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Managing experts behavior in large-scale consensus reaching processes with uninorm aggregation operators

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ABSTRACT

In many real-life large scale group decision making problems, it can be necessary and convenient a consensus reaching process, which is an iterative procedure aimed at seeking a high degree of agreement amongst experts' preferences before making a group decision. Although a wide variety of models and approaches have been proposed and developed to support consensus reaching processes, in large groups there are some important aspects that still require further study, such as the treatment of experts' behaviors that could hamper reaching the wanted agreement. More specifically, it would be necessary an approach to deal with experts properly, based on the overall behavior they present during the discussion process, as well as reinforcing repeated patterns of cooperative (or uncooperative) behavior adopted by experts. This paper presents an expert weighting methodology for consensus reaching processes in large-scale group decision making, that incorporates the use of uninorm aggregation operators. Such operators, which are characterized by their property of full reinforcement, are used in the proposed methodology to allow the experts' weighting based on their overall behavior during the consensus process and the behavior evolution across the time. This proposal is integrated in a consensus model for large-scale group decision making problems under uncertainty, and it is put in practice to show an illustrative example of its effectiveness and improvements with respect to other approaches.

1. Introduction

Decision making is a frequent process in human daily lives, in which there exist several alternatives and the best one/s shall be chosen. Group decision making (GDM) problems, characterized by the participation of multiple individuals or experts in such a process, have been subject of an extensive research in the last decades [1,2].

In the traditional resolution process for GDM problems [3], each expert provides his/her preferences over alternatives and the best alternative or subset of them is selected, disregarding the degree of agreement between experts' preferences. This often leads to the drawback that some experts may not accept the decision made [4], because they might consider that their opinions have not been heard. For this reason, the study of consensus reaching processes (CRPs), in which experts aim at reaching a collective agreement before making a decision [5], has become a prominent research topic in GDM [6–8]. CRPs are iterative discussion processes in which experts *must* accept a priori to collaborate bringing their opinions closer to each other in order to achieve an agreement [9].

Classically, GDM problems taking place in most organizations occur at a strategic level, in which a small number of people are responsible for making the decision. However, the expansion of technological paradigms such as social media and e-marketplaces, is causing that the so-called large-scale GDM problems [10–12], in which a larger number of experts can take part, attain a greater importance. In CRPs carried out in these contexts, it may occur that some experts or coalitions of them, who may have established a collaboration contract [9], try to break it at some stage in such processes. These experts might refuse to cooperate with the rest of the group to reach an agreement [13] and try to strategically bias the solution for the GDM problem [14], hence it is necessary to identify and deal with these non-cooperative experts' behaviors to ensure a normal CRP development.

Some early proposals to deal with strategic manipulation of preferences in classical GDM problems were proposed by Yager [14,15], where experts' preferences may be penalized before applying the alternatives selection process, by analyzing how drastic

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and biased their opinions are. Later on, an approach focused on consensus-based GDM problems was proposed in [13], where a consensus model for dealing with non-cooperative behaviors of experts in CRPs was presented. Such a model defines a methodology based on fuzzy clustering to identify non cooperating experts and subgroups, and applies a weight-based scheme to penalize them, according to the behavior they presented. In this penalizing scheme, importance weights of experts are updated if they show a non-cooperative behavior only, by penalizing their current value. Nevertheless, the values of experts' weights cannot be increased again, even though they change their mind and decide to adopt a more cooperating attitude from a specific discussion round onwards. Moreover, in [13] the weight updating applied on each expert's preferences at a given discussion round is based on his/her behavior at such a round only, not taking into account neither how his/her behavior was previously nor how it has evolved since the beginning of the CRP. Considering for instance a situation in which two experts present a currently cooperative behavior after four rounds of discussion, they should not be assigned the same importance weight if only one of them has kept cooperating since the beginning of the CRP, and the other one has not cooperated until now.

Regarding the previous cases, making use of the available historical information about experts' behavior in CRPs is, as far as we know, a challenge not properly addressed in this research field yet. If tackled properly, this aspect would allow a more accurate and appropriate management of such behaviors. Nevertheless, there exist several proposals for dynamic multi-criteria decision making in recent literature [16,17], in whose framework the set of alternatives varies over time and each alternative is assessed at multiple time instants (similarly to CRPs for GDM problems, in which experts must provide and revise their assessments over alternatives across several discussion rounds). In these frameworks, a global (dynamic) assessment value for each alternative is computed, so that historical information about previous assessments on that alternative is also considered. For instance, in [16] Campanella and Ribeiro proposed the use of associative aggregation operators to compute global assessments in dynamic multi-criteria decision making scenarios. More specifically, they illustrated the usefulness of uninorm aggregation operators [18,19], due to the interesting properties that they present to reinforce both positive and negative assessments on alternatives at successive time instants.

Inspired by the reinforcement-based aggregation techniques mentioned above to integrate historical information in dynamic decision making approaches, in this paper we propose a uninorm-based methodology for managing non-cooperative behaviors based on the overall behavior of each expert over the course of CRPs in large-scale GDM problems. To do this, we present a weighting scheme based on fuzzy set theory and the methodology of computing with words (CW) [20], that incorporates the use of uninorm aggregation operators, aiming at three goals:

(i) Assigning importance weights to experts based not only on their current behavior, but also on their patterns of behavior presented at previous consensus rounds.
(ii) Such weights are computed based on a linguistic modeling to represent the uncertainty related to the experts' behavior.
(iii) Exploiting the full reinforcement property of uninorm operators to reinforce repeated patterns of cooperative (or non-cooperative) behaviors by an expert at successive rounds.

A consensus model for large-scale GDM under uncertainty that incorporates the proposed weighting scheme is also introduced. Finally, an illustrative example is presented to show the properties of the weighting scheme in practice, as well as its advantages with respect to other consensus approaches.

This paper is set out as follows: in Section 2, some basic concepts about CRPs in GDM, uninorm aggregation operators and CW methodology for reasoning processes are reviewed. Straightaway, Section 3 presents in further detail the uninorm-based weighting scheme for experts in CRPs. A consensus model for managing experts' behaviors that integrates the proposed scheme is then presented in Section 4. Section 5 presents the illustrative example conducted, and some concluding remarks are finally drawn in Section 6.

2. Preliminaries

This section firstly revises some basic concepts about GDM problems and CRPs, followed by an overview of uninorm aggregation operators and the methodology of CW for reasoning processes, both of which are taken into account in the proposal presented in this paper.

2.1. Consensus reaching processes in GDM

GDM entails the participation of multiple experts who must make a collective decision to find a common solution to a problem. Decision processes in which several experts with different knowledge and experience take part, may usually lead to better decisions than those made by just one expert [2].

A GDM problem is formally characterized by the following elements [1]:

- The existence of a common problem to be solved.
- A set $X=\{x_1, \ldots, x_n\}$ $(n \geq 2)$, of *alternatives* or possible solutions to the problem.
- A set $E=\{e_1, \ldots, e_m\}$ $(m \geq 2)$, of individuals or *experts*, who express their opinions or preferences over alternatives X. As previously indicated, this paper focuses on large-scale GDM problems, such that $m \gg 2$.

In order to express their opinions over alternatives, each expert utilizes a preference structure. Fuzzy preference relations are one of the most widely utilized preference structures in many GDM approaches found in [21]. A fuzzy preference relation P_i associated to expert e_i, can be represented for X finite as a $n \times n$ matrix, as follows:

$$P_i = \begin{pmatrix} - & \ldots & p_i^{1n} \\ \vdots & \ddots & \vdots \\ p_i^{n1} & \ldots & - \end{pmatrix}$$

being each numerical *assessment* $p_i^{lk} = \mu_{P_i}(x_l, x_k) \in [0, 1]$, the degree of preference of the alternative x_l over x_k, $l, k \in \{1, \ldots, n\}$, $l \neq k$, according to e_i, such that:

- $p_i^{lk} > 0.5$ indicates e_i's preference of x_l over x_k.
- $p_i^{lk} < 0.5$ indicates e_i's preference of x_k over x_l.
- $p_i^{lk} = 0.5$ indicates e_i's indifference between x_l and x_k.

Fuzzy preference relations can accomplish diverse properties [22–24]. In [25,26], some of these properties have been studied and considered in a consensus model for GDM with fuzzy preference relations. Accordingly, in order to provide or facilitate the construction of consistent preference relations, in this work we assume the reciprocity property in fuzzy preference relations, i.e. assessments accomplish that if $p_i^{lk} = x$, $x \in [0, 1]$, $l \neq k$, then $p_i^{kl} = 1 - x$.

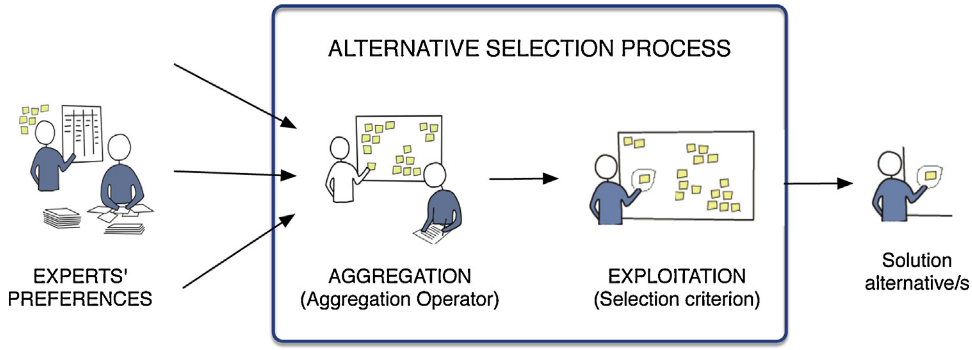

Fig. 1. Classical resolution process for GDM problems.

The solution for a GDM problem can be obtained by applying either a *direct approach* or an *indirect approach* [3]. In the former, a solution is directly obtained from the individual preferences of each expert, without constructing a social opinion first [27], whereas in the latter, a social opinion or *collective preference* is firstly computed from individual opinions and then used to obtain the solution for the problem. Regardless of the approach considered, the traditional selection process for reaching a solution to GDM problems is made up by two phases [28] (see Fig. 1): (i) an *aggregation phase*, in which preferences of experts are combined by using an aggregation operator, and (ii) an *exploitation phase*, where a selection criterion is applied to obtain an alternative or subset of them as the solution for the problem.

When a GDM problem is solved by applying the alternatives selection process only, it may occur that some experts feel that their opinions have not been taken into consideration to find the solution, therefore they would not accept it. Since a sufficient level of collective agreement is crucial in many real-life situations, it becomes necessary to apply a CRP, introducing an additional phase in the resolution process for GDM problems. CRPs aim at obtaining a high level of group agreement before making a decision [4,6].

The term *consensus* can be defined as the agreement produced by mutual consent between all members in a group or between several groups [5]. The process to reach consensus is a dynamic and iterative process, consisting of several rounds of discussion, and frequently coordinated by a human figure: the moderator. The moderator is a key figure in CRPs, being in charge of supervising and guiding experts across the discussion process [5]. Reaching consensus implies that experts *must* modify their initial opinions throughout the CRP, bringing them closer to the rest of the group's opinions. In this sense, they must comply with a collaboration contract [9], according to which they accept to collaborate with each other to search for a common agreed solution.

Fig. 2 shows a general CRP scheme followed in a large number of GDM approaches [6]. In the following, its main phases are briefly described:

1 *Gathering preferences*: Each expert e_i provides his/her preferences over alternatives to the moderator, e.g. by means of a fuzzy preference relation.
2 *Determine degree of consensus*: The current degree of consensus in the group, *cr*, is computed. Such a consensus degree is usually measured as a value in the [0,1] interval (where a value of 1 indicates full or unanimous agreement between all experts on all the alternatives). To do so, different consensus measures can be utilized, based on the use of similarity/distance metrics to calculate

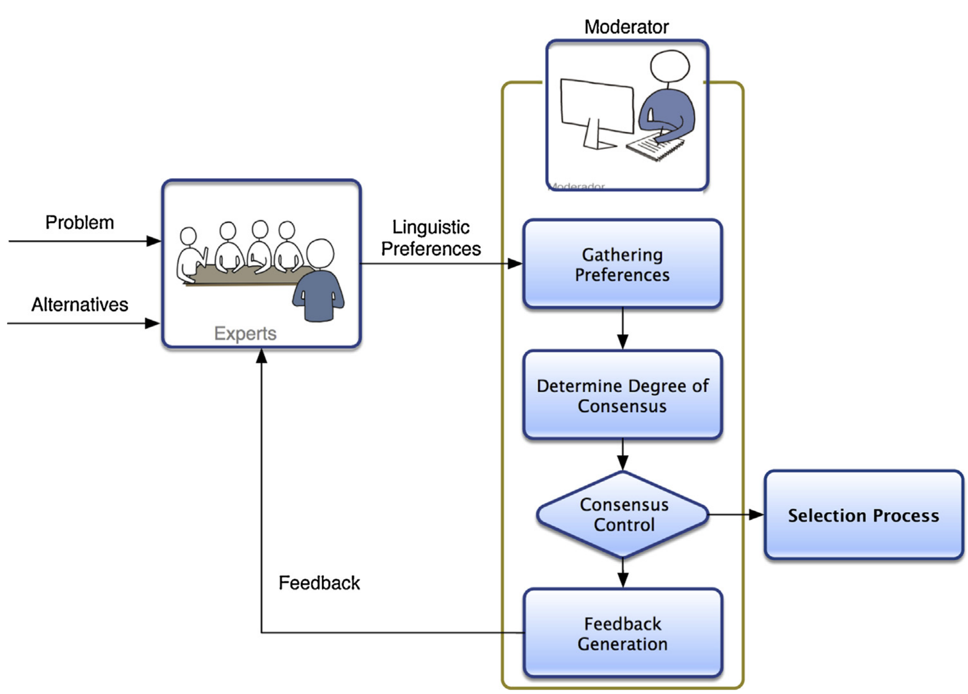

Fig. 2. General scheme for CRPs.

degrees of similarity between preferences of experts, and aggregation operators that obtain the degree of consensus in the group from such similarity values [6].

3 *Consensus control*: In this phase, the consensus degree *cr* previously obtained, is compared with a consensus threshold $\mu \in]0, 1]$ fixed a priori, which indicates the minimum level of agreement required. If $cr \geq \mu$ then consensus has been achieved and the group proceeds to the selection process; otherwise, it is necessary another discussion round. In order to limit the number of discussion rounds allowed, a parameter $Maxround \in \mathbb{N}$ can be introduced. If the number of consensus rounds applied exceeds *Maxround*, a different GDM strategy might be adopted. Some examples of such strategies have been provided in the literature [5], such as: (i) delegating the decision to a subgroup, (ii) conducting a community building session, (iii) applying a simple majority vote, or (iv) excluding experts who do not contribute to achieve consensus.

4 *Feedback generation*: The moderator computes a collective preference, P_c, by aggregating the individual preferences of all experts. Based on P_c, the moderator identifies the farthest experts' assessments from consensus, and advises them to modify such assessments with the aim of increasing the consensus degree in the following round. Each expert is responsible for modifying his/her own assessments (and, consequently, committing with the collaboration contract established), by increasing/decreasing assessment values and bringing them closer to P_c. Each piece of advice generated consists in a triplet $(e_i, (x_l, x_k), Direction)$ which indicates that the expert e_i *must* modify his/her assessment p_i^{lk} in the direction given by $Direction \in \{increase, decrease\}$.

2.2. Uninorm aggregation operators

Uninorm aggregation operators were introduced by Yager and Rybalov [18], and they provide an unification of *t*-norm and *t*-conorm operators. For this reason, before reviewing uninorm operators the formal definitions of *t*-norm and *t*-conorm are revised.

Definition 1. [18]. A triangular norm or *t*-norm *T* is a mapping,

$$T : [0, 1] \times [0, 1] \rightarrow [0, 1]$$

having the following properties for all $a, b, c, d \in [0, 1]$:

i) Commutativity: $T(a, b) = T(b, a)$.
ii) Monotonicity: $T(a, b) \geq T(c, d)$ if $a \geq c$ and $b \geq d$.
iii) Associativity: $T(a, T(b, c)) = T(T(a, b), c)$.
iv) Neutral element: $T(a, 1) = a$.

T-norms are conjunctive aggregation operators, therefore they exhibit the following property,

$$T(a_1, \ldots, a_n) \leq \min_i [a_i]$$

From this property, we can see that the aggregated value is never greater than the lowest a_i. Moreover, if all a_i's values are low, then such values shall reinforce each other so that the resulting aggregated value is even lower. This property is known as *downward reinforcement* [29].

Some well-known examples of *t*-norms are:

- Minimum: $T_{\min}(a, b) = \min(a, b)$.
- Product: $T_{prod}(a, b) = ab$.
- Łukasiewicz *t*-norm: $T_{Luk}(a, b) = \max\{0, a + b - 1\}$.

Definition 2. [18]. A triangular conorm or *t*-conorm *S* is a mapping,

$$S : [0, 1] \times [0, 1] \rightarrow [0, 1]$$

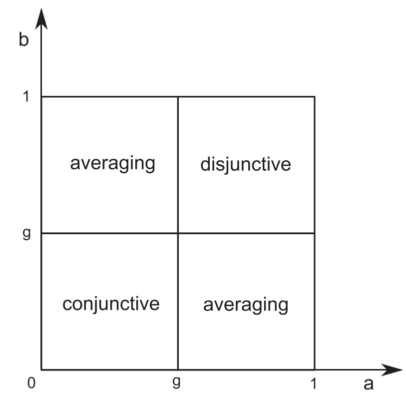

Fig. 3. Behavior of uninorms for different input values. Adapted from [16].

having the following properties for all $a, b, c, d \in [0, 1]$:

i) Commutativity: $S(a, b) = S(b, a)$.
ii) Monotonicity: $S(a, b) \geq S(c, d)$ if $a \geq c$ and $b \geq d$.
iii) Associativity: $S(a, S(b, c)) = S(S(a, b), c)$.
iv) Neutral element: $S(a, 0) = a$.

T-conorms are disjunctive aggregation operators, therefore they exhibit the following property,

$$S(a_1, \ldots, a_n) \geq \max_i [a_i]$$

From this property, we can see that the aggregated value is always at least as high as the largest a_i. Additionally, if all a_i's values are high, then such values shall reinforce each other, thus leading to an even higher aggregated value. This property is known as *upward reinforcement* [29].

Some well-known examples of *t*-conorms are:

- Maximum: $S_{\max}(a, b) = \max(a, b)$.
- Probabilistic sum: $S_{prob}(a, b) = a + b - ab$.
- Łukasiewicz *t*-conorm: $S_{Luk}(a, b) = \min\{a + b, 1\}$.

Uninorm operators were proposed by Yager and Rybalov to provide a generalization of the *t*-norm and the *t*-conorm, such that the neutral element can lie anywhere in the unit interval, and whose behavior varies depending on the values to aggregate being higher or lower than such a neutral element [18,19]. They are defined as follows:

Definition 3. [18] A uninorm is a mapping,

$$U : [0, 1] \times [0, 1] \rightarrow [0, 1]$$

having the following properties for all $a, b, c, d \in [0, 1]$:

i) Commutativity: $U(a, b) = U(b, a)$.
ii) Monotonicity: $U(a, b) \geq U(c, d)$ if $a \geq c$ and $b \geq d$.
iii) Associativity: $U(a, U(b, c)) = U(U(a, b), c)$.
iv) Neutral element: $\exists g \in [0, 1] : U(a,g)=a$.

Unlike *t*-norms and *t*-conorms, in which the neutral elements are 1 and 0 respectively, uninorms can take any value in the unit interval as the neutral element. Uninorms may have conjunctive, disjunctive or averaging behavior, depending on input values *a*, *b* being greater or lower than *g*, as illustrated in Fig. 3 [16].

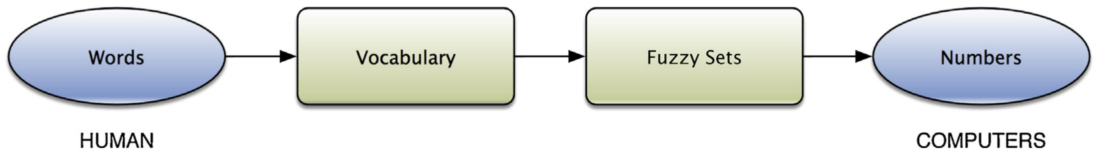

Fig. 4. Paradigm of man-machine understanding.

Taken from [34].

Two general families of uninorm operators with neutral element g were introduced by Fodor et al. [30]:

$$U(a,b)=\begin{cases}(a)\quad gT_U\left(\frac{x}{g},\frac{y}{g}\right) & \text{if } 0\le a,b\le g,\\ (b)\quad g+(1-g)S_U\left(\frac{x-g}{1-g},\frac{y-g}{1-g}\right) & \text{if } g\le a,b\le 1,\\ (c1)\quad \max(a,b) & \text{if } \min(a,b)\le g\le \max(a,b),\\ (c2)\quad \min(a,b) & \text{if } \min(a,b)\le g\le \max(a,b).\end{cases} \quad (1)$$

with T_U and S_U being any t-norm and any t-conorm operator, respectively. The difference between both families of uninorm operators are in the use of either item (c1), which defines the so-called $\mathcal{U}_{\max}$ family of uninorm operators, or item (c2), which defines the $\mathcal{U}_{\min}$ family of uninorm operators [31].

Some later works generalized the case in which $\min(a, b)\le g\le \max(a, b)$, by considering the use of any averaging aggregation operator M_U (see Fig. 3), such that $\min(a, b)\le M_U(a, b)\le \max(a, b)$ [16]. For instance, in [32] Ribeiro et al. is proposed an adaptation of uninorm operators that applies the OWA (*ordered weighted averaging*) operator [33] in such a case.

For any $g\in]0, 1[$, we can see that uninorms consider an upward reinforcement when aggregating high input values (above g), and a downward reinforcement when aggregating low input values (below g), hence they can be used as *full reinforcement* operators, as it will be shown in this paper to reinforce cooperative or non cooperative behaviors of experts over the course of a CRP.

Some examples of uninorms which shall be utilized in our proposal, are shown below:

Example 1. Consider the product t-norm and probabilistic sum t-conorm reviewed above. Then, the following uninorm is defined based on the general families introduced by Fodor et al. (Eq.(1))[30]:

$$U(a,b)=\begin{cases}\dfrac{ab}{g} & \text{if } 0\le a,b\le g,\\ \dfrac{a+b-ab-g}{1-g} & \text{if } g\le a,b\le 1,\\ M_U(a,b) & \text{if } \min(a,b)\le g\le \max(a,b).\end{cases} \quad (2)$$

with $M_U(a, b)$ being an averaging operator.

Example 2. The cross-ratio uninorm [22] is an example of continuous uninorm in $[0, 1]^2\setminus\{(0, 1), (1, 0)\}$, with neutral element $g=0.5$:

$$U(a,b)=\begin{cases}0 & \text{if } (a,b)\in\{(0,1),(1,0)\},\\ \dfrac{ab}{ab+(1-a)(1-b)} & \text{otherwise.}\end{cases} \quad (3)$$

2.3. Computing with words methodology for reasoning processes

In many real situations, human beings utilize natural language consisting of words and expressions, to communicate, reason and understand their environment. Computers, on the other hand, use much more formal symbols [34]. The paradigm of *Computing with Words* (CW) was proposed by Zadeh [20] to establish a comprehensive link of communication between human beings and computer systems, and to increase the use of natural language in communication, reasoning and decision making processes carried out by such systems. This methodology facilitates human–computer cooperation to a high degree, since it provides a framework in which concepts are modeled in an amenable way to both sides.

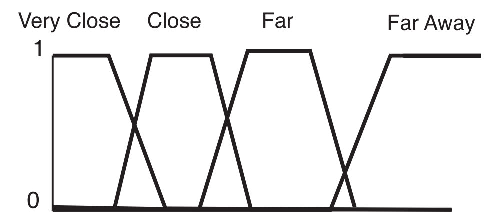

Fig. 5. Different linguistic terms for the attribute *distance*.

The methodology of CW is based on fuzzy sets theory [35], so that concepts belonging to a vocabulary can be modeled by means of fuzzy sets, thus being easily understood by human beings and computers (see Fig. 4). Some key elements in CW are the concepts of linguistic variable and linguistic term, formulated by Zadeh.

Definition 4. [36–38] A linguistic variable is characterized by a 5-tuple $(H, T(H), U, G, M)$, where H is the name of the variable; $T(H)$ symbolizes the set of linguistic terms or linguistic values of H, with each value being a fuzzy variable generically denoted as X that ranges over a universe of discourse U; G is a syntactic rule (normally given by a grammar) to generate the names of linguistic terms in H; and M is a semantic rule to associate each element in H with its meaning, $M(X)$, given by a fuzzy set in U.

Based on Zadeh's definition of linguistic variable, we can see that a linguistic term is a word or phrase, utilized to express the value of the variable. Aided by linguistic terms, human beings can better understand and reason about the different features of their environment. For example, considering the linguistic variable *distance*, some possible linguistic terms to express the value of such an attribute could be: "*very close*","*close*","*far*" and "*far away*".

Given the inherent vagueness and imprecision that the values of linguistic terms present, fuzzy sets [35] constitute a useful tool to formalize the concepts associated to them, as shown in Fig. 5, where the semantics of linguistic terms belonging to the variable *distance* are represented as fuzzy sets with trapezoidal membership functions [39]. Thus, by using fuzzy set theory computers are capable of understanding and carrying out computational and reasoning processes over such concepts. Let $\tau\in T(H)$ be a linguistic term (e.g. "close") belonging to a vocabulary associated to a linguistic variable H (e.g. *distance*). We can then express τ as a fuzzy subset in the domain $Y\in U$ of H. Given a value $y\in Y$, its membership degree to τ, $\mu_\tau(y)\in[0, 1]$ indicates the compatibility degree of the value y with the linguistic term τ. These ideas will be used in our proposal to determine how compatible an expert's behavior is with the concept of cooperativeness, which is given by a linguistic term.

3. Uninorm-based management of experts' behavior in CRPs

This section presents a novel expert weighting methodology to deal with non-cooperative behaviors of experts in CRPs carried out in large-scale GDM problems, aimed at overcoming the drawbacks

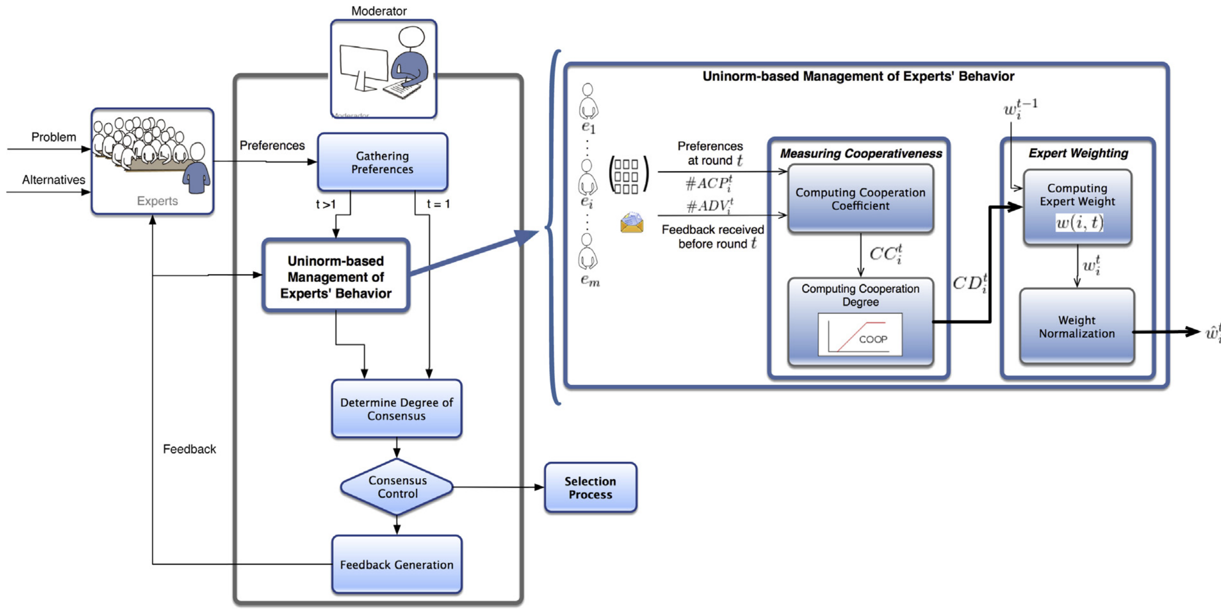

Fig. 6. Scheme of the uninorm-based method to weight experts based on their behavior in CRPs.

such behaviors might cause in these processes. Unlike classical strategic GDM problems with a few number of decision makers (less than ten generally), large-scale GDM problems with a larger number of them (from dozens to several thousands) present more difficulties to detect and deal with experts who try to manipulate the CRP, because such manipulations can be hidden in an easier way. The methodology consists of two main phases, depicted in Fig. 6:

- *Measuring cooperativeness*: A measure to evaluate the degree of cooperation of each expert at current CRP round, based on CW and uninorm operators, is defined (see Section 3.1).
- *Experts weighting*: Experts' importance weights are computed based on their behavior at the current and previous CRP rounds, i.e., taking into account their overall behavior since the beginning of the CRP and how such a behavior evolved at each round (see Section 3.2).

The management of non-cooperative behaviors of experts is applied from the second consensus round onwards, since its computations require information about the feedback received by experts at the end of the previous round.

Eventually, in Section 4 this methodology will be integrated in a consensus model as it is shown in Fig. 6

3.1. Measuring cooperativeness

In this phase, fuzzy set theory and the methodology of CW are utilized to measure the degree of cooperation that each expert presents at a given consensus round. To do this, the two following steps are conducted: (i) compute a cooperation coefficient for each expert, and (ii) compute a cooperation degree based on each cooperation coefficient value.

3.1.1. Computing cooperation coefficient

The objective in this step is to evaluate the behavior adopted by each expert, $e_i \in E$, in the current consensus round, $t \in \mathbb{N}$, by defining a coefficient that indicates how cooperative his/her behavior is. Thus, we introduce the so-called *cooperation coefficient*, which evaluates an expert's behavior based on the amount of feedback received and the amount of assessments he/she modified according to such feedback. This coefficient is defined as follows:

Definition 5. Let $\#ADV_i^t$ be the total number of preference degrees or assessments p_i^{lk} that e_i has been advised to modify and $\#ACP_i^t$ be the amount of assessments that he/she accepted to modify according to the feedback received at round t. The cooperation coefficient of e_i at round t, $CC_i^t \in [0, 1]$, is defined as:

$$CC_i^t = \begin{cases} 1 & \text{if } \#ADV_i^t = 0, \\ \eta \dfrac{\#ACP_i^t}{\#ADV_i^t} + (1-\eta)\left(1 - \dfrac{\#ADV_i^t - \#ACP_i^t}{n(n-1)}\right) & \text{otherwise.} \end{cases} \quad (4)$$

Remark 1. $n(n-1)$ is the total number of assessments in P_i. The higher the value of CC_i^t, the more cooperative e_i's behavior is at round t. Notice that the lower $\#ACP_i^t$, the more penalizing is applied on CC_i^t. On the other hand, for values of $\#ADV_i^t$ closer to zero, the resulting CC_i^t will be less penalized, even if $\#ACP_i^t$ is low, since a low $\#ADV_i^t$ means that most of e_i's assessments are close to consensus. Parameter $\eta \in [0, 1]$ is utilized to control the penalizing on CC_i^t attending to different criteria, e.g. the amount of advices accepted by the experts and the total number of advices received. If e_i does not receive any advice at a given round, then $CC_i^t = 1$ since in such a case all of his/her assessments values are close to consensus.

As will be later shown in the consensus model presented in Section 4, a parameter $\varepsilon \geq 0$ called acceptability threshold is utilized to identify which assessments should be modified by an expert. This parameter is also used here to decide whether the degree of change applied by an expert on an identified assessment, is enough or not to consider that the advice has been accepted by him/her. Let p_i^{lkt} and $p_i^{lk(t+1)}$ denote e_i's assessment on (x_l, x_k), before and after revising the changes suggested in the advice generation phase at round t, respectively. On the other hand, let ADV_i^t be the set of assessments that e_i has been advised to modify in such a round, so that $|ADV_i^t| = \#ADV_i^t$. Based on ε, the procedure illustrated in Algorithm 1 is applied for each expert, to determine the number of advices accepted by him/her, $\#ACP_i^t$. Notice here that an advice is considered as accepted if the degree of change applied by the expert exceeds ε.

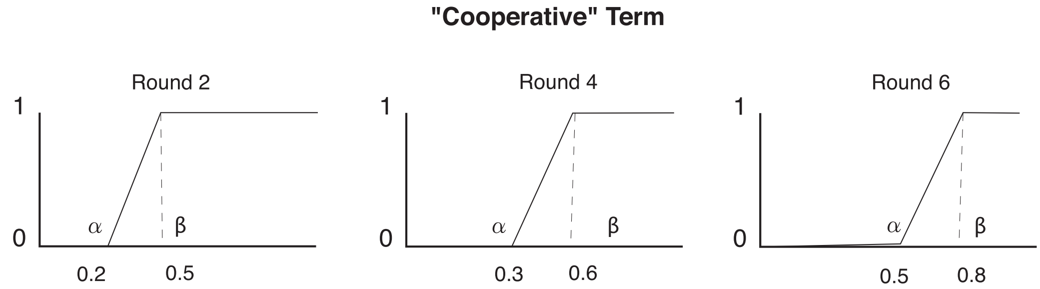

Fig. 7. Evolution of the fuzzy membership function associated to the linguistic term "cooperative" across the CRP.

Algorithm 1. Procedure to compute the number of changes accepted by an expert $e_i \in E$

1. Assign $\#ACP_i^t \leftarrow 0$.
2. **for** each e_i's assessment $p_i^{lkt} \in ADV_i^t$ **do**
3. **if** (e_i modified p_i^{lkt} in the direction advised) *AND* ($|p_i^{lkt} - p_i^{lk(t+1)}| > \varepsilon$) **then**
4. $\#ACP_i^t \leftarrow \#ACP_i^t + 1$.
5. **end if**
6. **end for**

The following example illustrates the computation of the cooperation coefficient.

Example 3. Consider a CRP round t in a GDM problem with four alternatives ($n = 4$). Two experts e_1 and e_2 received feedback to modify one and ten out of their assessments respectively, i.e. $\#ADV_1^t = 1$ and $\#ADV_2^t = 10$. If e_1 refuses to modify the only assessment she was advised to modify, i.e. $\#ACP_1^t = 0$, and e_2 modifies three out of his assessments bringing them closer to consensus, i.e. $\#ACP_2^t = 3$, then for $\eta = 0.5$ we have:

$$CC_1^t = 0.5\left(\frac{0}{1}\right) + 0.5\left(1 - \frac{1-0}{12}\right) = 0.458$$

$$CC_2^t = 0.5\left(\frac{3}{10}\right) + 0.5\left(1 - \frac{10-3}{12}\right) = 0.358$$

As we can see above, the cooperation coefficient regards the fact that, even though e_1 did not modify none of her assessments, she received feedback for one assessment only, which means that most of her preferences were already close to consensus. Therefore, her cooperation coefficient value is slightly higher than that of e_2, who received feedback on almost all of his assessments and did not apply most of them. Keep in mind that the higher η, the more penalized is e_2.

3.1.2. Computing cooperation degree

Based on the cooperation coefficient CC_i^t previously computed, in this phase the degree of cooperation at the current consensus round is determined for each expert. Such a degree of cooperation aims at reflecting the extent to which the value of CC_i^t satisfies the notion of cooperativeness established in the specific GDM problem. To do this, we consider that the concept of cooperativeness can be easily modeled linguistically, by defining it as a linguistic variable. Thus, we propose applying reasoning processes based on the CW methodology (see Section 2.3) to measure the cooperation degrees. The concept of cooperativeness is modeled as follows:

Definition 6. Let "cooperative" be a linguistic term, whose semantics are given by a fuzzy set *COOP* in [0,1], with the following non-decreasing membership function:

$$\mu_{COOP}(y) = \begin{cases} 0 & \text{if } y < \alpha, \\ \dfrac{y-\alpha}{\beta-\alpha} & \text{if } \alpha \le y < \beta, \\ 1 & \text{if } y \ge \beta. \end{cases} \quad (5)$$

with α, β, $y \in [0, 1]$, $\alpha < \beta$. The cooperation degree of e_i at round t, denoted by, CD_i^t corresponds to the membership degree of CC_i^t to the fuzzy subset *COOP*, i.e. $CD_i^t = \mu_{COOP}(CC_i^t) \in [0, 1]$.

The fuzzy membership function establishes how restrictive the notion of being cooperative is: the larger α and β, the more restrictive such a notion is, so that only the highest values of CC_i^t are assigned a maximum cooperation degree. In many real-life problems, this notion of cooperativeness may vary across the time, e.g. in a CRP an expert who only cooperates partially should be more penalized if such a process is at an advanced stage, after several discussion rounds; i.e., a given value of CC_i^t may imply distinct degrees of cooperation in two different GDM problems, or even in different phases of a CRP. In order to reflect this, we propose the flexible use of different membership functions for the semantics of the term "cooperative" at each consensus round, by increasing the value of α and β gradually as the CRP goes on, thus reducing the support [35] of the fuzzy subset *COOP* progressively. This changeable meaning of cooperativeness justifies the importance of conducting this step in our proposal, instead of computing experts' weights from CC_i^t directly.

The following example illustrates how to reflect an increasingly restrictive notion of cooperativeness in a CRP.

Example 4. Consider a fuzzy set *COOP*, whose membership function parameters have the initial values $\alpha = 0.2$ and $\beta = 0.5$ at the beginning of the CRP. After the fourth consensus round, $t \ge 4$, the value of both parameters will be increased by 0.1 per round, until α reaches a value of 0.9 and β reaches a value of 1. Thus, the approach is more restrictive with the behavior of experts as the CRP progresses, as shown in Fig. 7.

3.2. Expert weighting

Once the cooperation degrees of experts have been obtained, in this phase we firstly apply uninorm aggregation operators (Section 3.2.1) to assign each expert an importance weight that reflects both his/her current behavior and the behavior previously adopted since the beginning of the CRP. A normalization process is then applied on weights (Section 3.2.2) to allow recovery of importance in experts' preferences along the CRP.

Table 1
Cooperation degrees of two experts at each round.

t	1	2	3	4	5	6
CD_1^t	–	0.6	0.9	0.4	0.7	0.7
CD_2^t	–	0.2	0.1	0.8	0.9	0.75

Table 2
Weights assigned to experts at each round.

t	1	2	3	4	5	6
w_1^t	0.5	0.6	0.92	0.66	0.79	0.87
w_2^t	0.5	0.2	0.04	0.42	0.66	0.83

3.2.1. Computing experts' weights

Uninorm aggregation operators are utilized to compute importance weights of experts due to their full reinforcement property, which allows to reflect that: (i) if both the current and previous expert behaviors are highly cooperative, then his/her importance weight should be reinforced upwards, and (ii) if both the current and previous expert behaviors are highly non-cooperative, then his/her importance weight should be reinforced downwards. The following function is defined to compute importance weights:

Definition 7. The function $w(i, t)$ returns the importance weight of expert e_i at round t, denoted by $w_i^t \in [0, 1]$ and computed as follows:

$$w_i^t = w(i,t) = \begin{cases} g & \text{if } t = 1, \\ U(CD_i^t, w_i^{t-1}) & \text{if } t > 1. \end{cases} \tag{6}$$

being U an uninorm operator and $g \in]0, 1[$ its neutral element, according to which an input value above g is viewed as a good behavior in the aggregation, and vice versa.

Given any $t > 1$ and $i \in \{1, \ldots, m\}$, the function $w(i, t)$ shows the full reinforcement property, as a direct consequence of U being a uninorm operator.

In Eq. (6), at the beginning of the CRP ($t = 1$), all experts are assigned the same weight, $w_i^1 = g, \forall i$, since there is no available information about their behavior yet. On the other hand, when $t = 2$, experts have already revised their assessments for the first time, based on the feedback received, and $w_i^2 = U(CD_i^2, g) = CD_i^2$. Finally, when $t > 2$, both the current and previous behaviors of each expert are taken into account when computing his/her weight. Due to the full reinforcement property of uninorms, $w(i, t)$ increases or decreases when either a good or a bad behavior is present at successive consensus rounds, respectively. The following example illustrates this property.

Example 5. Consider two experts, $e_1, e_2 \in E$, with the cooperation degrees shown in Table 1, along a CRP consisting of six rounds, and the following operator $U(a, b)$ based on Fodor's general families of uninorms, with $g = 0.5$ [30]:

$$U(a,b) = \begin{cases} 2ab & \text{if } 0 \le a, b \le 0.5, \\ 2(a+b-ab-0.5) & \text{if } 0.5 \le a, b \le 1, \\ \dfrac{a+b}{2} & \text{if } \min(a,b) < 0.5 < \max(a,b). \end{cases} \tag{7}$$

Table 2 summarizes the importance weights assigned to e_1 and e_2 at each round computed by applying Eq. (6), the values obtained for e_1 are further detailed below:

- $w_1^1 = w(1.1) = g = 0.5$.
- $w_1^2 = w(1, 2) = U(CD_1^2, w_1^1) = U(0.6, 0.5) = 0.6$.
- $w_1^3 = w(1, 3) = U(0.9, 0.6) = 0.92$.
- $w_1^4 = U(0.4, 0.92) = 0.66$.
- $w_1^5 = U(0.7, 0.66) = 0.79$.
- $w_1^6 = U(0.7, 0.79) = 0.87$.

Notice that an upward reinforcement is applied on e_1's weight at $t = 3$, $t = 5$ and $t = 6$, since her current cooperation degree and her previous behavior (given by w_1^{t-1}) are high. Fig. 8 represents and compares such weights with cooperation degrees, CD_i^t, whose values reflect experts' behavior at the current CRP round only, thus disregarding experts' behavior in previous rounds. Both upward and downward reinforcement can be observed at different rounds for e_2's weight, whose behavior varies across the CRP.

The previous example shows that the use of uninorm operators allows to consider information about experts' behavior at previous discussion rounds when computing their importance weight, w_i^t, at a given round t. Thus, Fig. 8(a) shows that when an expert has a good behavior trend, her weight keeps being reinforced upwards, even though her behavior at a particular round is slightly worse than the behavior adopted in the previous round. On the other hand, if an expert has bad behavior trend, his weight suffers a downward reinforcement, but if his behavior improves significantly in the following rounds, he might finally be reinforced upwards (see Fig. 8(b)).

3.2.2. Weight normalization

In order to all experts can recover importance in their opinions along the CRP, in this phase a normalization is applied to weights w_i^t, as follows:

$$\hat{w}_i^t = \frac{w_i^t}{\sum_{i=1}^m w_i^t} \tag{8}$$

with $\hat{w}_i^t \in [0, 1]$ and $\sum_i \hat{w}_i^t = 1$. Once the weights have been normalized, they are taken into account in the current round of discussion to compute the collective preference P_c by aggregating experts' preferences, as well as in the computation of consensus degrees, as described in the consensus model presented below.

4. Consensus model

Keeping in mind our focus on large-scale GDM problems due to its difficulty to manage the strategic manipulations that might carry out the decision makers involved. In this section, we present a consensus model for large-scale GDM under uncertainty, aimed at the management of non-cooperative behaviors in CRPs based on uninorm operators. Therefore, in order to facilitate the treatment of such behaviors, the proposed consensus model incorporates the uninorm-based weighting scheme introduced in Section 3, as previously depicted in Fig. 6.

Remark 2. The consensus model described in this section focuses on the use of fuzzy preference relations by all experts. Nevertheless, the weight-based methodology presented in this paper to deal with experts' behaviors can be utilized in a variety of consensus models with feedback mechanism [6], regardless of the preference structures and information domains considered [40,41].

The weighting scheme computes the weights assigned to experts' preferences following the phases of the consensus model (see Fig. 9):

- The computation of the collective preference by aggregating the individual preferences of experts.
- The computation of consensus degrees by aggregating similarity degrees between all different pairs of experts in the group.

Further detail on the use of weights is given below, in the description of the consensus model, whose phases are based on

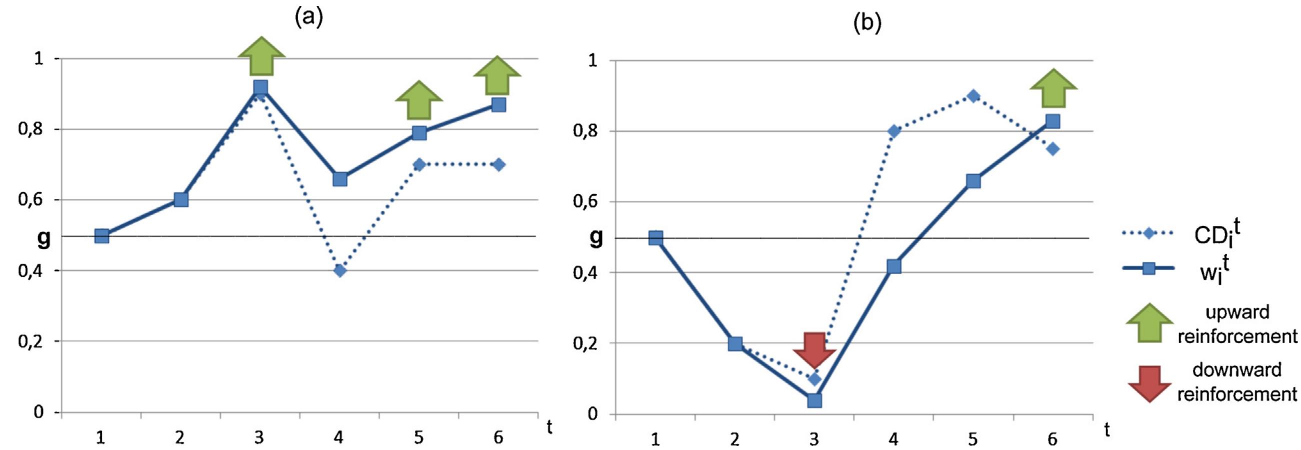

Fig. 8. Cooperation degrees and weights assigned to (a) e_1 and (b) e_2.

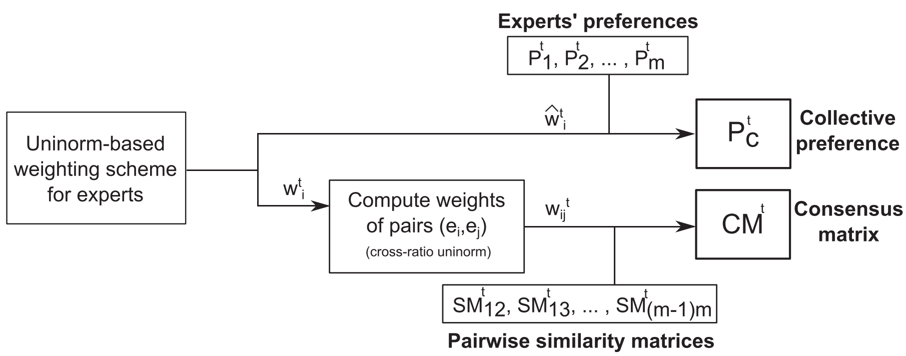

Fig. 9. Use of experts' weights in the consensus model.

the scheme previously shown in Fig. 6. These phases are applied sequentially, once at each CRP round t, until consensus is achieved.

(1) *Gathering preferences*: Each expert provides his/her preferences over alternatives in X, by means of a preference structure (e.g. a reciprocal fuzzy preference relation, $P_i^t = (p_i^{lkt})_{n\times n}$, such that if $p_i^{lkt} = x \in [0, 1]$, then $p_i^{klt} = 1 - x$). Notice that preferences are hereinafter denoted by P_i^t, to indicate the round t in which they are utilized.

(2) *Computing consensus degree*: The level of agreement in the group is computed, by means of the following steps:

(a) For each pair of experts e_i, $e_j (i<j)$, compute a similarity matrix $SM_{ij}^t = (sm_{ij}^{lkt})_{n\times n}$,

$$SM_{ij}^t = \begin{pmatrix} - & \dots & sm_{ij}^{1nt} \\ \vdots & \ddots & \vdots \\ sm_{ij}^{n1t} & \dots & - \end{pmatrix}$$

with $sm_{ij}^{lkt} \in [0, 1]$ being the degree of similarity between e_i and e_j's assessments on the pair of alternatives (x_l, x_k) at round t, computed as follows: [42]:

$$sm_{ij}^{lkt} = 1 - \left| p_i^{lkt} - p_j^{lkt} \right| \tag{9}$$

(b) Compute a consensus matrix $CM^t = (cm^{lkt})_{n\times n}$ by using a weighted averaging aggregation operator. Each element $cm^{lkt} \in [0, 1]$, $l \neq k$, is computed as follows:

$$cm^{lkt} = \frac{\sum_{i=1}^{m-1} \sum_{j=i+1}^{m} w_{ij}^t sm_{ij}^{lkt}}{\sum_{i=1}^{m-1} \sum_{j=i+1}^{m} w_{ij}^t} \tag{10}$$

Table 3
Computation of importance weights for pairs of experts.

w_{ij}^t	$w_2^t = 0.75$	$w_3^t = 0.5$	$w_4^t = 0.25$	$w_5^t = 0.6$
$w_1^t = 0.2$	0.43	0.2	0.08	0.27
$w_2^t = 0.75$	–	0.75	0.5	0.82
$w_3^t = 0.5$	–	–	0.25	0.6
$w_4^t = 0.25$	–	–	–	0.33

$w_{ij}^t \in [0, 1]$ represents the importance weight associated to a pair of experts (e_i, e_j), and it is obtained from individual weights w_i^t and w_j^t as $w_{ij}^t = U_r(w_i^t, w_j^t)$, being U_r the cross ratio uninorm operator shown in Eq. (3).

Remark 3. The aim of applying weights in Eq. (10) is to assign more importance to similarity values associated to pairs of experts whose degree of cooperation is higher. Thus, we aim at preventing a possible lack of convergence towards agreement due to non cooperating experts who present a strong disagreement with each other.

Example 6. Let $E = \{e_1, e_2, e_3, e_4, e_5\}$ be five experts with the following importance weights at round t: $w_1^t = 0.2$, $w_2^t = 0.75$, $w_3^t = 0.5$, $w_4^t = 0.25$ and $w_5^t = 0.6$. By applying the cross-ratio operator, the weights w_{ij}^t assigned to all different pairs of experts in E are shown in Table 3. From the table it can be observed that, given its neutral element $g = 0.5$, the cross-ratio operator also shows the full reinforcement property, which in our case will be used to reinforce positively the agreement values of those pairs of experts who highly cooperate to achieve consensus, and vice versa.

(a) Once obtained CM^t, consensus degrees are computed at three levels [43]:
i. *Level of pairs of alternatives* (cp^{lkt}): Obtained from CM^t as $cp^{lkt} = cm^{lkt}$, $l, k \in \{1, \ldots, n\}$, $l \neq k$.
ii. *Level of alternatives* (ca^{lt}): Degrees of consensus on each alternative $x_l \in X$ are computed as:

$$ca^{lt} = \frac{\sum_{k=1, k \neq l}^{n} cp^{lkt}}{n-1} \tag{11}$$

iii. *Level of preference relation* (cr^t):

$$cr^t = \frac{\sum_{l=1}^{n} ca^{lt}}{n} \tag{12}$$

(3) *Consensus control*: The consensus degree cr^t previously computed is checked to decide whether it is enough or not. If consensus is enough, the group moves on to the selection process. Otherwise, it is necessary to carry out another round of discussion. Two parameters, whose values are fixed a priori by the group, can be utilized in this phase:
- A consensus threshold $\mu \in]0, 1]$, whose value indicates the minimum level of agreement required amongst members in the group.
- A maximum number of discussion rounds allowed, $Maxround \in \mathbb{N}$. If the number of rounds carried out exceeds this value, then the CRP ends without having reached consensus, in which case a different decision strategy should be adopted by the group (see Section 2.1).

(4) *Advice generation*: If $cr^t < \mu$, the group moves onto this phase, in which farthest experts' assessments from consensus are identified, and a set of change recommendations on such assessments are provided to experts, with the aim of increasing consensus in the following rounds. The following steps are carried out in this phase:
(i) A collective preference P_c^t is obtained, by aggregating experts' assessments on each pair of alternatives:

$$p_c^{lkt} = \sum_{i=1}^{m} \hat{w}_i^t p_i^{lkt} \tag{13}$$

where $\hat{w}_i^t \in [0, 1]$ is the normalized importance weight assigned to the expert e_i according to his/her behavior (see Section 3.2.2).

Remark 4. Experts' weights are utilized in Eq. (13) to obtain a collective preference P_c^t, which better reflects the opinions of experts who contribute positively to achieve consensus, thus making a better group decision (in accordance with the main goal of our proposal).

(ii) A proximity matrix $PP_i^t = (pp_i^{lkt})_{n \times n}$ between each expert's preference relation and P_c^t, defined by

$$PP_i^t = \begin{pmatrix} - & \ldots & pp_i^{1nt} \\ \vdots & \ddots & \vdots \\ pp_i^{n1t} & \ldots & - \end{pmatrix}$$

is computed for each expert. Proximity values pp_i^{lkt} are used to identify the farthest preferences from the collective opinion, and they are obtained as follows:

$$pp_i^{lkt} = 1 - \left| p_i^{lkt} - p_c^{lkt} \right| \tag{14}$$

(iii) Pairs of alternatives (x_l, x_k) whose consensus degrees ca^{lt} and cp^{lkt} are not enough, are then identified:

$$CC^t = \{(x_l, x_k) | ca^{lt} < cr^t \wedge cp^{lkt} < cr^t\} \tag{15}$$

Afterwards, experts who should change their opinion on each of these pairs are identified, taking into account their proximity degrees to P_c^t. To do this, an average proximity $\bar{pp}^{lkt}$ is calculated:

$$\bar{pp}^{lkt} = \frac{\sum_{i=1}^{m} pp_i^{lkt}}{m} \tag{16}$$

As a result, experts e_i whose $pp_i^{lkt} < \bar{pp}^{lkt}$ will be advised to modify their assessment on the pair $(x_l, x_k) \in CC$.

(iv) A set of direction rules are applied to suggest the direction of changes proposed to experts, in order to increase the level of consensus. Such rules are based on the use of an acceptability threshold $\varepsilon \geq 0$ which may take a positive value close to zero and allows a margin of acceptability when p_i^{lkt} and p_c^{lkt} are close enough to each other.
- DIR.1: If $(p_i^{lkt} - p_c^{lkt}) < -\varepsilon$, then expert e_i should *increase* his/her assessment on the pair of alternatives (x_l, x_k).
- DIR.2: If $(p_i^{lkt} - p_c^{lkt}) > \varepsilon$, then expert e_i should *decrease* his/her assessment on the pair of alternatives (x_l, x_k).
- DIR.3: If $-\varepsilon \leq (p_i^{lkt} - p_c^{lkt}) \leq \varepsilon$, then expert e_i does not need to modify his/her assessment on the pair of alternatives (x_l, x_k).

Notice that, as previously illustrated in Section 3.1, the use of ε in our proposal is twofold: (i) as an acceptability threshold to identify assessments that should be modified, and (ii) in the computation of $\#ACP_i^t$ (see Section 3.1) to control whether the changes applied by the expert are significant enough or not to consider them as accepted.

As we have shown, experts weights are applied in Eqs. (10) and (13) to compute the consensus matrix, CM^t, and collective preference, P_c^t, respectively (see Fig. 9).

5. Illustrative example

In this section, we present an illustrative example based on the resolution of a large-scale GDM problem conducting a CRP. Although the group size in these problems may vary between a few dozens to thousands of experts, this example considers 30 experts for the sake of clarity in the results presented.

The uninorm-based consensus model presented in the previous section, is applied and compared with two other consensus models: (i) a model that utilizes a weighting scheme which only penalizes to manage non-cooperative behaviors [13], not allowing to recover importance in weights despite experts change their behavior, and (ii) a consensus model that does not use any weighting scheme on experts' preferences regarding their behavior.

A GDM problem is firstly formulated, in which experts have different behavioral patterns across the CRP (Section 5.1). A software simulation has been then applied and carried out with a multi-agent system presented in [44] for each of the three consensus models considered, in order to analyze the results obtained by applying the uninorm-based weighting scheme (Section 5.2) and compare them with the results obtained by the other two approaches (Section 5.3).

5.1. GDM problem formulation

Let us suppose a university government panel formed by 30 members, $E = \{e_1, \ldots, e_{30}\}$, who must reach an agreement about choosing a supportive action plan to be launched next year. There are four possible plans to contribute for, $X = \{x_1, x_2, x_3, x_4\}$:

- x_1: Helping typhoon victims.
- x_2: Supporting hospitalized victims.
- x_3: Protecting endangered species.
- x_4: Helping in reforestation tasks.

Table 4
Parameters of membership function for fuzzy set *COOP* at each consensus round.

t	2	3	4	5	6	7	8	9	10
(α, β)	(0.2,0.5)	(0.2,0.5)	(0.3,0.6)	(0.4,0.7)	(0.5,0.8)	(0.6,0.9)	(0.7,1)	(0.8,1)	(0.9,1)

Table 5
Consensus degrees and weight computation for e_{22}–e_{24}.

t Consensus degree		1 0.6714	2 0.7174	3 0.7572	4 0.7973	5 0.8319	6 0.8517
e_{22}	CC^t_{22}		1.0	1.0	1.0	1.0	
	CD^t_{22}		1.0	1.0	1.0	1.0	
	w^t_{22}	0.5	1.0	1.0	1.0	1.0	
	w^t_{22}	0.0333	0.0381	0.0381	0.0432	0.0449	
e_{23}	CC^t_{23}		0.3	0.3	0.3	0.2666	
	CD^t_{23}		0.3333	0.3333	0.0	0.0	
	w^t_{23}	0.5	0.3333	0.2222	0.0	0.0	
	w^t_{23}	0.0333	0.0127	0.0084	0.0	0.0	
e_{24}	CC^t_{24}		0.0	1.0	1.0	1.0	
	CD^t_{24}		0.0	1.0	1.0	1.0	
	w^t_{24}	0.5	0.0	0.5	1.0	1.0	
	w^t_{24}	0.0333	0.0	0.0191	0.0432	0.0449	

Experts elicit their preferences by using reciprocal fuzzy preference relations. The minimum agreement level required is $\mu = 0.85$, and the maximum number of discussion rounds allowed is $Maxround = 10$.

The membership function of the fuzzy set *COOP*, which is used to compute the cooperation degree of experts at each round, takes the following initial parameter values at the beginning of the CRP: $\alpha = 0.2$, $\beta = 0.5$. Then, from the third consensus round onwards, the value of both parameters increases by 0.1 per round, until $\alpha = 0.9$ and $\beta = 1$. Thus, the approach becomes more restrictive with the notion of cooperativeness as the CRP goes on. Table 4 shows the membership function parameters considered at each round.

The uninorm operators utilized in our proposal include:

- The operator based on Fodor general family of uninorm operators shown in Eq. (7) to compute individual weights w_i^t, with $g = 0.5$.
- The cross-ratio operator (see Eq. (3)) to compute pairwise experts' weights w_{ij}^t.

Experts adopted different patterns of behavior, that have been modeled by means of a simulation framework for GDM problems so-called AFRYCA [6]:

- *Cooperative behavior*: 22 out of the 30 panel members, e_1–e_{22} presented a full cooperative behavior throughout the CRP, in the sense that they applied all changes suggested on their assessments accordingly.
- *Mixed behavior*: The remaining eight panel members, e_{23}–e_{30} present a variable behavior across the CRP, i.e. some of them agreed to modify a few of their assessments towards consensus, ignoring some advice received or even modifying their assessments against consensus.

5.2. Results obtained by applying the uninorm-based approach

Table 5 summarizes the results of applying our uninorm-based proposal to manage experts' behaviors. Such results include the consensus degree at each round until consensus is achieved, the cooperation coefficients CC_i^t and cooperation degrees CD_i^t, and the weights (before and after normalizing) assigned to a representative subgroup of experts with different behavior, ranging from e_{22} to e_{24}. These three experts have been chosen to be analyzed in further detail in this example, since they adopt different behavior patterns of interests for our proposal throughout the CRP[1]:

- e_{22}: This expert always cooperates, applying all changes on his preferences as indicated in the feedback received.
- e_{23}: This expert presents a highly non-cooperative behavior across the whole CRP.
- e_{24}: This expert does not cooperate at the beginning of the CRP, but she then decides to change her behavior and cooperates in later rounds.

In order to provide a better insight of the CRP state at each round, a visual representation of experts' preferences and the collective preference is shown in Fig. 10. Such a visual representation has been generated with a graphical monitoring tool of preferences so-called MENTOR, presented in [45]. The collective preference is shown by a blue cross and labeled as 'P'. Preferences of e_{22}, e_{23} and e_{24} are also labeled to ease their visualization, and they are depicted by green, red and yellow crosses, respectively. The rest of experts' preferences are depicted by gray points.

The results observed in Table 5 and Fig. 10 are briefly analyzed below:

- e_{22} is always assigned the maximum importance weight for $t \geq 2$, since his behavior is completely cooperative across the CRP. We can observe in Fig. 10 that, despite e_{22}'s preferences are far from consensus at the beginning of the CRP, they quickly become closer to P_c, because his opinions are taken into account to a high degree.
- e_{23} shows a highly non-cooperative behavior over the course of the CRP, therefore his preferences end up quite far from the group preference when consensus is reached (see Fig. 10). Although his cooperation coefficient values are similar at all rounds, we can observe that his weight tends to decrease due to the downward reinforcement property of uninorm operators (see e.g.

[1] A large amount of information is utilized in this example, therefore it has not been included in the paper for space reasons. A supplementary material file including such information and the results of applying the proposed methodology, can be found in the datasets link at: http://sinbad2.ujaen.es/afryca

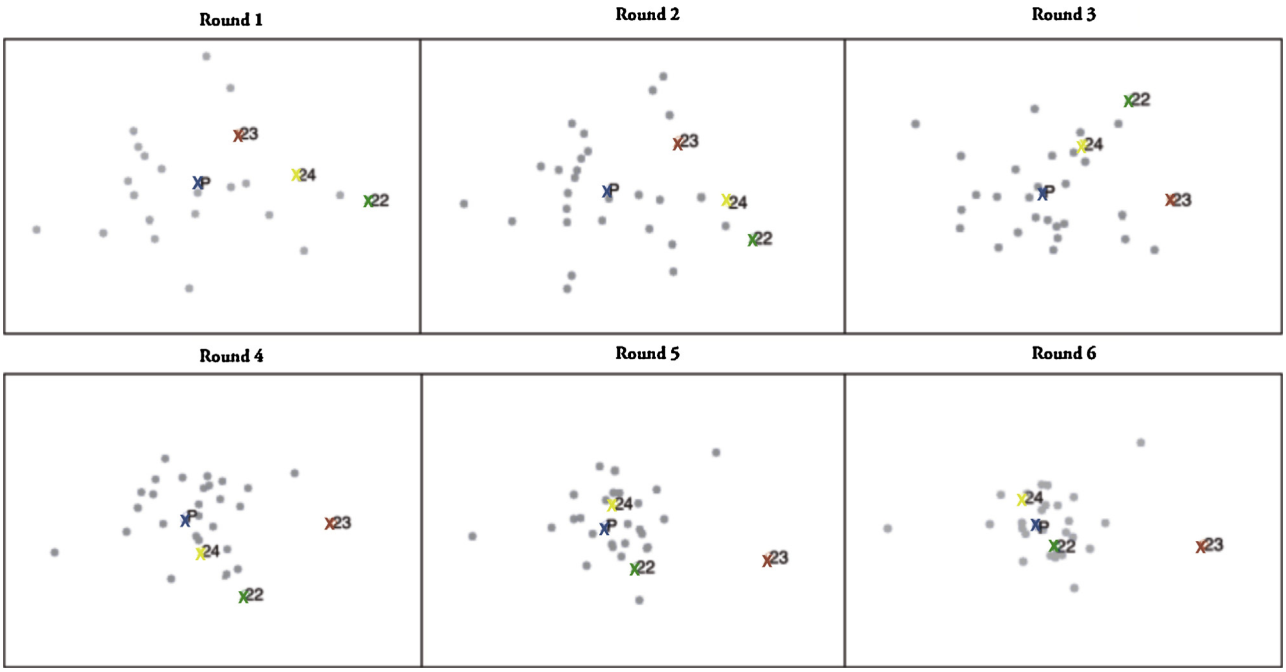

Fig. 10. Visual monitoring of preferences over the course of the CRP. (For interpretation of the references to color in the text, the reader is referred to the web version of this article.)

Table 6
Comparison of non-normalized weights computed with each proposal.

CRP round t	1	2	3	4	5	6	7
Consensus degrees							
No penalizing	0.6714	0.7145	0.7531	0.7831	0.8087	0.8301	0.8515
No importance recovery	0.6714	0.7150	0.7539	0.7859	0.8161	0.8375	**0.8525**
Importance recovery (uninorm-based)	0.6714	0.7174	0.7572	0.7973	0.8319	**0.8517**	

	Weight penalizing scheme	w_i^1	w_i^2	w_i^3	w_i^4	w_i^5	w_i^6
	No penalizing	1.0	1.0	1.0	1.0	1.0	1.0
e_{22}	No importance recovery	1.0	0.1696	0.1000	0.0958	0.0958	0.095
	Importance recovery (uninorm-based)	0.5	1.0	1.0	1.0	1.0	
	No penalizing	1.0	1.0	1.0	1.0	1.0	1.0
e_{23}	No importance recovery	1.0	0.3635	0.1520	0.0033	0.0	0.0
	Importance recovery (uninorm-based)	0.5	0.3333	0.2222	0.0	0.0	
	No penalizing	1.0	1.0	1.0	1.0	1.0	1.0
e_{24}	No importance recovery	1.0	0.0247	0.0111	0.0111	0.0111	0.0111
	Importance recovery (uninorm-based)	0.5	0.0	0.5	1.0	1.0	

$w_{23}^3 = 0.2222$). Furthermore, from the fourth round onwards, the cooperation degree becomes null, due to the change in the fuzzy membership function of *COOP*, therefore the resulting importance weights become null as well.

- e_{24} presents a completely non-cooperative behavior at the second round, therefore her opinions are not taken into account at all in such a round and they initially move further from the group opinion. However, when the expert realizes this, she decides to cooperate obeying all the advice received from the third CRP round onwards. For this reason, her importance weights are gradually increased until they reach the maximum value, and her opinions are finally brought closer to consensus.

These examples allow us to notice that the uninorm-based approach is capable of assigning experts a weight taking into account their overall behavior at previous CRP rounds, not only at the current one.

5.3. *Consensus reaching progress comparison with other proposals*

Once analyzed the behavior managing obtained in the proposal presented in this paper, we have considered adequate to compare the consensus reaching progress in the GDM problem when different consensus models are applied. To do so, we compare the CRP with the behavior managing based on uninorms presented in this paper, and the CRP penalizing model introduced in [13] that penalizes non-cooperative behaviors, but does not allow to improve experts importance, even though they eventually decide to cooperate. The results are also compared with those obtained by applying a CRP that does not manage experts' behavior (as presented in Fig. 2), i.e. all experts have equal importance $w_i^t = 1$ across the CRP. Experts' weights and the consensus degrees obtained with each of the three approaches are summarized in Table 6.

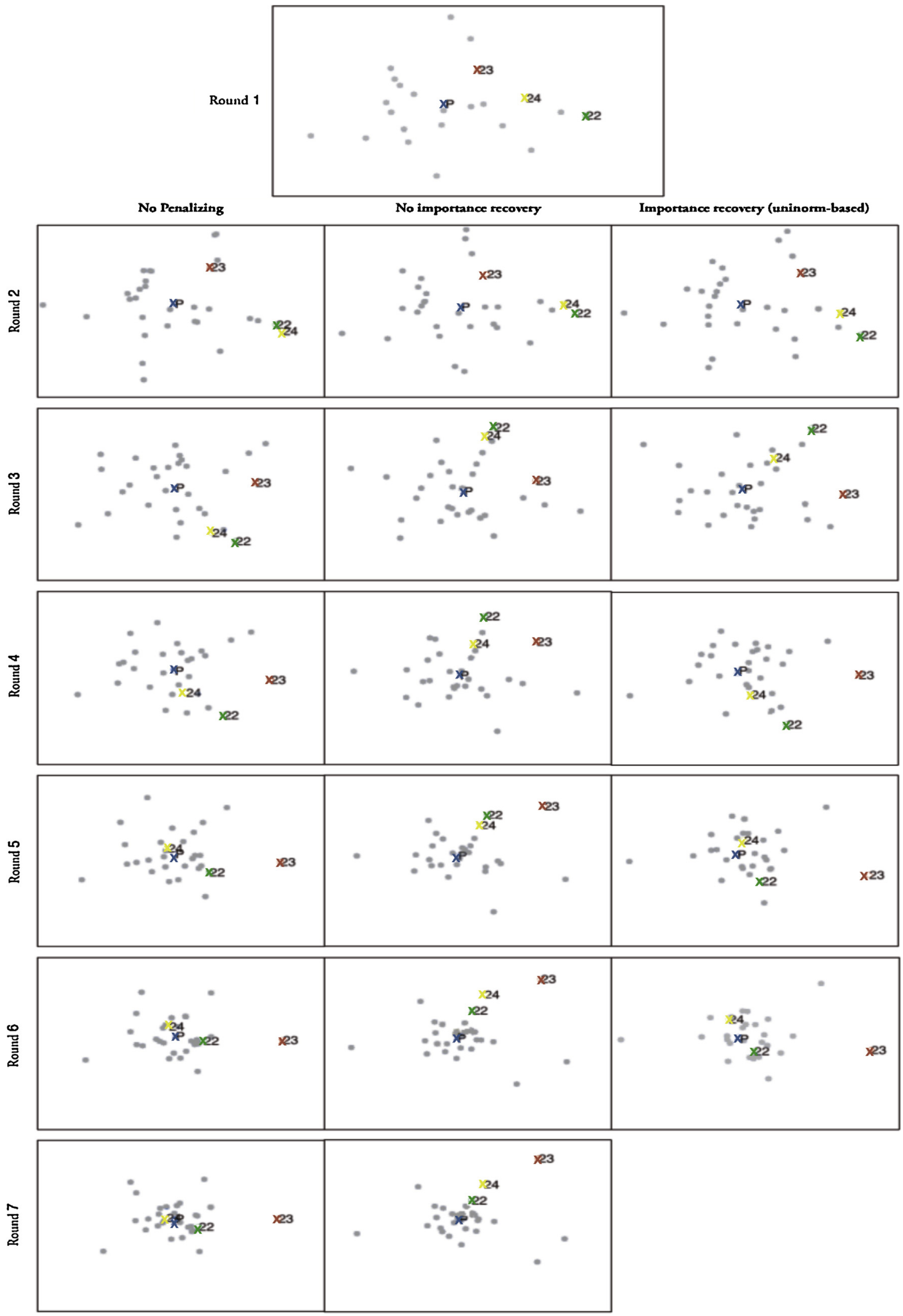

Fig. 11. Visual monitoring of preferences for the three consensus models compared. (For interpretation of the references to color in the text, the reader is referred to the web version of this article.)

Remark 5. In [13], weights are assigned based on the distance between experts' preferences, P_i^t, and the collective preference P_c^t. Moreover, if an expert is penalized due not to being close to P_c^t, then his/her importance weight cannot be recovered at later rounds, regardless of how close his opinions are later brought to P_c^t.

Remark 6. Since this problem involves 30 experts, and the normalized weight values of each expert might be low and difficult to analyze, for the sake of clarity and ease in comparisons Table 6 displays the absolute (non-normalized) weights $w_i^t \in [0, 1]$ obtained for each approach. Nevertheless, weights are internally normalized in the scheme, as shown in Section 3.2.2.

Fig. 11 shows the visual representation of preferences at each round, for the three consensus models compared. The following results can be observed regarding experts e_{22}, e_{23} and e_{24}, for each consensus model:

- Expert e_{22}
  - *No penalizing*: His preferences are equally important as the rest of the group preferences, therefore they are taken into account to obtain the group opinion.
  - *No importance recovery*: Since his initial preferences are far from the group opinion, in this model e_{22} is penalized at the beginning of the CRP. His highly cooperative behavior allows him to approach P_c^t at the end of the CRP, although his opinions are not as close to consensus as the opinions of other cooperating experts whose initial opinions where closer to P_c^t.
  - *Importance recovery*: In this case, preferences are brought much closer to consensus due to the full degree of cooperation presented, which is faithfully reflected in the high importance weights assigned and the closeness to P_c^t.
- Expert e_{23}.
  - *No penalizing*: His preferences are also taken into account to obtain the group opinion, despite his non-cooperative behavior along the CRP. In this model, e_{23}'s final position is slightly closer to P_c^t compared to the other two models, because e_{23} deviates the group solution in his favor.
  - *No importance recovery*: Due to his non-cooperative behavior, his preferences gradually become further from consensus, hence when consensus is achieved his opinions are far from the group opinion.
  - *Importance recovery*: Similarly to the model without importance recovery, his preferences become far from consensus as the CRP progresses.
- Expert e_{24}.
  - *No penalizing*: Her preferences are as important as the rest of the group preferences throughout the CRP, even though she does not cooperate at the first rounds.
  - *No importance recovery*: Since her importance is decreased at the first rounds, this expert's preferences remain far from the group due not to being able to improve her importance despite turning cooperative from the third round onwards.
  - *Importance recovery*: In this case, the uninorm-based weighting scheme allows to reflect e_{24} change of behavior at the third round. As a result, despite her preference becoming further from the rest of the group at the first rounds, they are eventually brought closer to consensus at the last rounds.

Moreover, in the weighting approach without importance recovery, we can see some other unlabeled experts who presented a mixed behavior and could not recover importance in their weight by the end of the CRP: these experts' preferences remain far from P_c^t at the last round.

To sum up, we can conclude that the proposal based on uninorms presented in this paper offers multiple advantages with respect to the other two:

- The weight assigned to an expert is computed based on his/her behavior at the current round of the CRP, as well as the evolution of such a behavior across previous rounds.
- The full reinforcement property of uninorm operators allows to reinforce the weight of experts if their behavior is highly cooperative (or uncooperative) at successive rounds.
- The cooperation coefficient provides a realistic measure of experts' behavior, regarding not only the amount of feedback they accepted to modify, but also the amount of feedback they received, which indicates how close to consensus their preferences are.
- Finally, we can observe that weighting pairs of experts to aggregate similarity values (see consensus model, Section 4), allows to maintain a reasonable convergence towards consensus, which is achieved at the sixth round of discussion with our proposed model, compared to the seven rounds required with the other two models.

6. Concluding remarks and future directions

Large-scale group decision making problems are becoming increasingly common in the last years. When a consensus reaching process must be conducted in these contexts to reach a collective agreement, the presence of individuals or subgroups of them who present a non-cooperative behavior and try to manipulate such a process in their favor is particularly frequent. In this paper, we have presented an approach based on uninorm aggregation operators, fuzzy sets and the methodology of computing with words, to detect and deal with experts' non-cooperative behavior. Such an approach applies a uninorm-based weighting scheme (inspired by previous works on dynamic multi-criteria decision making), to assign experts different importance weights according to their overall behavior across the CRP and the way such a behavior evolves. After integrating the weighting approach in a consensus model for group decision making under uncertainty, an example has been presented to illustrate its advantages with respect to other penalizing methodologies applied to consensus approaches.

Further research in the area is mainly oriented towards the study of new measures of cooperativeness, that facilitate detecting multiple types of behavior associated to the different ways of acting by experts, in order to deal with each type of behavior properly. We also aim at studying the practical application of the proposed methodology in collective intelligence systems, such as e-marketplaces, social networks and group recommender systems.

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