A Promising Framework Based on Multistep Continuous Newton Scheme for Developing Robust Power-Flow Methods

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Abstract: Solving Power-Flow problem of ill-conditioned systems is still a challenge in realistic power systems due to most of available techniques are quite inefficient. This paper aims to address this issue by introducing a novel Power-Flow solution paradigm. It basically reformulates the traditional Continuous Newton's method in a multistep scheme, so that, the variables are progressively refined each step using different numerical arrangements. The developed solution paradigm envisages a novel family of Power-Flow techniques. For the sake of exemplify, two novel methods based on the introduced solution paradigm are developed. They are tested in several large-scale realistic ill-conditioned systems under different stressing and demanding conditions. Up to eight well-known Power-Flow solution techniques are considered for comparison. Results shown that the introduced solution paradigm constitutes a promising framework for developing robust and efficient Power-Flow methods, which may be widespread used in industry applications.

1. Introduction

Typically, solution of the Power-Flow (PF) problem is found using the traditional Newton-Raphson technique (NR). Despite that solution of the PF problem has been profusely studied during decades (interesting readers can find an updated literature review about this topic in [1]), solving this problem in ill-conditioned cases is still not trivial and might be challenging in realistic systems [2].

In order to illustrate problematic derived of ill-conditioned systems, let us analyze the three situations depicted in Fig. 1. In order to simplify, we have considered a one-variable case. In Fig. 1a, the well-conditioned scenario is showed, in such case, a standard numerical algorithm (like the NR), properly finds the solution in very few iterations. However, NR and Newton-like methods may fail in the situations depicted in Fig. 1b. In such case, the Jacobian matrix is not continuously invertible which may lead to divergence. In the case showed in Fig. 1c, instability of the numerical algorithm is provoked by the guess considered for initializing the iterative procedure. Specifically, if the starting point is outside of the Region of Attraction, numerical mapping becomes unstable [3]. According to [4], this behavior is symptomatic of ill-conditioning when the initial guess is a flat start and NR is used as a solution technique.

In power systems, ill conditioning of PF equations are mainly due to large R/X ratio, operation near to the maximum loadability point (MLP) or incidence of large and short branches at the same bus [5]. Solving this kind of systems demand robustness properties of numerical methods. Various robust PF solution techniques have been specifically devoted to solve this kind of systems.

One of the most traditional robust PF approach is the Iwamoto's method [6]. It is really a step-size based Newton-like method, since it is essentially based on damped the Newton's increment vector. The damping is calculated solving an optimization problem, which aims to minimize the residual over the updating direction calculated. Iwamoto's method is often considered as benchmark in multiple references (see e.g. [7]) and its philosophy has been adopted for developing other robust PF solution methods [8-10]. Although quite robust, Iwamoto's method tends to be very slow, especially when the solution is approached, which limits its widespread usage in industry applications.

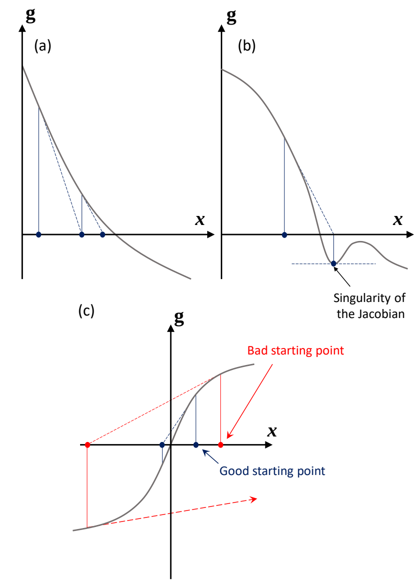

Fig. 1 Sketch of well (a) and ill-conditioned cases (b) and (c)

Solution of the PF problem as a dynamic system has been explored in several references [11, 12]. Essentially, the PF equations are reformulates as a set of autonomous ordinary differential equations. Then, for solving this system, any available routine (e.g. ode's solvers of Matlab), are used for computing the equilibrium points of the artificial dynamic system, which correspond with the solution of the original PF problem. It is demonstrated that resulting artificial model is stable and the proposed iterative procedure is barely affected by the starting point, nevertheless, calculation of dynamic equilibrium points is computationally expensive in large-scale systems as it was pointed out in [11].

In [13], a factorized solution of PF equations has been proposed. It is based on the factorized solution of the equality-constrained state estimation. The solution procedure is suitable for systems with bad initialization, since it is able to find a very good starting point in the sense of the weighted least squares problem. In addition, this methodology is able to find the solution of PF equations in the unsolvable region, which makes it very suitable for determining the MLP [14, 15].

Recently, the Levenberg-Marquardt's method has been profusely studied for solving the PF problem of ill-conditioned systems [5, 16]. These methodologies are quite robust, however, they may show slow convergence or produce an inaccurate solution [17].

Based on the Continuous Newton's (CN) method [18], any numerical technique can be adapted for solving the PF. This idea was firstly exploited in [4] and posteriorly used by the authors for developing several PF solvers [19-21]. Basically, the PF problem can be assimilated as an autonomous set of ordinary differential equations (ODE). On the basis of this analogy, any numerical integration scheme can be employed for solving the PF equations by just replacing the function evaluated by the Newton's increment vector. Since this kind of approaches are Newton-like methods, they still suffer of some issues (e.g. instability at turning points). Nevertheless, they constitute an interesting family of efficient and robust enough techniques especially suitable for large-scale ill-conditioned systems. Recently, a comparative study of several Runge-Kutta methods applied to PF problem was addressed by the authors in [22].

Despite that most of power systems are naturally well-conditioned, ill-conditioned ones are becoming more frequent [11]. Consequently, developing novel robust PF solution methods is interesting and well-received. As it was commented, most of available robust PF solvers are not efficient enough. This issue limits the usage of robust PF solution approaches in industry applications. This paper aims to address this issue by achieving the following targets:

- Proposing a novel paradigm for solving the PF equations. This paradigm basically consists of reformulating the traditional CN method in a multistep arrangement. In each step, the solution vector is refined using a numerical technique. Consequently, the developed PF solution framework allows to incorporate many numerical techniques as steps considered. This fact concludes in a very versatile structure. Since the solution is refined in various stages, more accuracy results are expected with respect to the traditional 1-step CN method.
- The introduced paradigm envisages a novel family of reliable and efficient PF solution methods. With the aim of exemplify, two novel PF methods based on the developed multistep paradigm are developed.
- Developed PF methods are tested in several realistic large-scale ill-conditioned systems under different conditions. Analysis is completed by including a comprehensive comparative study with other well-known PF techniques.

Besides the Introduction, this paper is structured in the following sections. Section 2 briefly explains the PF problem and the CN philosophy. The developed solution paradigm is presented in Section 3. Section 4 develops two novel PF methods based on the developed multistep scheme. Several numerical experiments are carried out in Section 5. Finally, the main conclusions are presented in Section 6.

2. Background

2.1. Outlines of the PF Problem

In this paper, we have limited our scope to the PF formulation based on power injections and polar coordinates, since this framework is typically preferred [23]. Nevertheless, despite that a huge variety of PF formulations are available in the literature (see e.g. [24, 25] and references therein), one should note that introduced formulation can be indistinctly used no matters the approach considered.

Hence, the PF problem in polar coordinates and based on power mismatches is established as a set of n nonlinear equations as follows [26]:

$$\mathbf{g}(\boldsymbol{x}) = \begin{cases} \Delta P_i = P_i^{sch} - \sum |V_i|\,|V_j||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j) \\ \Delta Q_i = Q_i^{sch} - \sum |V_i|\,|V_j||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j) \end{cases} \tag{1}$$

where, P_i^{sch} and Q_i^{sch} are the scheduled active and reactive power injections at i^{th} bus, respectively; $V_i\angle\delta_i$ is the complex voltage at i^{th} bus; and $Y_{ij}\angle\theta_{ij}$ is the ij^{th} element of the admittance matrix. On the other hand, the PF vector of variables $\boldsymbol{x} \in \mathbb{R}^n$ can be defined as follows:

$$\boldsymbol{x} = \left[\boldsymbol{\delta_{PV}}|\boldsymbol{\delta_{PQ}}|\boldsymbol{V_{PQ}}\right]^T \tag{2}$$

where $\boldsymbol{\delta_{PV}} \in \mathbb{R}^{n_g}$ is the vector of voltage angles at PV buses, $\boldsymbol{\delta_{PQ}} \in \mathbb{R}^{n_l}$ is the vector of voltage angles at PQ buses, $\boldsymbol{V_{PQ}} \in \mathbb{R}^{n_l}$ is the vector of voltage magnitudes at PQ buses; and n_l, n_g are the total number of PQ and PV buses, respectively.

Solution of (1) has to be calculated using some available iterative technique. There are multiple stopping criteria for determining when the iterative process has approached the solution. In this paper, solution of the PF is considered properly approached when the maximum absolute residual is lower than a given threshold. Formally, this criteria can be expressed as follows:

$$\|\mathbf{g}(\boldsymbol{x})\|_\infty \leq \varepsilon \tag{3}$$

where ε is the convergence tolerance. In the case of divergence, criteria (3) is never satisfied since algorithm leads (2) far away to the solution. In order to avoid an infinity loop in this situation, iterative process also stops if the total number of iterations is greater than a given limit. This condition might be established as follows:

$$k > k_{\max} \tag{4}$$

where k denotes the total number of iterations and $k_{\max}$ is the maximum number of iterations allowed.

2.2. The CN method and its application to the PF problem

Typically, solution of the PF is found using the traditional NR method. For the sake of self-sufficiency, the generic k^{th} if the NR technique for solving (1) is defined as follows:

$$\Delta \boldsymbol{x}^{(k)} = -\left[\mathbf{g}_{\boldsymbol{x}}^{(k)}\right]^{-1} \mathbf{g}\left(\boldsymbol{x}^{(k)}\right) \tag{5}$$

$$\boldsymbol{x}^{(k+1)} = \boldsymbol{x}^{(k)} + \Delta \boldsymbol{x}^{(k)} \tag{6}$$

where $\mathbf{g}_{\boldsymbol{x}} = \nabla_{\boldsymbol{x}}^{T} \mathbf{g}(\boldsymbol{x}) \in \mathbb{R}^{n \times n}$ is the Jacobian matrix of (1). Now, let us consider a set of n ODEs as follows:

$$\dot{\boldsymbol{x}} = \mathbf{f}(\boldsymbol{x}) \tag{7}$$

For a specified time instant t and an arbitrary time step Δt, the simplest method for integrating (7) is the Explicit-Euler technique, as follows:

$$\Delta \boldsymbol{x}^{(t)} = \Delta t \mathbf{f}\left(\boldsymbol{x}^{(t)}\right) \tag{8}$$

$$\boldsymbol{x}^{(t+\Delta t)} = \boldsymbol{x}^{(t)} + \Delta \boldsymbol{x}^{(t)} \tag{9}$$

As it is pointed out in [18], an analogy between (5)-(6) and (8)-(9) can be easily established if one defines:

$$\mathbf{f}(\boldsymbol{x}) = -[\mathbf{g}_{\boldsymbol{x}}]^{-1} \mathbf{g}(\boldsymbol{x}) \tag{10}$$

In other words, the traditional NR method for solving nonlinear equations can be viewed as the Explicit-Euler formula for solving ODEs. In this case, variable t can be considered dummy and it vanishes of the formulation.

This paradigm for solving nonlinear equations is coherent with the Newton's theorem. Accordingly, NR solves a nonlinear equation by approximation the area under the function by a rectangle [27]. Keeping this in mind, CN can be conceptually viewed as a generalization of the Newton's theorem to any other integration formula as the already described Explicit-Euler technique or the family of Runge-Kutta (RK) formulas [28].

3. The introduced solution paradigm

This paper aims to develop a novel solution paradigm for the PF problem based on the CN method. During this section, the introduced philosophy is firstly described and its formal formulation is developed, which aims to establish a common framework to develop PF solution methods. Then, several topics related with the solution of the PF problem using the developed approach are discussed.

3.1. Formulation

As PF solution based on RK formulas, function (10) is firstly evaluated in p intermediate points as follows [22]:

$$\boldsymbol{v}_{\mathbf{1}}^{(k)} = \mathbf{f}\left(\boldsymbol{x}^{(k)}\right)$$

$$\boldsymbol{v}_{\mathbf{2}}^{(k)} = \mathbf{f}\left(\boldsymbol{x}^{(k)} + h\left(a_{21}\boldsymbol{v}_{\mathbf{1}}^{(k)}\right)\right)$$

$$\boldsymbol{v}_{\mathbf{3}}^{(k)} = \mathbf{f}\left(\boldsymbol{x}^{(k)} + h\left(a_{31}\boldsymbol{v}_{\mathbf{1}}^{(k)} + a_{32}\boldsymbol{v}_{\mathbf{2}}^{(k)}\right)\right)$$

$$\vdots$$

$$\boldsymbol{v}_{\boldsymbol{p}}^{(k)} = \mathbf{f}\left(\boldsymbol{x}^{(k)} + h\left(a_{p1}\boldsymbol{v}_{\mathbf{1}}^{(k)} + a_{p2}\boldsymbol{v}_{\mathbf{2}}^{(k)} + \cdots + a_{p,p-1}\boldsymbol{v}_{\boldsymbol{p-1}}^{(k)}\right)\right) \tag{11}$$

where h is the discrete step size and a_{ij} is the ij^{th} element of the called RK matrix, which is properly defined for each numerical method as follows:

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & & & \\ a_{21} & a_{22} & & \\ \vdots & & \ddots & \\ a_{p-1,1} & \cdots & a_{p-1,p-2} & a_{p-1,p-1} \end{bmatrix} \in \mathbb{R}^{(p-1)\times(p-1)} \tag{12}$$

In this case, p denotes the order of the numerical technique. As it can be seen, function is evaluated p times. This is a critical aspect for solving nonlinear equations since each function evaluation implies a factorization of the Jacobian matrix. It is well-known that factorization of the Jacobian matrix is the heaviest part of any PF solution technique based on the Newton's method, thus, higher order normally implies higher computational burden.

Now, those PF solution methods based on the family of RK formulas update the state vector as follows:

$$\boldsymbol{x}^{(k+1)} = \boldsymbol{x}^{(k)} + h \textstyle\sum_{i=1}^{p} b_i \boldsymbol{v}_i^{(k)} \tag{13}$$

where b are the weights, which are properly defined for each numerical method.

The main modification introduced by the developed paradigm is to update the PF state vector using information of $K-1$ previously computed steps. Thus, (13) would be split in up to K intermediate steps as follows:

$$\hat{\boldsymbol{x}}_{\mathbf{1}}^{(k)} = \boldsymbol{x}^{(k)} + h \textstyle\sum_{i=1}^{p} b_{1i} \boldsymbol{v}_i^{(k)}$$

$$\hat{\boldsymbol{x}}_{\mathbf{2}}^{(k)} = \hat{\boldsymbol{x}}_{\mathbf{1}}^{(k)} + h \textstyle\sum_{i=1}^{p} b_{2i} \boldsymbol{v}_i^{(k)}$$

$$\vdots$$

$$\boldsymbol{x}^{(k+1)} = \hat{\boldsymbol{x}}_{\boldsymbol{K-1}}^{(k)} + h \textstyle\sum_{i=1}^{p} b_{Ki} \boldsymbol{v}_i^{(k)} \tag{14}$$

In this case, since up to K different numerical methods can be employed, coefficients b are properly defined in the weights matrix, as follows:

$$\boldsymbol{B} = \begin{bmatrix} b_{11} & \cdots & b_{1p} \\ \vdots & \ddots & \\ b_{K1} & \cdots & b_{Kp} \end{bmatrix} \in \mathbb{R}^{K \times p} \tag{15}$$

The main idea behind the introduced solution paradigm is to obtain a more refined value of the updated state vector, by using a recursive scheme which computes $K-1$ previous or predicted values of (2). This scheme is very versatile since allows us to develop a huge variety of different PF solution method by simply using up to K different numerical methods.

In that sense, introduced family of PF solution techniques can be viewed as the application of the Embedded RK formulas [29] for solving the PF. These techniques estimate the local truncation error by nesting two RK methods. In that case, one of the method has order p while the other has order $p-1$. However, introduced solution paradigm shows several important differences with respect the standard Embedded RK methods:

- Introduced solution paradigm is more flexible since it is possible to use K numerical methods with the same order or any other combination.

- In RK techniques, stability is guaranteed if $\sum_{i=1}^{p} b_{Ki} = 1$ [30]. However, it has been observed that PF solution approaches based on the introduced multistep scheme, are even stable and robust if this condition is not satisfied.
- In calculation of ODEs, typically higher order implies higher accuracy as it is showed in Fig. 2. However, highest order does not always implies the best computational performance in nonlinear problems. One should note that PF solution approaches based on RK formulas require as many factorizations as the order of the numerical method considered, moreover, the highest order does not always ensure the highest convergence rate [22]. In this case, rather than the order, higher computational performance is achieved with more steps, it means, higher K. This fact is demonstrated in Section 5.

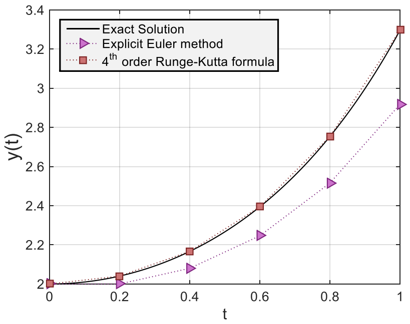

Fig 2 Accuracy of the Explicit Euler and 4th order Runge Kutta formulas for solving the ODE $\dot{y} = yt$, with initial condition $\dot{y}(0) = 2$ and $h = 0{,}2$

However, as family of RK formulas, PF solution techniques based on the introduced paradigm can be defined using the well-known Butcher's Tableau. This mnemonic device has, in this case, the structure shown in Fig. 3.

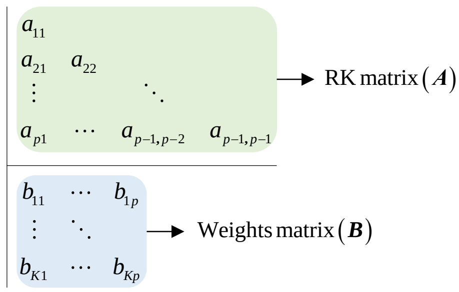

Fig 3 Structure of Butcher's Tableau for defining PF techniques based on the developed multistep paradigm.

One should note that nodes of the RK formulas (typically denotes by c), lack of sense in the case of autonomous functions such as (10).

3.2. Step Size control

The control of the step size is a critical aspect for solving ODEs. As example, In Fig. 4 it is showed the influence of the step size in the accuracy of the explicit Euler method. Clearly, the smallest step size implies the highest accuracy. However, higher computational burden is obtained since more intermediate points have to be calculated.

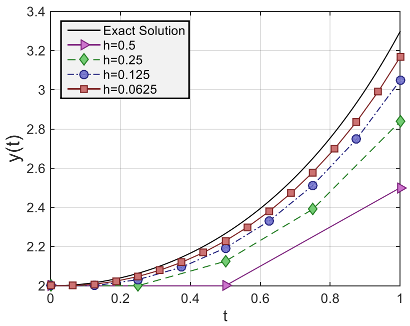

Fig 4 Influence of the step size for solving the ODE $\dot{y} = yt$, with initial condition $\dot{y}(0) = 2$

As it was commented, methods based on the introduced solution paradigm and the Embedded RK formulas have similar structure. These techniques allow an effective step size control scheme based on the estimation of the local truncation error. Consequently, we can take advantage of this particular characteristic for properly controlling the step size. Specifically, in reference [31], the step size is updated each iteration according to the following rule:

$$h = \min\left(h_{\max}, \max\left(h_{\min}, 0.9h\left(\frac{1}{\epsilon^{(k)}}\right)^{p+1}\right)\right) \tag{16}$$

where $h_{\max}$ and $h_{\min}$ are the maximum and the minimum step size allowed, respectively; and ϵ is the local truncation error which can be calculated as follows:

$$\epsilon^{(k)} = \left\|\boldsymbol{x}^{(k+1)} - \hat{\boldsymbol{x}}_{\boldsymbol{K-1}}^{(k)}\right\|_{\infty} \tag{17}$$

Nevertheless, the step size has to be initialized. In solution of nonlinear problems, the step size should be initialized according to the proximity of the starting point to the solution. Proximity of the starting point to the solution is difficult to know *a priori*, however, function (18) can be used to estimate it:

$$W\left(\boldsymbol{x}^{(k)}\right) = \frac{1}{2}\left[\mathbf{g}\left(\boldsymbol{x}^{(k)}\right)\right]^{T}\mathbf{g}\left(\boldsymbol{x}^{(k)}\right) \tag{18}$$

Making use of the expression (18), the step size can be properly initialized as follows:

$$h = \min\left(h_{\max}, \max\left(h_{\min}, \frac{1}{W\left(\boldsymbol{x}^{(0)}\right)^{0.06}}\right)\right) \tag{19}$$

3.3. Computational burden

The main computational burden of any PF calculation based on the Newton's method is the factorization of the Jacobian matrix [4]. Those techniques based on the introduced solution paradigm have to factorize the Jacobian matrix p times each iteration. Consequently, developed methodology does not imply higher computational burden than other PF solution techniques based on RK formulas [4, 22]. Nevertheless, as it is showed in Section 4, developed methodology achieves higher order of convergence.

In comparison with NR, developed methodology is always less efficient. Nevertheless, computational burden of both techniques is similar if one limits the usage of high order schemes. In fact, personal experience have showed that very good results can be achieved with $p = 2$. Consequently, higher values of p will be only justified if the convergence is outstandingly enhanced or a very high degree of robustness is achieved. Although these requirements are not typically necessary, this topic will be covered in future works.

3.4. Behavior at turning points

As NR, PF solution approaches based on the introduced paradigm fail at turning points. However, this issue can be properly solved by using some continuation or homotopy technique. The well-known Continuation Power Flow [32], has been profusely studied for avoiding the singularity of the Jacobian matrix at turning points. Developed methodology can be properly used at corrector stage of the Continuation Power Flow in order to overcome its potential issues provoking by the singularity of the Jacobian matrix. Stability of the introduced methodology at turning points can be also enhanced using regularization techniques [33]. Anyway, this topic is out of the scope of the present paper and it will be covered in future works.

3.5. Handling generators' reactive limits

Despite that developed formulation does not directly take into account equipment's reactive limits, they might been easily incorporated using some available mechanism for NR. One of the most common strategy [34, 35] proceeds as follows. Firstly, the PF is solved without considering the generators' reactive limits. When the solution is reached, it is checked if some reactive limit has been hit. In affirmative case, those PV buses which some reactive limit has been violated are converted to PQ. In converted buses, reactive power injections are replaced by the corresponding limit. Then, the PF is solved again. This process is repeated until a feasible solution is achieved or all system buses are PQ. In the latest case, the problem is declared unfeasible. For the sake of clarity, Fig. 5 shows the flowchart of the considered procedure for incorporating the generators' reactive limits in the proposed PF framework.

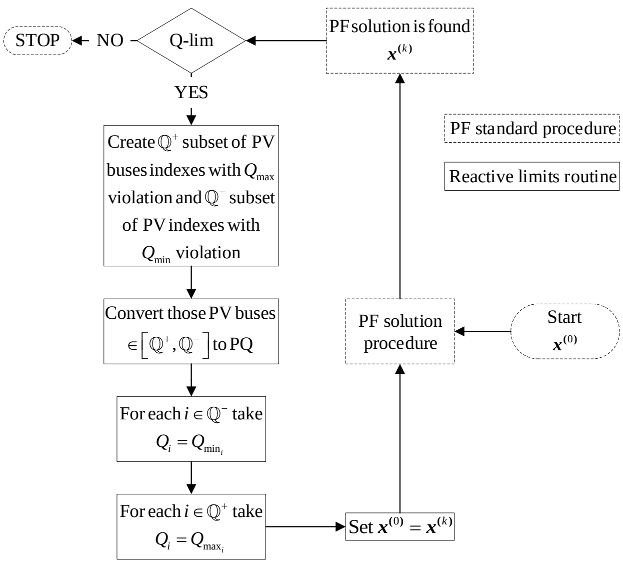

Fig 5 Flowchart of the considered strategy for handling the generators' reactive limits

Athough the strategy explained above is considered the most standard one and it has been used in this work, it is worth to mention that developed methodology is versatile enough for incorporating other strategies. For example, in [11] the generators' reactive limits are handled as inequality constraints. This mechanism might be adapted into the introduced methodology in the same way.

3.6. Multiple PF solutions

Due to the quadratic form of PF equations (1), they present two solutions or equilibrium points (called high and low voltage solutions). Frequently, the high voltage solution is aimed to be calculated since it corresponds with the stable operating point of the system. Reachability of each solution mainly depends on the starting point considered for initializing the iterative process.

3.7. Handling tap changers and phase shifters

Transformer's tap changers and phase shifters are not certainly taken into account in the developed formulation. However, it might be easily incorporated using some available model for the series admittance. For example, MATPOWER software package [35], handles the transformer as a series admittance, which already incorporates the model of tap changers and phase shifters. Hence, when some of these parameters has to be changed, the corresponding series admittance varies consequently.

Regarding how handling remote controlled taps or phase shifters for regulating the nodal voltages, it could be considered using a similar strategy described in Fig. 5. This strategy is not currently implemented in MATPOWER software, in addition, it leads to an optimization problem in order to determine which control should act each case. These topics will be covered in future works.

4. Developed PF solution methods

For the sake of exemplify, two novel PF techniques based on the solution paradigm introduced in Section 3 are developed.

4.1. Midpoint-Euler method (MPE) for solving the PF

This method combines the Midpoint rule at the first step and the Explicit Euler method at the second step, therefore, this technique has $p = 2$ and $K = 2$. The Butcher's Tableau of the developed MPE is showed in Fig. 6.

$$\begin{array}{c|cc} 1/2 & & \\ \hline & 0 & 1 \\ & 1/2 & 0 \end{array}$$

Fig 6 Butcher's Tableau of the developed MPE

On the other hand, the generic k^{th} iteration of the developed MPE is carried out as follows:

$$\boldsymbol{v}_1^{(k)} = \mathbf{f}\left(\boldsymbol{x}^{(k)}\right) \tag{20}$$
$$\boldsymbol{v}_2^{(k)} = \mathbf{f}\left(\boldsymbol{x}^{(k)} + {}^{h}/_{2}\, \boldsymbol{v}_1^{(k)}\right) \tag{21}$$
$$\widehat{\boldsymbol{x}}_1^{(k)} = \boldsymbol{x}^{(k)} + h\boldsymbol{v}_2^{(k)} \tag{22}$$
$$\boldsymbol{x}^{(k+1)} = \widehat{\boldsymbol{x}}_1^{(k)} + {}^{h}/_{2}\, \boldsymbol{v}_1^{(k)} \tag{23}$$

4.2. Trapezoidal-Midpoint-Euler method (TMPE) for solving the PF

Unlike the developed MPE, the TMPE combines the Trapezoidal rule, the Midpoint method and the Explicit Euler technique, consequently, it has $K = 3$ and $p = 2$. The Butcher's Tableau of the developed MPE is showed in Fig. 7.

$$\begin{array}{|cc} 1 & \\ \hline 1/2 & 1/2 \\ 0 & 1/2 \\ 1/2 & 0 \end{array}$$

Fig 7 Butcher's Tableau of the developed TMPE

On the other hand, the generic k^{th} iteration of the developed TMPE is carried out as follows:

$$\boldsymbol{v}_1^{(k)} = \mathbf{f}\left(\boldsymbol{x}^{(k)}\right) \tag{24}$$
$$\boldsymbol{v}_2^{(k)} = \mathbf{f}\left(\boldsymbol{x}^{(k)} + h\boldsymbol{v}_1^{(k)}\right) \tag{25}$$
$$\widehat{\boldsymbol{x}}_1^{(k)} = \boldsymbol{x}^{(k)} + {}^{h}/_{2}\left[\boldsymbol{v}_1^{(k)} + \boldsymbol{v}_2^{(k)}\right] \tag{26}$$
$$\widehat{\boldsymbol{x}}_2^{(k)} = \widehat{\boldsymbol{x}}_1^{(k)} + {}^{h}/_{2}\, \boldsymbol{v}_2^{(k)} \tag{27}$$
$$\boldsymbol{x}^{(k+1)} = \widehat{\boldsymbol{x}}_2^{(k)} + {}^{h}/_{2}\, \boldsymbol{v}_1^{(k)} \tag{28}$$

For the sake of clarity, Fig. 8 shows the proposed flowchart for solving the PF using either the developed MPE or TMPE. As it can be seen, once implemented, the proposed flowchart can be adapted for using any other technique based on the introduced paradigm with a few modifications. Moreover, programs which has already implemented the standard NR can easily incorporates the developed methods, by just adding the subroutine for initializing and updating the step size, besides replacing the NR iteration (defined by (5) and (6)), by equations (20)-(23) or (24)-(28). These aspects facilitate the implementation of the developed solution paradigm in industry applications.

It should be noted as both developed methods have $p = 2$. This implies that, despite that NR requires less computations, computational burden of developed method is not unaffordable and prohibitive for most of industry tools.

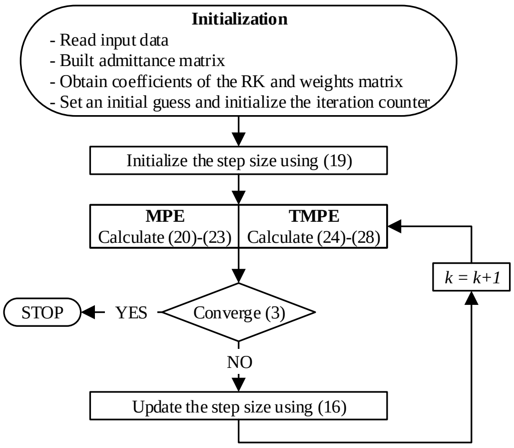

Fig 8 Proposed flowchart for solving the PF using the developed MPE or TMPE

5. Tests and results

With the aim to check the suitability of developed MPE and TMPE for solving the PF in ill-conditioned systems, the following PF solution methods have been compared:

- Standard NR.
- Iwamoto's method [6].
- High order Levenberg-Marquardt method (HLM) [5]. Let $\alpha \in \mathbb{R}$ be the so-called damping factor. In this paper, this parameter is updated at each iteration: $\alpha^{(k)} = \left\|\mathbf{g}\left(\boldsymbol{x}^{(k)}\right)\right\|_{\infty}^{1,3}$. Based on our personal experience, HLM achieves its optimal performance using this adaptive mechanism.
- PF solution technique based on the 2nd order Adams-Bashforth's method (AB2) [20].
- PF solution methods based on the midpoint rule (MP), accelerated 3rd order Runge-Kutta technique (ARK3) and 4th order Runge-Kutta formula (RK4) [22]. In this case, RK4 is implemented following the guidelines provided in [22].
- Reverse Bulirsch-Stoer approach (RBS) [21].
- Developed MPE and TMPE.

The NR can be considered as the standard PF solution approach. On the other hand, Iwamoto's method is maybe the most popular robust PF solver while HLM is a newfangled method for solving ill-conditioned power systems. Remainder considered methodologies are PF solution approaches based on the CN method, which makes them very suitable for contrasting the main features of the developed techniques.

In all cases, the iterative solution has been initialized using a flat start (i.e. all voltage magnitudes equal to 1 pu at PQ buses and all voltage angles equal to 0 deg at PQ and PV buses). Moreover, $\varepsilon = 10^{-6}$ has been imposed as convergence criteria.

For solving the PF using the developed techniques, $h_{\max}$ and $h_{\min}$ have to be tuned. In fact, there are not formal rules to tune these parameters. However, the following recommendations based on our personal experience can be adopted:

- We have observed that developed MPE and TMPE becomes unstable when $h_{\max} \geq 1$. On the other hand, it is advantageous enlarging the step size when the convergence is ensured in order to speed up the iterative procedure. Consequently, $h_{\max} = 1$ has been considered in the developed MPE and TMPE.

- Certainly, if $h_{\min}$ is tuned too short, a high degree of robustness can be achieved. However, developed PF solution methods might be unnecessary slow since convergence features become very linear. In this paper, $h_{\min} = 0{,}4$ has been considered for the developed MPE and TMPE. Anyway, this parameter can be tuned as short as needed.

The ideas introduced above are not universal, however, they worked quite well in the studied systems. Nevertheless, they can be adapted according each scenario, for instance, in well-conditioned systems it might be recommendable set $h_{\min}$ as large as possible.

For remainder considered approaches, parameters provided in the correspondent references have been considered. In the case of AB2, its acceleration strategy based on switching to the standard NR when $\|\mathbf{g}(\boldsymbol{x})\|_\infty < 10^{-1}$ has been disabled, in order to test all methodologies in similar conditions.

All considered methodologies along the developed methods have been coded in MATPOWER v7.0 [35]. All simulations have been run under Windows 10 on a 3.4 GHz Intel Core i7-8750H CPU 2.2 GHz personal laptop (16.00 GB RAM). It is worth to mention that all reported solution times have been calculated as the average value of 100 simulations, with the aim of avoiding potential influence of parallel computer activities.

For the sake of checking the goodness of results reported, the solution found by the standard NR using the default start provided by MATPOWER has been considered as benchmark solution. Thus, comparing this solution with that obtained by remainder considered techniques, it is concluded that the solution reported is accurate enough, if the difference between both values is not greater than 0.1 pu in voltage magnitudes and 0.01 deg in voltage angles.

5.1. Studied systems

Developed PF solution approaches besides remainder considered methodologies have been tested in the following systems:

- The 3012-bus snapshot of the Polish transmission system at 2007-08 winter evening peak. This system is available in the MATPOWER database [36].
- The 3374-bus snapshot of the Polish transmission system at 2007-08 winter evening peak. This system is available in the MATPOWER database [36].
- The 13659-bus portion of the European transmission system from the EU Pegase project [37, 38]. This system is available in the MATPOWER database [36].
- A duplicated version of the 13659-bus portion of the European transmission system from the EU Pegase project [37, 38]. The resulting system has 27318 buses.

For the sake of completeness, Table 1 reports the main characteristics of studied systems. Considered systems have been reported as ill-conditioned [21]. In fact, NR shows convergence difficulties when a flat start is used, which is symptomatic of ill-conditioning [4].

Table 1 Main characteristics of studied systems

Characteristic		3012-bus	3374-bus	13659-bus	27318-bus
Buses		3012	3374	13659	27318
Branches		3572	4161	20467	40935
Generators		502	596	4092	8184
Load	MW	27169	48363	381431	762863
	MVar	10200	19527	98523	197046
n		5725	6355	23225	46451
Cond. number		$2{,}9 \times 10^6$	$4{,}3 \times 10^6$	$1{,}5 \times 10^8$	$1{,}6 \times 10^8$

5.2. Base Load scenario

Table 2 reports the total number of iterations employed by considered methodologies for solving the studied systems at base load scenario.

Table 2 Total number of iterations for base load scenario

Method	3012-bus	3374-bus	13659-bus	27318-bus
NR	Diverge	Diverge	Diverge	Diverge
Iwamoto	2310	1396	49*	46*
HLM	13	12	33	33
AB2	48	49	Diverge	69
MP	35	36	33*	33*
ARK3	23	23	46*	46*
RK4	27	28	Diverge	Diverge
RBS	14	14	15	15
MPE	6	6	6	6
TMPE	5	5	5	5

* *Inaccurate solution*

As expected, NR diverged in all studied systems. AB2 and RK4 diverged in the 13659-bus system while RK4 also diverged in the 27318-bus system. Iwamoto's method, MP and ARK3 occasionally achieved an inaccurate solution (the low voltage one in this case). HLM, RBS and developed approaches successfully solved all considered systems. Developed MPE and TMPE required many less iterations than remainder techniques. TMPE was slightly faster than MPE, employing an iteration less than MPE, which confirms that higher order of convergence can be achieved employing more steps, it means, with higher values of *K*.

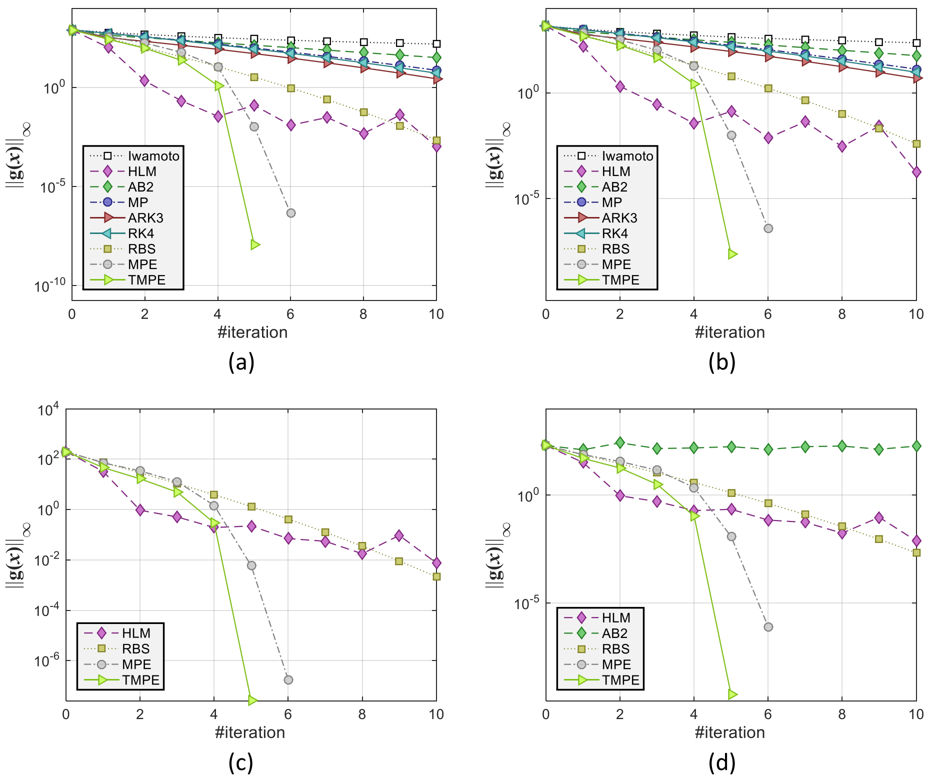

Fig 9 Convergence characteristics of different PF methods in the studied systems: 3012-bus (a), 3374-bus (b), 13659-bus (c) and 27318-bus (d)

Fig. 9 shows the convergence characteristics of considered techniques in the studied systems at base load scenario. As it can be seen, developed methods showed outstanding convergence performance. HLM and RBS normally showed smaller residual than developed MPE until 4th iteration, beyond that, residual produced by developed PF methods is always smaller. These features are distinctive of good robustness properties.

Table 3 reports the execution time in seconds consumed by considered PF solution techniques in the studied systems at base load scenario.

Table 3 Execution time [s] for base load scenario

Method	3012-bus	3374-bus	13659-bus	27318-bus
NR	--	--	--	--
Iwamoto	45,28	29,89	4,32*	8,29*
HLM	0,58	0,65	7,53	15,87
AB2	0,88	0,99	--	11,87
MP	1,21	1,39	5,36*	11,37*
ARK3	0,84	0,93	7,63*	16,27*
RK4	1,83	2,11	--	--
RBS	0,85	0,94	4,22	9,13
MPE	0,23	0,24	1,00	2,11
TMPE	0,18	0,20	0,84	1,77

* *Inaccurate solution*

As it can be seen, developed methods were much faster than remainder considered methodologies. These results are coherent if one analyzes the computational effort employed by each technique for solving the studied systems. Fig. 10 shows the total number of matrix factorizations required by each methodology for solving the studied systems at base load scenario. As it can be easily checked, developed methods required less factorizations than remainder PF solution approaches.

It is remarkable the case of HLM. Despite this method and developed techniques (MPE and TMPE) required similar amount of matrix factorizations in the 3012-, and 3374-bus systems, the former was significantly slower. This is explained, in the fact that factorization required by HLM might be much heavier than that required by the other considered methods due to the product $[\mathbf{g}_x]^T\mathbf{g}_x$, which produces a very dense matrix. It also explains why RBS was faster in the 13659-bus system and its duplicated version.

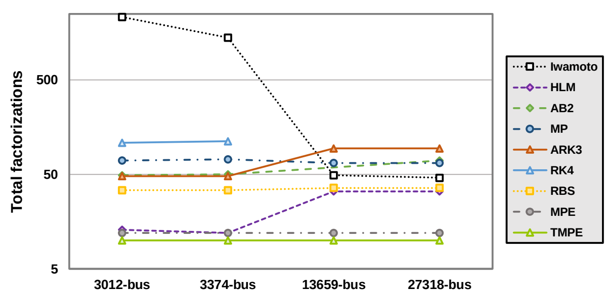

Fig 10 Total matrix factorizations required for solving the studied systems at base load scenario

5.3. Limit Load scenario

Now, let us analyze the effect of the loading level in the performance of the developed approaches. To do that, the loading level of the system has been increased as follows:

$$P_i^{sch} = \lambda P_i^{sch} \quad \text{at PQ and PV buses} \tag{29}$$

$$Q_i^{sch} = \lambda Q_i^{sch} \quad \text{at PQ buses} \tag{30}$$

where $\lambda \in \mathbb{R}^+$ is the loading level. Thus, the loading level has been increased in steps of 0,0001 pu until all considered methodologies failed. This situation is considered as a limit case since the PF has to be solved when λ is close to the MLP. Table 4 reports the total number of iterations at limit cases.

Table 4 Total number of iterations for limit load scenario

Method	3012-bus $\lambda = 1{,}2734$	3374-bus $\lambda = 1{,}1586$	13659-bus $\lambda = 1{,}0017$	27318-bus $\lambda = 1{,}0017$
NR	Diverge	Diverge	Diverge	Diverge
Iwamoto	2130	1260	47	46
HLM	46	45	170	175*
AB2	55	56	Diverge	Diverge
MP	35	36	33*	33*
ARK3	23	23	Diverge	Diverge
RK4	27	28	Diverge	Diverge
RBS	14	14	Diverge	Diverge
MPE	10	10	8	8
TMPE	8	8	6	6

* *Inaccurate solution*

One should note that this scenario is more demanding. Proximity of the MLP normally increases the degree of ill-conditioning and the solution of the PF is harder to reach. Thus, AB2, ARK3, RK4 and RBS diverged in the 13659-bus and its duplicated version while MP achieved an inaccurate solution in these systems. On the other hand, HML found an inaccurate solution in the 27318-bus system. This was expected since fixed points found by those methods based on the Levenberg's technique might not be solutions of the original PF equations (1) [17]. On the other hand, Iwamoto's method and developed PF solution approaches were able to successfully solve all studied systems. As for the base load scenario, developed approaches employed less iterations than remainder considered methods.

Table 5 reports the execution time in seconds at limit scenario. Again, developed methods were faster than remainder methodologies.

Table 5 Execution time [s] for limit load scenario

Method	3012-bus $\lambda = 1{,}2734$	3374-bus $\lambda = 1{,}1586$	13659-bus $\lambda = 1{,}0017$	27318-bus $\lambda = 1{,}0017$
NR	--	--	--	--
Iwamoto	41,55	27,12	4,14	8,26
HLM	1,95	2,15	38,91	78,67*
AB2	0,99	1,12	--	--
MP	1,21	1,39	5,36*	11,37*
ARK3	0,84	0,93	--	--
RK4	1,83	2,11	--	--
RBS	0,85	0,94	--	--
MPE	0,35	0,39	1,32	2,80
TMPE	0,29	0,32	1,00	2,10

* *Inaccurate solution*

5.4. Reactive limits enforcement

In this case, we have analyzed the performance of the developed methods when the generators' reactive limits are enabled. To do that, the default strategy implemented in MATPOWER and previously described in Section 3.5 has been considered. Table 6 shows the total number of iterations (labelled TI), total PF solutions required for achieving a feasible solution (labelled TS) and the average number of iterations (labelled AI) of the considered methods for solving the PF with generators' reactive limits enforced.

One should note that this scenario is more difficult to solve that base load one, due to, after a PF solution, size of the state vector grows since some PV buses are converted to PQ. Thus, NR and AB2 diverged in all studied systems. Iwamoto's method, MP, ARK3 and RK4 failed in the 27318-bus system. RK4 also diverged in the 13659-bus system. As in the base load scenario, Iwamoto's method, MP and ARK3 achieved an inaccurate solution in the 13659-bus system. On the other hand, HLM, RBS and developed techniques successfully solved all studied systems. As previous experiments, developed methods required less iterations than remainder techniques for reaching a feasible solution.

Table 7 reports the total execution time consumed by different methods for solving the PF in the studied systems with reactive limits enforced. As expected, developed methods were faster than remainder methodologies.

Table 6 Comparison of different solution methods with generators' reactive limits enforced

Method	3012-bus			3374-bus			13659-bus			27318-bus		
	TI	TS	AI	TI	TS	AI	TI	TS	AI	TI	TS	AI
NR	Div.	--	--	Div.	--	--	Div.	--	--	Div.	--	--
Iwamoto	5064	3	1688	3541	4	885,25	84*	2	42	Div.	--	--
HLM	18	3	6	17	4	4,25	36	2	18	36	2	18
AB2	Div.	--	--	Div.	--	--	Div.	--	--	Div.	--	--
MP	76	3	25,33	87	4	21,75	61*	2	30,50	Div.	--	--
ARK3	57	3	19,00	66	4	16,50	67*	2	33,50	Div.	--	--
RK4	63	3	21,00	74	4	18,50	Div.	--	--	Div.	--	--
RBS	33	3	11,00	38	4	9,50	26	2	13,00	26	2	13,00
MPE	10	3	3,33	11	4	2,75	8	2	4,00	8	2	4,00
TMPE	7	3	2,33	8	4	2,00	6	2	3,00	7	2	3,50

* *Inaccurate solution*
TI: total number of iterations
TS: total number of PF solutions required for achieving a feasible solution
AI: average number of iterations ($AI = TI/TS$)

Table 7 Execution time [s] with generators' reactive limits enforced

Method	3012-bus	3374-bus	13659-bus	27318-bus
NR	--	--	--	--
Iwamoto	100,02	77,44	7,41*	--
HLM	0,82	0,90	8,36	17,83
AB2	--	--	--	--
MP	2,66	3,38	9,82	--
ARK3	2,10	2,74	11,12	--
RK4	4,27	5,62	--	--
RBS	2,07	2,71	7,35	15,78
MPE	0,38	0,46	1,34	2,87
TMPE	0,27	0,35	1,02	2,48

* *Inaccurate solution*

6. Conclusions

A novel solution paradigm for ill-conditioned power systems has been presented. It basically reformulates the traditional CN method [4] in a multistep scheme, so that, the updated PF state vector is progressively refined each step by using different numerical arrangements. Formal formulation of the developed paradigm has been introduced. The developed solution paradigm envisages a novel family of PF solution techniques.

For the sake of exemplify, two novel PF solution methods based on the introduced solution paradigm have been developed. They have been tested in four large-scale realistic ill-conditioned systems. Three scenarios have been considered:

- Base load.
- Limit loading level.
- Generators' reactive limits enforcement.

Developed methods have been compared with up to eight well-known PF solution approaches. Superiority of developed techniques has been manifested in a high degree of robustness and efficiency, notably outperforming remainder considered methodologies.

Other PF solution approaches based on the introduced solution paradigm should be developed and analyzed in future works, besides the suitability of the developed solution arrangement in other related problems like the Continuation Power Flow [32].

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