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Review

Developed Newton-Raphson based Predictor-Corrector load flow approach with high convergence rate

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ABSTRACT

In this paper, a new methodology called *Newton-Raphson-Predictor-Corrector* (NR-PC) is applied to solve the load-flow (LF) problem of well and ill-conditioned power systems. In the proposed LF method, the Predictor-Corrector mechanism is developed to achieve convergence rate of order $1 + \sqrt{2} \approx 2.4$ instead of 2 for the standard Newton Raphson (NR). The proposed NR-PC LF method is validated on different test systems; IEEE 30-bus, 57-bus, 118-bus and 300-bus systems as well-conditioned test cases, 13-bus and 20-bus systems as naturally ill-conditioned test systems, 1354-bus, 2869-bus and 9241-bus systems as realistic very large-scale test systems. The sensitivity of the proposed method with different R/X transmission line ratios and loading conditions is validated and compared with well-known methods. The simulation results show that the proposed LF method has better convergence characteristics and low computation time compared with benchmark methods.

1. Introduction

The load-Flow (LF) calculation is considered one of the most common computational tools used in power system analysis. LF study is essential for planning expansion of power systems as well as determining the best operation of existing systems. LF solution is considered the starting point for many studies such as; continuation power flow, optimal power flow and on-line applications, which required fast computational time [1–3]. Consequently, there is a continuing search for new methods with fast convergence characteristics and robust solution, especially for ill-conditioned systems.

Gauss-Seidel method is considered the earliest method used to solve the LF problem [1,4,5]. This method suffers a slow convergence and high number of required iterations. Later, NR was proposed to solve the convergence problems of GS [4,6]. NR is considered as the standard LF method that widely used in industry applications. However, NR method started to lose its ability to converge fast with dramatically increasing in power systems size. In addition, the Jacobian elements of NR method have to be updated during the iterative process. Consequently, Fast Decoupled (FD) method was proposed to improve the computational speed of NR [7]. This method gives bad performance in case of ill-conditioned systems which have high R/X ratios or high loading values [8,9].

New approach based on current injection formulation has been developed to accelerate the convergence rate of NR method and avoid the updating of Jacobian elements [10]. However, this approach is more suitable in case of load buses (PQ-type) and exhibits low convergence rate in case of generation buses (PV-type). In [9,11–14], distinct efforts have been made to efficiency represent the PV buses and improve the convergence characteristics of the previous approach.

Ill-conditioned cases may provoke convergence problems in most of standard LF methods [15]. Several robust LF methods have been developed in order to solve the ill-conditioned systems. Robust LF methods can be broadly classified as; methods based on second order formulation [16–19] and other based on Continuous Newton's method [15]. Ref. [16], has presented Iwamoto's method (IM) which based on second order Taylor series. IM calculates an optimal multiplier each iteration, which minimizes the least squares cost function. This multiplier is then used to modify the corrector vector in order to avoid the divergence. This principle has been used by other authors in order to develop other robust LF solvers [17–19]. In [15], the Continuous Newton's method has been presented and applied to LF problem. This method establishes an analogy relationship between the LF and a set of autonomous ordinary differential equations. Therefore, any well-assessed numeric method can be used for solving the LF problem. In [15], the 4th order Runge-Kutta (RK4) formula has been successfully applied to LF problem and its robustness has been demonstrated. Results reported in [15], demonstrate that the RK4 is much more efficient than

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IM.

In [20], a simple modification of NR method has been proposed to achieve a convergence rate of order $1+\sqrt{2}\approx 2.4$ instead of 2 for the standard NR. In this approach, a Predictor-Corrector mechanism (NR-PC) has been proposed to accelerate the convergence characteristic of standard NR method. In this paper, load flow problem is efficiently solved by applying the NR-PC approach. The proposed NR-PC LF method is used to find the load flow solution of well and ill-conditioned small, medium and large-scale systems. The performance of LF method has been compared with different benchmark load-flow methods. The proposed NR-PC LF method gives accurate results with fast convergence characteristics and low computation time compared with the previous load flow methods.

The rest of the paper is organized as follows: Section 2 presents a brief description about LF problem. The developed NR-PC load flow method is presented in Section 3. Simulation results are given in Section 4. The main features of the proposed load flow approach are summarized in Section 5, followed by the conclusions in Section 6.

2. LF problem formulation

LF problem models the nonlinear relationships among the injected power at system buses, the power demands, the bus voltages and the circuit parameters. The basic information of LF solution are voltage magnitude and phase angle at each bus, real and reactive power flow in each transmission line. From these values, other additional information such as; current flow and power losses can be calculated. The active and reactive power mismatches for each bus can be given as:

$$\Delta P_i = P^{sch} - \sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\cos(\delta_i-\delta_j-\theta_{ij}) \tag{1}$$

$$\Delta Q_i = Q^{sch} - \sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\sin(\delta_i-\delta_j-\theta_{ij}) \tag{2}$$

where, ΔP_i and ΔQ_i are the active and reactive power mismatches at bus i respectively, P^{sch} and Q^{sch} are the injected active and reactive powers at bus i respectively, $V_i\angle\delta_i$ is the complex voltage at bus i, $Y_{ij}\angle\theta_{ij}$ is the $ijth$ element of admittance matrix and n is the total number of buses. The unknown vector of LF problem is defined as:

$$\boldsymbol{x} = \{\boldsymbol{\delta_{PV}} \cup \boldsymbol{\delta_{PQ}} \cup \boldsymbol{V_{PQ}}\} \tag{3}$$

where $\boldsymbol{\delta_{PV}}$ and $\boldsymbol{\delta_{PQ}}$ are the voltage angle vector of PV and PQ buses, respectively. $\boldsymbol{V_{PQ}}$ is the voltage magnitude vector of PQ buses. The unknown vector size is given as:

$$n_x = n_{PV} + 2n_{PQ} \tag{4}$$

where, n_{PV} and n_{PQ} are the number of PV buses and PQ buses, respectively. The LF process is repeated until the convergence condition is satisfied as:

$$\max\{|\Delta P_i| \cup |\Delta Q_i|\} \le \varepsilon \ \forall\ i \tag{5}$$

This means that the iterative process will be stopped when the maximum mismatch (1) and (2), is lower than the preset convergence tolerance ε. In order to simplify the notation, compact version of (1) and (2) will be used onwards:

$$g(\boldsymbol{x}) = 0 \tag{6}$$

3. Proposed NR-PC load flow method

In this section, the proposed NR-PC load flow method is presented. This method is based on the set of nonlinear equations solver developed in [20]. The section is divided into two subsections; standard NR and proposed NR-PC methods.

3.1. Standard NR method

Let us consider the solution of the generic nonlinear equation $f(x)=0$ with x as a variable. If x^0 is an initial estimate of the solution, $f(x)$ can be expanded around x^0 in Taylor series yields;

$$f(x) = f(x^0) + f'(x^0)(x-x^0) + \frac{1}{2!}f''(x^0)(x-x^0) + \dots + \frac{1}{n!}f^n(x^0)(x-x^0) = 0 \tag{7}$$

By neglecting the second and higher order terms, Eq. (7) can be rewritten as:

$$f(x) \approx f(x^0) + f'(x^0)(x-x^0) = 0$$

The value of x can be obtained from

$$x = x^0 - \frac{f(x^0)}{f'(x^0)} \tag{8}$$

In general, the new estimated value of x for each iterative (k) is given as:

$$x^{k+1} = x^k - \frac{f(x^k)}{f'(x^k)} \tag{9}$$

or

$$x^{k+1} = x^k - \Delta x \tag{10}$$

where

$$\Delta x = \frac{f(x^k)}{f'(x^k)} \tag{11}$$

The above equations can be extended to solve the set of simultaneously non-linear equations in (6) as:

$$\Delta\boldsymbol{x}^{(k)} = \boldsymbol{J_x}(\boldsymbol{x}^{(k)})^{-1} g(\boldsymbol{x}^{(k)})$$
$$\boldsymbol{x}^{(k+1)} = \boldsymbol{x}^{(k)} + \Delta\boldsymbol{x}^{(k)} \tag{12}$$

where $\boldsymbol{J_x}$ is the Jacobian matrix of the system, which is the first-order partial derivatives of (6) with respect to unknown vector. Eq. (12) is a generic kth iteration of the NR process, which is repeated until the convergence criteria (5) is satisfied.

The performance of standard NR method is shown in Fig. 1. For the sake of simplicity, a problem with $n_x = 1$ is analysed. It can be observed that the iterative process starts in an initial guess point $x^{(0)}$, as derivative of $f(x^{(k)})$ at each point, the process is repeated until point x^*, which verifies $f(x^*)=0$.

3.2. Proposed NR-PC LF method

NR-PC proposes a slight modification of standard NR, which is able to reach a convergence rate of order $1+\sqrt{2}\approx 2.4$. The following are

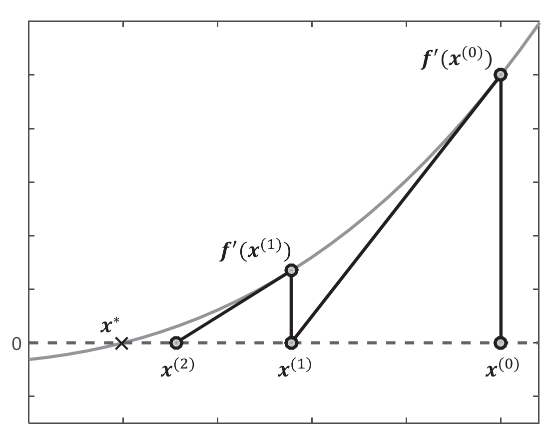

Fig. 1. Sketch of standard NR method.

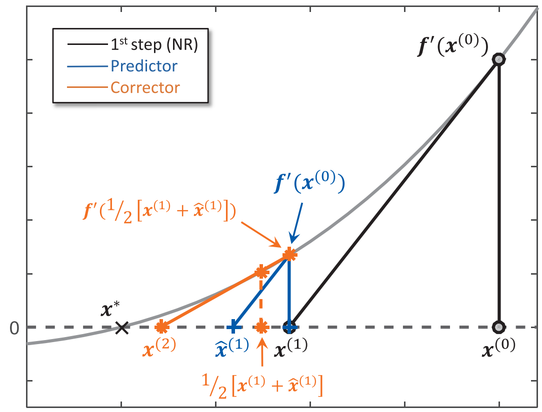

Fig. 2. Sketch of NR-PC method.

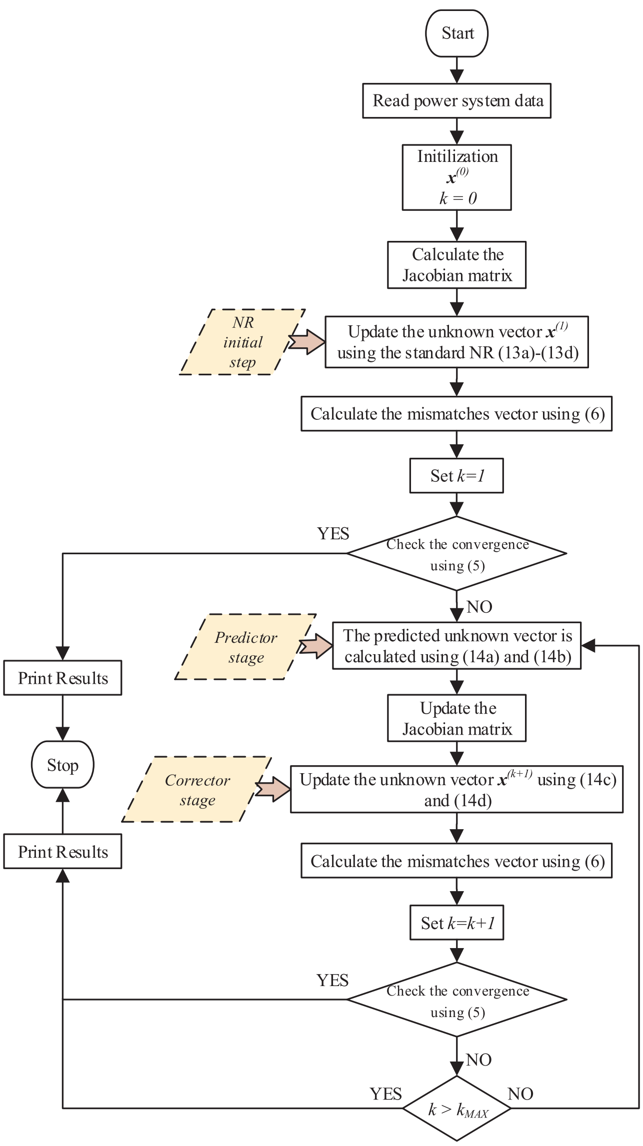

Fig. 3. Flowchart of proposed NR-PC LF algorithm.

considered the main advantages of NR-PC; the computation of superior order derivatives is not required, it does not need excessive data storage, and the value of J_x of current iteration can be reutilized for the following iteration. NR-PC suggests a Predictor-Corrector mechanism, which can be summarized as follows:

At $k = 0$

$$\hat{\boldsymbol{x}}^{(0)} = \boldsymbol{x}^{(0)} \tag{13a}$$

$$\Delta\boldsymbol{x}^{(0)} = \frac{g(\boldsymbol{x}^{(0)})}{J_x\left(\frac{1}{2}[\boldsymbol{x}^{(0)} + \hat{\boldsymbol{x}}^{(0)}]\right)} \Rightarrow \Delta\boldsymbol{x}^{(0)} = J_x(\boldsymbol{x}^{(0)})^{-1} g(\boldsymbol{x}^{(0)}) \tag{13b}$$

$$\boldsymbol{x}^{(1)} = \boldsymbol{x}^{(0)} + \Delta\boldsymbol{x}^{(0)} \tag{13c}$$

For $k \geqslant 1$

$$\Delta\hat{\boldsymbol{x}}^{(k)} = \frac{g(\boldsymbol{x}^{(k)})}{J_x\left(\frac{1}{2}[\boldsymbol{x}^{(k-1)} + \hat{\boldsymbol{x}}^{(k-1)}]\right)} \Rightarrow \Delta\hat{\boldsymbol{x}}^{(k)} = J_x\left(\frac{1}{2}[\boldsymbol{x}^{(k-1)} + \hat{\boldsymbol{x}}^{(k-1)}]\right)^{-1} g(\boldsymbol{x}^{(k)}) \tag{14a}$$

$$\hat{\boldsymbol{x}}^{(k+1)} = \boldsymbol{x}^{(k)} + \Delta\hat{\boldsymbol{x}}^{(k)} \tag{14b}$$

$$\Delta\boldsymbol{x}^{(k)} = \frac{g(\boldsymbol{x}^{(k)})}{J_x\left(\frac{1}{2}[\boldsymbol{x}^{(k)} + \hat{\boldsymbol{x}}^{(k)}]\right)} \Rightarrow \Delta\boldsymbol{x}^{(k)} = J_x\left(\frac{1}{2}[\boldsymbol{x}^{(k)} + \hat{\boldsymbol{x}}^{(k)}]\right)^{-1} g(\boldsymbol{x}^{(k)}) \tag{14c}$$

$$\boldsymbol{x}^{(k+1)} = \boldsymbol{x}^{(k)} + \Delta\boldsymbol{x}^{(k)} \tag{14d}$$

where the superscript ($\wedge$) refers to the predicted values.

Fig. 2 shows the performance of NR-PC. From this figure, it can be observed that the first iteration is carried out similar to standard NR. Moreover, the followings steps should be carried out as:

(1) The value of $J_x\left(\frac{1}{2}[\boldsymbol{x}^{(0)} + \hat{\boldsymbol{x}}^{(0)}]\right) \Rightarrow J_x(\boldsymbol{x}^{(0)})$ is reused to find an intermediate predicted value $\hat{\boldsymbol{x}}^{(1)}$. This stage is called “Predictor”.
(2) Calculate $J_x\left(\frac{1}{2}[\boldsymbol{x}^{(1)} + \hat{\boldsymbol{x}}^{(1)}]\right)$.
(3) The following value of the unknown vector $\boldsymbol{x}^{(2)}$, is found in a stage called “Corrector”.
(4) Each iteration, only the value of $J_x\left(\frac{1}{2}[\boldsymbol{x}^{(k)} + \hat{\boldsymbol{x}}^{(k)}]\right)$ is computed while $J_x\left(\frac{1}{2}[\boldsymbol{x}^{(k-1)} + \hat{\boldsymbol{x}}^{(k-1)}]\right)$ is reused from the previous iteration.
(5) The process is repeated until the desirable convergence is reached or the maximum number of iterations k_{MAX} is surpassed.

For the sake of clarity, the solution procedure for load flow problem using NR-PC is summarized in the flowchart of Fig. 3.

4. Simulations and results

The effectiveness of the proposed NR-PC LF method is tested in a wide range of scenarios. Various small, medium and large well and ill-conditioned systems are considered. However, the systems employed in this paper are indicated below:

- Standard well-conditioned systems: IEEE 30-bus, 57-bus, 118-bus and 300-bus systems are considered as well-conditioned test cases. The bus and line data of these systems are given in [21].
- Naturally ill-conditioned systems: 13-bus and 20-bus systems are taken as naturally ill-conditioned test systems. The details of these systems are given [22,23]. It is worth to mention that different LF methods may converge with some difficulty in these cases.
- Realistic very large-scale systems: 1354-bus, 2869-bus and 9241-bus systems are taken as realistic very large-scale test systems. These systems are based on portions of European transmission system from the PEGASE project. The details of these systems are given in [24,25].

Furthermore, the performance of the proposed NR-PC LF method is compared with the following benchmark LF methods:

- NR method [6]: NR method is considered one of the most used LF

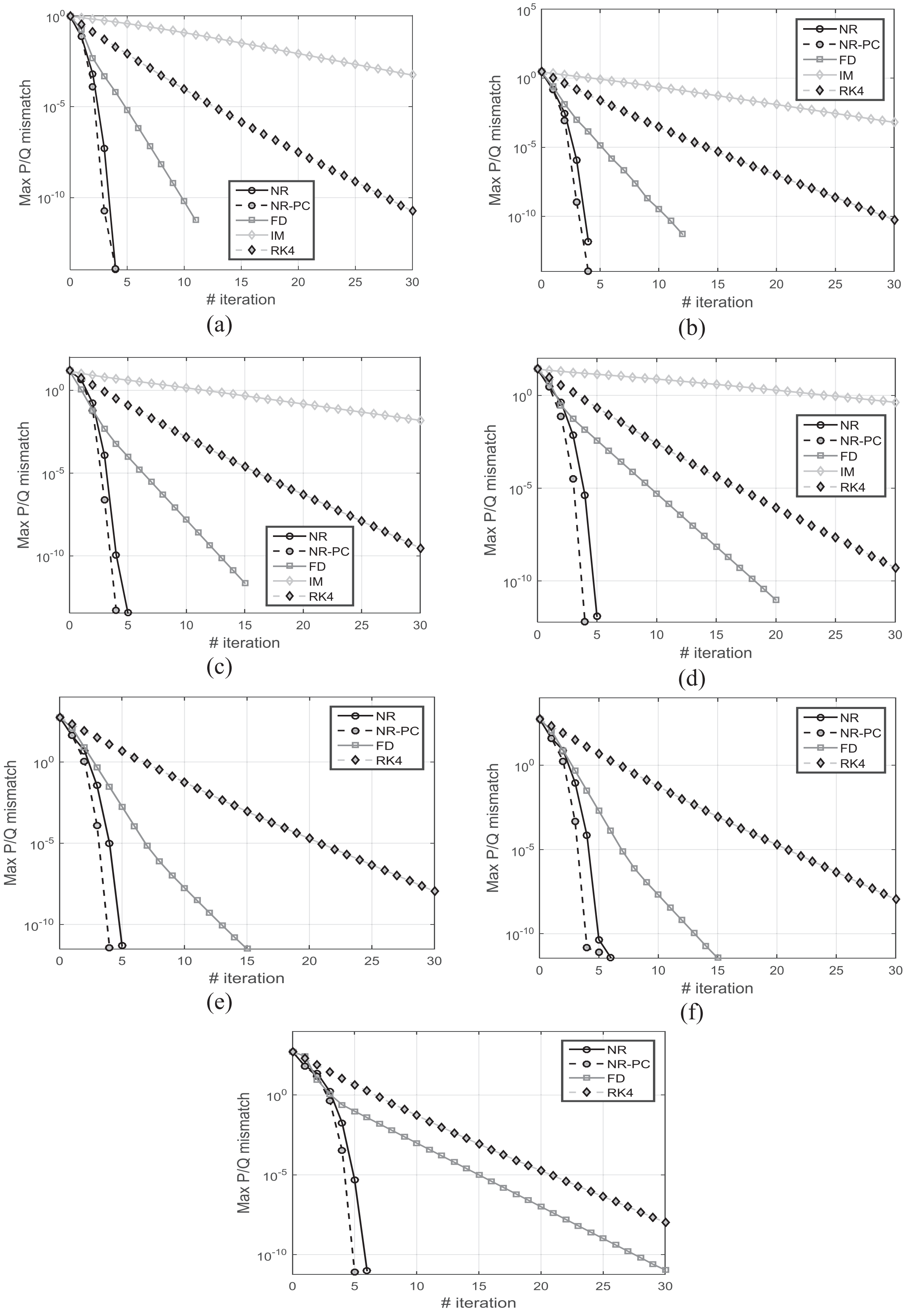

Fig. 4. Convergence characteristics of studied cases using different LF methods; (a) IEEE 30-bus system, (b) IEEE 57-bus system, (c) IEEE 118-bus system, (d) IEEE 300-bus system, (e) PEGASE 1354-bus system, (f) PEGASE 2869-bus system and (g) PEGASE 9241-bus system.

Table 1
Required number of iterations and computation time for different LF methods.

System	Number of iterations					Computation time (second)				
	NR	NR-PC	FD	IM	RK4	NR	NR-PC	FD	IM	RK4
IEEE 30-bus	4	4	11	96	31	0.008	0.005	0.006	0.106	0.093
IEEE 57-bus	4	4	12	91	33	0.009	0.006	0.007	0.115	0.101
IEEE 118-bus	5	4	15	123	35	0.011	0.008	0.009	0.201	0.164
IEEE 300-bus	5	4	20	188	36	0.019	0.012	0.016	0.548	0.295
PEGASE 1354-bus	5	4	15	NC	40	0.058	0.040	0.050	NC	1.359
PEGASE 2869-bus	6	5	15	NC	40	0.146	0.112	0.122	NC	3.027
PEGASE 9241-bus	6	5	31	NC	40	0.505	0.401	0.478	NC	9.86

NC = no convergence.

Table 2
Error of the proposed NR-PC method with respect to the standard NR.

System	Maximum error with respect to NR	
	Voltage magnitude	Voltage angle
IEEE 30-bus	2.2×10^{-15}	9.2×10^{-14}
IEEE 57-bus	2.2×10^{-12}	1.2×10^{-10}
IEEE 118-bus	4.4×10^{-16}	1.2×10^{-13}
IEEE 300-bus	1.3×10^{-13}	1.5×10^{-11}
PEGASE 1354-bus	3.5×10^{-14}	2.3×10^{-12}
PEGASE 2869-bus	4.7×10^{-14}	3.3×10^{-12}
PEGASE 9241-bus	2.2×10^{-13}	3.4×10^{-12}

methods. NR normally converges in few iterations in case of well-conditioned systems. However, it can find some difficulties when the system is ill-conditioned.

- FD method [7]: This method is usually faster than NR, nevertheless, it often employs many more iterations than standard NR. Occasionally, FD has better convergence properties than standard NR [15].
- Iwamoto's method (IM) [16]: It can be considered as the most-referenced robust LF method, especially for ill-conditioned systems.
- 4th order Runge-Kutta method (RK4): The robustness of this method has been proven in [15]. Moreover, it is normally more efficient than IM.

Among all methods that based on the second order formulation, IM can be considered as the most robust LF method. This method has been widely referenced in the literature [15,26–28]. On the other hand, RK4 is considered the robust LF method that based on the Continuous Newton's method. Therefore, the performance of the proposed NR-PC LF method is compared with these two methods.

All simulations have been carried out using a Personal Computer Intel Core i5-7500 3.4 GHz using the package software MATPOWER 6.0 [29]. In all cases, the flat initial guess point is used, the maximum number of iterations (k_{MAX}) is 500 and the convergence tolerance (ε) is 10^{-12}.

4.1. Well-conditioned systems

The load flow solution for well-conditioned test systems are obtained by the proposed NR-PC, NR, FD, IM and RK4 methods. The convergence characteristics of these methods are shown in Fig. 4. The total number of iterations and computation time of the different LF methods are presented in Table 1.

From this table, it can be observed that the proposed NR-PC LF method has the fastest convergence rate among all studied methods. In addition, it has the lowest computation time in all cases. It is worth to mention that IM has convergence problems in case of very low convergence tolerance ε.

In order to verify the accuracy of the proposed LF method, the maximum errors of the results obtained by NR-PC and standard NR load flow methods are presented in Table 2. From this table, it can be observed that the results obtained by the proposed LF method match with a high degree of those obtained by standard NR.

4.2. Ill-conditioned systems

4.2.1. Naturally ill-conditioned systems

In this case, 13-bus and 20-bus ill-conditioned systems are used to validate the ability of the proposed NR-PC LF method. Moreover, the condition of the 20-bus system has been deteriorated opening the branch between the buses 15 and 20 (weakly meshed 20-bus system). The required number of iterations and computation time of different LF methods are presented in Table 3. Moreover, the convergence characteristics of LF methods for these systems are shown in Fig. 5. From the results obtained, it can be observed that the proposed NR-PC LF method is able to converge with less number of iterations and low computation time compared with the other methods.

4.2.2. Heavy loading conditions

The reliability of proposed NR-PC LF method is tested at different loading values. The loading level is gradually increased close to their maximum loading values (λ_{max}) which obtained using continuation power flow analysis (CPF) [30].

Table 4 presents the total number of iterations obtained by different load flow methods at the maximum loadability points, for 30-bus, 57-bus, 118-bus, 300-bus, 1354-bus, 2869-bus and 9241-bus systems. It can be observed that the proposed NR-PC LF method has the lowest

Table 3
Total Number of iterations and computation time for different LF methods in naturally ill-conditioned systems.

System	Number of iterations					Computation time (second)				
	NR	NR-PC	FD	IM	RK4	NR	NR-PC	FD	IM	RK4
13-bus system	5	4	12	56	31	0.007	0.004	0.008	0.194	0.083
20-bus system	5	4	23	124	32	0.009	0.006	0.010	0.128	0.092
20-bus system[a]	5	4	27	124	33	0.009	0.006	0.013	0.128	0.096

[a] Weakly meshed case.

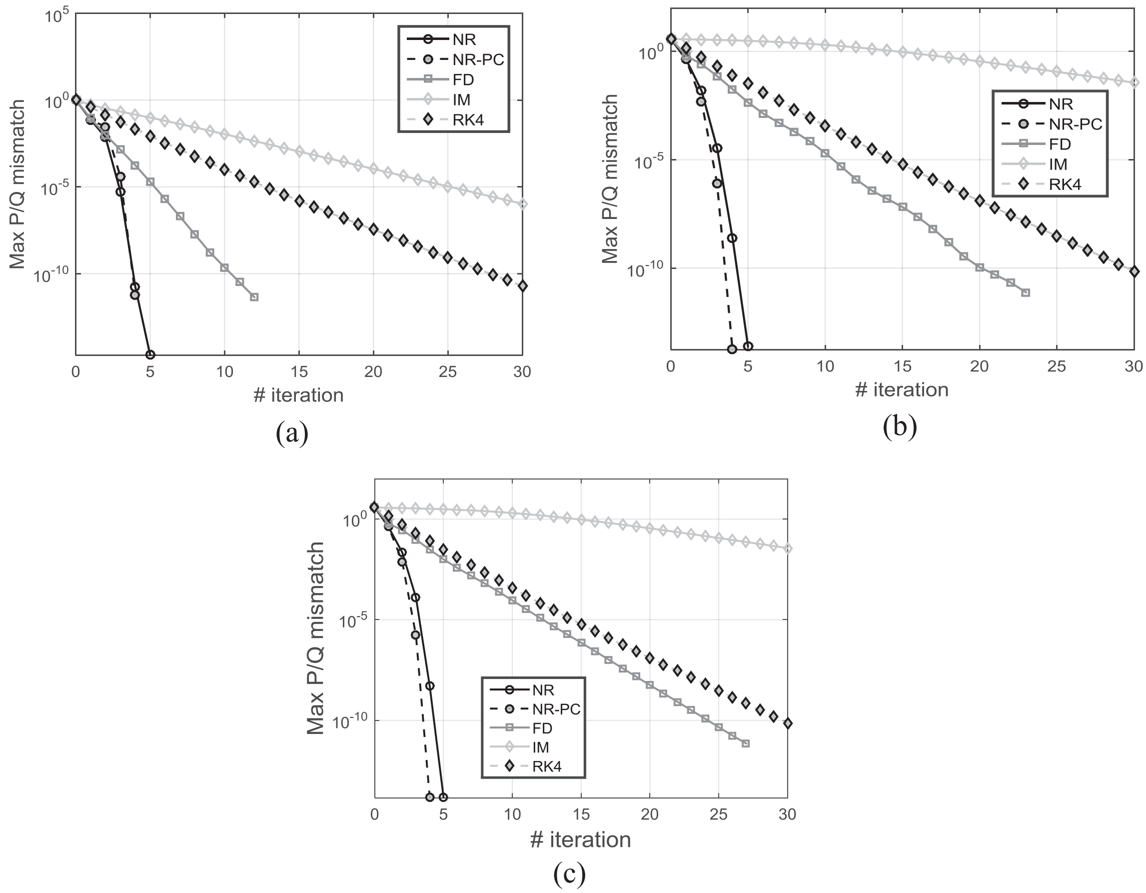

Fig. 5. Convergence characteristics of studied ill-conditioned cases using different LF methods; (a) 13-bus system, (b) 20-bus system and (c) weakly meshed 20-bus system.

Table 4
Required number of iterations at the maximum loadability points of different LF methods.

System	λ_{max}	Number of iterations				
		NR	NR-PC	FD	IM	RK4
IEEE 30-bus	2.681	6	5	32	58	33
IEEE 57-bus	1.5972	5	5	20	70	33
IEEE 118-bus	1.8664	5	4	17	81	35
IEEE 300-bus	1.055	5	4	23	181	36
PEGASE 1354-bus	1.313	6	5	28	NC	40
PEGASE 2869-bus	1.141	6	5	15	NC	40
PEGASE 9241-bus	1.076	7	6	32	NC	40

number of iterations compared with the other methods.

4.2.3. High R/X ratios

The sensitivity of the proposed NR-PC LF method with different R/X ratios of test systems is validated and compared with the other LF methods.

Tables 5 presents the number of iterations of different LF methods at different R/X ratios for IEEE 30-bus, 57-bus, 118-bus and 300-bus, 1354-bus, 2869-bus and 9241-bus systems. From this table, it can be observed that the convergence rate of the proposed NR-PC LF method is better than the other methods.

4.2.4. Critical ill-conditioned systems

In this subsection, several critical ill-conditioned systems are used to validate the proposed NR-PC LF method. These systems are obtained by modifying the loading conditions and R/X ratios and opening one branch simultaneously for the well-conditioned systems (30-bus, 57-bus and 118-bus). The critical open branches in the test systems have been determined and used to deteriorate the condition of the systems with keeping their intrinsic stability. However, these branches have been determined based on the value of condition numbers ($Cond.\ number = \frac{max\ \ eigenvalue}{min\ \ eigenvalue}$) of the Jacobian matrices of the test systems when several branches are opened. The branch with higher condition number respect to its value before the branch opening has been selected to be opened. With this selectin criteria, the stability is kept but the condition of the system is lightly poor. Table 6 presents the conditions of these critical ill-conditioned cases whereas Table 7 presents the total number of iterations of NR and NR-PC.

From Tables 6 and 7, it can be observed that the benefits of the proposed NR-PC are more noticeable in case of the critical ill-conditioned systems. In all cases, the NR-PC has been able to converge in two less iterations than the standard NR.

5. Outstanding features of NR-PC LF method

In this section, the main features of the proposed NR-PC LF method compared with standard NR LF method are summarized. As previously mentioned, the proposed NR-PC has higher convergence rate compared with the standard NR. This feature can be more clarified by calculating the efficiency index of both methods. This index takes into consideration the convergence order and number of functions or derivative evaluations of each iteration. The efficiency index of the two iterative methods are given in Table 8. From this table, it can be observed that the efficiency index of NR-PC method is 10% larger than that of the standard NR method.

Due to higher convergence order, the proposed NR-PC usually

Table 5
Required number of iterations of different LF methods at different R/X ratios.

R_{New}	X_{New}	NR	NR-PC	FD	IM	RK4
IEEE 30-bus system						
$1 \times R_{Old}$	$0.5 \times X_{Old}$	4	3	21	187	32
$3 \times R_{Old}$	$1.0 \times X_{Old}$	5	4	52	137	31
$4 \times R_{Old}$	$1.0 \times X_{Old}$	NC	NC	NC	NC	NC
IEEE 57-bus system						
$1 \times R_{Old}$	$0.5 \times X_{Old}$	4	4	21	187	33
$3 \times R_{Old}$	$1.0 \times X_{Old}$	5	4	39	135	33
$4 \times R_{Old}$	$1.0 \times X_{Old}$	NC	NC	NC	NC	NC
IEEE 118-bus system						
$1 \times R_{Old}$	$0.5 \times X_{Old}$	5	4	28	290	36
$3 \times R_{Old}$	$1.0 \times X_{Old}$	5	4	57	207	35
$4 \times R_{Old}$	$1.0 \times X_{Old}$	NC	NC	NC	NC	NC
IEEE 300-bus system						
$1 \times R_{Old}$	$0.5 \times X_{Old}$	5	4	30	428	37
$2 \times R_{Old}$	$1.0 \times X_{Old}$	7	5	55	260	36
$3 \times R_{Old}$	$1.0 \times X_{Old}$	NC	NC	NC	NC	NC
PEGASE 1354-bus system						
$1 \times R_{Old}$	$0.5 \times X_{Old}$	6	5	35	NC	41
$3 \times R_{Old}$	$1.0 \times X_{Old}$	6	5	70	NC	40
$4 \times R_{Old}$	$1.0 \times X_{Old}$	NC	NC	NC	NC	NC
PEGASE 2869-bus system						
$1 \times R_{Old}$	$0.5 \times X_{Old}$	6	5	36	NC	41
$3 \times R_{Old}$	$1.0 \times X_{Old}$	7	6	70	NC	40
$4 \times R_{Old}$	$1.0 \times X_{Old}$	NC	NC	NC	NC	NC
PEGASE 9241-bus system						
$1 \times R_{Old}$	$0.5 \times X_{Old}$	6	5	30	NC	40
$2 \times R_{Old}$	$1.0 \times X_{Old}$	8	6	82	NC	40
$3 \times R_{Old}$	$1.0 \times X_{Old}$	NC	NC	NC	NC	NC

NC = no convergence.

Table 6
Conditions of the considered critical ill-conditioned systems.

Case	λ	R_{New}	X_{New}	Open branch
30-bus system	1.54	$3 \times R_{Old}$	$1 \times X_{Old}$	15–18
57-bus system	1.23	$3 \times R_{Old}$	$1 \times X_{Old}$	7–8
118-bus system	1.63	$3 \times R_{Old}$	$1 \times X_{Old}$	16–17

Table 7
Number of iterations of NR and NR-PC in the considered critical ill-conditioned systems.

Case	NR	NR-PC	FD	IM	RK4
30-bus system	8	6	337	110	32
57-bus system	9	7	NC	108	33
118-bus system	8	6	221	149	35

NC = no convergence.

Table 8
Efficiency index of NR-PC and standard NR.

Method	Convergence order	Number of functions or derivative evaluations	Efficiency index
NR	2	2	$2^{1/2} \approx 1.4142$
NR-PC	$1 + \sqrt{2} \approx 2.4$	2	$(1 + \sqrt{2})^{1/2} \approx 1.5538$

employs one or two less iterations to converge compared with the standard NR method. Moreover, the proposed method is able to improve the computational time compared with the standard NR due to higher efficiency index. However, the robustness of NR-PC LF method is validated using several well and ill-conditioned systems. In all studied cases, the proposed NR-PC LF method is able to successfully converge ensuring a higher degree of efficiency compared with the standard NR method.

Table 9
Reduction percentage in the required average number of iterations for all studied cases.

Cases	Average number of iterations		
	NR	NR-PC	Reduction percentage (%)
Well-conditioned systems	5	4.29	14.2
Ill-conditioned systems	5	4	20
Heavy loading conditions	5.71	4.86	15
High R/X ratios	5.64	4.57	19
Critical Ill-conditioned systems	8.33	6.33	24

Table 10
Number of iterations for NR and NR-PC in the critical ill-conditioned 118-bus system with deteriorated initial guess.

	error = 0.4	**error** = 0.42	**error** = 0.44
GS	Fail[a]	Fail[a]	Fail[a]
GS-NR	Fail[a]	Fail[a]	Fail[a]
NR	Diverge	Diverge	Diverge
NR-PC	8	8	8

[a] It did not converge in 500 iterations.

For the sake of clarity, Table 9 reports a comparison of the NR-PC with respect to the NR. This comparison is achieved by calculating the average number of iterations employing by the two methods for the different studied cases in the previous section (well, ill and critical ill-conditioned cases). The average number of iterations has been calculated by summing the required number of iterations of all the test systems then divided by the total number of the used test systems of each case.

From Table 9, it can be observed that the proposed NR-PC LF method reduce the required number of iterations compared with the standard NR in all studied cases, especially in case of ill-conditioned systems.

In addition, we have observed good robustness features for the proposed NR-PC LF method in case of the flat initial guess deterioration. Let us assume that the flat initial guess is deteriorated by adding an error factor to the initial voltage magnitude of PQ buses in the aforementioned critical ill-conditioned 118-bus case. Table 10 presents the required number of iterations of NR, NR-PC LF, Gauss-Seidel (GS), and Hybrid GS-NR approach. It's well known that the GS method always converges to the stable equilibrium point if it exists. Disadvantage of the GS method is the hyperbolic characteristic of convergence i.e. initially this method converges very fast but later when approaching solution becomes very slow. This slow convergence is the reason why the GS method is not widely used. However, thanks to its fast initial convergence the GS method can be used as the method to find good starting point for the NR method. Hence, this hybrid GS-NR approach is compared with the proposed method. In this case, the LF solution is started by GS and changed to the standard NR when the maximum power mismatch is smaller than 10^{-2}. From Table 10, it can be observed that both of GS and hybrid GS-NR methods have failed owing to it has not been able to converge in less of 500 iterations. The proposed NR-PC is able to converge with different error factors while the NR method has diverged.

The main merit the proposed NR-PC LF (reducing the required iterations number by one or two and computation time compared with NR) can be more clarified if this method used in Optimal Power Flow (OPF) or Continuation Power Flow (CPF) algorithms, where the load flow is repeatedly solved several times.

6. Conclusions

In this paper, LF problem of well and ill-conditioned power systems has been solved using novel NR-PC methodology. This methodology is able to improve the convergence rate of standard NR method. The proposed NR-PC LF method can be used to find fast load flow solution without the need of computing superior order derivatives. The proposed method has been validated using different small, medium and large-scale well and ill-conditioned systems. The simulation results show the effectiveness and the superiority of the proposed method compared with the different benchmark methods; Iwamoto, NR, FD, and RK4. The future work will focus on employing NR-PC in those pathology cases which have slow convergence such as systems including voltage control devices. In addition, the proposed NR-PC method can be considered in continuation and optimal power flow analysis.

Appendix A. Supplementary material

Supplementary data to this article can be found online at https://doi.org/10.1016/j.ijepes.2018.09.021.

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