Computing With Comparative Linguistic Expressions and Symbolic Translation for Decision Making: ELICIT Information

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***Abstract*—Many real-world decision making (DM) problems present changing contexts in which uncertainty or vagueness appear. Such uncertainty has been often modeled based on the linguistic information by using single linguistic terms. Dealing with linguistic information in DM demands processes of computing with words whose main characteristic is to emulate human beings reasoning processes to obtain linguistic outputs from linguistic inputs. However, often single linguistic terms are limited or do not express properly the expert's knowledge, being necessary to elaborate richer linguistic expressions easy to understand and able to express greater amount of knowledge, as it is the case of the comparative linguistic expressions based on hesitant fuzzy linguistic terms sets. Nevertheless, current computational models for comparative linguistic expressions present limitations both from understandability and precision points of view. The 2-tuple linguistic representation model stands out in these aspects because of its accuracy and interpretability dealing with linguistic terms, both related to the use of the symbolic translation, although 2-tuple linguistic values are still limited by the use of single linguistic terms. Therefore, the aim of this article is to present a new fuzzy linguistic representation model for comparative linguistic expressions that takes advantage of the goodness of the 2-tuple linguistic representation model and improve the interpretability and accuracy of the results in computing with words processes, resulting the so-called extended comparative linguistic expressions with symbolic translation. Taking into account the proposed model, a new computing with words approach is presented and then applied to a DM case study to show its performance and advantages in a real case by comparing with other linguistic decision approaches.**

***Index Terms*—Computing with words, comparative linguistic expressions, decision making (DM), hesitant fuzzy linguistic term set, symbolic translation.**

Manuscript received February 15, 2019; revised July 30, 2019; accepted September 3, 2019. Date of publication September 9, 2019; date of current version October 6, 2020. This work was supported in part by the Spanish Ministry of Economy and Competitiveness through the Spanish National Research Projects TIN2015-66524-P and PGC2018-099402-B-I00 and in part by the Postdoctoral Fellowship Ramón y Cajal (RYC-2017-21978). *(Corresponding author: Luis Martínez.)*

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This article has supplementary downloadable material available at http://ieeexplore.ieee.org, provided by the authors.

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Digital Object Identifier 10.1109/TFUZZ.2019.2940424

I. Introduction

NOWADAYS, human beings need to deal with problems characterized by multiple alternatives or options and decide which one is the best one/s as solution of the problem, this process is known as *decision making* (DM). DM problems may take place in changing environments, in which the uncertainty and vagueness are common, i.e., DM under uncertainty [1]–[3]. In such conditions, the use of linguistic information based on the fuzzy linguistic approach [4] has given successful results leading to *linguistic decision making* (LDM) [5]. Nevertheless, the use of linguistic information implies to accomplish computations with it. Computing with Words (CW) [6]–[9] is one of the most used methodologies, which carries out on processes where words (in a natural or artificial language) and not numbers are used for computing, emulating in this way, human beings cognitive processes [7], [10]–[13]. CW processes obtain linguistic outcomes from linguistic inputs, obtaining results, which are easily understandable and properly represented.

Consequently, several linguistic computational models have been developed to accomplish such linguistic computations within CW, that can be classified into two groups: 1) *linguistic computational model based on membership functions models* (semantic models) [2], [14]–[16] and 2) *linguistic symbolic computational model based on ordinal scales* (symbolic models) [5], [17]–[19]. However, the symbolic models stand out because their simplicity and high interpretability. Among them, *the 2-tuple linguistic computational model* [11], [20] is a symbolic model that extends the use of indexes, modifying the fuzzy linguistic approach representation by adding a parameter, so-called *symbolic translation*, to the basic linguistic representation in order to improve the accuracy of the linguistic computations, keeping the CW scheme and the interpretability of the results.

Nonetheless, the elicitation of linguistic information by either linguistic terms or 2-tuple linguistic values are still limited because such information must be always expressed by single linguistic terms, defined *a priori*. This limitation is especially important in LDM problems, since experts face uncertain decision situations in which they might hesitate among several linguistic terms and would prefer to employ more complex linguistic expressions to elicit their own knowledge. In order to overcome such a limitation, several proposals aimed at improving the elicitation of the linguistic information [21], but the expressions generated by those proposals were not close to human beings cognitive process or they did not provide a formalization method explaining how to obtain the linguistic expressions. On the other hand, Rodríguez *et al.* [22] proposed a linguistic model to construct linguistic expressions based on the

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hesitant fuzzy linguistic term sets (HFLTS). Later on, Rodríguez *et al.* [23] proposed the use of HFLTS and context-free grammars to build in a formal way *comparative linguistic expressions* (CLEs) that are more flexible and richer than single linguistic terms and also closer to the language used by human beings in real-world decision problems.

Many DM models have used CLEs and their corresponding HFLTS transformation to model and compute experts' information. Some of them have used a symbolic approach, in which the CLEs are transformed into linguistic terms intervals, losing information during the process [22], [24]–[27]. Others have utilized a symbolic approach in which the *fuzzy envelope* [28] of each CLE is computed [28]–[31]. Following with fuzzy envelope, a collection of proposals including *trapezoidal fuzzy numbers* [32], *discrete fuzzy numbers* [33], *possibility distribution* [34], [35], *proportional hesitant fuzzy linguistic term set* [36], and *probabilistic linguistic term set* [37] have provided successful results to carry out computations with HFLTS. However, although the latter models keep a fuzzy representation or have improved the precision in the results, the interpretability remains their weakness, since the representation of the results is difficult to understand because is far from human beings cognitive process.

CLEs are closer to the way of thinking of human beings but the results obtained in CW with them are still limited both from the point of view of interpretability and precision due to its discrete expression domain. As it was aforementioned, symbolic translation solves this problem for single linguistic terms. Therefore, the combination of CLEs and symbolic translation might lead to an improved CW processes for CLEs. Several proposals that combine both concepts have been introduced in the specialized literature. Some approaches have considered the use of HFLTSs and 2-tuple linguistic model independently [38]. Others proposals define CLEs based on 2-tuple linguistic term set to express experts' hesitancy [39], [40], while others are based on 2-tuple representation whose symbolic translation is formed by several values that represent the experts' hesitancy [41]. However, the abovementioned proposals present limitations and/or drawbacks from CW point of view (see Section II-D).

Taking into account previous drawbacks for computing with CLEs and keeping in mind the CW methodology, this article proposes a new fuzzy linguistic representation for CLEs together with a linguistic computational model that will keep understandability of results and improve their precision. Such a new linguistic model extends the CLEs by using the concept of symbolic translation introduced by the 2-tuple linguistic model [17] resulting the so-called *Extended Comparative LInguistiC Expressions with SymbolIc Translation* (ELICIT) information. These expressions extend the representation of CLEs generated by a context-free grammar into a continuous domain to perform CW processes without any kind of approximation. For sake of clarity, the main novelties of this article are enumerated as follows.

1) A new linguistic model, so-called ELICIT, which represents linguistic information through the generation of ELICIT information, an extension of CLEs in a continuous domain by using the symbolic translation concept related to the 2-tuple linguistic model.
2) A CW approach based on the ELICIT, which takes advantage of the main characteristics of the ELICIT information, interpretability, and precision.
3) A linguistic computational model for ELICIT information composed by several basic operations such as negation and aggregation together with a fuzzy comparison operator.

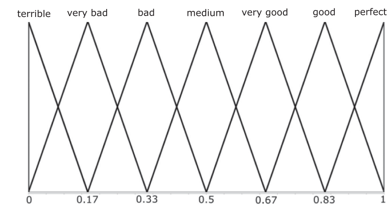

Fig. 1. Seven-term with its semantic.

The new representation and its computational model based on fuzzy arithmetic will be applied in DM research area, in which linguistic expressions have been widely used [5], [11], [42], [43]. Eventually, a real-world case study will be solved by using ELICIT information and a comparison with other previous models will show its advantages.

The rest of article is organized as follows. Section II reviews basic concepts about the DM, CW, and CLEs. Furthermore, a short literature review with the main related approaches to the proposed method is carried out. Section III introduces the new linguistic representation model along with the generation of ELICIT expressions through of a context-free grammar also defined. Furthermore, a new linguistic computational model based on such expressions is also described. Section IV shows the performance of the CW approach based on the ELICIT information in a real-world LDM problem and a comparison with previous models. Finally, Section V concludes this article.

II. Preliminaries

This section reviews some concepts related to fuzzy linguistic approach, DM, CW, and CLEs in order to understand easily this contribution. Moreover, several proposals conceptually related to the proposed model are also revised, to show the originality, novelty of the proposal and for further comparisons in real-world DM problems.

A. Fuzzy Linguistic Approach

Most of real-world problems present incomplete information, vagueness, and uncertainty that often cannot be modeled by probabilistic models. Under these conditions, the use of linguistic information has obtained successful results [44] in different fields [10], [45]–[47]. Several approaches have been presented in the literature to model linguistic information [4], [12], [48]–[51], among them, the *fuzzy linguistic approach* uses fuzzy set theory [52] to manage uncertainty and model linguistic information by using linguistic variables described by Zadeh [4] as *"A variable whose values are not numbers but words or sentences in a natural or artificial language"*. A linguistic variable is characterized by a syntactic value or *label* and a semantic value (see Fig. 1). Whereas the label is a word that belongs to a set of linguistic terms, semantics is provided by a fuzzy set in a discourse universe. Due to words are less precise than numbers, the concept of linguistic variable seems suitable to model complex and uncertain information.

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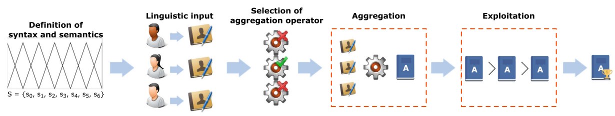

Fig. 2. Linguistic DM scheme.

Fig. 3. CW scheme.

Remark 1: Note that Appendix A related to fuzzy concepts has been introduced for a better understanding of the proposal.

The fuzzy linguistic approach provides the basis to model information linguistically by using a fuzzy representation. Furthermore, one of the main contributions of this approach, is a methodology for CW that merges natural languages and computations with fuzzy variables. Although the fuzzy linguistic approach and the CW methodology have been applied in several fields [10], [45]–[47], this contribution focuses on the DM research area (see Section II-B) due to their convenience and suitability to deal with DM problems under uncertainty [20].

B. Linguistic DM Under Uncertainty

DM is a quotidian process [53]–[58] characterized by a set of alternatives and the need to decide, which one/s is/are the best. In order to solve DM problems, experts provide their knowledge about the set of such alternatives and make decisions by means of reasoning processes [59]–[61]. Formally, a DM problem is characterized by a set of experts $E = \{e_1, \ldots, e_k\}$ who express their assessments over a sequence of alternatives or options $X = \{x_1, \ldots, x_n\}$, defined by a finite set of criteria $C = \{c_1, \ldots, c_m\}$. Due to the fact that, most of the real-world DM problems involve incomplete information, vagueness, and uncertainty that often cannot be modeled by probabilistic models, classical decision theory models cannot be applied. Therefore, under these conditions, the use of linguistic information and its modeling using linguistic variables [4] has obtained successful results. The use of linguistic variables to elicit knowledge and preferences about either alternatives or criteria is often used by decision makers involved in DM problems, raising the concept on LDM. The phases that compose the LDM scheme [5] are *definition of syntax and semantics*, *selection of an aggregation operator*, *aggregation,* and *exploitation*. All of them are graphically shown in Fig. 2.

The linguistic resolution scheme shows the necessity of operating with linguistic information to find a solution in LDM problems. The CW methodology mimics the human beings reasoning, that is, compute and reason by means of words, obtaining linguistic results from linguistic premises. Zadeh [7] defined CW as *"A methodology in which words are used in place of numbers for computing, reasoning, and DM"*.

In recent years, CW methodology has been intensively applied in DM [11], [46], [62] and, thus, multiple CW schemes have been proposed in the literature [13], [63], [64]. These schemes emphasize the need of obtaining accurate linguistic results easy to compute and understand. Fig. 3 shows the CW scheme introduced by Yager in [13], [64], in which the importance of the processes of *translation* and *retranslation* in CW was pointed out. The former translates the linguistic inputs into a machine-manipulate format based on fuzzy tools in which the computations are carried out. The latter consists of converting the computing results into linguistic information again to facilitate the human comprehension.

There are several computational models developed to perform linguistic computations [2], [46] based on the fuzzy linguistic approach [4], [50], [51]. However, the 2-tuple linguistic model proposed by Herrera and Martínez [17] is notable for its interpretability and accuracy [3].

Remark 2: Note that, due to the key role of the 2-tuple linguistic model in our contribution, concepts related to such a linguistic model have been included in Appendix B.

C. Hesitant Fuzzy Linguistic Term Sets and Comparative Linguistic Expressions

In spite of the fuzzy linguistic approach has been successfully applied in DM, the modeling of linguistic information is limited when experts provide their preferences by using just single terms. Experts might face situations in which they hesitate among several linguistic terms at the same time. Therefore, to overcome such a limitation, the concept HFLTS [22] was introduced. HFLTSs are based on the fuzzy linguistic approach that will serve as bases to increase the flexibility of the elicitation of linguistic information.

HFLTS can be directly used by the experts to elicit several linguistic values for a linguistic variable, but they are not close to the expressions used by the human beings. Therefore, it is necessary to define linguistic sentences closer to human beings expressions. Rodríguez *et al.* [21] reviewed the broadest linguistic approaches for modeling complex linguistic preferences. Although there are several proposals that obtain richer linguistic expressions than single linguistic terms, the one presented by Rodríguez *et al.* [22], [23] stands out because it provides a formalization process to generate linguistic expressions close to the common language used by human beings in DM problems. Such expressions, so-called CLEs are based on HFLTSs, that model the decision maker's hesitancy.

CLEs are built by using context-free grammars G_H. In [23] a basic context-free grammar for generating CLEs for eliciting DM preferences was introduced.

Definition 1. (see [23]): Let G_H be a context-free grammar and $S = \{s_0, \ldots, s_g\}$ a linguistic term set. The elements of $G_H = (V_N, V_T, I, P)$ are defined as follows.

$$V_N = \{\langle\text{primary term}\rangle, \langle\text{composite term}\rangle,$$
$$\langle\text{unary relation}\rangle, \langle\text{binary relation}\rangle, \langle\text{conjunction}\rangle\}$$
$$V_T = \{\text{at least, at most, between, and}, \ s_0, \ s_1, \ldots, s_g\}$$
$$I \in V_N.$$

The production rules defined in an extended Backus–Naur form are

$$P = \{I \ ::= \ \langle\text{primary term}\rangle | \langle\text{composite term}\rangle$$
$$\langle\text{composite term}\rangle \ ::= \ \langle\text{unary relation}\rangle\langle\text{primary term}\rangle|$$
$$\langle\text{binary relation}\rangle\langle\text{primary term}\rangle\langle\text{conjunction}\rangle$$
$$\langle\text{primary term}\rangle$$
$$\langle\text{primary term}\rangle \ ::= \ s_0|s_1|\ldots|s_g$$
$$\langle\text{unary relation}\rangle \ ::= \ \text{at least}|\text{at most}$$
$$\langle\text{binary relation}\rangle \ ::= \ \text{between}$$
$$\langle\text{conjunction}\rangle \ ::= \ \text{and}\}.$$

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Similar to the simple linguistic terms, the use of CLEs in DM implies processes of CW. To accomplish the computations on linguistic expressions, a transformation function E_G was defined to transform such expressions into HFLTSs.

Definition 2. (see [23]): Let E_{G_H} be a function that transforms CLEs, $ll \in S_{ll}$, obtained by G_H, into HFLTSs, H_S. S is the linguistic term set used by G_H and S_{ll} is the expression domain generated by G_H

$$E_{G_H} : S_{ll} \rightarrow H_S. \quad (1)$$

The CLEs generated by the context-free grammar G_H introduced in Definition 1 are transformed into HFLTSs H_S by means of the following transformations:

$$E_{G_H}(s_i) = \{s_i | s_i \in S\}$$
$$E_{G_H}(\text{ at most } s_i) = \{s_j | s_j \leq s_i \text{ and } s_j \in S\}$$
$$E_{G_H}(\text{ at least } s_i) = \{s_j | s_j \geq s_i \text{ and } s_j \in S\}$$
$$E_{G_H}(\text{ between } s_i \text{ and } s_j) = \{s_k | s_i \leq s_k \leq s_j \text{ and } s_k \in S\}.$$

Once the CLEs are transformed into a HFLTS, different computational models have been proposed [28], [31], [38], mainly based on the fusion of HFLTS by means of an *envelope* that can be obtained in different ways [22], [28]. Nevertheless, the proposal presented by Liu and Rodriguez [28] stands out. This proposal consists of a *fuzzy envelope* for HFLTSs, which represents the semantics of the CLEs by fuzzy membership functions obtained by aggregating the linguistic terms, which belong to the HFLTS, keeping in mind that the CW processes translate the linguistic input into a format based on fuzzy arithmetic to carry out the computations.

Definition 3. (see [28]): The *fuzzy envelope* $\text{env}_F(H_S)$ is defined as a trapezoidal fuzzy membership function as follows:

$$\text{env}_F(H_S) = T(a, b, c, d) \quad (2)$$

where H_S is a HFLTS and $T(a, b, c, d)$ is a fuzzy membership function (see [28] for further detail).

HFLTS fuzzy envelope is obtained by aggregating different fuzzy memberships functions with the OWA operator [65]. One of the most relevant characteristic of such operator is the possibility to set distinct importance to the linguistic terms that compose the HFLTS by means of weights assignment. Such importance will depend on the *optimism degree* related to the weights, which can be measured by the *orness measure*. This measure plays a key role in our contribution, since, thanks to it, we will able to compute fuzzy envelopes, which preserve as much information as possible.

Remark 3: Note that Appendix C related to the orness measure and its influence in our proposal has been included for a better understanding of it.

The CLEs are close to the linguistic structures used by human beings for eliciting preferences, specially in the real-world DM contexts, and improve preferences elicitation regarding single linguistic terms, but still the results obtained in CW with CLEs need improve precision and interpretability because of its current discrete expressions domain. Look at the LDM literature [20], the use of the symbolic translation solved an analogous problem with single linguistic terms. Consequently, it seems reasonable and promising to combine the CLEs and the symbolic translation to improve the CW processes with CLEs. Different attempts have been presented in the literature [38]–[41] that are briefly revised in the following section in which their drawbacks are pointed out in order to clarify the novelty advantages and need of our proposal.

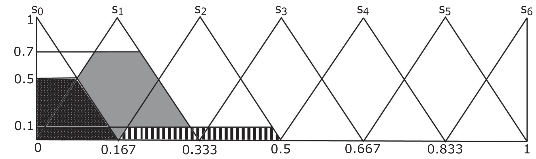

Fig. 4. Tan *et al.* proposal visualization.

D. Related Works

This section reviews several proposals that have combined to some extent the concepts of HFLTSs and the 2-tuple linguistic model to develop new representation models. We analyze such approaches taking into account the way of combining the CLEs and the 2-tuple linguistic model, also from the CW scheme point of view, pointing out their limitations and/or drawbacks.

1) Proposals that consider the use of the HFLTSs and the 2-tuple linguistic model independently.
   a) Zulueta *et al.* [38] presented an *environmental impact significance assessment* approach that allows to manage heterogeneous information, that is, the input information can be provided by the experts through CLEs, crisp numbers, interval-valued or hesitant fuzzy sets, and to carry out the computations with these different types of information, such information is unified into 2-tuple linguistic values. The results are represented by 2-tuple linguistic values. This approach aims at combining information but does not consider the CLEs as output; hence, it loses information in the unification process as well as reduces the expressiveness of assessments to just one term, although the output is linguistic and easy to understand.
2) Proposals define CLEs based on 2-tuple linguistic term sets to express decision makers' hesitancy.
   a) Tang *et al.* [39] introduced a linguistic approach that allows the construction of CLEs based on HFLTSs and the 2-tuple linguistic model, so-called *2-tuple hesitant linguistic fuzzy set* (2-TLFHS). The 2-TLFHS can be defined as follows.

      Definition 4. (see [39]): Let $S = \{(s_0, \alpha_0), (s_1, \alpha_1), \ldots, (s_g, \alpha_g)\}$ be a 2-tuple linguistic term set. A 2-tuple linguistic hesitant fuzzy set (2-TLHFS), LH, in S can be expressed as follows:

      $$LH = \{((s_{\theta_i}, \alpha_{\theta_i}), lh(s_{\theta_i}, \alpha_{\theta_i})) | (s_{\theta_i}, \alpha_{\theta_i}) \in S\}$$

      where $lh((s_{\theta_i}, \alpha_{\theta_i})) = \{r_1, r_2, r_3, \ldots, r_m\}$ is a set with m_i values in [0,1], denoting the possible membership degrees of the elements $x \in X$ to the set LH.

      Example 1: Supposing that one decision maker evaluates the quality of a wine, by using $S = \{s_0$: disgusting, s_1 : very bad, s_2 : bad, s_3 : normal, s_4 : good, s_5 : very good, s_5 : excellent$\}$, and provides the value 0.5 for *disgusting*, the value 0.7 for very bad and the value 0.1 for bad, the 2-TLHFS would be $LH = ((s_0, 0), 0.5), ((s_1, 0), 0.7), ((s_2, 0), 0.1)$ (graphically represented in Fig. 4).

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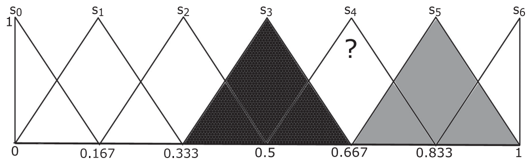

Fig. 5. Wei and Liao proposal visualization.

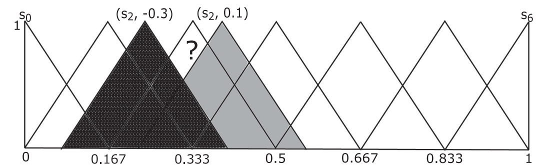

Fig. 6. Beg and Rashid proposal visualization.

These expressions model the decision maker's uncertainty by using hesitant fuzzy sets and the 2-tuple linguistic model. Nevertheless, the expressions present an important lack of interpretability, because they are not close to a human beings cognitive process, and it is really hard that decision makers provide their opinions by means of this type of expressions. Finally, although the computations are carried by using 2-TLHFS to avoid loss of information, the result is transformed into a 2-tuple linguistic value that implies lack of precision and reduces the expressiveness.

b) Wei and Liao [40] presented the concept of hesitant 2-tuple sets together with three kinds of operators to aggregate multigranularity hesitant fuzzy linguistic information without loss of information. The aim is to transform HFLTSs into hesitant 2-tuple sets in order to avoid loss of information. A hesitant 2-tuple set is defined as follows.

Definition 5. (see [40]): Let $S = \{s_0, s_1, \ldots, s_\tau\}$ be a linguistic term set, and (b_i, α_i) be a 2-tuple linguistic value on S, $i = 1, 2, \ldots, n$. If $(b_i, \alpha_i) < (b_j, \alpha_j)$ for any $i < j$, then is $\{(b_1, \alpha_1), (b_2, \alpha_2), \ldots, (b_l, \alpha_l)\}$, a hesitant 2-tuple set on S.

Example 2: $T^2_{S^7} = \{(s^7_3, 0), (s^7_5, 0)\}$ (graphically represented in Fig. 5).

This type of expressions are difficult to understand, because it consists of several 2-tuple linguistic values. Furthermore, there is no a definition that explains formally what happens with the information between two 2-tuple linguistic values whose linguistic terms are not consecutive, thus it is not clear whether such intermediate information is taken into account.

3) Other approaches are based on the 2-tuple representation whose symbolic translation is composed by several values that determine the decision makers' hesitancy.

a) Beg and Rashid [41] proposed a new linguistic representation model based on *hesitant 2-tuple linguistic information* defined as follows.

Definition 6. (see [41]): Let *X* be an universe of discourse and $S = \{s_1, \ldots, s_t\}$ be a linguistic term set, a hesitant linguistic term set in *X* is an expression A given by $A = \{(x, h(x)) | x \in X\}$ where $h(x) = (s_i, \beta_{ij}) \,\forall\, x \in X$.

This model represents the hesitant linguistic information by means of 2-tuple linguistic values, (s_i, β_{ij}), where s_i is a linguistic label and β_{ij} is a finite subset of $[-0.5, 0.5)$ that represents the possible symbolic translations of s_i. It is noted that the cardinality of β may be different for each x.

Example 3: $(\, s_2,\ (\,-0.3,\ 0.1\,)\,)$ (graphically represented in Fig. 6).

As in previous proposals, these expressions are far from structures used by human beings for eliciting preferences. Therefore, they present an important lack of interpretability and it is very difficult for decision makers to express their opinions and preferences through these expressions, which generate more uncertainty, from a semantic point of view, because the multiple values to represent the symbolic translation. Moreover, the results are numeric values, which imply loss of information and interpretability in the output.

III. ELICIT: Comparative Linguistic Expressions With Symbolic Translation

The lack of representation and computational models for CLEs that provide accurate and/or interpretable linguistic results manifests the need of a new approach able to keep the interpretability and accuracy of the results in CW processes dealing with CLEs. Taking into account the previous premises, this section presents the ELICIT linguistic *model*, a new fuzzy linguistic representation model that extends the CLEs by using the concept of symbolic translation used by the 2-tuple linguistic model.

This section is organized as follows. Section III-A introduces the ELICIT linguistic representation model and the ELICIT expressions. Section III-B presents a new CW approach based on the ELICIT linguistic model. Finally, Section III-C presents a computational model to accomplish the processes of CW by using ELICIT information.

A. Representation Model

The ELICIT linguistic model represents the linguistic information through the generation of ELICIT information, an extension of CLEs generated by a context-free grammar into a continuous domain by using the symbolic translation. The ELICIT information takes advantage of the main feature of CLEs that is their interpretability and, when it is necessary, it replaces the linguistic terms of the expressions by 2-tuple linguistic terms. In this way, the CW processes are performed without any kind of approximation, providing accurate results easy to understand.

Definition 7: Let G_H be a context-free grammar and $S = \{s_0, \ldots, s_g\}$ a linguistic term set. The elements of $G_H = (V_N, V_T, I, P)$ are defined as follows:

$$V_N = \{\langle\text{continuous primary term}\rangle,\ \langle\text{composite term}\rangle$$
$$\langle\text{unary relation}\rangle, \langle\text{binary relation}\rangle, \langle\text{conjunction}\rangle\}$$
$$V_T = \{\text{at least},\ \text{at most},\ \text{between},\ \text{and},\ \boldsymbol{(s_0, \alpha)^\gamma}$$
$$\boldsymbol{(s_1, \alpha)^\gamma}, \ldots, \boldsymbol{(s_g, \alpha)^\gamma}\}$$
$$I \in V_N.$$

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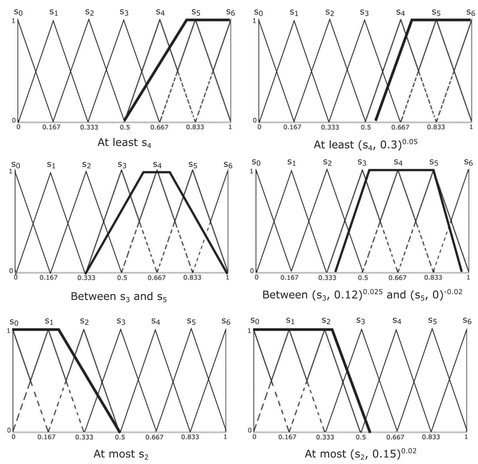

Fig. 7. Fuzzy comparison representation between CLEs and ELICIT.

The new production rules defined in an extended Backus–Naur form are

$$P = \{I ::= \langle\text{continuous primary term}\rangle |$$
$$\langle\text{composite term}\rangle$$
$$\langle\text{composite term}\rangle ::= \langle\text{unary relation}\rangle$$
$$\langle\text{continuous primary term}\rangle |$$
$$\langle\text{binary relation}\rangle\langle\text{continuous primary term}\rangle$$
$$\langle\text{conjunction}\rangle\langle\text{continuous primary term}\rangle$$
$$\langle\text{continuous primary term}\rangle \boldsymbol{::= (s_0, \alpha)^\gamma |}$$
$$\boldsymbol{(s_1, \alpha)^\gamma | \ldots | (s_g, \alpha)^\gamma}$$
$$\langle\text{unary relation}\rangle ::= \text{at least} | \text{at most}$$
$$\langle\text{binary relation}\rangle ::= \text{between}$$
$$\langle\text{conjunction}\rangle ::= \text{and}\}.$$

Thus, the possible ELICIT expressions generated according to the new definition of the context-free grammar are: "at least $(s_i, \alpha)^\gamma$," "at most $(s_i, \alpha)^\gamma$," and "between $(s_i, \alpha_1)^{\gamma_1}$ and $(s_j, \alpha_2)^{\gamma_2}$" (Fig. 7 shows an example of their fuzzy representation). Note that an additional parameter so-called *adjustment* and noted as γ has been included in each continuous primary term in the expression. This parameter provides essential information to carry out accurate computations with the ELICIT information as is further detailed in Section III-B4.

B. CW Approach for ELICIT Information

ELICIT representation model provides a new and flexible way to model linguistic information but it is also necessary to obtain interpretable and precise results in a CW processes by using ELICIT information and overcoming the drawbacks of previous linguistic models pointed out in Section II-D. Consequently, the definition of a new CW approach that takes advantage of these new expressions is logical and necessary. Here, a new CW approach is introduced for ELICIT information (see Fig. 8) based on the fuzzy linguistic approach (see Section II-A) that carries out CW computations in a precise way and provides linguistic results represented by ELICIT information by obtaining interpretable results. This section also describes the different processes carried out in such scheme that fulfil the general CW scheme showed in Fig. 3.

Fig. 8. ELICIT CW approach.

1) Linguistic Input and Output: As it was aforementioned, in a CW approach both inputs and outcomes have to be represented by linguistic information. The outputs will be logically represented by ELICIT expressions that should be generated from linguistic inputs close to the way of thinking of human beings and that facilitate the elicitation of their preferences. Section II-C presented the CLEs as linguistic expressions close to the common language of human beings, especially in DM problems, and that are able to model the hesitancy of the experts. For this reason, for the proposed CW approach, CLEs will be used to represent the linguistic input. Furthermore, ELICIT information previously computed can also be incorporated in the input of the CW approach.

2) Translation: For our proposal, the translation process consists of transforming the initial CLEs and ELICIT expressions into fuzzy numbers. Such transformations are carried out in different ways depending of the type of expression, CLE or ELICIT. CLEs are transformed into fuzzy numbers by computing their *fuzzy envelopes* (See Eq. (2)). Fuzzy envelopes are obtained by means of the aggregation of the linguistic terms that belong to the representative HFLTS of each CLE by using the OWA operator.

Remark 4: Note that the way to compute fuzzy envelopes is a key in our proposal. The OWA operator behavior regarding the importance of the linguistic terms belonging to a HFLTS determines the shape of the fuzzy envelopes and, consequently, the amount of information that fuzzy envelopes preserve. For our proposal, the more information is kept, the more accurate the results are. Such behavior is directly related to the *orness* measure of the OWA operator, thus, the orness measure plays at the same time a pivotal role in our proposal. For this reason, Appendix C introduces several necessary conditions regarding orness measure for preserving as much information as possible in the fuzzy envelopes computation.

On the other hand, the transformation of ELICIT information into fuzzy numbers is carried out by means of a function, noted as ζ^{-1}.

Definition 8: Let x_{el} be an ELICIT expression and $T(a, b, c, d)$ a trapezoidal fuzzy number. The function ζ^{-1} is defined as

$$\zeta^{-1} : x_{\text{el}} \to T(a, b, c, d). \tag{3}$$

Such that, from an ELICIT expression, it returns its equivalent trapezoidal fuzzy number.

The *adjustment* γ of the ELICIT expression, plays a key role in ζ^{-1}, since this parameter is used to obtain the points that define the corresponding fuzzy number, by preserving as much information as possible in the fuzzy representation of the ELICIT information (the *adjustment* computation for each ELICIT expression is introduced in Section III-B4 and further detailed in Appendix D). Depending on the ELICIT expression, the ζ^{-1} function is defined in different ways.

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1) *At least expression:* The function ζ^{-1} for an ELICIT expression whose relation is "at least" is defined as follows.

Proposition 1: Let at least $(s_i, \alpha)^\gamma$ be an ELICIT expression and $T_{\text{ELICIT}}(a,' b,' 1, 1)$ the fuzzy envelope of such ELICIT expression. There is a function ζ^{-1}

$$\begin{aligned} &\zeta^{-1}(\text{at least } (s_i, \alpha)^\gamma) = T(a, b, 1, 1) \\ &a = a' + \gamma \\ &b = b'. \end{aligned} \tag{4}$$

2) *At most expression:* The function ζ^{-1} for an ELICIT expression whose relation is "at most" is defined as follows.

Proposition 2: Let at most $(s_i, \alpha)^\gamma$ be an ELICIT expression and $T_{\text{ELICIT}}(0, 0, c,' d')$ the fuzzy envelope of such ELICIT expression. There is a function ζ^{-1}

$$\begin{aligned} &\zeta^{-1}(\text{at most } (s_i, \alpha)^\gamma) = T(0, 0, c, d) \\ &c = c' \\ &d = d' + \gamma. \end{aligned} \tag{5}$$

3) *Between expression:* The function ζ^{-1} for an ELICIT expression whose relation is "between" is defined as follows.

Proposition 3: Let between $(s_i, \alpha_1)^{\gamma_1}$ and $(s_j, \alpha_2)^{\gamma_2}$ be an ELICIT expression and $T_{\text{ELICIT}}(a,' b,' c,' d')$ the fuzzy envelope of such ELICIT expression. There is a function ζ^{-1}

$$\begin{aligned} &\zeta^{-1}(\text{between } (s_i, \alpha_1) \text{ and } (s_j, \alpha_2)) = T(a, b, c, d) \\ &a = a' + \gamma_1 \\ &b = b' \\ &c = c' \\ &d = d' + \gamma_2. \end{aligned} \tag{6}$$

3) Manipulation: The manipulation phase consists of carrying out fuzzy arithmetic computations with the fuzzy envelopes previously obtained, by resulting new fuzzy numbers noted as $\tilde{\beta}$. The fuzzy arithmetic operations have to keep the fuzzy representation to guarantee that the resulting fuzzy numbers $\tilde{\beta}$ can be represented in the initial fuzzy linguistic domain used in the input and, subsequently, transformed into ELICIT information.

Rezvani and Molani [66] proved that, by means of the fuzzy numbers shape function and $\alpha -$ cuts (see Appendix A), it is possible to compute different arithmetic operations keeping the fuzzy parametric representation (triangular or trapezoidal). This section presents the addition and subtraction fuzzy operations with ELICIT expressions represented by their envelopes.

Definition 9: Let $T_{\tilde{A}}(a_1, b_1, c_1, d_1)$ and $T_{\tilde{B}}(a_2, b_2, c_2, d_2)$ be two fuzzy envelopes modeled by two trapezoidal fuzzy numbers (TrFNs). Suppose the normal shape functions of $\tilde{A}, \tilde{B}$ as follows:

$$\mu_{\tilde{A}} = \begin{cases} \left(\dfrac{x - a_1}{b_1 - a_1}\right)^n, & \text{when } x \in [a_1, b_1) \\ 1, & \text{when } x \in [b_1, c_1] \\ \left(\dfrac{d_1 - x}{d_1 - c_1}\right)^n, & \text{when } x \in (c_1, d_1] \\ 0, & \text{otherwise} \end{cases}$$

$$\mu_{\tilde{B}} = \begin{cases} \left(\dfrac{x - a_2}{b_2 - a_2}\right)^n, & \text{when } x \in [a_2, b_2) \\ 1, & \text{when } x \in [b_2, c_2] \\ \left(\dfrac{d_2 - x}{d_2 - c_2}\right)^n, & \text{when } x \in (c_2, d_2] \\ 0, & \text{otherwise.} \end{cases}$$

And supposing $\tilde{A}_{\overline{\alpha}}$, $\tilde{B}_{\overline{\alpha}}$ are the $\alpha -$ cuts (see Definition 22) of $\tilde{A}$ and $\tilde{B}$, respectively

$$\begin{aligned} \tilde{A}_{\overline{\alpha}} &= [a_1 + \overline{\alpha}^{1/n}(b_1 - a_1), d_1 - \overline{\alpha}^{1/n}(d_1 - c_1)] \\ \tilde{B}_{\overline{\alpha}} &= [a_2 + \overline{\alpha}^{1/n}(b_2 - a_a), d_2 - \overline{\alpha}^{1/n}(d_2 - c_2)]. \end{aligned}$$

It should be noted that, in our proposal, the computational processes deal with normal and complete TrFN, hence $n = 1$ and $\overline{\alpha} = 1$.

Definition 10. (see [66]): The addition of two fuzzy envelopes modeled by two TrFNs $\tilde{A}$, $\tilde{B}$ can be defined with a shape function $\mu_{\tilde{A}+\tilde{B}}$ as

$$\mu_{\tilde{A}+\tilde{B}} = \begin{cases} \dfrac{(x - (a_1 + a_2))^n}{(b_1 + b_2) - (a_1 + a_2)}, & a_1 + a_2 \leq x \leq b_1 + b_2 \\ 1, & b_1 + b_2 \leq x \leq c_1 + c_2 \\ \dfrac{((d_1 + d_2) - x)^n}{(d_1 + d_2) - (c_1 + c_2)}, & c_1 + c_2 \leq x \leq d_1 + d_2 \\ 0, & \text{otherwise.} \end{cases}$$

Definition 11. (see [66]): The subtraction of two fuzzy envelopes modeled by two TrFNs $\tilde{A}$, $\tilde{B}$ can be defined with a shape function $\mu_{\tilde{A}-\tilde{B}}$ as

$$\mu_{\tilde{A}-\tilde{B}} = \begin{cases} \dfrac{(x - (a_1 - d_2))^n}{(b_1 - a_1 + d_2 - c_2)}, & a_1 - d_2 \leq x \leq b_1 - c_2 \\ 1, & b_1 - c_2 \leq x \leq c_1 - b_2 \\ \dfrac{((d_1 - a_2) - x)^n}{(d_1 - c_1 + b_2 - a_2)}, & c_1 - b_2 \leq x \leq d_1 - a_2 \\ 0, & \text{otherwise.} \end{cases}$$

Example 4: Let $T_A(0.2, 0.33, 0.45, 0.5)$ and $T_B(0.15, 0.2, 0.3, 0.4)$ be two fuzzy envelopes modeled by two TrFNs, the normal shape functions $\mu_{\tilde{A}+\tilde{B}}$ and $\mu_{\tilde{A}-\tilde{B}}$ are defined, respectively, as

$$\mu_{\tilde{A}+\tilde{B}} = \begin{cases} \dfrac{(x - 0.35)^n}{0.18}, & 0.35 \leq x \leq 0.53 \\ 1, & 0.53 \leq x \leq 0.75 \\ \dfrac{(0.9 - x)^n}{0.15}, & 0.75 \leq x \leq 0.9 \\ 0, & \text{otherwise} \end{cases}$$

$$\mu_{\tilde{A}-\tilde{B}} = \begin{cases} \dfrac{(x - 0.35)^n}{0.18}, & -0.2 \leq x \leq 0.03 \\ 1, & 0.03 \leq x \leq 0.25 \\ \dfrac{(0.9 - x)^n}{0.15}, & 0.25 \leq x \leq 0.35 \\ 0, & \text{otherwise.} \end{cases}$$

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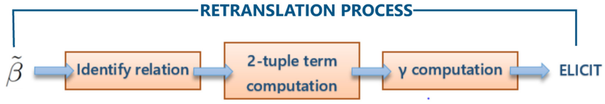

Fig. 9. Steps to build ELICIT expressions.

Remark 5: Note that Appendix A introduces the necessary fuzzy concepts.

4) Retranslation: Last but not least, the resulting fuzzy numbers $\overline{\beta}$ are transformed into ELICIT expressions in the *retranslation* process. This section explains in further detail the necessary steps to build ELICIT expressions from the results obtained by fuzzy computations on CLEs and ELICIT expressions, graphically represented in Fig. 9.

Starting from a fuzzy number $\tilde{\beta}$, the different steps to build an ELICIT expression are:

1) *Identify the relation:* Regarding Definition 7, the possible relations for ELICIT expressions are "at least," "at most," and "between". The relation is determined by the fuzzy number $\tilde{\beta}$ and the ζ function, defined as follows.

Definition 12: Let $S = \{s_0, \ldots, s_g\}$ be a set of linguistic terms and $\tilde{\beta}$ a fuzzy number. The function ζ is given by

$$\zeta(\tilde{\beta}) = x_{\text{el}},$$

$$\text{where} \begin{cases} x_{\text{el}} = \text{at least } (s_i, \alpha)^{\gamma} \text{ if } \tilde{\beta} = T(a, b, 1, 1) \\ x_{\text{el}} = \text{at most } (s_i, \alpha)^{\gamma} \text{ if } \tilde{\beta} = T(0, 0, c, d) \\ x_{\text{el}} = \text{between } (s_i, \alpha_1)^{\gamma_1} \text{and } (s_j, \alpha_2)^{\gamma_2} \\ \text{if } \tilde{\beta} = T(a, b, c, d). \end{cases}$$

Henceforth, for sake of clarity, it is assumed that the ELICIT expression is composed by a "between" relation, others are analogously developed in Appendix D. Thus, according to Definition 12, for a "between" relation the fuzzy number $\overline{\beta}$ is represented by $\tilde{\beta} = T(a, b, c, d)$ and consequently, the ELICIT expression is "between $(s_i, \alpha_1)^{\gamma_1}$ and $(s_j, \alpha_2)^{\gamma_2}$".

2) *2-tuple linguistic terms computation:* The ELICIT expression with the relation "between" is composed by two continuous primary terms $(s_i, \alpha_1)^{\gamma_1}$ and $(s_j, \alpha_2)^{\gamma_2}$. The process of obtaining the terms is divided into different steps.

a) *Compute linguistic terms (see Fig. 10):* To select the linguistic terms s_i and $s_j \in S, i, j \in \{0, \ldots g\}$, whose distance between the coordinates x of their respective centroids [67], $\overline{x}_i$ and $\overline{x}_j$, and the points b and c belonging to $\tilde{\beta}$ is minimal

$$i = \arg\min_h |b - \overline{x}_h|, \; h \in \{0, \ldots, g\}$$

$$j = \arg\min_h |c - \overline{x}_h|, \; h \in \{0, \ldots, g\}. \quad (7)$$

When this process finishes, the ELICIT expression so far is "between $(s_i, ?)^?$ and $(s_j, ?)^?$".

b) *Compute symbolic translations (see Fig. 11):* According to [20], [68], $1/2g$ represents the distance equivalent to a symbolic translation equal to 0.5 in S, where $g + 1$ is the cardinality of S

$$\alpha_1 = g \cdot (b - \overline{x}_i) \;\; \alpha_1 \in [-0.5, 0.5)$$

$$\alpha_2 = g \cdot (c - \overline{x}_j) \;\; \alpha_2 \in [-0.5, 0.5). \quad (8)$$

When this process finishes, the ELICIT expression so far is "between $(s_i, \alpha_1)^?$ and $(s_j, \alpha_2)^?$".

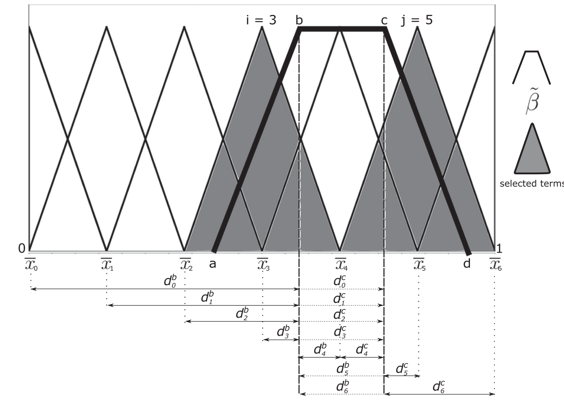

Fig. 10. Select linguistic terms.

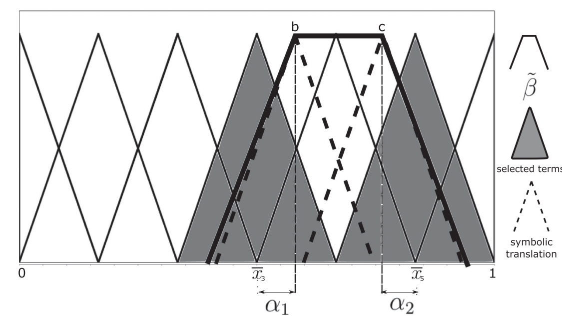

Fig. 11. Symbolic translation.

3) *Compute adjustments:* The *adjustment* is an additional parameter included in the ELICIT expression, which allows to keep information related to the fuzzy number $\tilde{\beta}$. This parameter will be used to obtain the fuzzy number $\tilde{\beta}$ from an ELICIT expression by using its inverse function, ζ^{-1} (see Section III-B2), preserving as much information as possible in the fuzzy representation and facilitating that accurate computations can be carried out in the manipulation phase. The steps to compute the adjustments for the ELICIT expression are as follows.

a) *Compute HFLTS (see Fig. 12):* The HFLTS of an ELICIT expression whose relation is "between" would be composed by

$$E_{\text{ELICIT}}(\text{between } (s_i, \alpha) \text{ and } (s_j, \alpha)) = \{s_k | (s_i, \alpha) \text{ and } (s_j, \alpha) \text{ and } s_i < s_k < s_j \text{ where } s_k \in S\}.$$

b) *Compute fuzzy envelope (see Fig. 13):* Applying (2), the fuzzy envelope of the computed HFLTS is noted as $T_{\text{ELICIT}} = T(a,' b,' c,' d')$. To compute b' and c', different weights for the linguistic terms, which compose the previous HFLTS are assigned

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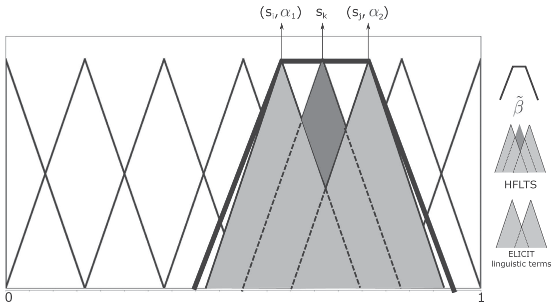

Fig. 12. HFLTS in "between" expression.

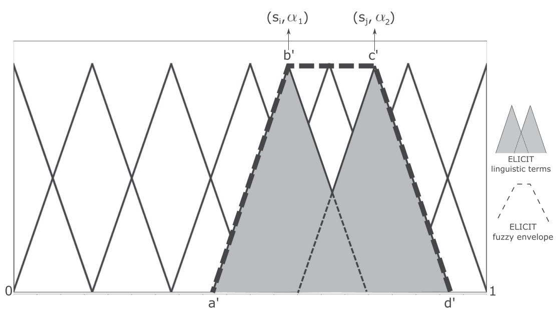

Fig. 13. Fuzzy envelope in "between" expression.

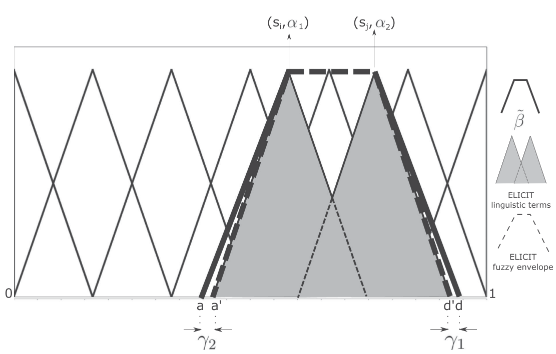

Fig. 14. γ_1 and γ_2 in "between" expression.

by using a parameter noted as ϵ and introduced in Appendix C. Specifically, to compute b', the value of ϵ_1 will be equal to 0 and to compute c', the value of ϵ_2 will be equal to 1. These values are provided to preserve as much information as possible in the ELICIT expressions for CW processes.

c) *Compute adjustments γ_1 and γ_2 (see Fig. 14):* The adjustments γ_1 and γ_2 are determined by the subtraction between the points a and d of $\tilde{\beta} = T(a, b, c, d)$ and the points a' and d' of $T_{\text{ELICIT}}(a,' b,' c,' d')$, so that

$$\begin{aligned} \gamma_1 &= a - a'\gamma_1 \in [0, 1] \\ \gamma_2 &= d - d'\gamma_2 \in [0, 1]. \end{aligned} \tag{9}$$

When this process finishes, the ELICIT expression is completed "between $(s_i, \alpha_1)^{\gamma_1}$ and $(s_j, \alpha_2)^{\gamma_2}$".

C. ELICIT Computational Model

This section introduces basic operations to accomplish the processes of CW with ELICIT information.

1) Comparison Operator: This section introduces a comparison operator for ELICIT information. The comparison among ELICIT expressions is carried out from their respective fuzzy envelopes. Therefore, in order to compare ELICIT expressions, a method for ranking fuzzy numbers is applied. There are several proposals that allow to rank fuzzy numbers [69]–[72] but, the method presented by Abbasbandy and Hajjari in [69] stands out, since it is far simpler and easier than other proposals. According to such a method, the ordering of ELICIT expressions is carried out by raking their respective fuzzy numbers $\tilde{\beta}$ by using the concept of *magnitude*.

Definition 13: Let x_{el1} and x_{el2} be two ELICIT expressions, $\tilde{\beta}_1$ and $\tilde{\beta}_2$ their equivalent fuzzy numbers obtained from $\zeta^{-1}(x_{\text{el1}})$ and $\zeta^{-1}(x_{\text{el2}})$ are as follows:

1) $\text{Mag}(\tilde{\beta}_1) > \text{Mag}(\tilde{\beta}_2) \iff \tilde{\beta}_1 \succ \tilde{\beta}_2$;
2) $\text{Mag}(\tilde{\beta}_1) < \text{Mag}(\tilde{\beta}_2) \iff \tilde{\beta}_1 \prec \tilde{\beta}_2$;
3) $\text{Mag}(\tilde{\beta}_1) = \text{Mag}(\tilde{\beta}_2) \iff \tilde{\beta}_1 \sim \tilde{\beta}_2$;

where $\text{Mag}(\beta)$ is the magnitude of the fuzzy number β.

Remark 6: Note that Appendix E related to ranking of fuzzy numbers and magnitude concept has been introduced for a better understanding of the comparison operator for ELICIT information.

2) Negation Operator: The concept of negation is also essential to deal with linguistic information. Therefore, its definition is relevant for dealing with ELICIT information. Depending on the linguistic expression, several negation operators are defined.

Definition 14: Let "at least $(s_i, \alpha)^\gamma$" be an ELICIT expression and $S = \{s_0, \ldots s_g\}$ a linguistic term set, the negation operator is defined as:

$$\text{Neg}(\text{at least } (s_i, \alpha)^\gamma) = \text{at most } \Delta(g - (\Delta^{-1}(s_i, \alpha)))^\gamma.$$

Definition 15: Let "at most $(s_i, \alpha)^\gamma$" be an ELICIT expression and $S = \{s_0, \ldots s_g\}$ a linguistic term set, the negation operator is defined as

$$\text{Neg}(\text{at most } (s_i, \alpha)^\gamma) = \text{at least } \Delta(g - (\Delta^{-1}(s_i, \alpha)))^\gamma.$$

Definition 16: Let "between $(s_i, \alpha_1)^{\gamma_1}$ and $(s_j, \alpha_2)^{\gamma_2}$" be an ELICIT expression and $S = \{s_0, \ldots s_g\}$ a linguistic term set, the negation operator is defined as

$$\begin{aligned} &\text{Neg}(\text{between } (s_i, \alpha_1)^{\gamma_1} \text{ and } (s_j, \alpha_2)^{\gamma_2}) \\ &= \text{between } \Delta(g - (\Delta^{-1}(s_i, \alpha_1)))^{\gamma_1} \text{ and} \\ &\Delta(g - (\Delta^{-1}(s_j, \alpha_2)))^{\gamma_2}. \end{aligned}$$

3) Aggregation Operators: The aggregation of multiple values into a single one is an essential process for any discipline based on the data processing, for instance DM. The aggregation process for ELICIT expressions is accomplished by using the fuzzy numbers $\tilde{\beta}$ obtained from the function ζ^{-1} of each expression. The operations carried out with the $\tilde{\beta}$ fuzzy numbers are based on the fuzzy arithmetic operations (see Section III-B3) defined by Rezvani and Molani [66]. Such fuzzy operations allow to keep the fuzzy representation and obtain new $\tilde{\beta}$ fuzzy numbers that can be again transformed into ELICIT expressions. A general aggregation operator is defined as follows.

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TABLE I
ENVELOPES OF HFLTS

		Criterion		
Expert	Alternative	c_1	c_2	c_3
e_1	a_1	T(0.67, 0.83, 1)	T(0.33, 0.5, 0.67, 0.83)	T(0.83, 1, 1)
	a_2	T(0.33, 0.5, 0.67)	T(0.67, 0.83, 1)	T(0.5, 0.67, 1, 1)
	a_3	T(0.33, 0.5, 0.67)	T(0.17, 0.33, 0.5)	T(0.17, 0.33, 0.5)
	a_4	T(0.67, 0.83, 1)	T(0.67, 0.83, 1, 1)	T(0.83, 1, 1)
e_2	a_1	T(0.33, 0.5, 0.67)	T(0, 0, 0.33, 0.5)	T(0.5, 0.67, 0.83)
	a_2	T(0.67, 0.83, 1)	T(0.83, 1, 1)	T(0.5, 0.67, 1, 1)
	a_3	T(0, 0.17, 0.33)	T(0, 0.17, 0.33)	T(0, 0, 0.33, 0.5)
	a_4	T(0.5, 0.67, 0.83)	T(0.17, 0.33, 0.5)	T(0.33, 0.5, 0.67)
e_3	a_1	T(0.83, 1, 1)	T(0.5, 0.67, 0.83, 1)	T(0.5, 0.67, 1, 1)
	a_2	T(0.17, 0.33, 0.5)	T(0.5, 0.67, 0.83)	T(0.33, 0.5, 0.67)
	a_3	T(0, 0.17, 0.33)	T(0, 0, 0.17, 0.33)	T(0, 0.17, 0.33)
	a_4	T(0.5, 0.67, 0.83)	T(0.17, 0.33, 0.5)	T(0.17, 0.33, 0.5)

Definition 17: Let $\{x_{\text{el}1}, \ldots, x_{elk}\}$ be a set of ELICIT expressions and $\{\tilde{\beta}_1, \ldots, \tilde{\beta}_k\}$ their equivalent fuzzy numbers obtained from $\{\zeta^{-1}(x_{\text{el}1}), \ldots, \zeta^{-1}(x_{elk})\}$, a fuzzy aggregation operator F is defined as

$$F(\tilde{\beta}_1, \ldots, \tilde{\beta}_k) = \tilde{\beta} = T(a, b, c, d) = x_{\text{el}}. \tag{10}$$

IV. CASE STUDY

This section presents a case study to show the usefulness of the proposed fuzzy linguistic representation model. First, a LDM problem is described. Afterwards, the LDM problem is solved by means of the ELICIT CW approach. Finally, the results are compared with another CW approach [20] to show the advantages of the proposal.

A. Definition of DM Problem

Let us suppose a prestigious university that wants to hire a Ph.D. student among four possible candidates $X = \{a_1, a_2, a_3, a_4\}$. The final decision is made for a group of three renowned professors $E = \{e_1, e_2, e_3\}$ who have to evaluate the candidates according to three criteria $C = \{c_1, c_2, c_3\}$, which are, respectively: *communication skills*, *research experience,* and *academic record*.

B. Resolution of DM Problem

In order to solve the DM problem, the ELICIT CW approach is applied. This section is divided into several sections that describes the different processes carried out in such approach (see Fig. 8).

1) Linguistic Input and Output: Due to the DM problem implies uncertainty and imprecision, the set of criteria will be evaluated by means of linguistic information. The experts provide their preferences through a linguistic domain based on such knowledge composed by 7 labels, $S_7 =$ {Horrible (H), Very bad (VB), Bad (B), Medium (M), Good (G), Very good (VG), Excellent (E)} (see Fig. 1), by using CLEs and single linguistic terms. For the sake of space, the preferences have been included as a supplementary material document, which is available online[1].

2) Translation: The experts' preferences are transformed into fuzzy numbers by computing their fuzzy envelopes, which are shown in Table I.

[1] [Online]. Available: https://sinbad2.ujaen.es/flintstones/en/study-cases/phdStudentSelection.pdf

TABLE II
RESULTING β FUZZY NUMBERS

Alternative	β
a_1	T(0.5, 0.65, 0.76, 0.88)
a_2	T(0.5, 0.67, 0.75, 0.86)
a_3	T(0.08, 0.21, 0.26, 0.43)
a_4	T(0.45, 0.62, 0.63, 0.76)

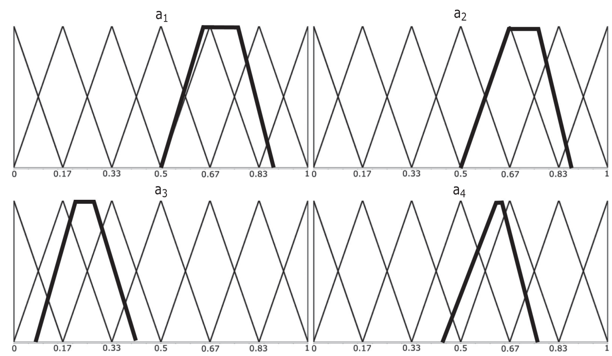

Fig. 15. β fuzzy representation for each alternative.

TABLE III
RESULTING ELICIT EXPRESSIONS

Alternative	ELICIT
a_1	$Between(Good, -0.11)^{0.018}$ and $(\textit{Very good}, -0.44)^{-0.056}$
a_2	$Between(Good, 0)^{0}$ and $(Good, 0.44)^{-0.055}$
a_3	$Between(\textit{Very bad}, 0.22)^{0.037}$ and $(Bad, -0.44)^{-0.001}$
a_4	$Between(Good, -0.33)^{-0.001}$ and $(Good, -0.22)^{-0.037}$

3) Manipulation: The fuzzy envelopes are aggregated by using an aggregation operator to obtain a collective value for each alternative (see Fig. 2). As it was aforementioned in Section III-B3, the fuzzy arithmetic operations carried out have to keep the fuzzy representation to guarantee that the resulting fuzzy numbers $\tilde{\beta}$ can be represented. For sake of simplicity and without losing generality the aggregation operator used in this case study is the *fuzzy arithmetic mean*.

Definition 18: Let $\{\tilde{A}_1, \ldots, \tilde{A}_n\}$ be a set of fuzzy numbers, the fuzzy arithmetic mean $\overline{x}$ is computed as

$$\overline{x}\{\tilde{A}_1, \ldots, \tilde{A}_n\} = \frac{1}{n}(\mu_{\tilde{A}_1+\tilde{A}_2+\ldots+\tilde{A}_n}). \tag{11}$$

The results of the aggregation process are showed in Table II and graphically represented in Fig. 15.

4) Retranslation: The resulting β are transformed into ELICIT expressions following the scheme represented in Fig. 9 (see Table III). In this way, the ELICIT model, on the contrary of others representation models (see Section II-D), provides linguistic results easily interpretable represented by ELICIT information, which is closer to the way of thinking of human beings and facilitates the understanding of the results by decision makers. Furthermore, the use of symbolic translation in ELICIT information guarantees that the linguistic computations have been carried out in a continuous domain, which means that the results have been obtained without any kind of approximation. Consequently, the results are more precise and reliable.

To conclude, the ranking of alternatives is obtained as solution of the problem and shown in Table IV. To do so, the approach introduced by Abbasbandy and Hajjari is used (see Appendix E).

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TABLE IV
RANKING OF ALTERNATIVES

Alternative	Ranking
a_1	1
a_2	2
a_3	4
a_4	3

TABLE V
RESULTING 2-TUPLE LINGUISTIC VALUES

Alternative	2-tuple	Ranking
a_1	$(Good, 0.14)$	2
a_2	$(Good, 0.16)$	1
a_3	$(Very\ bad, 0.42)$	4
a_4	$(Good, -0.33)$	3

Remark 7: Note that the function $f(r)$ defined to compute the ranking in this proposal is $f(r) = r$.

Thus, the best Ph.D. student to hire among the four possible candidates is a_1.

C. Comparison With Previous Models

Previous section reveals the advantages of the ELICIT linguistic model: fuzzy linguistic representation in a continuous domain, precision in CW processes and improved interpretability in the results. However, it would be convenient to compare such results with another proposed CW scheme to stand out the features that make this model innovative. To do this, we propose a CW approach introduced in [20]. This approach has been selected because it presents a CW scheme similar to our proposal. The experts can provide their preferences by using CLEs and the CW processes are carried out by transforming the initial linguistic preferences into fuzzy envelopes and finally transformed into 2-tuple linguistic values. The results of applying this approach to the case study are shown in Table V.

Remark 8: Note that to obtain the aggregated results for each alternative the arithmetic mean aggregation operator has been used.

Notice that the approaches provide different rankings of alternatives, whereas ELICIT linguistic model chooses a_1 as the best solution of the problem, the 2-tuple based approach [20] selects a_2. However, the former provides results with a greater amount of information that leads to a greater level of discrimination and, hence, greater accuracy, so it can be guaranteed that the solution provided by the ELICIT linguistic model is more precise and robust. To show the latter, a sensitive analysis will be carried out. In this case, one aspect of sensitive analysis is conducted: the analysis about the criteria weight evolution.

The previous results obtained from both approaches have been computed by considering the same weights for all the criteria (0.333) in which the sum of such weights have to be equal to 1. To carry out the sensitive analysis, such weights are modified. Fig. 16 shows the changes that have to take place in the criteria weights (x-axis) for two alternatives to exchange their positions according to the final ranking provided by the CW approach presented in [20] and ELICIT CW approach. Note that, for the former, the pair of alternatives $a_1 - a_2$ exchange their positions in the final ranking with slight changes (-0.067, 0.04, and -0.142) in the weights of the criteria. Concretely, c_2 is the most critical criterion, since with the slightest variation of its weight (0.04), there is a change in the ranking among the alternatives

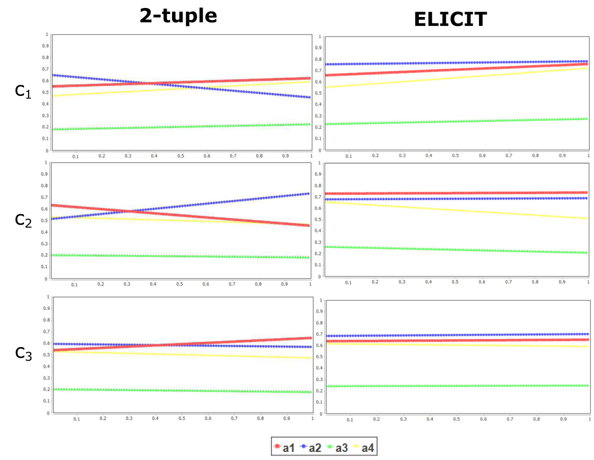

Fig. 16. Sensitive analysis for weights' evolution.

a_1 and a_2, that happens when the weight of c_2 is equal to 0.293 (0.333−0.04), being the weights of the remaining two 0.3535. There are also possible changes in the ranking with the pair of alternatives $a_2 - a_4$ for the criteria c_1 and c_2 but, in this case, the changes have to be more significant (-0.552 and 0.281). For the rest of the cases, the sensitive analysis provides non feasible solutions (lines never intersect in Fig. 16). Therefore, according to the sensitive analysis, the results obtained from the CW approach proposed in [20] are not enough robust since, by applying slight changes in the criteria weights, the final ranking is modified.

On the other hand, Fig. 16 shows that the weights evolution of the criteria does not imply any exchange in any pair of alternatives for the ELICIT CW approach, since for all the cases, sensitive analysis provides non feasible solutions. Therefore, the solution provided by the ELICIT CW approach is more robust than the previous one, since the final ranking remains unchanged for any variation of the criteria weights. Such robustness is determined by the ELICIT expressions, that represent more amount of information and allow to obtain more precise results and with a greater level of discrimination.

To conclude, ELICIT CW approach does not only provides more robust and precise results but, in addition, these are represented by ELICIT expressions, which are not limited by single linguistic terms as in the 2-tuple linguistic values and are closer to the linguistic structures used by human beings.

V. CONCLUSION

In this article, we have presented a new fuzzy linguistic representation model, which represents the linguistic information by means of ELICIT information, an extension of CLEs that takes advantage of the concept of symbolic translation used by the 2-tuple linguistic model. Such expressions are generated by a context-free grammar and composed by the relation of continuous primary terms represented by 2-tuple linguistic values. This novel approach has not lost of information when CW processes are applied since information is managed as a continuous range instead of a discrete one and provides linguistic results represented by ELICIT expressions close to the common language used by human beings. This new representation model has been applied in a DM problem and lately compared with another model to show its validity and advantages.

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To conclude, as future research, we will study the use of the ELICIT linguistic representation in multigranulariy and combination of linguistic-numerical information contexts and the proposal of new aggregation operators for ELICIT information.

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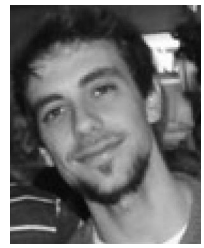

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