Study on Nonlocal Damage Localisation Under Pure Bending

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ABSTRACT

When non-linear material behaviour is modelled by means of the finite element method, the results usually depend on the size of finite elements, which is a very typical problem to be addressed in damage mechanics. When the material is mathematically described based on pure continuum models, i.e., with no displacement discontinuities, the formulation makes use of constitutive laws describing softening. Then, if the standard local continuum approach is used, the thickness of the localisation band depends directly on the element size.

Nonlocal formulations are a useful tool to remove such pathological mesh dependence. The main idea consists of using a nonlocal field to evaluate damage at each point, which means that damage development depends not just on the local value at each integration point, but on a weighted average value computed using the local value and other values at neighbouring integration points. The nonlocal field is computed as

$$\bar{\varepsilon}(\boldsymbol{x}) = \int_V \alpha(\boldsymbol{x}, \boldsymbol{\xi}) \tilde{\varepsilon}(\boldsymbol{\xi}) \, \mathrm{d}\boldsymbol{\xi} \tag{1}$$

where $\boldsymbol{x}$ stands for the point where the nonlocal value is computed, $\boldsymbol{\xi}$ for each of the points that contribute to the nonlocal value and $\tilde{\varepsilon}(\boldsymbol{\xi})$ for the equivalent strain at $\boldsymbol{\xi}$. Function α is a weight function which decreases with increasing distance between points $\boldsymbol{x}$ $\boldsymbol{\xi}$ and contains a parameter R with the dimension of length, which corresponds to the internal length (characteristic length) of the material (related to its heteregeneous microstructure). An overview of nonlocal models for softening materials can be found in [1].

When a nonlocal formulation is used near a boundary of the body of interest, certain numerical issues arise, since some of the contributing points are missing. To address this, alternative formulations have been proposed in the literature, such as the local complement suggested by Borino et al. [3], or alternative distance-based and stress-based approaches [2, 5]. In a recent study [4], the performance of such techniques for notched specimens has been evaluated.

This contribution is focused on studying the localisation properties of nonlocal damage models under bending. In order to allow an easy interpretation of the results, the uniform bending case is studied. Therefore, a beam with a constant cross section subjected to pure bending is analysed.

Initially, the solution remains uniform at the bottom face of the beam, up to a critical state after which damage tends to localise in equally space bands, as Figure 1 shows. Finally, only one of these bands will fully develop, which leads to the final failure of the beam.

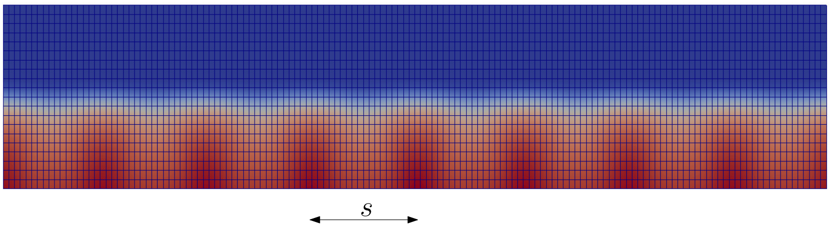

Figure 1: Localisation bands spaced at a distance s.

To study the effect of different averaging schemes, as well as the effect of the characteristic length in this process, a numerical model has been used, representing a periodic cell of the beam under pure bending. Damage evolution is described by using an isotropic damage material model, and the same material is used for all elements. To trigger localisation, a random perturbation in the displacement field is induced after each time step, which ensures a non-biased formation of the localisation bands. The dependence of the spacing of localisation bands and their evolution on the characteristic length and on the details of the nonlocal formulation is studied in depth. The results can be useful for identification of the most realistic nonlocal formulation that reflects the actual physical processes near a boundary.

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