Distributed resilient fusion filtering for nonlinear systems with multiple missing measurements via dynamic event-triggered mechanism ☆

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ARTICLE INFO

Keywords:
Nonlinear time-varying multi-sensor networked systems
Multiple missing measurements
Dynamic event-triggered communication
Distributed resilient fusion filtering
Inverse covariance intersection

ABSTRACT

This paper investigates the distributed resilient fusion filtering (DRFF) issue under inverse covariance intersection (ICI) fusion criterion and dynamic event-triggered mechanisms (DETMs), where the physical plant is described by stochastic nonlinear multi-sensor networked systems (MSNSs) with time-varying system parameters and multiple missing measurements (MMMs). The measurements from various sensor nodes to the fusion center may undergo the missing data, where this phenomenon is depicted by means of random variables governed by certain statistical principles. In addition, the DETM is adopted to regulate the communication process from each sensor node to fusion center, which can alleviate the network transmission situations with communication overload and energy consumption limitation. The purpose of the addressed issue is to construct a set of local resilient filters (LRFs) for stochastic nonlinear MSNSs with MMMs via the DETM, which can guarantee that the minimized upper bounds are derived and the desirable filter gain with easy-to-implementation form is given. Subsequently, via the obtained LRFs, a unified framework of the DRFF approach is formulated through using the ICI fusion criterion. In addition, the monotonicity analysis of the obtained upper bound in regard to the triggered parameter is examined by providing rigorous theoretical proof. Finally, the simulations with comparison experiment are provided to illustrate the validity of presented DRFF technique.

☆ This work was supported in part by the National Natural Science Foundation of China under Grant 12171124, the Natural Science Foundation of Heilongjiang Province of China under Grant ZD2022F003, the Key Foundation of Educational Science Planning in Heilongjiang Province of China under Grant GJB1422069, the "Ministerio de Ciencia e Innovación, Agencia Estatal de Investigación" of the Spanish Government and the European Regional Development Fund (ERDF) under Grant PID2021-124486NB-I00, and the Alexander von Humboldt Foundation of Germany.

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1. Introduction

During the past two decades, the quantity and category of sensors have been highly equipped in multi-sensor networked systems (MSNSs) [1,2,4,8]. Under the development of the related technology, the MSNSs could be effectively used to tackle a variety of problems in complicated application domains [9–11]. Specifically, in order to receive more accurate data and reduce costs, the study on the multi-sensor fusion estimation issues has stirred increasing research attention with extensive applications including signal processing, robot technology and target tracking and so forth [27,43]. Significantly, there are mainly two types of fusion structure, i.e., the centralized fusion and distributed fusion (DF) [1,2]. The latter one is the most widely used method in the existing literature due to its ease of utilization, robustness and fault tolerance [36]. For the fusion estimation algorithm, some fusion rules have been put forward under different conditions including the approaches of the matrix-weighted (MW) criterion, covariance intersection (CI) criterion as well as inverse covariance intersection (ICI) criterion and so on [3,34]. To mention a few, the distributed optimal fusion estimator has been designed in [37] for linear systems based on the MW fusion approach. Compared with the MW fusion method, it is noteworthy that the CI or ICI fusion strategy only requires the covariances of the estimation error rather than cross-covariances. For instance, the fusion estimation algorithm by using the CI fusion approach has been developed in [38] for uncertain systems subject to stochastic nonlinearities. Recently, the researchers have focused on handling various fusion issues for nonlinear MSNSs [21]. For instance, in [21], the authors presented a new fusion filtering strategy with distributed format, where the fusion filtering technique has been reported via the sequential CI fusion criterion.

During the implementation of existing filtering algorithms, it is commonly assumed that the real measurement can always be received by estimator [7,17,42]. However, such an assumption has certain conservatism, which might be deviated due to some reasons see example the missing data [44,45]. In fact, the received information in the estimator side might include the noises only in networked environment. Consequently, a good deal of effort has been done to examine the fusion filtering problems for MSNSs with missing measurements, see e.g. [6,32,35]. Among them, the fusion filtering technique with centralized processing way has been established in [32] for linear stochastic systems, where both the multiplicative noises and the missing measurements have been well examined and its influences have been discussed. Based on the weighted measurement fusion approach, the fusion filtering algorithm has been reported in [6] for time-invariant linear stochastic systems handling bandwidth constraint and missing measurements, where the quantized fusion filtering scheme has been derived in terms of the quantized measurements. Moreover, the issue of robust fusion estimation has been addressed in [35] for linear MSNSs subject to the transmission delay and missing measurements, where the fusion filter has been constructed by using the integrated CI fusion method. So far, an efficient state estimation method with distributed format has been presented in [14] for stochastic switching dynamical networks with time-varying parameters and multiple missing measurements (MMMs), which extends the actual application scope. However, the design problem of fusion filtering algorithm for nonlinear MSNSs has rarely taken the MMMs into account, which constitutes one of the research motivations.

The measurement information from large amount of sensor nodes generally needs to be transmitted and exchanged in MSNSs at each sampling time, which causes the case that the burden of network transmission and energy consumption would be increased [19,31,41]. To tackle this issue, some network scheduling schemes have been proposed naturally in the communication process, such as the Round-Robin scheduling protocol, random access protocol, event-triggered mechanism (ETM) and so on [5,47]. Among them, the ETM has been presented to reduce the block of transmission and network resources, whose main rule is that the ETM is established to achieve a certain goal only when the premier triggering condition is satisfied [16,25,26]. In fact, the general methods of the ETM can be classified as static event-triggered mechanism (SETM) and dynamic event-triggered mechanism (DETM) [12,15,39]. On the one hand, the SETM-based estimation and fusion estimation problems have got much attention over the past few years, and many corresponding approaches have been proposed to relieve the network transmission pressure [23,30,40,46]. For example, the distributed estimation and sequential fusion estimation problems have been addressed in [23,40] for linear MSNSs under the SETM, respectively. Furthermore, the DF estimation (DFE) problem has been discussed in [22] for nonlinear MSNSs with SETM and random transmission delays, where the fusion filter has been constructed by using the CI fusion method. On the other hand, compared with the SETM, the DETM can further reduce the triggering times by using a dynamic regulation equation, which is regarded as an effective method to improve the utilization of network resources. In the light of the DETM, the state estimation issues have been examined in [24,33] for linear singularly perturbed systems and MSNSs, respectively. Up to now, a little effort has been made to cope with the DFE problem via the DETM, even though it is of practical significance.

Another useful discussion is that the resilient filter can be taken into account during the design of the state estimation algorithm, which can overcome the sensitivity to the parameter variations/drifts during the filter implementation. Most of filtering approaches depend on a general assumption that the filter is capable of implementing exactly by means of the presented filter gain. However, it is worth noting that this assumption cannot be entirely satisfied in actual application. The major reason is that the low quality of the sensor with limited computing power may lead to calculation error, which further affects the estimation accuracy. Therefore, the problems of resilient estimation have been investigated and a number of relevant methods have been provided [13,28,29]. For instance, a resilient event-based filtering approach has been given in [28] for nonlinear time-invariant systems (NTISs), where the filter gain variations have been considered to characterize the resilient perturbation of proposed filter. Compared with the above single sensor system, in [29], the authors provided new distributed resilient estimation (DRE) strategy for stochastic nonlinear systems with sensor degradation and time-varying system parameters. Based on the method in [29], the DRE approach has been reported in [13] for time delay systems with nonlinear disturbances and stochastic perturbations, where the norm-bounded uncertainty and Bernoulli distributed random variable have been adopted to design the resilient estimator. Very recently, in [18], the authors designed new distributed resilient fusion filtering (DRFF) scheme to handle the influences induced by random sensor delays and communication constraint, where the static communication rule under fixed framework has been used to regulate the information transmission. Until now, however, the corresponding DRFF problem for time-varying nonlinear MSNSs has not been examined under the DETM yet, not to mention the case that the tackled nonlinear MSNSs undergo the MMMs.

In this paper, it is of important meaning to develop new DRFF approach for time-varying nonlinear MSNSs with MMMs via the DETM. To be specific, the nonlinear MSNSs undergo the MMMs phenomenon under the DETM, which are considered to further reflect the uncertainty of network channels. Moreover, the filter gain perturbation is taken into account in the design of the local filter with hope to enhance the robustness of desirable filtering method. Accordingly, the main difficulties are: (i) how to develop an effective approach to mitigate the impacts from the MMMs, filter gain perturbations and DETM thus improving the estimation performance; (ii) how to design a suitable DRFF algorithm to assure the minimization of the obtained upper bound (UB) with regard to the estimation error covariance (EEC) and choose the suitable fusion rule when the cross-covariance information from any two local filters cannot easily be obtained; and (iii) how to tackle the monotonicity analysis problem of upper bound regarding the EEC for addressed MSNSs from the theoretical viewpoint. The major contributions are as follows: 1) the issue of the DRFF under the DETM is tackled for time-varying nonlinear MSNSs with MMMs; 2) the local filter gain variation is introduced to describe the uncertainties for each estimator during the construction of the local filter; 3) the DRFF approach is presented with the help of the ICI fusion rule, where local UB of EEC is obtained and minimized by selecting the appropriate filter gain; and 4) the monotonicity analysis from theoretical perspective regarding the UB of EEC is given, which reflects the influence of the DETM onto the estimation performance from the energy constraint viewpoint.

Notations: $\mathbb{R}^m$ represents the m-dimensional Euclidean space. O^T, O^{-1} and $\mathrm{tr}(O)$ depict the transpose, inverse and trace of the matrix O. I represents an identity matrix with proper dimension. $\lambda_{\max}(O)$ denotes the largest eigenvalue of the matrix O. δ_{tk} stands for the Kronecker delta function. $\mathrm{Prob}\{a\}$ stands for probability of random variable a. $\mathbb{E}\{b\}$ denotes mathematical expectation of random variable b. The symbol "$\circ$" denotes the Hadamard product. Besides, we assume that the matrices have compatible dimensions in the matrix algebraic calculations.

2. Problem statement and preliminaries

Consider the following stochastic nonlinear MSNSs with time-varying system parameters and MMMs:

$$\mathrm{x}_{\mathrm{r}+1} = f(\mathrm{x}_\mathrm{r}) + B_\mathrm{r}\omega_\mathrm{r}, \tag{1}$$

$$y_{i,\mathrm{r}} = \Upsilon_{i,\mathrm{r}} C_{i,\mathrm{r}} \mathrm{x}_\mathrm{r} + v_{i,\mathrm{r}},\ \ i = 1, 2, \cdots, L \tag{2}$$

where $\mathrm{x}_\mathrm{r} \in \mathbb{R}^n$ is the state vector and its initial value is x_0 with mean $\bar{\mathrm{x}}_0$ and covariance $P_{0|0} > 0$, $y_{i,\mathrm{r}} \in \mathbb{R}^{m_i}$ denotes the measurement output of the ith sensor node. The additive noises $\omega_\mathrm{r} \in \mathbb{R}^p$ and $v_{i,\mathrm{r}} \in \mathbb{R}^{m_i}$ are zero-mean white noises with covariances $Q_\mathrm{r} > 0$ and $R_{i,\mathrm{r}} > 0$. Here, the nonlinearity $f(\cdot)$ is continuous differentiable. B_r and $C_{i,\mathrm{r}}$ are known matrices of suitable dimensions. $\Upsilon_{i,\mathrm{r}} := \mathrm{diag}\{\varsigma_{i,\mathrm{r}}^{(1)}, \varsigma_{i,\mathrm{r}}^{(2)}, \cdots, \varsigma_{i,\mathrm{r}}^{(m_i)}\}$ with $\varsigma_{i,\mathrm{r}}^{(q)}$ $(q = 1, 2, \cdots, m_i)$ being m_i independent random variables.

The phenomenon of MMMs is described by utilizing Bernoulli distributed random variables $\varsigma_{i,\mathrm{r}}^{(q)} \in \mathbb{R}$ $(q = 1, 2, \cdots, m_i)$ with:

$$\mathrm{Prob}\left\{\varsigma_{i,\mathrm{r}}^{(q)} = 1\right\} = \mathbb{E}\left\{\varsigma_{i,\mathrm{r}}^{(q)}\right\} = \bar{\varsigma}_{i,\mathrm{r}}^{(q)},$$

$$\mathrm{Prob}\left\{\varsigma_{i,\mathrm{r}}^{(q)} = 0\right\} = 1 - \bar{\varsigma}_{i,\mathrm{r}}^{(q)},$$

with $\bar{\varsigma}_{i,\mathrm{r}}^{(q)} \in [0, 1]$.

Remark 1. During the characterization of the measurements through multiple channels, the missing measurements are considered and missing probabilities of each channel may not have the same results in reality. To cater to the actual situation that each network channel may have different missing probabilities, the diagonal matrix $\Upsilon_{i,\mathrm{r}}$ with m_i independent random variables has been utilized to characterize the MMMs case. In particular, the expectations of each diagonal element are assumed to be known, which can be generally collected by resorting the statistical test.

For each sensor node, the DETM is introduced from the purpose of reducing communication burden and saving network resource. Specifically, the triggering instant sequences are expressed by $0 \le s_0^i < s_1^i \cdots < s_l^i < \cdots$, where s_{l+1}^i satisfies the following criterion:

$$s_{l+1}^i = \min\left\{\mathrm{r} \in \mathbb{N} | \mathrm{r} > s_l^i,\ \frac{1}{\beta_i}\rho_{i,\mathrm{r}} + \varrho_i - \|\varepsilon_{i,\mathrm{r}}\| \le 0\right\}, \tag{3}$$

with $\varrho_i > 0$ and $\beta_i > 0$ being scalars. Moreover, $\varepsilon_{i,\mathrm{r}} = y_{i,\mathrm{r}} - y_{i,s_l^i}$, where y_{i,s_l^i} represents the latest transmitted measurement at time r, and $\rho_{i,\mathrm{r}}$ stands for an internal dynamic variable obeying

$$\rho_{i,\mathrm{r}+1} = \varpi_i \rho_{i,\mathrm{r}} + \varrho_i - \|\varepsilon_{i,\mathrm{r}}\|,\ \ \rho_{i,0} = \rho_0^i, \tag{4}$$

where $\varpi_i > 0$ is a given scalar and $\rho_0^i \ge 0$ is known initial value.

Remark 2. When $\beta_i \to +\infty$, the condition (3) is transformed into the traditional SETM as in [14]. Consequently, the adopted DETM can be reduced to the SETM and then the criterion (3) can be described as $s_{l+1}^i = \min\{\mathrm{r} \in \mathbb{N} | \mathrm{r} > s_l^i,\ \varrho_i \le \|\varepsilon_{i,\mathrm{r}}\|\}$. Based on the criterion of the SETM, it is easy to see that the overall filtering performance may degrade with the increase of the fixed threshold ϱ_i because the received signals of the estimator only contain a small amount of information to be utilized. As such, the desirable performance cannot be guaranteed even though the occupation of the network bandwidth can be reduced by using the SETM. Naturally, the DETM has been proposed to overcome the above difficulty by means of adopting a dynamic variable to the SETM as shown in (3) and this variable meets the condition (4). According to the condition (4), the variation of the information can be easily detected and can further reflect that the threshold does make corresponding changes. In this regard, the numbers of the triggering events with respect to the DETM are lower than that with respect to the SETM. In addition, compared with the SETM, we can effectively save the network resources by using the DETM in the transmission process between the local sensor and estimator.

Based on the measurement, the following local nonlinear filter with recursive form is designed:

$$\hat{x}_{i,r+1|r} = f(\hat{x}_{i,r|r}), \tag{5}$$

$$\hat{x}_{i,r+1|r+1} = \hat{x}_{i,r+1|r} + \mathcal{K}_{i,r+1}(y_{i,s_l^i} - \bar{\Upsilon}_{i,r+1}C_{i,r+1}\hat{x}_{i,r+1|r}), \tag{6}$$

where $\hat{x}_{i,r+1|r}$ and $\hat{x}_{i,r|r}$ stand for the local one-step prediction and updated estimation of the state based on the measurements of the ith sensor node, respectively. The initial value of the local estimation is $\hat{x}_{i,0|0} = \bar{x}_0$. $\bar{\Upsilon}_{i,r+1} = \mathbb{E}\{\Upsilon_{i,r+1}\} = \text{diag}\{\bar{\varsigma}_{i,r}^{(1)}, \bar{\varsigma}_{i,r}^{(2)}, \cdots, \bar{\varsigma}_{i,r}^{(m_i)}\}$. $\mathcal{K}_{i,r+1} = K_{i,r+1} + \zeta_{i,r+1}\bar{K}_{i,r+1}$, where $K_{i,r+1}$ represents the local filter gain to be determined. $\zeta_{i,r+1}\bar{K}_{i,r+1}$ characterizes the ith filter gain perturbation. Here, $\zeta_{i,r+1}$ is a random variable which has zero mean and unity variance and $\bar{K}_{i,r+1}$ stands for a given real matrix. In the following text, we assume that $\varsigma_{i,r}^{(q)}$, ω_r, $v_{i,r}$, $\zeta_{i,r}$ and x_0 are mutually independent.

Remark 3. From the filter in (5) and (6), one can apparently know that the filter is established in terms of the classical method as in the extended Kalman filtering, which can be usually adopted to deal with the filtering problem of nonlinear stochastic system. Furthermore, the filter is expressed by means of recursion formulas, which is capable of handling multidimensional random processes by means of the online calculations. In addition, the gain perturbation is considered during the design of the nonlinear filter with hope to further improve the robustness of the filtering method, which can reflect that the resilient filter is insensitive to the potential interferences/disturbances. In this paper, $\zeta_{i,r+1}\bar{K}_{i,r+1}$ is exploited to describe the perturbation term with $\zeta_{i,r+1}$ being a random variable, and the innovation sequence is redesigned with the help of the transmitted measurement under the DETM, which are different from the conventional filtering method.

3. Design of fusion filtering algorithm

In this section, both local filtering error covariance (FEC) and prediction error covariance (PEC) are provided and expressed with the help of corresponding formulas firstly. Next, their UBs are obtained and local UB of the FEC is minimized at every sampling point by designing proper filter gain. Furthermore, in accordance with the local filters from all sensors, the resilient fusion filter is derived in terms of the ICI fusion strategy.

3.1. Design of the local filter

To begin, the corresponding errors regarding the local estimator are computed as follows. Specifically, denote the prediction error as $\tilde{x}_{i,r+1|r} = x_{r+1} - \hat{x}_{i,r+1|r}$. Then, subtracting (5) from (1), we have

$$\tilde{x}_{i,r+1|r} = f(x_r) - f(\hat{x}_{i,r|r}) + B_r\omega_r. \tag{7}$$

Using the method of the Taylor expansion around $\hat{x}_{i,r|r}$, the nonlinearity $f(\cdot)$ can be depicted as

$$f(x_r) = f(\hat{x}_{i,r|r}) + A_{i,r}\tilde{x}_{i,r|r} + o(|\tilde{x}_{i,r|r}|), \tag{8}$$

where $A_{i,r} = \partial f(x_r)/\partial x_r|_{x_r=\hat{x}_{i,r|r}}$ and the high-order term is denoted by $o(|\tilde{x}_{i,r|r}|)$. As in [20], it can be obtained that $o(|\tilde{x}_{i,r|r}|) = F_{i,r}U_{i,r}M_{i,r}\tilde{x}_{i,r|r}$, where $F_{i,r}$ stands for the problem-dependent known scaling matrix. $M_{i,r}$ represents a known matrix, which is used to regulate the filter. $U_{i,r}$ denotes an unknown matrix, which is used to describe the linearization error with $U_{i,r}U_{i,r}^T \leq I$. According to (1), (5) and (8), we have

$$\tilde{x}_{i,r+1|r} = (A_{i,r} + F_{i,r}U_{i,r}M_{i,r})\tilde{x}_{i,r|r} + B_r\omega_r. \tag{9}$$

Similarly, using (1) and (6), the filtering error is characterized by:

$$\begin{aligned}\tilde{x}_{i,r+1|r+1} = &(I - \mathcal{K}_{i,r+1}\bar{\Upsilon}_{i,r+1}C_{i,r+1})\tilde{x}_{i,r+1|r} + \mathcal{K}_{i,r+1}\varepsilon_{i,r+1}\\ &-\mathcal{K}_{i,r+1}\tilde{\Upsilon}_{i,r+1}C_{i,r+1}x_{r+1} - \mathcal{K}_{i,r+1}v_{i,r+1},\end{aligned} \tag{10}$$

where $\tilde{\Upsilon}_{i,r+1} = \Upsilon_{i,r+1} - \bar{\Upsilon}_{i,r+1}$ with $\mathbb{E}\{\tilde{\Upsilon}_{i,r+1}\} = 0$.

In order to facilitate later derivations, we need some basic results.

Lemma 1 *([38]). For $c, d \in \mathbb{R}^n$, one has*

$$cd^T + dc^T \le \chi cc^T + \chi^{-1} dd^T, \tag{11}$$

where $\chi > 0$.

Lemma 2 *([38]). Consider $\mathcal{U}$, $\mathcal{V}$, $\mathcal{W}$ and $\mathcal{Z}$ of suitable dimensions with $\mathcal{Z}\mathcal{Z}^T \le I$. For $\mathcal{F} > 0$ and $\kappa > 0$ satisfying $\kappa^{-1} I - \mathcal{W}\mathcal{F}\mathcal{W}^T > 0$, one has*

$$(\mathcal{U} + \mathcal{V}\mathcal{Z}\mathcal{W})\mathcal{F}(\mathcal{U} + \mathcal{V}\mathcal{Z}\mathcal{W})^T \le \mathcal{U}(\mathcal{F}^{-1} - \kappa \mathcal{W}^T \mathcal{W})^{-1}\mathcal{U}^T + \kappa^{-1}\mathcal{V}\mathcal{V}^T. \tag{12}$$

Lemma 3 *([14]). For a real-valued matrix $S = [s_{i,j}]_{n\times n}$ and a random matrix $G = \mathrm{diag}\{g_1, g_2, \cdots, g_n\}$, one has*

$$\mathbb{E}\{GSG^T\} = \begin{bmatrix} \mathbb{E}\{g_1^2\} & \mathbb{E}\{g_1 g_2\} & \cdots & \mathbb{E}\{g_1 g_n\} \\ \mathbb{E}\{g_2 g_1\} & \mathbb{E}\{g_2^2\} & \cdots & \mathbb{E}\{g_2 g_n\} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{E}\{g_n g_1\} & \mathbb{E}\{g_n g_2\} & \cdots & \mathbb{E}\{g_n^2\} \end{bmatrix} \circ S,$$

where $\circ$ denotes the Hadamard product.

Lemma 4 *([24]). For all $i = 1, 2, \cdots, L$, suppose that $\varpi_i \beta_i \ge 1$ is true and let $\gamma_{i,r}^1 > 0$ and $\gamma_{i,r}^2 > 0$ be given scalars. The covariance matrices $\Psi_{i,r} \triangleq \mathbb{E}\{\rho_{i,r}^2\}$ obey $\Psi_{i,r} \le \bar{\Psi}_{i,r}$, where*

$$\begin{aligned} \bar{\Psi}_{i,r+1} &= \Gamma_{i,r}(\bar{\Psi}_{i,r}) \\ &= \left((1+\gamma_{i,r}^1)(1+\gamma_{i,r}^2)\varpi_i^2 + (1+\gamma_{i,r}^{-1})(1+\beta_i)/\beta_i^2)\right)\bar{\Psi}_{i,r} \\ &\quad + \left((1+\gamma_{i,r}^1)(1+\gamma_{i,r}^{-2}) + (1+\gamma_{i,r}^{-1})(1+\beta_i^{-1})\right)\varrho_i^2 \end{aligned} \tag{13}$$

with the initial value $\bar{\Psi}_{i,0} = (\rho_0^i)^2$.

Based on the above Lemmas and some definitions, one can derive that the formulations with regard to the local PEC and the FEC are given.

Theorem 1. *For the ith estimator, the local PEC $D_{i,r+1|r}$ is expressed by:*

$$D_{i,r+1|r} = (A_{i,r} + F_{i,r}U_{i,r}M_{i,r})D_{i,r|r}(A_{i,r} + F_{i,r}U_{i,r}M_{i,r})^T + B_r Q_r B_r^T. \tag{14}$$

Proof. Based on (9) and the covariance definition, it is easy to have the conclusion that (14) is true. Then, it is evident to see that this theorem holds. ■

Theorem 2. *For the ith estimator, the local FEC $D_{i,r+1|r+1}$ can be characterized as*

$$\begin{aligned} D_{i,r+1|r+1} = {} & \mathbb{E}\left\{(I - \mathcal{K}_{i,r+1}\bar{\Upsilon}_{i,r+1}C_{i,r+1})D_{i,r+1|r}(I - \mathcal{K}_{i,r+1}\bar{\Upsilon}_{i,r+1}C_{i,r+1})^T\right\} \\ & + \mathbb{E}\left\{\mathcal{K}_{i,r+1}\varepsilon_{i,r+1}\varepsilon_{i,r+1}^T\mathcal{K}_{i,r+1}^T\right\} + \mathbb{E}\left\{\mathcal{K}_{i,r+1}\tilde{\Upsilon}_{i,r+1}C_{i,r+1}\mathrm{x}_{r+1}\mathrm{x}_{r+1}^T C_{i,r+1}^T\tilde{\Upsilon}_{i,r+1}^T\mathcal{K}_{i,r+1}^T\right\} \\ & + \mathbb{E}\left\{\mathcal{K}_{i,r+1}v_{i,r+1}v_{i,r+1}^T\mathcal{K}_{i,r+1}^T\right\} + \mathfrak{H}_1 + \mathfrak{H}_1^T - \mathfrak{H}_2 - \mathfrak{H}_2^T - \mathfrak{H}_3 - \mathfrak{H}_3^T, \end{aligned} \tag{15}$$

where

$$\begin{aligned} \mathfrak{H}_1 &= \mathbb{E}\left\{(I - \mathcal{K}_{i,r+1}\bar{\Upsilon}_{i,r+1}C_{i,r+1})\tilde{\mathrm{x}}_{i,r+1|r}\varepsilon_{i,r+1}^T\mathcal{K}_{i,r+1}^T\right\}, \\ \mathfrak{H}_2 &= \mathbb{E}\left\{\mathcal{K}_{i,r+1}\varepsilon_{i,r+1}\mathrm{x}_{r+1}^T C_{i,r+1}^T\tilde{\Upsilon}_{i,r+1}^T\mathcal{K}_{i,r+1}^T\right\}, \\ \mathfrak{H}_3 &= \mathbb{E}\left\{\mathcal{K}_{i,r+1}\varepsilon_{i,r+1}v_{i,r+1}^T\mathcal{K}_{i,r+1}^T\right\}. \end{aligned} \tag{16}$$

Proof. By means of (10) and covariance definition, the expression of the covariance matrix $D_{i,r+1|r+1}$ can be written as

$$\begin{aligned} D_{i,r+1|r+1} &= \mathbb{E}\{\tilde{\mathrm{x}}_{i,r+1|r+1}\tilde{\mathrm{x}}_{i,r+1|r+1}^T\} \\ &= \mathbb{E}\left\{(I - \mathcal{K}_{i,r+1}\bar{\Upsilon}_{i,r+1}C_{i,r+1})D_{i,r+1|r}(I - \mathcal{K}_{i,r+1}\bar{\Upsilon}_{i,r+1}C_{i,r+1})^T\right\} \\ &\quad + \mathbb{E}\left\{\mathcal{K}_{i,r+1}\varepsilon_{i,r+1}\varepsilon_{i,r+1}^T\mathcal{K}_{i,r+1}^T\right\} + \mathbb{E}\left\{\mathcal{K}_{i,r+1}\tilde{\Upsilon}_{i,r+1}C_{i,r+1}\mathrm{x}_{r+1}\mathrm{x}_{r+1}^T C_{i,r+1}^T\tilde{\Upsilon}_{i,r+1}^T\mathcal{K}_{i,r+1}^T\right\} \end{aligned}$$

$$
\begin{aligned}
&+\mathbb{E}\left\{\mathcal{K}_{i,\mathrm{r}+1}v_{i,\mathrm{r}+1}v_{i,\mathrm{r}+1}^T\mathcal{K}_{i,\mathrm{r}+1}^T\right\}+\mathfrak{H}_1+\mathfrak{H}_1^T-\mathfrak{H}_2-\mathfrak{H}_2^T-\mathfrak{H}_3-\mathfrak{H}_3^T-\mathfrak{D}_1-\mathfrak{D}_1^T-\mathfrak{D}_2\\
&-\mathfrak{D}_2^T+\mathfrak{D}_3+\mathfrak{D}_3^T,
\end{aligned}
\tag{17}
$$

where

$$
\begin{aligned}
\mathfrak{D}_1&=\mathbb{E}\left\{(I-\mathcal{K}_{i,\mathrm{r}+1}\bar{\Upsilon}_{i,\mathrm{r}+1}C_{i,\mathrm{r}+1})\tilde{\mathrm{x}}_{i,\mathrm{r}+1|\mathrm{r}}\mathrm{x}_{\mathrm{r}+1}^TC_{i,\mathrm{r}+1}^T\tilde{\Upsilon}_{i,\mathrm{r}+1}^T\mathcal{K}_{i,\mathrm{r}+1}^T\right\},\\
\mathfrak{D}_2&=\mathbb{E}\left\{(I-\mathcal{K}_{i,\mathrm{r}+1}\bar{\Upsilon}_{i,\mathrm{r}+1}C_{i,\mathrm{r}+1})\tilde{\mathrm{x}}_{i,\mathrm{r}+1|\mathrm{r}}v_{i,\mathrm{r}+1}^T\mathcal{K}_{i,\mathrm{r}+1}^T\right\},\\
\mathfrak{D}_3&=\mathbb{E}\left\{\mathcal{K}_{i,\mathrm{r}+1}\tilde{\Upsilon}_{i,\mathrm{r}+1}C_{i,\mathrm{r}+1}\mathrm{x}_{\mathrm{r}+1}v_{i,\mathrm{r}+1}^T\mathcal{K}_{i,\mathrm{r}+1}^T\right\}.
\end{aligned}
$$

It is obvious that $\mathfrak{D}_h=0$ $(h=1,2,3)$, which yields (15). ■

Remark 4. Following from the definitions of the local estimation errors, we have attained the recursive expressions with respect to the local PEC and FEC in Theorems 1 and 2, respectively. It should be noted that these expressions contain some uncertain/cross terms, which are generated due to the influence of the gain perturbation, MMMs and the linearization of nonlinear function. Thus, the accurate values of the local FEC and PEC cannot be obtained directly, which can be replaced by using their UBs commonly. Moreover, there is a need to minimize the local UB of the FEC and design filter gain at each instant properly.

Next, we will present the local UBs regarding the EEC, which can be expressed in iterative forms in the following theorem for each processor.

Theorem 3. *For each sensor subsystem, let $\gamma_{i,r}^u$ $(u=3,4,\cdots,7)$ be positive scalars. If*

$$
\Xi_{i,r+1|r}=A_{i,r}(\Xi_{i,r|r}^{-1}-\gamma_{i,r}^3M_{i,r}^TM_{i,r})^{-1}A_{i,r}^T+(\gamma_{i,r}^3)^{-1}F_{i,r}F_{i,r}^T+B_rQ_rB_r^T,
\tag{18}
$$

$$
\Xi_{i,r+1|r+1}=(1+\gamma_{i,r+1}^4)\Omega_{i,r+1}^2\Xi_{i,r+1|r}(\Omega_{i,r+1}^2)^T+K_{i,r+1}\Omega_{i,r+1}^3K_{i,r+1}^T+\bar{K}_{i,r+1}\Omega_{i,r+1}^4\bar{K}_{i,r+1}^T,
\tag{19}
$$

with the initial value $D_{i,0|0}\le\Xi_{i,0|0}$ and the condition

$$
(\gamma_{i,r}^3)^{-1}I-M_{i,r}\Xi_{i,r|r}M_{i,r}^T>0
\tag{20}
$$

have positive definite solutions $\Xi_{i,r+1|r}$ and $\Xi_{i,r+1|r+1}$, where

$$
\begin{aligned}
\Omega_{i,r+1}^0&=(1+\beta_i)\bar{\Psi}_{i,r+1}/\beta_i^2+(1+\beta_i^{-1})\varrho_i^2,\\
\bar{\Omega}_{i,r+1}^1&=(1+\gamma_{i,r+1}^7)\Xi_{i,r+1|r}+(1+(\gamma_{i,r+1}^7)^{-1})\hat{\mathrm{x}}_{i,r+1|r}\hat{\mathrm{x}}_{i,r+1|r}^T,\\
\Omega_{i,r+1}^2&=I-K_{i,r+1}\bar{\Upsilon}_{i,r+1}C_{i,r+1},\\
\Lambda_{i,r+1}&=\mathrm{diag}\left\{\bar{\varsigma}_{i,r+1}^{(1)}(1-\bar{\varsigma}_{i,r+1}^{(1)}),\bar{\varsigma}_{i,r+1}^{(2)}(1-\bar{\varsigma}_{i,r+1}^{(2)}),\cdots,\bar{\varsigma}_{i,r+1}^{(m_i)}(1-\bar{\varsigma}_{i,r+1}^{(m_i)})\right\},\\
\Omega_{i,r+1}^3&=\mu_{i,r+1}\Omega_{i,r+1}^0I+(1+(\gamma_{i,r+1}^5)^{-1})\Lambda_{i,r+1}\circ(C_{i,r+1}\bar{\Omega}_{i,r+1}^1C_{i,r+1}^T)+(1+(\gamma_{i,r+1}^6)^{-1})R_{i,r+1},\\
\Omega_{i,r+1}^4&=(1+\gamma_{i,r+1}^4)\bar{\Upsilon}_{i,r+1}C_{i,r+1}\Xi_{i,r+1|r}C_{i,r+1}^T\bar{\Upsilon}_{i,r+1}^T+\Omega_{i,r+1}^3,\\
\mu_{i,r+1}&=1+(\gamma_{i,r+1}^4)^{-1}+\gamma_{i,r+1}^5+\gamma_{i,r+1}^6
\end{aligned}
\tag{21}
$$

then we can get

$$
D_{i,r+1|r+1}\le\Xi_{i,r+1|r+1},\ \ D_{i,r+1|r}\le\Xi_{i,r+1|r}.
\tag{22}
$$

Moreover, if the local filter gain is selected as follows:

$$
K_{i,r+1}=(1+\gamma_{i,r+1}^4)\Xi_{i,r+1|r}C_{i,r+1}^T\bar{\Upsilon}_{i,r+1}^T(\Omega_{i,r+1}^4)^{-1},
\tag{23}
$$

then the minimized local UB for the FEC is obtained at every moment.

Proof. Firstly, in view of the above results, this theorem can be obtained by means of the mathematical induction. Taking the initial condition into account, we have $D_{i,0|0}\le\Xi_{i,0|0}$. Supposing $D_{i,\mathrm{r}|\mathrm{r}}\le\Xi_{i,\mathrm{r}|\mathrm{r}}$, then there is a need to check $D_{i,\mathrm{r}+1|\mathrm{r}+1}\le\Xi_{i,\mathrm{r}+1|\mathrm{r}+1}$. Next, using Lemma 2 and $(\gamma_{i,\mathrm{r}}^3)^{-1}I-M_{i,\mathrm{r}}\Xi_{i,\mathrm{r}|\mathrm{r}}M_{i,\mathrm{r}}^T>0$, we have

$$
(A_{i,\mathrm{r}}+F_{i,\mathrm{r}}U_{i,\mathrm{r}}M_{i,\mathrm{r}})D_{i,\mathrm{r}|\mathrm{r}}(A_{i,\mathrm{r}}+F_{i,\mathrm{r}}U_{i,\mathrm{r}}M_{i,\mathrm{r}})^T\le A_{i,\mathrm{r}}(D_{i,\mathrm{r}|\mathrm{r}}^{-1}-\gamma_{i,\mathrm{r}}^3M_{i,\mathrm{r}}^TM_{i,\mathrm{r}})^{-1}A_{i,\mathrm{r}}^T+(\gamma_{i,\mathrm{r}}^3)^{-1}F_{i,\mathrm{r}}F_{i,\mathrm{r}}^T,
\tag{24}
$$

where $\gamma_{i,\mathrm{r}}^3>0$ is a scalar. By substituting (24) into (14), we obtain

$$
D_{i,\mathrm{r}+1|\mathrm{r}}\le A_{i,\mathrm{r}}(D_{i,\mathrm{r}|\mathrm{r}}^{-1}-\gamma_{i,\mathrm{r}}^3M_{i,\mathrm{r}}^TM_{i,\mathrm{r}})^{-1}A_{i,\mathrm{r}}^T+(\gamma_{i,\mathrm{r}}^3)^{-1}F_{i,\mathrm{r}}F_{i,\mathrm{r}}^T+B_\mathrm{r}Q_\mathrm{r}B_\mathrm{r}^T.
\tag{25}
$$

Secondly, we will check that $D_{i,\mathrm{r}+1|\mathrm{r}+1} \le \Xi_{i,\mathrm{r}+1|\mathrm{r}+1}$ holds. According to the fact $\varepsilon_{i,\mathrm{r}+1}\varepsilon_{i,\mathrm{r}+1}^T \le \varepsilon_{i,\mathrm{r}+1}^T\varepsilon_{i,\mathrm{r}+1} I$, we obtain

$$\begin{aligned}\mathbb{E}\left\{\varepsilon_{i,\mathrm{r}+1}\varepsilon_{i,\mathrm{r}+1}^T\right\} &\le \mathbb{E}\left\{\varepsilon_{i,\mathrm{r}+1}^T\varepsilon_{i,\mathrm{r}+1}\right\} I \\ &\le \left((1+\beta_i)\mathbb{E}\{\rho_{i,\mathrm{r}}^2\}/\beta_i^2 + (1+\beta_i^{-1})\varrho_i^2\right) I \\ &\le \left((1+\beta_i)\bar{\Psi}_{i,\mathrm{r}+1}/\beta_i^2 + (1+\beta_i^{-1})\varrho_i^2\right) I \\ &:= \Omega_{i,\mathrm{r}+1}^0 I. \end{aligned} \tag{26}$$

Subsequently, the uncertain terms in (15) will be handled via Lemma 1, it follows that

$$\begin{aligned}\mathfrak{H}_1 + \mathfrak{H}_1^T \le{}& \gamma_{i,\mathrm{r}+1}^4 \mathbb{E}\left\{(I - \mathcal{K}_{i,\mathrm{r}+1}\bar{\Upsilon}_{i,\mathrm{r}+1}C_{i,\mathrm{r}+1})D_{i,\mathrm{r}+1|\mathrm{r}}(I - \mathcal{K}_{i,\mathrm{r}+1}\bar{\Upsilon}_{i,\mathrm{r}+1}C_{i,\mathrm{r}+1})^T\right\} \\ &+(\gamma_{i,\mathrm{r}+1}^4)^{-1}\mathbb{E}\left\{\mathcal{K}_{i,\mathrm{r}+1}\varepsilon_{i,\mathrm{r}+1}\varepsilon_{i,\mathrm{r}+1}^T\mathcal{K}_{i,\mathrm{r}+1}^T\right\}, \end{aligned} \tag{27}$$

$$\begin{aligned}-\mathfrak{H}_2 - \mathfrak{H}_2^T \le{}& \gamma_{i,\mathrm{r}+1}^5 \mathbb{E}\left\{\mathcal{K}_{i,\mathrm{r}+1}\varepsilon_{i,\mathrm{r}+1}\varepsilon_{i,\mathrm{r}+1}^T\mathcal{K}_{i,\mathrm{r}+1}^T\right\} + (\gamma_{i,\mathrm{r}+1}^5)^{-1}\mathbb{E}\left\{\mathcal{K}_{i,\mathrm{r}+1}\tilde{\Upsilon}_{i,\mathrm{r}+1}C_{i,\mathrm{r}+1}\mathrm{x}_{\mathrm{r}+1}\right. \\ &\left.\times \mathrm{x}_{\mathrm{r}+1}^T C_{i,\mathrm{r}+1}^T\tilde{\Upsilon}_{i,\mathrm{r}+1}^T\mathcal{K}_{i,\mathrm{r}+1}^T\right\}, \end{aligned} \tag{28}$$

$$-\mathfrak{H}_3 - \mathfrak{H}_3^T \le \gamma_{i,\mathrm{r}+1}^6 \mathbb{E}\left\{\mathcal{K}_{i,\mathrm{r}+1}\varepsilon_{i,\mathrm{r}+1}\varepsilon_{i,\mathrm{r}+1}^T\mathcal{K}_{i,\mathrm{r}+1}^T\right\} + (\gamma_{i,\mathrm{r}+1}^6)^{-1}\mathbb{E}\left\{\mathcal{K}_{i,\mathrm{r}+1}v_{i,\mathrm{r}+1}v_{i,\mathrm{r}+1}^T\mathcal{K}_{i,\mathrm{r}+1}^T\right\}, \tag{29}$$

with the positive scalars $\gamma_{i,\mathrm{r}+1}^4$, $\gamma_{i,\mathrm{r}+1}^5$ and $\gamma_{i,\mathrm{r}+1}^6$. Applying Lemma 1 again, we have

$$\mathbb{E}\left\{\mathrm{x}_{\mathrm{r}+1}\mathrm{x}_{\mathrm{r}+1}^T\right\} \le (1+\gamma_{i,\mathrm{r}+1}^7)D_{i,\mathrm{r}+1|\mathrm{r}} + (1+(\gamma_{i,\mathrm{r}+1}^7)^{-1})\hat{\mathrm{x}}_{i,\mathrm{r}+1|\mathrm{r}}\hat{\mathrm{x}}_{i,\mathrm{r}+1|\mathrm{r}}^T := \Omega_{i,\mathrm{r}+1}^1, \tag{30}$$

where $\gamma_{i,\mathrm{r}+1}^7 > 0$ is a scalar.

Substituting (26)–(30) into (15), we obtain

$$\begin{aligned}D_{i,\mathrm{r}+1|\mathrm{r}+1} \le{}& (1+\gamma_{i,\mathrm{r}+1}^4)\mathbb{E}\left\{(I - \mathcal{K}_{i,\mathrm{r}+1}\bar{\Upsilon}_{i,\mathrm{r}+1}C_{i,\mathrm{r}+1})D_{i,\mathrm{r}+1|\mathrm{r}}(I - \mathcal{K}_{i,\mathrm{r}+1}\bar{\Upsilon}_{i,\mathrm{r}+1}C_{i,\mathrm{r}+1})^T\right\} \\ &+(1+(\gamma_{i,\mathrm{r}+1}^4)^{-1} + \gamma_{i,\mathrm{r}+1}^5 + \gamma_{i,\mathrm{r}+1}^6)\mathbb{E}\left\{\mathcal{K}_{i,\mathrm{r}+1}\varepsilon_{i,\mathrm{r}+1}\varepsilon_{i,\mathrm{r}+1}^T\mathcal{K}_{i,\mathrm{r}+1}^T\right\} \\ &+(1+(\gamma_{i,\mathrm{r}+1}^5)^{-1})\mathbb{E}\left\{\mathcal{K}_{i,\mathrm{r}+1}\tilde{\Upsilon}_{i,\mathrm{r}+1}C_{i,\mathrm{r}+1}\mathrm{x}_{\mathrm{r}+1}\mathrm{x}_{\mathrm{r}+1}^T C_{i,\mathrm{r}+1}^T\tilde{\Upsilon}_{i,\mathrm{r}+1}^T\mathcal{K}_{i,\mathrm{r}+1}^T\right\} \\ &+(1+(\gamma_{i,\mathrm{r}+1}^6)^{-1})\mathbb{E}\left\{\mathcal{K}_{i,\mathrm{r}+1}v_{i,\mathrm{r}+1}v_{i,\mathrm{r}+1}^T\mathcal{K}_{i,\mathrm{r}+1}^T\right\} \\ ={}& (1+\gamma_{i,\mathrm{r}+1}^4)(I - K_{i,\mathrm{r}+1}\bar{\Upsilon}_{i,\mathrm{r}+1}C_{i,\mathrm{r}+1})D_{i,\mathrm{r}+1|\mathrm{r}}(I - K_{i,\mathrm{r}+1}\bar{\Upsilon}_{i,\mathrm{r}+1}C_{i,\mathrm{r}+1})^T \\ &+K_{i,\mathrm{r}+1}\check{\Omega}_{i,\mathrm{r}+1}^3 K_{i,\mathrm{r}+1}^T + \bar{K}_{i,\mathrm{r}+1}\check{\Omega}_{i,\mathrm{r}+1}^4\bar{K}_{i,\mathrm{r}+1}^T, \end{aligned} \tag{31}$$

where

$$\begin{aligned}\check{\Omega}_{i,\mathrm{r}+1}^3 ={}& \mu_{i,\mathrm{r}+1}\mathbb{E}\left\{\varepsilon_{i,\mathrm{r}+1}\varepsilon_{i,\mathrm{r}+1}^T\right\} + (1+(\gamma_{i,\mathrm{r}+1}^5)^{-1})\mathbb{E}\left\{\tilde{\Upsilon}_{i,\mathrm{r}+1}C_{i,\mathrm{r}+1}\mathrm{x}_{\mathrm{r}+1}\mathrm{x}_{\mathrm{r}+1}^T C_{i,\mathrm{r}+1}^T\tilde{\Upsilon}_{i,\mathrm{r}+1}^T\right\} \\ &+(1+(\gamma_{i,\mathrm{r}+1}^6)^{-1})R_{i,\mathrm{r}+1}, \end{aligned} \tag{32}$$

$$\check{\Omega}_{i,\mathrm{r}+1}^4 = \left((1+\gamma_{i,\mathrm{r}+1}^4)\bar{\Upsilon}_{i,\mathrm{r}+1}C_{i,\mathrm{r}+1}D_{i,\mathrm{r}+1|\mathrm{r}}C_{i,\mathrm{r}+1}^T\bar{\Upsilon}_{i,\mathrm{r}+1}^T + \check{\Omega}_{i,\mathrm{r}+1}^3\right), \tag{33}$$

with $\mu_{i,\mathrm{r}+1}$ mentioned in (21). In the light of Lemma 3 and (30), it follows that

$$\begin{aligned}\mathbb{E}\left\{\tilde{\Upsilon}_{i,\mathrm{r}+1}C_{i,\mathrm{r}+1}\mathrm{x}_{\mathrm{r}+1}\mathrm{x}_{\mathrm{r}+1}^T C_{i,\mathrm{r}+1}^T\tilde{\Upsilon}_{i,\mathrm{r}+1}^T\right\} &= \mathbb{E}\left\{\tilde{\Upsilon}_{i,\mathrm{r}+1}\tilde{\Upsilon}_{i,\mathrm{r}+1}^T\right\}\circ\mathbb{E}\left\{C_{i,\mathrm{r}+1}\mathrm{x}_{\mathrm{r}+1}\mathrm{x}_{\mathrm{r}+1}^T C_{i,\mathrm{r}+1}^T\right\} \\ &\le \Lambda_{i,\mathrm{r}+1}\circ(C_{i,\mathrm{r}+1}\Omega_{i,\mathrm{r}+1}^1 C_{i,\mathrm{r}+1}^T), \end{aligned} \tag{34}$$

where $\Lambda_{i,\mathrm{r}+1}$ is derived as (21). By substituting (26), (30) and (34) into (31), we have

$$\begin{aligned}D_{i,\mathrm{r}+1|\mathrm{r}+1} \le{}& (1+\gamma_{i,\mathrm{r}+1}^4)(I - K_{i,\mathrm{r}+1}\bar{\Upsilon}_{i,\mathrm{r}+1}C_{i,\mathrm{r}+1})D_{i,\mathrm{r}+1|\mathrm{r}}(I - K_{i,\mathrm{r}+1}\bar{\Upsilon}_{i,\mathrm{r}+1}C_{i,\mathrm{r}+1})^T \\ &+K_{i,\mathrm{r}+1}\left(\mu_{i,\mathrm{r}+1}\Omega_{i,\mathrm{r}+1}^0 I + (1+(\gamma_{i,\mathrm{r}+1}^5)^{-1})\Lambda_{i,\mathrm{r}+1}\circ(C_{i,\mathrm{r}+1}\Omega_{i,\mathrm{r}+1}^1 C_{i,\mathrm{r}+1}^T)\right. \\ &\left.+(1+(\gamma_{i,\mathrm{r}+1}^6)^{-1})R_{i,\mathrm{r}+1}\right)K_{i,\mathrm{r}+1}^T + \bar{K}_{i,\mathrm{r}+1}\left((1+\gamma_{i,\mathrm{r}+1}^4)\bar{\Upsilon}_{i,\mathrm{r}+1}C_{i,\mathrm{r}+1}D_{i,\mathrm{r}+1|\mathrm{r}}C_{i,\mathrm{r}+1}^T\bar{\Upsilon}_{i,\mathrm{r}+1}^T\right. \\ &+\mu_{i,\mathrm{r}+1}\Omega_{i,\mathrm{r}+1}^0 I + (1+(\gamma_{i,\mathrm{r}+1}^5)^{-1})\Lambda_{i,\mathrm{r}+1}\circ(C_{i,\mathrm{r}+1}\Omega_{i,\mathrm{r}+1}^1 C_{i,\mathrm{r}+1}^T) \\ &\left.+(1+(\gamma_{i,\mathrm{r}+1}^6)^{-1})R_{i,\mathrm{r}+1}\right)\bar{K}_{i,\mathrm{r}+1}^T. \end{aligned} \tag{35}$$

Based on the induction method as well as (18), (19), (25) and (35), it is easy to see that $D_{i,\mathrm{r}+1|\mathrm{r}+1} \le \Xi_{i,\mathrm{r}+1|\mathrm{r}+1}$.

Next, the filter gain is designed, which can be deduced by minimizing local UB concerning on the FEC at every instant. In particular, we arrive at

$$\frac{\partial \mathrm{tr}(\Xi_{i,r+1|r+1})}{\partial K_{i,r+1}} = -2(1+\gamma^4_{i,r+1})(I - K_{i,r+1}\bar{\Upsilon}_{i,r+1}C_{i,r+1})\Xi_{i,r+1|r}C^T_{i,r+1}\bar{\Upsilon}^T_{i,r+1} + 2K_{i,r+1}\Omega^3_{i,r+1}, \tag{36}$$

where $\Omega^3_{i,r+1}$ is defined in (21). Then, the filter gain can be given as in (23) by letting the above equation be zero, and the proof is complete. ■

In the light of the analyses above, one can derive the local filter gain, which can guarantee the obtained UB of the local FEC to be minimized for each processor at every moment. In addition, it is remarkable to note that some scalars $\gamma^u_{i,r}$ $(u = 3, \cdots, 7)$ in Theorem 3 are utilized during the process of tackling the uncertain/cross terms by using some mathematical methods. When the developed approach is carried out, these scalars need to be regulated and fixed such that the constraint condition (20) can be satisfied, which can further strengthen the flexibility of the proposed algorithm. In the following subsection, in the light of the local filters, a DRFF criterion is provided in terms of the ICI fusion strategy.

3.2. Fusion filtering method

Due to the DETM influence, the expression of the estimation error cross-covariance matrices between any two local processors is difficult to be calculated. Consequently, the DRFF algorithm is carried out in terms of the ICI fusion rule, which only needs the information of the local FEC [34]. The detailed contents of the ICI fusion method are shown below.

In this subsection, based on the local estimation $\hat{x}_{i,r|r}$ $(i = 1, \cdots, L)$ as well as the UB of the FEC $\Xi_{i,r|r}$, the ICI fusion estimator $\hat{x}^{ICI}_{r|r}$ and covariance $\Xi^{ICI}_{r|r}$ are

$$\hat{x}^{ICI}_{r|r} = \sum_{i=1}^{L} \Pi^{ICI}_{i,r|r}\hat{x}_{i,r|r}, \tag{37}$$

$$\Xi^{ICI}_{r|r} = \left(\sum_{i=1}^{L}\Xi^{-1}_{i,r|r} - \left(\sum_{i=1}^{L} W_i\Xi_{i,r|r}\right)^{-1}\right)^{-1}, \tag{38}$$

where the weighted coefficients $0 \le W_i \le 1$ satisfy

$$\sum_{i=1}^{L} W_i = 1. \tag{39}$$

The fusion gains $\Pi^{ICI}_{i,r|r}$ are obtained by:

$$\Pi^{ICI}_{i,r|r} = \Xi^{ICI}_{r|r}\left(\Xi^{-1}_{i,r|r} - W_i\left(\sum_{i=1}^{L} W_i\Xi_{i,r|r}\right)^{-1}\right). \tag{40}$$

The aforementioned DRFF algorithm can be transformed into an optimization problem which can be described as follows:

$$\min_{W_i}\left\{\mathrm{tr}(\Xi^{ICI}_{r|r})\right\}, \quad \text{s.t.} \sum_{i=1}^{L} W_i = 1,\ W_i \in [0,\ 1]. \tag{41}$$

The solutions of (41) can be obtained by means of using "fmincon" function in Matlab software.

Next, the DRFF approach is checked whether it can achieve the goal based on the ICI fusion rule.

Theorem 4. *For the MSNSs (1) and (2), the DRFF rule (37)–(41) is true, one has*

$$P^{ICI}_{r|r} = \mathbb{E}\left\{(x_r - \hat{x}^{ICI}_{r|r})(x_r - \hat{x}^{ICI}_{r|r})^T\right\} \le \Xi^{ICI}_{r|r}. \tag{42}$$

Proof. Based on the fact that $D_{i,r|r} \le \Xi_{i,r|r}$ holds, the proof can be easily derived in terms of the proposed results in [34]. ■

Remark 5. As in (41), a minimization problem with the constraint has been considered to propose the DRFF algorithm. The corresponding solutions can be calculated, which are locally optimal one. In addition, more efforts and work can be done to develop the DRFF algorithm for the purpose to provide better fusion result in near future.

Remark 6. Notice that the DETM has been applied in MSNSs, which can influence each measurement output of the sensor, and then the local filter and UBs of the EEC have been respectively derived in (5), (6) and Theorem 3. In a general way, the above results are transmitted to the fusion center, which can flexibly provide new estimation by using the proper fusion rule. In this paper, the

Algorithm 1 DETMBDRFF Algorithm.

Step 1: Set $\mathrm{r}=0$ and initialize parameters.
Step 2: Calculate $\hat{x}_{i,\mathrm{r}+1|\mathrm{r}}$ by (5).
Step 3: Compute $\Xi_{i,\mathrm{r}+1|\mathrm{r}}$ by (18).
Step 4: Compute the filter gain $K_{i,\mathrm{r}+1}$ in accordance with (23).
Step 5: Calculate $\hat{x}_{i,\mathrm{r}+1|\mathrm{r}+1}$ with the help of (6).
Step 6: Compute $\Xi_{i,\mathrm{r}+1|\mathrm{r}+1}$ via (19).
Step 7: Obtain $\hat{x}^{ICI}_{\mathrm{r}|\mathrm{r}}$ by (37).
Step 8: Obtain the fusion EEC $\Xi^{ICI}_{\mathrm{r}|\mathrm{r}}$ from (38).
Step 9: Let $\mathrm{r}=\mathrm{r}+1$. Return to *Step* 2.

cross-covariances from any two sensors cannot be easily calculated because the received measurement outputs from each sensor have some differences under the DETM. As such, the MW criterion cannot be utilized in terms of the case. Accordingly, the ICI criterion has been used, whose features are that the cross-covariance matrices are unnecessary to compute, and the computation amount can be reduced.

In accordance with the local recursive formulations in Theorem 3 as well as the fusion method, the DETM-based DRFF (DETMB-DRFF) algorithm can be presented as follows.

Remark 7. According to the above implementation process of the DETMBDRFF algorithm, it is easy to see that the proposed approach can be performed with a recursive form. To be specific, in time domain, the local filter and UBs of the EEC have been respectively descried in (5), (6), (18), and (19). Based on the above expressions from each signal processor, the fusion results have been obtained in (37) and (38). Accordingly, the newly presented DETMBDRFF algorithm is capable to carry out in computer by using the algorithm step. It could be noted that the fusion estimation approach still has some limitations and defects in the existing literature. To obtain better performance, it is necessary to establish other novel fusion strategies which can fit different scenes in future work.

4. Monotonicity analysis

In this section, our intention is to discuss how the triggering parameter of the DTEM affects the estimation performance from a theoretical perspective. In the light of the Theorem 3, the monotonic relationship between the triggering parameter β_i and the local UB of the FEC is provided as follows.

Denoting the local UBs of $D_{i,\mathrm{r}|\mathrm{r}}$ and $D_{i,\mathrm{r}+1|\mathrm{r}}$ by $\Xi^{\beta_i}_{i,\mathrm{r}|\mathrm{r}}$ and $\Xi^{\beta_i}_{i,\mathrm{r}+1|\mathrm{r}}$ based on the triggering parameter β_i, respectively, we set

$$\begin{aligned}\mathfrak{C}_i(\Xi_{i,\mathrm{r}|\mathrm{r}}) &= \Xi_{i,\mathrm{r}+1|\mathrm{r}}\\ &= A_{i,\mathrm{r}}(\Xi^{-1}_{i,\mathrm{r}|\mathrm{r}} - \gamma^3_{i,\mathrm{r}} M^T_{i,\mathrm{r}} M_{i,\mathrm{r}})^{-1} A^T_{i,\mathrm{r}} + (\gamma^3_{i,\mathrm{r}})^{-1} F_{i,\mathrm{r}} F^T_{i,\mathrm{r}} + B_{\mathrm{r}} Q_{\mathrm{r}} B^T_{\mathrm{r}},\end{aligned} \tag{43}$$

$$\begin{aligned}\mathrm{r}_i(\beta_i,\ K_{i,\mathrm{r}+1},\ \Xi_{i,\mathrm{r}|\mathrm{r}}) &= (1+\gamma^4_{i,\mathrm{r}+1})\Omega^2_{i,\mathrm{r}+1}\mathfrak{C}_i(\Xi_{i,\mathrm{r}|\mathrm{r}})(\Omega^2_{i,\mathrm{r}+1})^T + K_{i,\mathrm{r}+1}\Omega^3_{i,\mathrm{r}+1}K^T_{i,\mathrm{r}+1}\\ &\quad + \bar{K}_{i,\mathrm{r}+1}\Omega^4_{i,\mathrm{r}+1}\bar{K}^T_{i,\mathrm{r}+1}.\end{aligned} \tag{44}$$

Then, according to Theorem 3, it follows that $\Xi^{\beta_i}_{i,\mathrm{r}+1|\mathrm{r}} = \mathfrak{C}_i(\Xi_{i,\mathrm{r}|\mathrm{r}})$, $\Xi^{\beta_i}_{i,\mathrm{r}+1|\mathrm{r}+1} = \mathrm{r}_i(\beta_i,\ K^{\beta_i}_{i,\mathrm{r}+1},\ \Xi^{\beta_i}_{i,\mathrm{r}|\mathrm{r}})$ with $K^{\beta_i}_{i,\mathrm{r}+1} = (1+\gamma^4_{i,\mathrm{r}+1})\Xi^{\beta_i}_{i,\mathrm{r}+1|\mathrm{r}} C^T_{i,\mathrm{r}+1} \bar{\Upsilon}^T_{i,\mathrm{r}+1} \Omega^{-4}_{\beta_i,\mathrm{r}+1}$, where the only difference between matrix $\Omega^4_{\beta_i,\mathrm{r}+1}$ and $\Omega^4_{i,\mathrm{r}+1}$ is that $\Xi_{i,\mathrm{r}+1|\mathrm{r}}$ in the latter one is replaced by $\Xi^{\beta_i}_{i,\mathrm{r}+1|\mathrm{r}}$.

Theorem 5. *For two scalars $\underline{\beta}_i > 0$ and $\bar{\beta}_i > 0$, if the scalars satisfy $\underline{\beta}_i < \bar{\beta}_i$, we have*

$$\Xi^{\underline{\beta}_i}_{i,r+1|r+1} \ge \Xi^{\bar{\beta}_i}_{i,r+1|r+1}. \tag{45}$$

Proof. It is simple to see that the inequality (45) is true for $\mathrm{r}=0$. Assuming $\Xi^{\underline{\beta}_i}_{i,\mathrm{r}|\mathrm{r}} \ge \Xi^{\bar{\beta}_i}_{i,\mathrm{r}|\mathrm{r}}$, we have

$$\begin{aligned}\Xi^{\underline{\beta}_i}_{i,\mathrm{r}+1|\mathrm{r}+1} &= \mathrm{r}_i(\underline{\beta}_i,\ K^{\underline{\beta}_i}_{i,\mathrm{r}+1},\ \Xi^{\underline{\beta}_i}_{i,\mathrm{r}|\mathrm{r}})\\ &\ge \mathrm{r}_i(\underline{\beta}_i,\ K^{\bar{\beta}_i}_{i,\mathrm{r}+1},\ \Xi^{\underline{\beta}_i}_{i,\mathrm{r}|\mathrm{r}})\\ &\ge \mathrm{r}_i(\bar{\beta}_i,\ K^{\bar{\beta}_i}_{i,\mathrm{r}+1},\ \Xi^{\bar{\beta}_i}_{i,\mathrm{r}|\mathrm{r}})\\ &= \Xi^{\bar{\beta}_i}_{i,\mathrm{r}+1|\mathrm{r}+1},\end{aligned} \tag{46}$$

because of the monotonicity of $\mathrm{r}_i(\beta_i,\ K_{i,\mathrm{r}+1},\ \Xi_{i,\mathrm{r}|\mathrm{r}})$ in β_i and $\Xi_{i,\mathrm{r}|\mathrm{r}}$, thus ending the proof regarding the monotonicity analysis. ■

Remark 8. In this paper, the DETM is introduced for nonlinear MSNSs to improve the utilization of the network resource. The theoretical analysis of the fact that a change of the triggering parameter influences the estimation performance is shown in Theorem 5. In engineering practice, if a larger value of the triggering parameter β_i is taken, then it would increase the number of triggering times, which may yield a higher precision of the estimation since more useful information can be transmitted. Moreover, in view of the recursive forms of the $\Xi_{i,r+1|r}$ and $\Xi_{i,r|r}$, the UBs would be affected as the system noises and gain perturbations increase.

Remark 9. So far, the DRFF issue has been handled for stochastic nonlinear MSNSs with time-varying system parameters and MMMs via the DETM. From Theorem 3, the local UBs of the FEC and PEC have been provided for each processor. It is shown that the derived local UB of the FEC is minimized by providing proper filter gain. In particular, we can observe that the corresponding information of time-varying system parameters, MMMs and DETM can be explicitly reflected in the proposed fusion filtering method. Moreover, we provide the algorithm analysis from the theoretical perspective, where the relationship between the triggering parameter of DTEM and estimation performance is clarified. In addition, based on the deduced UB on the FEC in Theorem 3, the boundedness analysis problem of the filtering error can also be discussed. During this process, it is noteworthy that some assumptions should be provided before. For example, the parameter matrices $A_{i,r}$, $B_{i,r}$, $C_{i,r}$, $F_{i,r}$, $M_{i,r}$, $\bar{K}_{i,r}$, and other noise dependent information $R_{i,r}$, $Q_{i,r}$, $\bar{\Upsilon}_{i,r}$ can be assumed to be bounded in certain range. Moreover, other scalars ϱ_i, β_i, ϖ_i, $\gamma_{i,r}^l$ $(l = 1, 2, \cdots, 7)$ are fixed values, and the constraint condition (20) can be ensured. In view of the above assumptions, the boundedness of filtering error can be achieved and corresponding theoretical results will be given in future investigations.

5. An illustrative example

In this section, we provide the simulation tests with comparison experiments to demonstrate the feasibility and advantages of new DETMBDRFF algorithm. In particular, both the MMMs with different missing probabilities and DETM with different triggering rates are discussed. Moreover, the estimation accuracy based on ICI fusion filter and CI fusion filter is compared and demonstrated.

Consider the stochastic nonlinear system (1)–(2) including three sensors, where the relevant time-varying system parameters are expressed as follows:

$$B_r = \begin{bmatrix} 0.065 \\ 0.064 + 0.01\cos(0.2r) \end{bmatrix}, \quad C_{1,r} = \begin{bmatrix} 1.1 & 1.2 + 0.1\cos(0.1r) \end{bmatrix},$$

$$C_{2,r} = \begin{bmatrix} 1.52 & 1.7 + 0.2\cos(0.2r) \end{bmatrix}, \quad C_{3,r} = \begin{bmatrix} 1.22 & 1.2 + 0.3\cos(0.1r) \end{bmatrix}.$$

Choose the following nonlinear function $f(x_r)$:

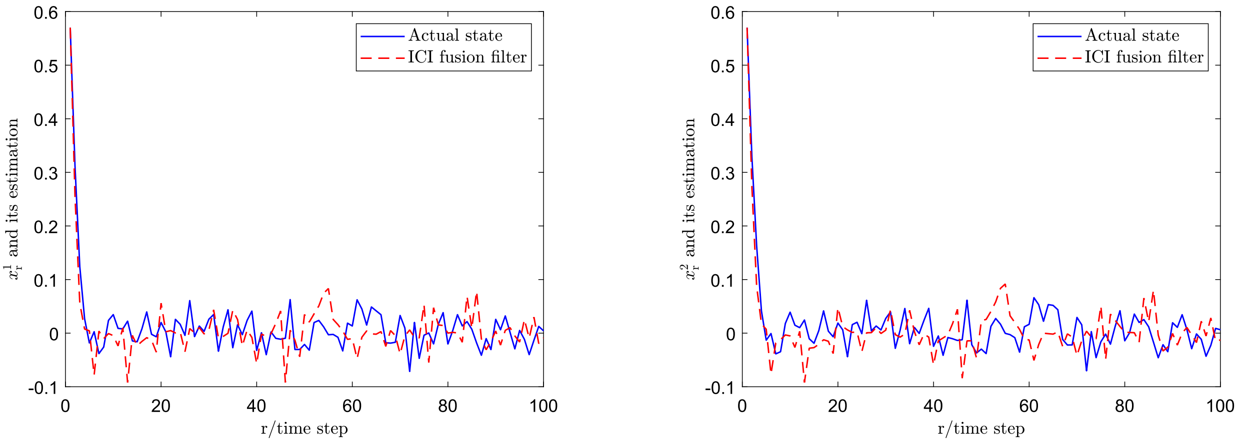
$$f(x_r) = \begin{bmatrix} -0.415x_r^1 + 0.445x_r^2 + 0.8x_r^1 x_r^2 \\ -0.326x_r^1 + 0.556x_r^2 + 0.6\sin(r)x_r^1 x_r^2 \end{bmatrix},$$

where $x_r = \begin{bmatrix} x_r^1 & x_r^2 \end{bmatrix}^T$ is system state.

In this simulation, the variances of Gaussian noise sequences are $Q_r = 0.151$, $R_{i,r} = 0.22$ with $i = 1, 2, 3$ respectively. Choose some related parameters as $\gamma_{i,r}^1 = \gamma_{i,r}^4 = \gamma_{i,r}^5 = \gamma_{i,r}^6 = 0.21$, $\gamma_{i,r}^2 = \gamma_{i,r}^7 = 1$, $\bar{\varsigma}_{i,r}^{(1)} = 0.75$. In addition, the initial values are $\bar{x}_0 = \hat{x}_{i,0|0} = \begin{bmatrix} 0.57 & 0.57 \end{bmatrix}^T$, $P_{0|0} = \Xi_{i,0|0} = 2.5I_2$, $F_{i,r} = 0.1I_2$, $M_{i,r} = 0.1I_2$, $\bar{K}_{i,r} = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}^T$ and $\gamma_{i,r}^3 = \left(1.5\lambda_{\max}(M_{i,r}\Xi_{i,r|r}M_{i,r}^T) + 1\right)^{-1}$.

In view of the dynamic triggering condition (3) and (4), the thresholds are selected by $\varrho_1 = 0.47$, $\varrho_2 = 0.52$ and $\varrho_3 = 0.42$. Moreover, the other parameters are taken as $\varpi_i = 0.1$, $\beta_i = 15$ and the initial values of the internal dynamic variable are set as $\rho_{i,0} = 1$. Therefore, in the light of the parameters, the distributed resilient fusion filter $\hat{x}_{r|r}^{ICI}$ can be derived at each instant. The results of the simulation are presented in Figs. 1-8. Among them, the sub-figures (a) and (b) represent the two state components in Figs. 1-5. In particular, the newly designed fusion filter is plotted in Fig. 1, in which the solid curves represent actual states and the dashed curves plot the state estimation value by using proposed fusion filters. Fig. 2 shows the comparison of traces of the UB for the local FEC and the fusion FEC, it can be easily seen that the estimation performance of new developed fusion filter is better than local filters. This observation further shows the advantages of fusion filtering scheme compared with the local filtering method. In order to further check the validity of presented DRFF approach, the mean-square errors (MSEs) curves are plotted in Fig. 3, where MSEs $= \left(\sum_{l=1}^{100}(x_r^{\tau(l)} - \hat{x}_{i,r|r}^{\tau(l)})^2\right)/100$ $(\tau = 1, 2)$ and the symbol τ denotes state component. From the above simulations, we can see that the ICI-based fusion filtering algorithm can carry out well to estimate the actual state and the UBs are above than their MSEs.

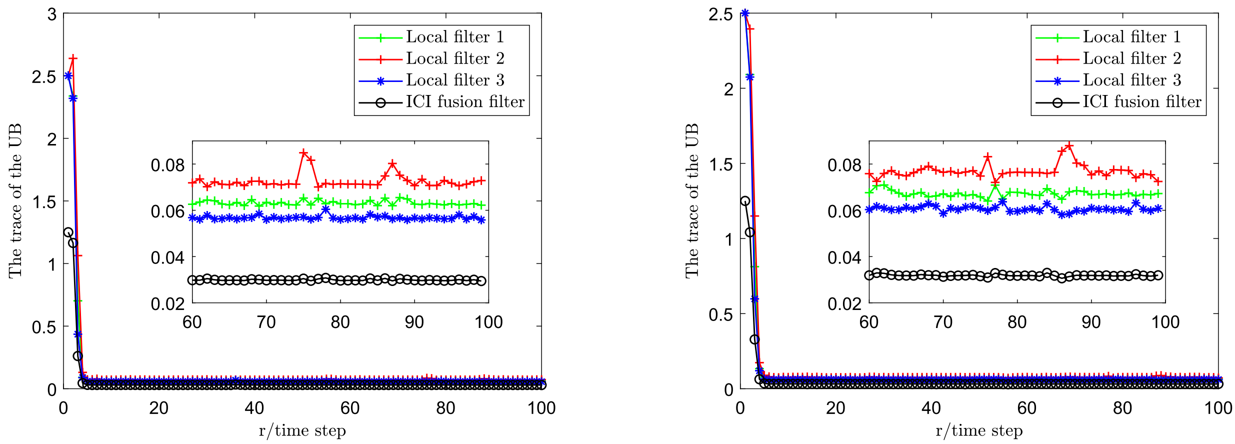
In order to further demonstrate the effect from the MMMs, the traces of the UB concerning on the FEC are shown in Fig. 4 under four different cases, i.e., $\bar{\varsigma}_i^{(1)}$ are 0.25, 0.5, 0.75, and 1, respectively. From the curves, we can see that the UBs decrease when the probability of the MMMs increases, i.e., when more useful signals are transmitted to the filter in the process of information transmission. Hence, it follows from the simulation results that new fusion filtering algorithm performs well. Furthermore, we present Fig. 5 to reflect the influence of the triggering parameter β_i on estimation accuracy when the values of β_i are 15, 25, 100 and 10^4. It is observed that the traces of UB increase as β_i decreases from Fig. 5, and the UB is minimum when β_i tends to the largest number. Moreover, Table 1 displays the triggering rate with regard to different β_i, which is concluded that the triggering rate increases monotonically when β_i increases. From Table 1 and Fig. 5, it can be known that the presented DETM relieves the communication load. Therefore, we reach the conclusion that the DETM is more efficient to be used in comparison with SETM. Moreover, for each sensor node, Fig. 6 shows the triggering times under the DETM when the value of triggering parameter β_i is 15. In addition, from the purpose of comparison regarding different fusion methods, Fig. 7 displays the estimation accuracy comparison of the developed DETMBDRFF algorithm in terms of new fusion method in our paper and the CI fusion approach from [3], respectively. It is easy to observe that the estimation accuracy of proposed fusion filter is superior to the CI fusion filter.



(a) The first state component

(b) The second state component

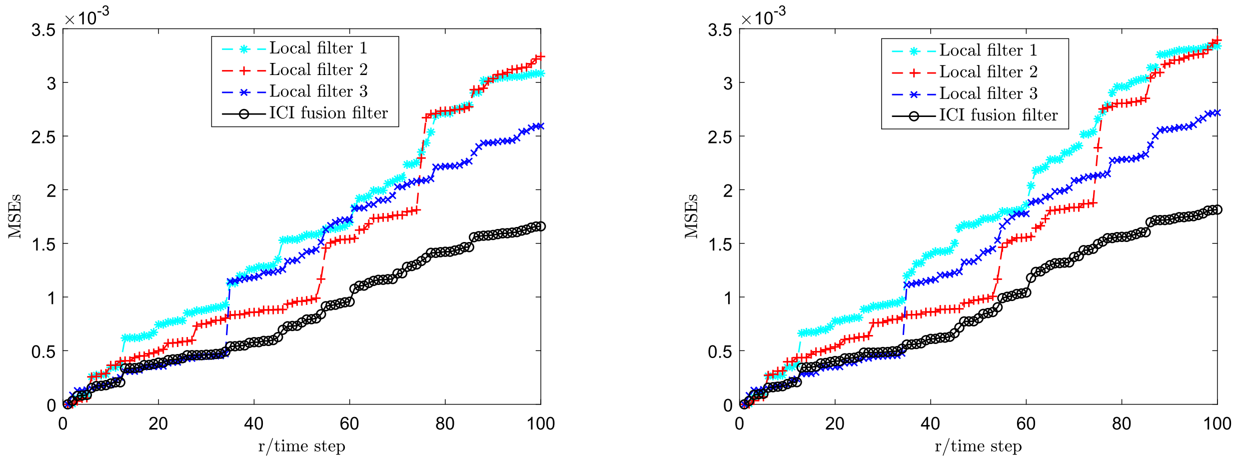
Fig. 1. State x_r^1, x_r^2 and their fusion filters.



(a) The first state component

(b) The second state component

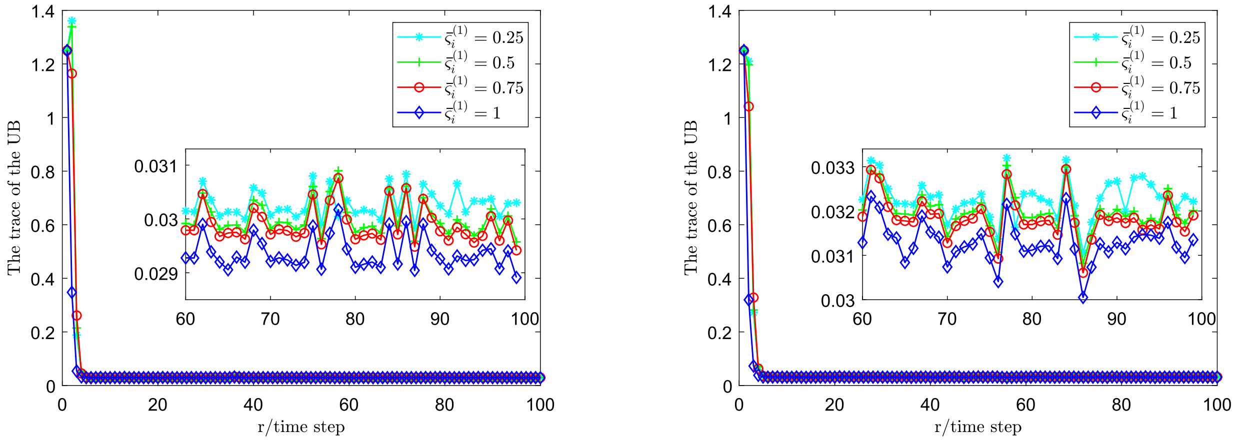
Fig. 2. The traces of the UB for the ICI fusion filter and local filters 1, 2, 3.



(a) The first state component

(b) The second state component

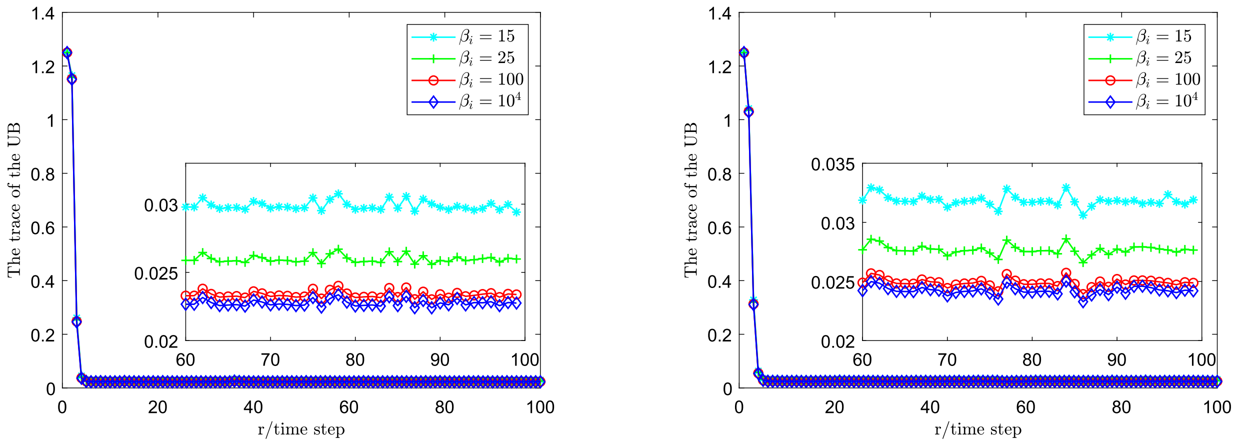
Fig. 3. Comparison of the MSEs between the ICI fusion filter and local filters 1, 2, 3.



(a) The first state component

(b) The second state component

Fig. 4. The traces of the UB of the fusion FEC under different missing probabilities.



(a) The first state component

(b) The second state component

Fig. 5. The traces of the UB of the fusion FEC under different values of β_i.

Table 1
Triggering rates of sensor node i $(i = 1, 2, 3)$.

	Sensor node 1	Sensor node 2	Sensor node 3
DETM ($\beta_i = 15$)	27%	32%	35%
DETM ($\beta_i = 25$)	31%	32%	35%
DETM ($\beta_i = 100$)	31%	37%	36%
DETM ($\beta_i = 10^4$)	32%	39%	38%

In the simulation, the value of β_i is also taken as 10^4 for the comparison purpose. So far, we select different values of β_i to further explain the influence of different parameters on estimation accuracy. According to the Fig. 5 and Table 1, one can see that the aforementioned results have been respectively shown based on different parameters β_i. For instance, the variation range of the UB is gradually smaller with the increasingly larger of the parameter β_i based on the results in Fig. 5. In Table 1, when the parameter β_i takes a very large number, the triggering rate seems to be unchanged. In view of the above analysis, we can obtain that the DETM can further relieve the consumption of the communication resources, and the estimation accuracy of the presented approach can be still ensured.

In order to present an explicit geometrical interpretation regarding accuracy relations of the local filters and fusion filters, the covariance ellipses of the local FECs Ξ_1, Ξ_2, Ξ_3 and the fusion FECs Ξ_{ICI} and Ξ_{CI} are shown in Fig. 8. The simulation result is that the inclusion relation between the ellipses of the fusion FECs and the ellipses of the local FECs is $\Xi_{ICI} \leq \Xi_{CI} \leq \Xi_i$ $(i = 1, 2, 3)$, which

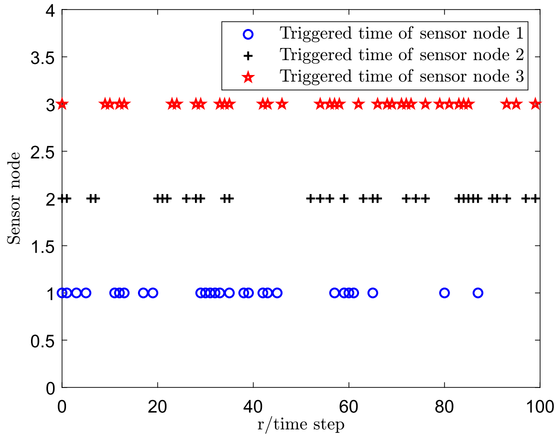


Fig. 6. Triggering times of three sensor nodes.

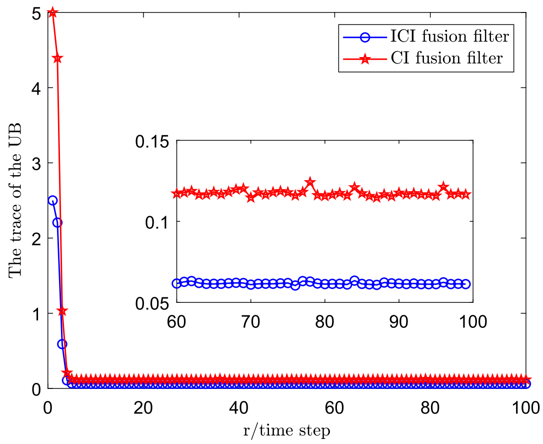


Fig. 7. The traces of UB for the ICI fusion filter and CI fusion filter.

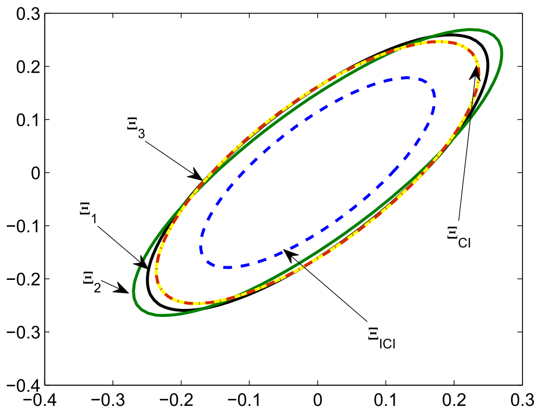


Fig. 8. The accuracy comparison of the FECs Ξ_i $(i = 1, 2, 3)$, Ξ_{ICI} and Ξ_{CI}.

demonstrates that the accuracy of the ICI fusion filter is superior to the CI fusion filter and the local filters when the iterative step r is selected as 100.

Remark 10. So far, we make some comparison experiments in order to better demonstrate the validity and advantages of new DETMBDRFF algorithm, where the MMMs, DETM and fusion accuracy are all well discussed and examined. To be more specific, we conduct the simulation comparisons with regard to MMMs under different occurrence probabilities, which illustrate the influences from MMMs with different cases. Moreover, the impacts from the DETM are taken into account, where some communication situations with different triggering parameters are shown to regulate the transmission frequency. Furthermore, the so-called fusion estimation accuracy is compared by using newly proposed fusion method and the CI fusion approach. The above comparisons and discussions further demonstrate the advantages of main results.

In accordance with the above results, the DRFF problem of this paper has been carried out for time-varying nonlinear MSNSs subject to MMMs under the DETM, and the corresponding advantages compared with other methods can be summarized from three aspects. 1) A novel approach has been developed for nonlinear MSNSs, which might provide higher precision than that for nonlinear systems with a single sensor; 2) Under the dynamical communication rule, the presented DRFF approach can reach a balance, which can regulate the relationship between network resource and estimation accuracy of this approach; 3) An optimized upper bound has been obtained in contrast with the CI fusion filter.

6. Conclusion remarks

In this paper, we have made great efforts to address the DRFF problem for stochastic nonlinear MSNSs subject to MMMs as well as DETMs. For each sensor node, the phenomenon of the MMMs has been depicted in the light of the Bernoulli distribution and local gain perturbation has been discussed. Moreover, the DETM has been used to save the consumption of network resources. With the help of the induction approach, the local UBs regarding the FEC and PEC have been derived, where the obtained local UB of the FEC has been minimized for each processor by properly determining local filter parameter. Besides, a rigorous theoretical proof has been shown, where we discussed the monotonicity relationship between the triggering parameter and obtained UB of the FEC. The major features of proposed DRFF mechanism lie in that an optimized filtering method with distributed format has been given and the desirable performance analysis has been discussed via theoretical derivations. Moreover, in terms of the recursive characteristic of the presented DRFF method, it is suitable to the online computation. Finally, the simulations with certain comparisons have testified the effectiveness and correctness of presented DRFF strategy. Furthermore, the study extensions of the provided approach may involve the research on the other communication protocols including the random access protocol, weighted try-once-discard protocol as in [47,48].

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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