to export power and obtain a monetary profit during those hours when the local energy prices are still high (see Fig. 4). However, when renewable generation is disabled in Scenario 3, this industry forces to complete the duty cycle earlier, in order to avoid that deferrable consumption coincides with the night peak demand provoked by inflexible loads (see Fig. 3).

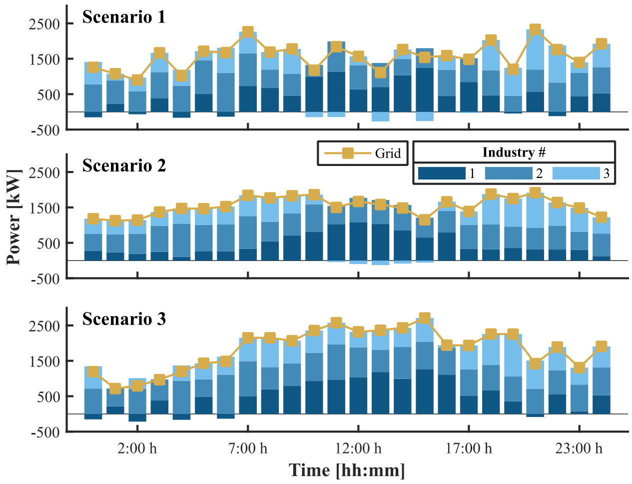

Fig. 4 – Power imported (positive)/exported (negative) for each industry and the park

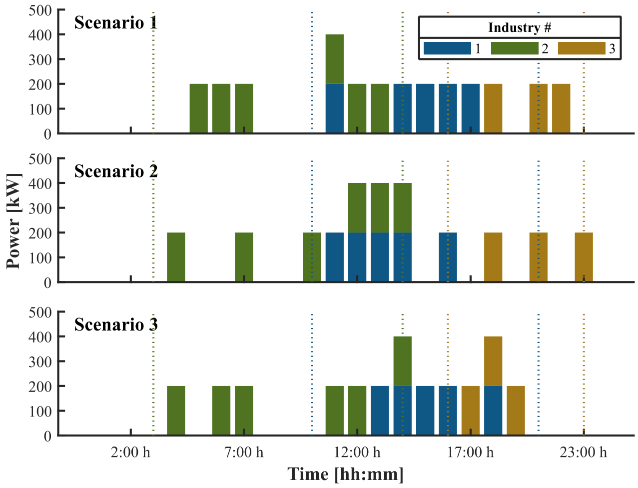

Fig. 5 – Scheduling of deferrable loads in the studied scenarios

Table 4 reports the total energy imported/exported by industries under different scenarios. As expected, imported energy was higher in Scenario 3, due to the absence of local generation. This issue provokes that industries were unable to export power under this scenario. When comparing scenarios 1 and 2, it is observed that total imported power is similar, however, the exportable capacity is notably reduced when BESs are disabled in scenario 2.

Table 4 – Total energy imported/exported by industries under different scenarios

Energy [kWh]	**Scenario #**		
	1	**2**	**3**
Imported	36,900	36,980	43,620
Exported	130.1	85.1	0

On the other hand, Table 5 shows some indicators related to interruptible and curtailable loads. As observed, interruptible loads are more frequently disconnected in scenarios 2 and 3, however, the opposite trend is observed for curtailed demand, which in turn decreases in scenarios 2 and 3. It indicates that, under stressful conditions (i.e. no storage or RES), the industries recur to the interruptible mechanism to not only improve their economy through flexibility provision, but also to reduce the net energy demand. In contrast, curtailable loads are not capable to provide as much flexibility and, in consequence, this mechanism is less frequently leveraged by industries.

Table 5 - Analysis of various indicators related to flexible loads under different scenarios

Indicator	**Scenario #**		
	1	**2**	**3**
Interrupted hours	11	14	15
Curtailed load [kWh]	1241	1073	664

5.5 – Comparison with Non-flexible scenarios

Now, we analyse two extreme scenarios in which flexibility is disabled in different degrees. Thus, we label the scenario ‘noFlex’ under which loads are inflexible and therefore demand cannot be neither curtailed nor interrupted. Moreover, we assume that deferrable loads must start their duty cycles at the lowest bound of their time windows (i.e. L_i^D) and have to be operated continuously. Note that this scenario corresponds with the conventional local market mechanism in which end users do not receive a monetary benefit for providing flexibility. On the other hand, the scenario labelled as ‘0Flex’ is identical to the former but disabling batteries, thus resulting in a scenario where flexibility cannot be provided any case.

The resulting local energy price under these non-flexible scenarios is plotted in Fig. 6. As seen, prices draw similar trends to those prices in Fig. 4. Actually, the absence of local storage avoids shaving peak pricing during night. However, this issue is also observed now at morning peak in the 'noFlex' scenario. It demonstrates that flexible loads have also a direct impact on local energy prices.

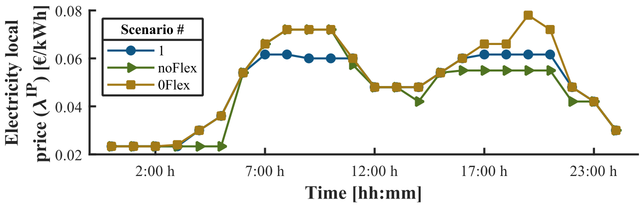

Fig. 6 - Resulting local energy prices (i.e. λ^{IP}) under non-flexible scenarios

Next, Table 6 reports different energy and economic indicators related to non-flexible scenarios. It is clear that disabling flexibilities impacts notably on the monetary balance of users. This is especially due to the reduction of incomes for flexibility provision, which are reduced until zero in the '0Flex' case. Nevertheless, total cost for industries in the 'noFlex' scenario is still lower than that observed for Scenario 3 (see Table 3). It indicates that economy of industries is more impacted by local generation than flexible loads. Regarding energy indicators, it is remarkable that, despite renewable local generation is enabled yet in both non-flexible cases, exportable capacity of industries resulted null. As expected, imported energy was higher in the '0Flex' scenario due to the absence of local storage capacity.

Table 6 - Various results under non-flexible scenarios

Indicator	**Scenario #** 'noFlex'	**Scenario #** '0Flex'
Energy cost [€]	1919	2114
Flexibility cost [€]	34	0
Total cost [€]	1885	2114
Energy imported [kWh]	39,630	40,060
Energy exported [kWh]	0	0

5.6 – Comparison with Keys of Repartition (KoR)

Next, we analyse the results obtained in the present case study under a cost allocation method based on Keys of Repartition (KoR). This mechanism shares the total cost of the park among industries according to their overall consumption [30]. This way, retailer energy prices are directly transferred to users. In this regard, we assume that the whole park is centrally managed by the park manager. Note that under this scenario, industries are not incentive to provide flexibility. Thus, we consider two different cases. On the one hand, we apply KoR to the case 'noFlex' analysed in the previous section. On the other hand, we apply the same idea but enabling flexibility through flexible loads. Note that this second case may entail social barriers as industries may discourage to provide flexibility if they do not receive a monetary counterpart.

Table 7 compares these two cases. As seen, if flexibility is disabled, the total cost for industries is higher than that reported in Table 3. As expected, the industries ~155 € more in this case, which is in fact lower than the incomes for flexibility provision in the corresponding base case. This different in payments is absorbed by the park manager in the proposed market structure, which demonstrates that this agent is not budget balance as discussed in Section 3.3.

As expected, the most favourable case is when industries are centrally managed and therefore a central entity has access to individual loads and assets. As seen, in this case industries pay 255 € less than in the considered base case in Table 3, which is exactly the payment received by industries for flexibility provision. This result indicates that extra monetary payment in which industries incur is absorbed by the park manager, who pursues in this case her individual benefit. This particular behaviour of the manager is further remarked observing the total exported energy, which was null in this case. Thereby, the park manager seeks for incrementing her monetary benefit by exporting to the grid in the considered base case, while under KoR industries are not interested in exporting energy to the grid.

Table 7 - Various results under a payment allocation method based on KoR

Indicator	**KoR**	
	With Flex.	**w.o. Flex**
Total cost [€]	1307	1718
Energy imported [kWh]	31,250	39,663
Energy exported [kWh]	0	0

As observed, the most favourable case for industries is allowing central management of own assets under KoR. However, this situation entails some important drawbacks that limit its implementation in real cases:

- Under this mechanism, industries are exposed to volatile energy prices fixed by the retailer, incrementing their risk.
- A market mechanism based on KoR requires the park manager to be budget balance, neglecting the possibility of obtaining a monetary profit. It may suppose a barrier for implantating a central entity to coordinate the park.
- In case of enabling flexibility from industries, the park manager needs full access to own loads and assets, which entails privacy concerns. In the face of this situation, industries may be reluctant to provide full access if they are not faithfully compensated through proper market products.

5.7 – Sensitivity Analysis

Finally, we provide a sensitivity analysis regarding renewable generation potential. To this end, we multiply the base renewable generation in Fig. 3 by a real number called ‘scaled factor’. In this regard, Fig. 7 analyses some energy indicators for different scaled factors. As expected, imported energy is reduced with the renewable potential for both industries and IP. Actually, energy imported by the park is always lower than that demanded by industries due to the presence of local storage and P2P exchanges. In contrast, exportable energy increases due for the same reasons.

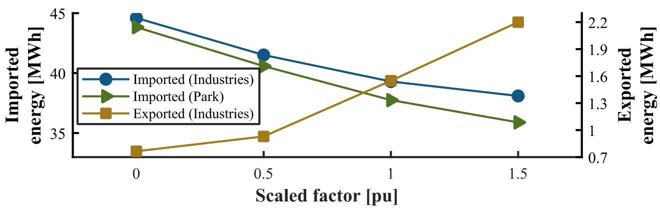

Fig. 7 – Energy analysis for different renewable potential

On the other hand, Fig. 8 analyses the total cost for industries, which clearly decreases with the scaled factor due to local generation allows industries to reduce their dependency of imported power, but also increases their exportable capacity, thus improving the economy of users. However, incomes for flexibility increases until the scaled factor equals 0.5 pu, decreasing beyond this point. This result indicates that under high PV penetration, economy of industries is driven by PV panels rather than flexible loads. In this sense, industries focus on improving their monetary balance by optimal exchanging power with the park rather than managing flexible loads. This aspect is especially relevant when implanting monetary mechanisms for flexibility provision, since under high local generation, the flexibility mechanism might be neglected by industries, which would rather focus on maximizing their profit via P2P trading.

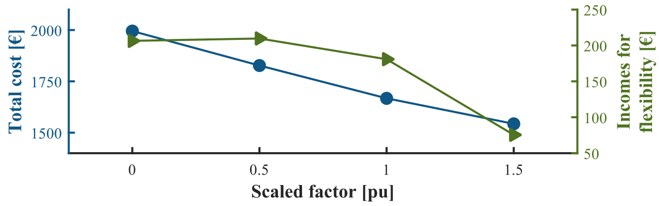

Fig. 8 – Economy analysis for different renewable potential

6 – Concluding Remarks

A novel market model for industrial parks has been developed in this paper. The main novelty of the new proposal lies in the inclusion of a fair pricing mechanism for flexibility provision, which aims at encouraging end-users to provide flexibility through optimally managing local resources (i.e. storage and flexible loads).

The new market model was raised as a bi-level price-setting game framework, which is solved using C&CGS in order to accommodate binary variables.

A case study was conducted to validate the new proposal. Results obtained allowed us to conclude that the developed model effectively encourages flexibility provision while resulting economically attractive for the industries partaking in the park. The effect of local prices was also analysed, showing that local storage turned out to be very effective to shave peak prices while energy balance could be notably improved through optimally managing flexible loads. Other relevant aspects such as the effects of disabling flexibility and incrementing local generation were also studied and discussed.

In future works, we will focus on applying the proposed solution mechanism to other local markets models such as energy communities, where the optimal management of controllable appliances can be incentive through proper market products.

Appendix A – Estimation of Net Demand without Flexibility

In the considered model, flexibility is provided by either optimally charging-discharging batteries as well as managing flexible loads. In this sense, when flexibility is disabled, batteries are neglected and curtailed loads are assumed to be non-flexible (i.e. $p_{i,t}^{C} = \hat{p}_{i,t}^{C}; \forall t \in \mathcal{T}$). Moreover, the rest of flexible loads (i.e. interruptible and deferrable) are assumed to be inelastic. Thus, interruptible loads cannot be actually interrupted, as said (A1), while deferrable loads are assumed to start their duty cycled at the preferred time slot (i.e. the lower bound of their time windows in this paper), as seen in (A2).

$$\hat{u}_{i,t}^{S} = 1; \forall t \in \mathcal{T} \tag{A1}$$

$$\hat{u}_{i,t}^{D} = 1; \forall t \in \left[L_i^D, \dots, L_i^D + \frac{\delta_i^D}{\Delta\tau}\right] \tag{A2}$$

On the basis of the assumptions above, the net demand without flexibility for the i^{th} industry partaking in the park can be estimated, as follows:

$$\hat{E}_i^{Net} = \sum_t \{\hat{p}_{i,t}^{L} - \hat{p}_{i,t}^{RES} + \hat{p}_{i,t}^{C} + \hat{u}_{i,t}^{S} \cdot \hat{p}_{i,t}^{S} + \hat{u}_{i,t}^{D} \cdot \hat{p}_{i,t}^{D}\} \tag{A3}$$

Appendix B – KKT Conditions for Industries

The KKT conditions for (3) assuming binary variables as parameters are given below:

$$\frac{\partial \mathcal{L}_i}{\partial p_{i,t}^{IP,b}} = \Delta\tau\left(\lambda_t^{IP} + \lambda_i^{DR}\right) + \phi_{i,t} - \underline{\mu}_{i,t}^{IP,b} + \overline{\mu}_{i,t}^{IP,b} = 0; \forall i \in \mathcal{I} \wedge t \in \mathcal{T} \tag{B1}$$

$$\frac{\partial \mathcal{L}_i}{\partial p_{i,t}^{IP,s}} = -\Delta\tau\left(\nu^{s}\lambda_t^{IP} + \lambda_i^{DR}\right) - \phi_{i,t} - \underline{\mu}_{i,t}^{IP,s} + \overline{\mu}_{i,t}^{IP,s} = 0; \forall i \in \mathcal{I} \wedge t \in \mathcal{T} \tag{B2}$$

$$\frac{\partial \mathcal{L}_i}{\partial p_{i,t}^{C}} = -\phi_{i,t} - \Delta\tau\underline{\mu}_i^{Z} - \underline{\mu}_{i,t}^{C} + \overline{\mu}_{i,t}^{C} = 0; \forall i \in \mathcal{I} \wedge t \in \mathcal{T} \tag{B3}$$

$$\frac{\partial \mathcal{L}_i}{\partial p^{B,c}_{i,t}} = -\phi_{i,t} - \Delta\tau\eta^B_i\phi^B_{i,t} - \underline{\mu}^{B,c}_{i,t} + \overline{\mu}^{B,c}_{i,t} = 0; \forall i \in \mathcal{I} \wedge t \in \mathcal{T} \tag{B4}$$
$$\frac{\partial \mathcal{L}_i}{\partial p^{B,d}_{i,t}} = \phi_{i,t} + \Delta\tau\phi^B_{i,t}/\eta^B_i - \underline{\mu}^{B,d}_{i,t} + \overline{\mu}^{B,d}_{i,t} = 0; \forall i \in \mathcal{I} \wedge t \in \mathcal{T} \tag{B5}$$
$$\frac{\partial \mathcal{L}_i}{\partial \epsilon^B_{i,t}} = \phi^B_{i,t} - \phi^B_{i,t-1} - \underline{\mu}^{\epsilon}_{i,t} + \overline{\mu}^{\epsilon}_{i,t} = 0; \forall i \in \mathcal{I} \wedge t \in \mathcal{T}\backslash\{t = 1 \wedge t = |\mathcal{T}|\} \tag{B6}$$
$$\frac{\partial \mathcal{L}_i}{\partial \epsilon^B_{i,1}} = \phi^{B(0)}_i - \underline{\mu}^{\epsilon}_{i,1} + \overline{\mu}^{\epsilon}_{i,1} = 0; \forall i \in \mathcal{I} \tag{B7}$$
$$\frac{\partial \mathcal{L}_i}{\partial \epsilon^B_{i,|\mathcal{T}|}} = \phi^B_{i,|\mathcal{T}|} - \phi^B_{i,|\mathcal{T}|-1} + \phi^{B(end)}_i - \underline{\mu}^{\epsilon}_{i,|\mathcal{T}|} + \overline{\mu}^{\epsilon}_{i,|\mathcal{T}|} = 0; \forall i \in \mathcal{I} \tag{B8}$$
$$0 \le p^{IP,x}_{i,t} \perp \underline{\mu}^{IP,x}_{i,t} \ge 0; \forall i \in \mathcal{I} \wedge t \in \mathcal{T} \wedge x \in \{b, s\} \tag{B9}$$
$$0 \le \overline{p}^{IP}_i - p^{IP,x}_{i,t} \perp \overline{\mu}^{IP,x}_{i,t} \ge 0; \forall i \in \mathcal{I} \wedge t \in \mathcal{T} \wedge x \in \{b, s\} \tag{B10}$$
$$0 \le p^C_{i,t} - \underline{p}^C_{i,t} \perp \underline{\mu}^C_{i,t} \ge 0; \forall i \in \mathcal{I} \wedge t \in \mathcal{T} \tag{B11}$$
$$0 \le \hat{p}^C_{i,t} - p^C_{i,t} \perp \overline{\mu}^C_{i,t} \ge 0; \forall i \in \mathcal{I} \wedge t \in \mathcal{T} \tag{B12}$$
$$0 \le p^{B,x}_{i,t} \perp \underline{\mu}^{B,x}_{i,t} \ge 0; \forall i \in \mathcal{I} \wedge t \in \mathcal{T} \wedge x \in \{c, d\} \tag{B13}$$
$$0 \le \overline{p}^B_i - p^{B,x}_{i,t} \perp \overline{\mu}^{B,x}_{i,t} \ge 0; \forall i \in \mathcal{I} \wedge t \in \mathcal{T} \wedge x \in \{c, d\} \tag{B14}$$
$$0 \le \epsilon^B_{i,t} - \underline{\epsilon}^B_i \perp \underline{\mu}^{\epsilon}_{i,t} \ge 0; \forall i \in \mathcal{I} \wedge t \in \mathcal{T} \tag{B15}$$
$$0 \le \overline{\epsilon}^B_i - \epsilon^B_{i,t} \perp \overline{\mu}^{\epsilon}_{i,t} \ge 0; \forall i \in \mathcal{I} \wedge t \in \mathcal{T} \tag{B16}$$
$$0 \le \Delta\tau \textstyle\sum_t p^C_{i,t} - \mathrm{Z}^C_i \perp \underline{\mu}^Z_i \ge 0; \forall i \in \mathcal{I} \tag{B17}$$
$$\phi_{i,t}, \phi^B_{i,t}, \phi^{B(0)}_i, \phi^{B(end)}_i: \text{free}; \forall i \in \mathcal{I} \wedge t \in \mathcal{T} \tag{B18}$$
$$\begin{matrix} \underline{\mu}^{IP,b}_{i,t}, \overline{\mu}^{IP,b}_{i,t}, \underline{\mu}^{IP,s}_{i,t}, \overline{\mu}^{IP,s}_{i,t}, \underline{\mu}^C_{i,t}, \overline{\mu}^C_{i,t}, \\ \underline{\mu}^{B,c}_{i,t}, \overline{\mu}^{B,c}_{i,t}, \underline{\mu}^{B,d}_{i,t}, \overline{\mu}^{B,d}_{i,t}, \underline{\mu}^{\epsilon}_{i,t}, \overline{\mu}^{\epsilon}_{i,t}, \underline{\mu}^Z_i \end{matrix} \ge 0; \forall i \in \mathcal{I} \wedge t \in \mathcal{T} \tag{B19}$$

Above, (B1)-(B8) are the stationary conditions, resulting from deriving the Lagrangian of (3) w.r.t. $\boldsymbol{x}_i$. On the other hand, (B9)-(B17) are the complementarity conditions linked to the inequality constraints in (3), while (B18) and (B19) ensure dual feasibility.

Appendix C – Linearization of (6a)

Nonlinearities arise in the second term of (6a) due to the product of continuous variables. To linearize this term, we apply the Strong Duality Theorem (SDT), which states that the value of a primal objective is equal to its dual at the optimum if the Slater's conditions hold [33].

In our particular problem, the second term in (6a) is actually the objective function of industries (lower level). When binary variables are taken as parameters (which is indeed assumed in the sub-problem (6)), the industries' problems become linear and therefore the Slater's conditions hold. This way, the nonlinear term in (6a) can be replaced by its linear dual counterpart, which reads as:

$$F_i = \begin{matrix} -\Delta\tau \sum_t \left\{ \begin{matrix} \overline{p}^{IP}_i \left(\overline{\mu}^{IP,b}_{i,t} + \overline{\mu}^{IP,s}_{i,t}\right) + \overline{p}^B_i \left(\overline{\mu}^{B,c}_{i,t} + \overline{\mu}^{B,d}_{i,t}\right) - \\ \underline{\mu}^{\epsilon}_{i,t}\underline{\epsilon}^B_i + \overline{\mu}^{\epsilon}_{i,t}\overline{\epsilon}^B_i - \underline{\mu}^C_{i,t}\underline{p}^C_{i,t} + \overline{\mu}^C_{i,t}\hat{p}^C_{i,t} + \\ \phi_{i,t}\left(\hat{p}^{RES}_{i,t} - \hat{p}^L_{i,t} - \tilde{u}^S_{i,t}p^S_{i,t} - \tilde{u}^D_{i,t}p^D_{i,t}\right) \end{matrix} \right\} + \\ \overline{\epsilon}^B_i \left(\phi^{B(0)}_i + \phi^{B(end)}_i\right) + \underline{\mu}^Z_i \mathrm{Z}^C_i \end{matrix} \tag{C1}$$

References

[1] X. Chen, M. Dong, L. Zhang, X. Luan, X. Cui, Z. Cui. Comprehensive evaluation of environmental and economic benefits of industrial symbiosis in industrial parks. *Journal of Cleaner Production* 2022; 354: 131635. 10.1016/j.jclepro.2022.131635.

[2] European Environment Agency. Sectoral greenhouse gas emissions by IPCC sector. Online, available at: https://www.eea.europa.eu/data-and-maps/daviz/change-of-co2-eq-emissions-2/#tab-chart_4, (accessed on 5 Oct. 2023).

[3] Council of the EU. European climate law: Council and Parliament reach provisional agreement. Press release 5 May, 2021. Online, available at: https://www.consilium.europa.eu/en/press/press-releases/2021/05/05/european-climate-law-council-and-parliament-reach-provisional-agreement/, (accessed on 5 Oct. 2023).

[4] Urban Europe. Europe Towards Positive Energy Districts. 2020. Online, available at: https://jpi-urbaneurope.eu/wp-content/uploads/2020/06/PED-Booklet-Update-Feb-2020_2.pdf, (accessed on 5 Oct. 2023).

[5] R.R. Heeres, W.J.V. Vermeulen, F.B. de Walle. Eco-industrial park initiatives in the USA and the Netherlands: first lessons. *Journal of Cleaner Production* 2004; 12(8-10): 985-95. 10.1016/j.jclepro.2004.02.014.

[6] Y.U. Ban, J.H. Jeong, S.K. Jeong. Assessing the performance of carbon dioxide emission reduction of commercialized eco-industrial park projects in South Korea. *Journal of Cleaner Production* 2016; 114: 124-31. 10.1016/j.jclepro.2015.09.083.

[7] Business Norway. These industrial parks lead the way for a circular economy. Published 10 Sept. 2023. Online, available at: https://businessnorway.com/articles/these-industrial-parks-lead-the-way-for-a-circular-economy, (accessed on 5 Oct. 2023).

[8] Pôle Metropolitain de l'Artois. Online, available at: https://investinartois.fr/en/focus-on-renewable-energies/, (accessed on 5 Oct. 2023).

[9] Green Investment Group. Ulsan Floating Offshore Wind Project. Online, available at: https://www.greeninvestmentgroup.com/en/projects-and-perspectives/ulsan-floating-offshore-wind-project.html, (accessed on 5 Oct. 2023).

[10] Y. Zhou. Transition towards carbon-neutral districts based on storage techniques and spatiotemporal energy sharing with electrification and hydrogenation. *Renewable and Sustainable Energy Reviews* 2022; 162: 112444. 10.1016/j.rser.2022.112444.

[11] T. Maes et al. Energy management on industrial parks in Flanders. *Renewable and Sustainable Energy Reviews* 2011; 15(4): 1988-2005. 10.1016/j.rser.2010.11.053.

[12] Y. Zhao, K. Peng, B. Xu, H. Li, Y. Liu, X. Zhang. Bilevel Optimal Dispatch Strategy for a Multi-Energy System of Industrial Parks by Considering Integrated Demand Response. *Energies* 2018; 11: 1942. 10.3390/en11081942.

[13] N. Liu, L. Zhou, C. Wang, X. Yu, X. Ma. Heat-Electricity Coupled Peak Load Shifting for Multi-Energy Industrial Parks: A Stackelberg Game Approach. *IEEE Transactions on Sustainable Energy* 2020; 11(3): 1858-69. 10.1109/TSTE.2019.2945557.

[14] W. Xu, D. Zhou, X. Huang, B. Lou, D. Liu. Optimal allocation of power supply systems in industrial parks considering multi-energy complementarity and demand response. *Applied Energy* 2020; 275: 115407. 10.1016/j.apenergy.2020.115407.

[15] M.Z. Oskouei, S. Zeinal-Kheiri, B. Mohammadi-Ivatloo, M. Abapour, H. Mehrjerdi. Optimal Scheduling of Demand Response Aggregators in Industrial Parks Based on Load Disaggregation Algorithm. *IEEE Systems Journal* 2022; 16(1): 945-53. 10.1109/JSYST.2021.3074308.

[16] M.Z. Oskouei, B. Mohammadi-Ivatloo, M. Abapour, M. Shafiee and A. Anvari-Moghaddam. Strategic Operation of a Virtual Energy Hub With the Provision of Advanced Ancillary Services in Industrial Parks. *IEEE Transactions on Sustainable Energy* 2021; 12(4): 2062-73. 10.1109/TSTE.2021.3079256.

[17] J. Guo et al. Low-Carbon Robust Predictive Dispatch Strategy of Photovoltaic Microgrids in Industrial Parks. *Frontiers in Energy Research* 2022; 10. 10.3389/fenrg.2022.900503.

[18] European Commission. Project Call (2018): Energy efficiency of industrial parks through energy cooperation and mutualised energy services. Online, available at: https://cordis.europa.eu/programme/id/H2020_EE-18-2017/es, (accessed on 5 Oct. 2023).

[19] B. Crowley, J. Kazempour, L. Mitridati. Dynamic Pricing in an Energy Community Providing Capacity Limitation Services. 2023; arxiv-2309.05363. Online, available at: https://arxiv.org/pdf/2309.05363, (accessed on 5 Oct. 2023).

[20] N. Vespermann, T. Hamacher, J. Kazempour. Risk Trading in Energy Communities. *IEEE Transactions on Smart Grid* 2021; 12(2): 1249-63. 10.1109/TSG.2020.3030319.

[21] S.A. Habeeb, M. Tostado-Véliz, H.M. Hasanien, R.A. Turky, W.K. Meteab, F. Jurado. DC Nanogrids for Integration of Demand Response and Electric Vehicle Charging Infrastructures: Appraisal, Optimal Scheduling and Analysis. *Electronics* 2021; 10(20): 2484. 10.3390/electronics10202484.

[22] M. Tostado-Véliz, A.R. Jordehi, L. Fernández-Lobato, F. Jurado. Robust energy management in isolated microgrids with hydrogen storage and demand response. *Applied Energy* 2023; 345: 121319. 10.1016/j.apenergy.2023.121319.

[23] L. Canale, A.R. di Fazio, M. Russo, A. Frattolillo, M. Dell'Isola. An Overview on Functional Integration of Hybrid Renewable Energy Systems in Multi-Energy Buildings. *Energies* 2021; 14(4): 1078. 10.3390/en14041078.

[24] G.E. Constante-Flores, A.J. Conejo, R.M. Lima. Stochastic scheduling of generating units with weekly energy storage: A hybrid decomposition approach. *International Journal of Electrical Power & Energy Systems* 2023; 145: 108613. 10.1016/j.ijepes.2022.108613.

[25] M.S. Javadi, M. Lotfi, A.E. Nezhad, A. Anvari-Moghaddam, J.M. Guerrero, J.P.S. Catalão. Optimal Operation of Energy Hubs Considering Uncertainties and Different Time Resolutions. *IEEE Transactions on Industry Applications* 2020; 56(5): 5543-52. 10.1109/TIA.2020.3000707.

[26] gridX. Dynamic Electricity Pricing. Online, available at: https://www.gridx.ai/knowledge/dynamic-electricity-pricing#:~:text=Dynamic%20electricity%20pricing%20is%20defined,varies%20widely%20between%20different%20types., (accessed on 5 Oct. 2023).

[27] M. Tostado-Véliz, H.M. Hasanien, A.R. Jordehi, R.A. Turky, F. Jurado. Risk-averse optimal participation of a DR-intensive microgrid in competitive clusters considering response fatigue. *Applied Energy* 2023; 339: 120960. 10.1016/j.apenergy.2023.120960.

[28] I. Alsaidan, A. Khodaei, W. Gao. A Comprehensive Battery Energy Storage Optimal Sizing Model for Microgrid Applications. *IEEE Transactions on Power Systems* 2018; 33(4): 3968-80. 10.1109/TPWRS.2017.2769639.

[29] C. Feng, F. Wen, S. You, Z. Li, F. Shahnia, M. Shahidehpour. Coalitional Game-Based Transactive Energy Management in Local Energy Communities. *IEEE Transactions on Power Systems* 2020; 35(3): 1729-40. 10.1109/TPWRS.2019.2957537.

[30] A.D. Mustika, R. Rigo-Mariani, V. Debusschere, A. Pachurka. A two-stage management strategy for the optimal operation and billing in an energy community with collective self-consumption. *Applied Energy* 2022; 310: 118484. 10.1016/j.apenergy.2021.118484.

[31] M. Tostado-Véliz, A.R. Jordehi, H.M. Hasanien, R.A. Turky, F. Jurado. A novel stochastic home energy management system considering negawatt trading. *Sustainable Cities and Society* 2023; 97: 104757. 10.1016/j.scs.2023.104757.

[32] X. Wang, F. Li, Q. Zhang, Q. Shi, J. Wang. Profit-Oriented BESS Siting and Sizing in Deregulated Distribution Systems. *IEEE Transactions on Smart Grid* 2023; 14(2): 1528-40. 10.1109/TSG.2022.3150768.

[33] A. J. Conejo, C. Ruiz. Complementarity, Not Optimization, is the Language of Markets. *IEEE Open Access Journal of Power and Energy* 2020; 7: 344-53. 10.1109/OAJPE.2020.3029134.

[34] J. Kazempour. Advanced Optimization and Game Theory for Energy Systems. Online, available at: https://www.jalalkazempour.com/teaching, (accessed on 5 Oct. 2023).

[35] C. Ruiz, A.J. Conejo. Robust transmission expansion planning. *European Journal of Operational Research* 2015; 242(2): 390-401. 10.1016/j.ejor.2014.10.030.

[36] S. Siddiqui and S.A. Gabriel. An SOS1-Based Approach for Solving MPECs with a Natural Gas Market Application. *Networks and Spatial Economics* 2013; 13(2): 205-27. 10.1007/s11067-012-9178-y.

[37] J. Fortuny-Amat, B. McCarl. A representation and economic interpretation of a two-level programming problem. *The Journal of the Operational Research Society* 1981; 32(9): 783-92. 10.2307/2581394.

[38] S. Pineda, J.M. Morales. Solving Linear Bilevel Problems Using Big-Ms: Not All That Glitters Is Gold. *IEEE Transactions on Power Systems* 2019; 34(3): 2469-71. 10.1109/TPWRS.2019.2892607.

[39] B. Zeng, L. Zhao. Solving two-stage robust optimization problems using a column-and-constraint generation method. *Operational Research Letters* 2013; 41(5): 457-61. 10.1016/j.orl.2013.05.003.

[40] Gurobi Optimization L.L.C. Gurobi Optimizer Reference Manual, 2021. Online, available at: https://www.gurobi.com, (accessed on 5 Oct. 2023).

[41] Engie. Historical data reports. Online, available at: https://www.engieresources.com/historical-data#reports_anchor, (accessed on 5 Oct. 2023).

[42] C. Carmona-Delgado, E. Romero-Ramos, J. Riquelme-Santos. Fast and reliable distribution load and state estimator. *Electric Power Systems Research* 2013; 101: 110-24. 10.1016/j.epsr.2013.03.004.

[43] European Comission. Photovoltaic Geographical Information System. Online, available at: https://re.jrc.ec.europa.eu/pvg_tools/en/tools.html, (accessed on 5 Oct. 2023).

[44] S. Mandal, B.K. Das, N. Hoque. Optimum sizing of a stand-alone hybrid energy system for rural electrification in Bangladesh. *Journal of Cleaner Production* 2018; 200: 12-27. 10.1016/j.jclepro.2018.07.257.