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Consensual Group-AHPSort: Applying consensus to GAHPSort in sustainable development and industrial engineering

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ABSTRACT

Multi-Criteria Group Decision Making (MCGDM) deals with the process of taking decisions among a number of decision makers who evaluate different alternatives on several criteria that may be conflicting. In such a type of problems conflicts may are not only related to the criteria but also with the decision makers that could result in unsatisfactory solutions because of disagreements or deadlocks to achieve a decision. To avoid these undesirable results, *consensus* processes have been applied to group decision. The application of MCGDM has been mainly focused on decision problems for ranking and choice alternatives but, similarly to multi-criteria decision making, other types of problems such as *sorting* and *description* can be formulated. Although, a few number of MCGDM-sorting models, comparing with ranking and choice ones, have been proposed their importance and impact is quite significant nowadays. Hence, our aim is to consider the recent proposal on Group-AHPSort that did not consider any disagreement measure across the sorting decision process and study how the use of consensus processes can avoid some *misclassifications* or disagreements among the group of decision makers taking part in the MCGDM problem. To illustrate our approach, a case study focusing on the sorting of five European Union countries according to their sustainable development performance regarding social issues will show the importance and utility of detecting and smoothing disagreements in Group-AHPSort.

1. Introduction

Decision Making is a common daily task for human beings. A common and natural formulation of decision-making problems is Multi-Criteria Decision Making (MCDM) in which a decision among different alternatives should be taken according to multiple criteria that may be in conflict (Greco, Figueira, Greco, & Ehrgott, 2016).

MCDM problems can be applied to different situations (Roy, 1981): choice, sorting, ranking and description problems and also elimination (Bana e Costa 1996), design (Keeney, 1992) and cognitive problems (Zamarrón-Mieza, Yepes et al. 2017).

Most of MCDM approaches in the literature have been mainly applied to ranking and choice decision problems (Ishizaka and Nemery, 2013a,b, Figueira, Greco, Figueira, & Ehrgott, 2016, Kwok & Lau, 2019): PROMETHEE, ELECTRE, AHP, TOPSIS, etc. Among them, that Analytic Hierarchical Process (AHP) (Saaty, 2003, Ishizaka & Labib, 2011) is one of the most spread and useful MCDM methods (Lolli, Ishizaka et al. 2014, Ishizaka, Siraj et al. 2016, Ho & Ma, 2018), often applied to ranking problems, occasionally for choice ones and recently adapted to deal with MCDM sorting problems with AHPSort (Ishizaka, Nemery et al. 2012).

The necessity of involving multiple decision makers to have a comprehensive view and knowledge about the problem is nowadays quite common, because of complexity, heterogeneity and just-in-time required solution. Such decisional situations with several decision makers involved and multiple criteria evaluated, are called Multi-Criteria Group Decision Making (MCGDM) problems.

Group Decision Making (GDM) has been applied to different application areas such as politics (Eklund, Rusinowska et al. 2008, Filippakou & Tapper, 2015), management (Ishizaka and Nemery, 2013a,b, Banaeian, Mobli et al. 2018), engineering (Xu, Li et al. 2007, Kwong, Ye et al. 2011), etc. Classical GDM resolution schemes (Lu, Zhang et al. 2006) obtain the solution by using aggregating processes and disregarding the level of agreement among the decision makers involved in it. Therefore, often emerge situations in which one or several decision makers do not accept the obtained solution, because they consider that their preferences have not been sufficiently taken into account. To avoid such situations, consensus measures (Ben-Arieh & Chen, 2006, García-

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Lapresta, 2008) and Consensus Reaching Processes (CRPs) (Dong, Zhang et al. 2010, Kacprzyk, 2011, Dong, Xu et al. 2013, Dong & Saaty, 2014, Palomares, Martinez et al. 2014, Dong & Cooper, 2016, Rodríguez, Labella, Liu, Rodríguez, & Martínez, 2018, Wang, Shuai et al. 2019, Wu, Jin et al. 2019) have been widely applied to GDM.

Most of MCGDM approaches have been formulated for choice and ranking problems (Lu & Ruan, 2007, Qin, Liu et al. 2017) but recently group sorting approaches have been proposed (Lolli, Ishizaka et al. 2015, López & Ishizaka, 2017). This paper is focused on the AHPSort approach for sorting MCGDM problems, Group-AHPSort (GAHPSort) (López & Ishizaka, 2017) that aims at sorting alternatives into classes, according to all decision makers' knowledge. If the level of agreement among decision makers is ignored, it can result in unsuccessful solutions. Because the information elicited by the decision makers could drive to significant differences and conflicts in the sorting process.

Therefore, our aim is to develop a *Consensual GAHPSort* method that achieves at least a minimum degree of consensus, across the GAHPSort method before the aggregation processes and show the influence of disagreements in the results in sorting MCGDM. Among the different possibilities to achieve the consensus in the group decision process (Palomares, Estrella et al. 2014). Our proposal will apply a minimum cost consensus model based on non-linear programming (Zhang, Dong et al. 2011) for supporting decision makers to achieve a satisfactory consensual solution for all members involved in the sorting MCGDM problem.

Consensual GAHPSort process adds a minimum-cost consensus process for achieving a level of agreement in each one of the key elements of the *GAHPSort* namely, *profiles, weights* and *pairwise comparisons.*

Consequently, Consensual GAHPSort introduces the following novelties:

1. It applies CRPs in the resolution of sorting MCGDM problems. So far, there is no another sorting MCGDM approach that includes a CRP.
2. It obtains agreed and more reliable solutions than other approaches that do not consider CRPs, since its solutions avoid frustrated decision makers due to lack of consideration of their opinions and avoid conflicts among them.
3. The minimum cost model obtains rapidly solutions keeping decision makers' preferences as much as possible and avoids common problems in CRPs such as time consuming or deadlocks.

Finally, to show the reliability and performance of the Consensual GAHPSort; it will be applied to a case study focuses on sorting five European Union (EU) countries according to their sustainable development achievements regarding their social policies. To evaluate the performance of the selected countries, several *Sustainable Development Goals* adopted on the *2030 Agenda* (Resolution, 2015) have been considered.

The remaining of this paper is set up as follows: Section 2 provides the necessary preliminaries about AHPSort/GAHPSort methods, CRPs and minimum-cost consensus. Section 3 presents the *Consensual* GAHPSort for MCGDM sorting problems. Section 4 develops an illustrative case study of consensual GAHPSort application showing its results and comparing them with the ones obtained by GAHPSort approach. Section 5 concludes this paper.

2. Preliminaries

This section revises the main AHPSort approaches and concepts related to CRPs necessary to understand our proposal.

2.1. AHPSort/GAHPSort

Sorting problems are part of the problems solved by MCDM methods. According to Roy (1981), sorting problems (P • β) are oriented *"towards the characterization of the conditions for assigning actions to predefined categories and each category must be devised in relation to a specific subsequent treatment of the actions which it contains"*. AHPSort and its derivatives are an adaptation of the AHP method (Saaty, 1999) for sorting MCDM problems, although it could be also used for ranking alternatives (for example with a medium/large set of alternatives).

The AHPSort (Ishizaka, Nemery et al. 2012) aims at sorting alternatives into classes that are ordered from *most* to *least* preferred, according to the following three phases composed by 8 different steps (Ishizaka, Nemery et al. 2012):

A) *Phase 1*: **Problem definition**
   1) The criteria c_j, $j = 1,\ldots,m$, the alternatives a_k, $k = 1,\ldots,l$ and the goal of the problem are established.
   2) It is defined the ***classes*** C_i, $i = 1,\ldots,n$, that are ordered and can have a linguistic descriptor (e.g. excellent, good, medium, bad, poor)
   3) The ***profiles*** of each class, C_i, are defined by means either a local limiting profiles lp_{ij} (minimum performance that should have a criterion c_j to be in the class C_i), or with local central profiles cp_{ij} (characteristic example of an element in the class C_i on c_j). Fig. 1 shows a representation of both profiles.

D) *Phase 2:* ***Evaluations***
   1) The decision maker compares pairwise the importance of the criteria i and j on a scale 1 to 9 (Table 1). All pairwise comparisons a_{ij} are gathered in a comparison matrix A. The formula (3) can be used to calculate the consistency of the evaluations. If the matrix is consistent or near consistent (i.e. the evaluations are not contradictory), we can calculate the weights. To calculate the weight w_j of each criteria c_j from the comparison matrix, we use the eigenvalue method (1) (Saaty, 1972, Saaty, 1980).

$$\boldsymbol{A}\cdot\boldsymbol{w} = \lambda\cdot\boldsymbol{w} \tag{1}$$

where
A is the comparison matrix
w is the priorities/weights vector
λ is the maximal eigenvalue

$$\boldsymbol{CI} = \frac{\lambda_{max}}{\boldsymbol{n}-1} \tag{2}$$

where
n is the dimension of the matrix
λ_{max} is the maximal eigenvalue

$$\boldsymbol{CR} = \boldsymbol{CI}/\boldsymbol{RI} \tag{3}$$

CR is the consistency ratio
RI is the random index
RI represents the average CI of 500 randomly filled matrices. Saaty carried out this experiment in order to identify if decision makers provide their preferences randomly and detect inconsistencies. If CR is less than 10% (the inconsistency is less than 10% of 500 randomly filled matrices), then the matrix is of an acceptable consistency. Saaty (1977) calculated the following random indices (see Table 2):

1) Each alternative a_k is pairwise compared with the limiting profiles lp_{ij} or central profiles cp_{ij} for each criterion c_j (gathered in a matrix). As in the standard AHP, the question is: how many times the alternative a_k is better/preferred/more important than the limiting or central profile.
2) The local priority p_{kj} of each alternative a_k and the local priority of the limiting lp_{kij} or central profiles, cp_{kij} for such specific alternative a_k are computed for each criterion c_j by means of the previous matrices with Eq. (1).

E) *Phase 3:* **Assignment to classes**
1) First, it is computed global priorities p_k, lp_{ki} or cp_{ki} for respectively alternative a_k, limiting and central profiles by aggregating local priorities.

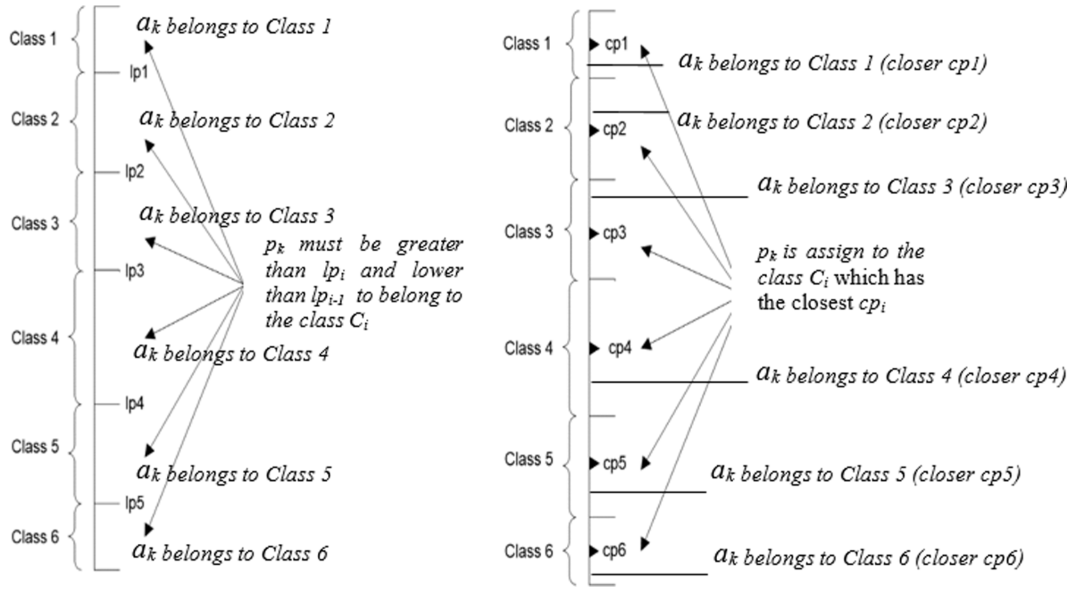

Fig. 1. Sorting with limiting/central profiles.

Table 1
Evaluation scale.

Numeric scale	Meaning
1	Equal importance
2	Equally to moderate
3	Moderate importance
4	Moderately to strong
5	Strong importance
6	Strongly to very strong
7	Very strong importance
8	Very strong to extremely
9	Extreme importance

Table 2
Random index.

N	3	4	5	6	7	8	9	10
RI	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

$$p_k = \sum_{j=1}^{m} p_{kj} \cdot w_j \tag{4}$$

$$lp_{ki} or cp_{ki} = \sum_{j=1}^{m} p_{kij} \cdot w_j \tag{5}$$

where p_{kij} refers to the local priority of the limiting or central profiles for the alternative a_k, class C_i and criterion c_j.

1) The assignment of an alternative a_k to a class C_i is accomplished by the comparison of p_k with lp_{ki} or cp_{ki} (see Fig. 1).
2) Steps 5) to 8) are repeated for each alternative to be classified.

In (López & Ishizaka, 2017) the AHPSort approach was updated, to deal with GDM problems. The GAHPSort (scheme represented in Fig. 2) was introduced by modifying the previous algorithm. The main differences are:

- In the *problem definition*, there is a group of decision makers $DM = \{dm_s, s = 1, \cdots, h\}$ in addition to the goal, criteria and alternatives. The ordered classes C_i, $i = 1,\ldots,n$ are defined by the group. Then, each decision-maker solve the sorting problem with AHPSort individually

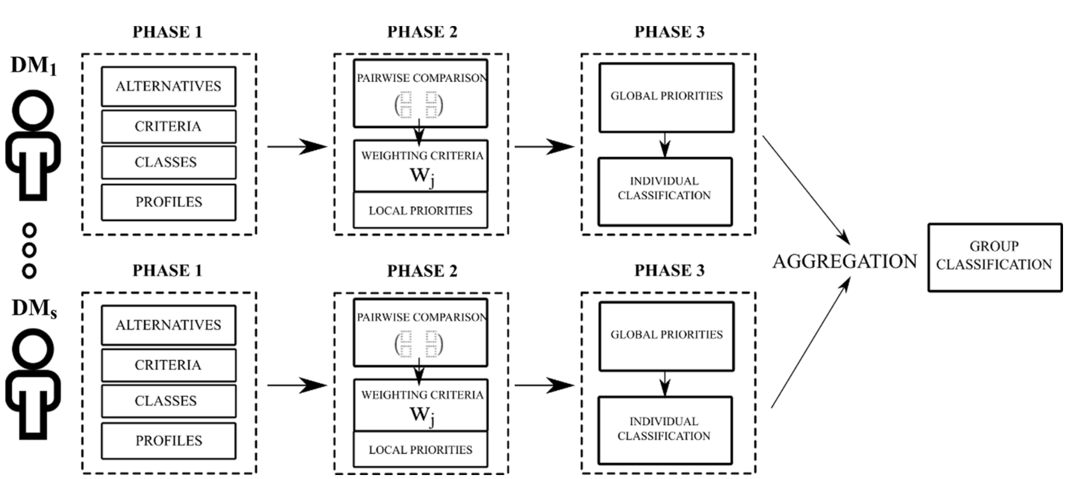

Fig. 2. GAHPSort: Group sorting process.

- Finally, in the global assignment to classes there are three possible scenarios: (i) all decision makers assign a_k to the same class C_i (unanimity), (ii) the majority of the decision makers assign a_k to the class C_i (simple majority) and (iii) an equal number of decision makers assign a_k to two or more different classes. For the latter case, the assignment is carried out by considering two different views, optimistic and pessimistic. If an optimistic view is applied, the alternative a_k is classified in the best class, otherwise, the alternative a_k is classified in the worst class (see (López & Ishizaka, 2017) for further details).

2.2. Consensus reaching processes

GDM problems are usually solved by a selection process that obtains the best alternative(s) as solution of the problem. However, sometimes the goal of the problem is not to obtain the best solution, but an accepted one for all involved decision makers in the problem. This task is as key as it is complex. Key because there are many real decision situations (Saint & Lawson, 1994, Palomares, Estrella et al. 2014) in which a consensual solution is more valued and necessary, and complex because in many cases the decision makers' opinions may diverge enormously due to the different levels of decision makers' knowledge and even their own interests. For this reason, it seems necessary to apply a CRP. The concept of *consensus* has been widely discussed in the specialized literature. Some researchers consider consensus as unanimity (Kline, 1972) (almost impossible to reach in real MCDM problems) but others have a softer view, closer to the perception that human beings have about it. According to the definition proposed by Saint and Lawson (1994), consensus is *"a state of mutual agreement among the members of a group, where all legitimate concerns of individuals have been addressed to the satisfaction of the group"*. In this contribution, we consider the latter definition of consensus. Therefore, a consensus process requires that decision makers modify their opinions making them closer to each other and in this way to obtain a collective opinion, which is satisfactory for all of them.

In Fig. 3, a graphical general scheme for CRP is shown. CRP is often a dynamic and iterative process, frequently coordinated by a human figure known as moderator who is responsible for supervision and guiding the discussion between decision makers, in which decision makers must modify their opinions and judgments in order to increase the level of agreement between them and reach an agreement (Saint & Lawson, 1994). In the specialized literature, consensus approaches have been proposed during the past decades (Dong, Chen, & Herrera, 2015; Dong, Luo, & Liang, 2015; Dong, Zhang, & Herrera-Viedma, 2016; Labella, Liu, Rodríguez, & Martínez, 2018; Quesada, Palomares, & Martinez, 2015; Wu, Jin, Fujita, & Xu, 2019; W. Xu, Chen, Dong, & Chiclana, 2020; X.H. Xu, Du, & Chen, 2015; Cao et al., 2019; Zhang, Dong, & Zhang, 2020).

Palomares, Estrella et al. (2014) presented a comprehensive taxonomy for the categorization of such approaches by considering two types of consensus measures and two classes of consensus procedures (see Fig. 4):

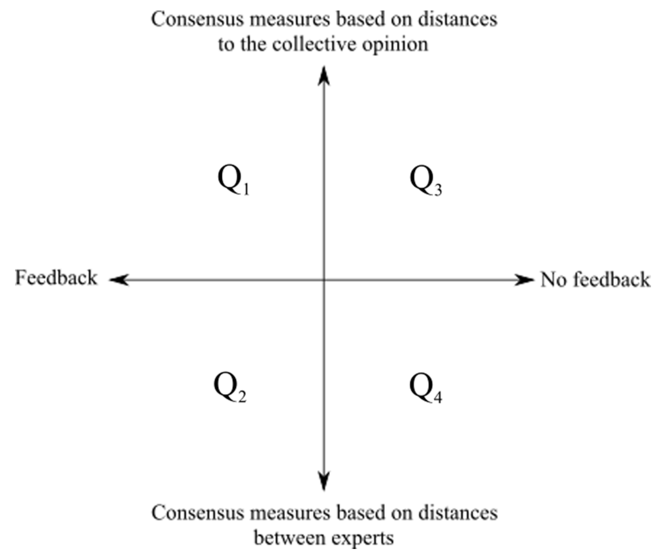

Fig. 4. Taxonomy for CRPs.

- Q1: Consensus reaching processes that provide a feedback to decision makers to guide their opinions and use consensus measures by computing distances to the group collective preference.
- Q2: Similar feedback to Q1 but the consensus measure is done by computing pairwise similarities.
- Q3: Consensus models that do not provide a feedback to decision makers but modify automatically their opinions and a similar consensus measure to Q1.
- Q4: Similar automatic modification process to Q3 and consensus measures like in Q2.

For further detail about CRPs and their categorization see (Herrera-Viedma, García-Lapresta et al. 2011, Palomares, Estrella et al. 2014)

Remark 1.. *Note that the former taxonomy identifies two different consensus models approaches based on their feedback or no feedback process. Whereas the former require the frequent involvement of the decision makers, the latter apply automatic changes in decision makers' preferences in order to achieve consensus as soon as possible. This proposal makes use of a no feedback approach but either of them could be applied.*

The modification of decision makers' preferences by means of feedback processes may lead to endless negotiation processes whose resulting cost to achieve the consensus is too high. The latter issue is not trivial, since the resources for consensus building are usually limited. For this reason, several proposals focus on minimizing as much as possible the changes in the initial decision makers' preferences (Ben-Arieh & Easton, 2007, Dong, Xu et al. 2010, Zhang, Dong et al. 2011) were proposed.

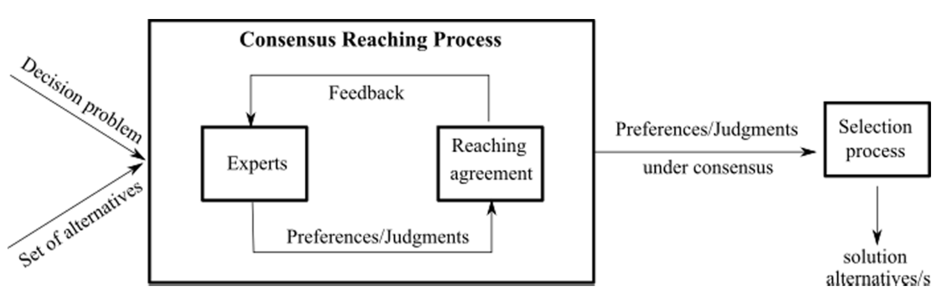

Fig. 3. General scheme for CRP.

2.3. Minimum-cost consensus

In the taxonomy showed in Fig. 4, the CRPs in Q1 and Q2 use a feedback mechanism to guide decision makers to bring their opinions closer together. However, this feedback process may imply and excessive change in the original decision makers' preferences and, consequently, a high cost consumption and a very time-consuming process.

Dong, Xu et al. (2010) introduced a CRP that preserved as much as possible the original decision makers' preferences by computing the distance among them and the collective opinion. In this case, the consensus degree of each decision maker regarding his/her modified opinion should be acceptable.

$$\min \sum_{s=1}^{h} d(\overline{o}_s, o_s) \tag{M-1}$$

$$\overline{o} = F(\overline{o}_1, \overline{o}_2, \cdots, \overline{o}_h)$$

$$\left|\overline{o}_s - \overline{o}\right| \leq \varepsilon, s = 1, \cdots h$$

where o_s represents the original decision maker dm_s's opinion, $\overline{o}_s$ represents his/her adjusted opinion, $d(\cdot)$ is the deviation between $\overline{o}_s$ and o_s, $\overline{o}$ represents the collective opinion and ε is the maximum acceptable distance between each adjusted decision maker's opinion and the collective one.

On the other hand, Ben-Arieh and Easton (2007) introduced the concept of consensus cost as *"the cost of changing the original decision makers' preferences"*. Although Ben-Arieh and Easton designed a linear-time algorithm to obtain the minimum total cost, a specific optimization model was not given (Zhang, Zhao et al. 2020). For this reason, in (Zhang, Dong et al. 2011) was proposed an optimization cost model (M−2) to minimize the overall cost of modifying decision makers' opinions.

$$\min \sum_{s=1}^{h} c_s \left|\overline{o}_s - o_s\right|$$

$$\left|\overline{o}_s - \overline{o}\right| \leq \varepsilon, s = 1, \cdots h \tag{M-2}$$

where c_s is the cost of changing decision maker dm_s's opinion 1 unit, o_s represents the original decision maker dm_s's opinion, $\overline{o}_s$ represents his/her adjusted opinion, $\overline{o}$ represents the collective opinion and ε is the maximum acceptable distance between each adjusted decision maker's opinion and the collective one.

Lately, Zhang, Dong et al. (2011), taking advantage of the benefits of the models (M−1) and (M−2), proposed a new optimization model to study the influence of different aggregation operators in computing collective opinion in a minimum cost model.

$$\min \sum_{s=1}^{h} c_s \left|\overline{o}_s - o_s\right|$$

$$\overline{o} = F(\overline{o}_1, \overline{o}_2, \cdots, \overline{o}_h)$$

$$\left|\overline{o}_s - \overline{o}\right| \leq \varepsilon, s = 1, \cdots h \tag{M-3}$$

where F is an aggregation operator.

Afterwards, many proposals based on minimum cost models were proposed in the specialized literature (Dong, Li, Xu, & Gu, 2015; G. Zhang, Dong, & Xu, 2012; H. Zhang, Dong, Chiclana, & Yu, 2019; Zhang, Dong, & Zhang, 2020).

The minimum-cost model (M−3) provides an optimal solution in which the initial decision makers' preferences are modified as less as possible. Therefore, normally, a real CRP will always demand more changes from decision makers than the minimum cost model. In addition, the decision makers' preferences are modified automatically, which avoids problems related to real feedback processes such as time consumption or deadlocks. For this reason, we will make use of the non-linear programming model (M−3) to compute consensus in different steps of the sorting MCDM method GAHPSort and evaluate its influence in the resolution of sorting MCDM problems.

3. Consensual-GAHPSort

The importance of MCDM problems in real world and their increasing complexity demand a comprehensive view of the decision problem that hardly can be managed by just one decision maker. Therefore, MCGDM problems are often requested by companies and institutions mainly for choice and ranking issues. However, the need of sorting alternatives is necessary nowadays among other reasons because, due to the information available for the decision problems may imply many alternatives that initially should be sorted to make further decision later on.

Among the different sorting MCGDM approaches for group decision, GAHPSort approach (López & Ishizaka, 2017) sorts alternatives in classes ordered from *least* to *most* preferred according to the scheme presented in Fig. 2. It has been pointed out in Section 1 that the GAHPSort ignores the level of agreement among the decision makers involved in the decision problem to achieve the sorting of alternatives. This might result in unsatisfactory results for the decision makers because they do not share the solution achieved.

Therefore, in this section a *Consensual*-GAHPSort method is introduced in order to smooth the possible disagreements among decision makers and look for a satisfactory solution for all of them with a minimum cost. This novel method adds three new steps, for increasing the consensus, to the original GAHPSort method (López & Ishizaka, 2017) that are showed below and highlighted inside the dotted box in Fig. 5:

i. *Agreeing profiles*: the GAHPSort originally allows the decision makers involved in the MCGDM to fix different central/limiting profiles for the different classes ignoring level of agreement among them. It may result in important differences in the sorting process. Therefore, the first new step of the consensual GAHPSort consists of obtaining consensual central/limiting profiles for the classes.
ii. *Agreed criteria weights:* each decision maker during the GAHPSort approach can provide different preferences among the criteria to obtain their weights that may result again in disagreements and conflicting sorting results. Hence, the second novel step added to the GAHPSort will apply a consensus process to obtain agreed weights for all the decision makers.
iii. *Agreed pairwise comparisons:* the GAHPSort requires decision makers to provide pairwise comparisons between alternatives, a_k, and the central/limiting profiles of the different classes ignoring either agreement or disagreement among them. Consequently, the third step added in the *Consensual* GAHPSort consists of detecting disagreements in the pairwise comparisons among the decision makers and bring their opinions closer, smoothing such disagreements.

Among the different CRPs revised in Section 2.2, that may be applied to achieve the agreement in the different steps previously enumerated, this proposal takes advantage of the *minimum-cost consensus* models proposed in the literature (Ben-Arieh, Easton et al. 2009, Zhang, Dong et al. 2011, Liu, Chan et al. 2012, Zhang, Dong et al. 2014) and includes the use of a minimum cost consensus model (Zhang, Dong et al. 2011) (see Section 2.3) that facilitates the consensus achievement with a minimum cost from the modification of opinions point of view. The two previous steps are based on this consensus model and each one further detailed in the coming subsections.

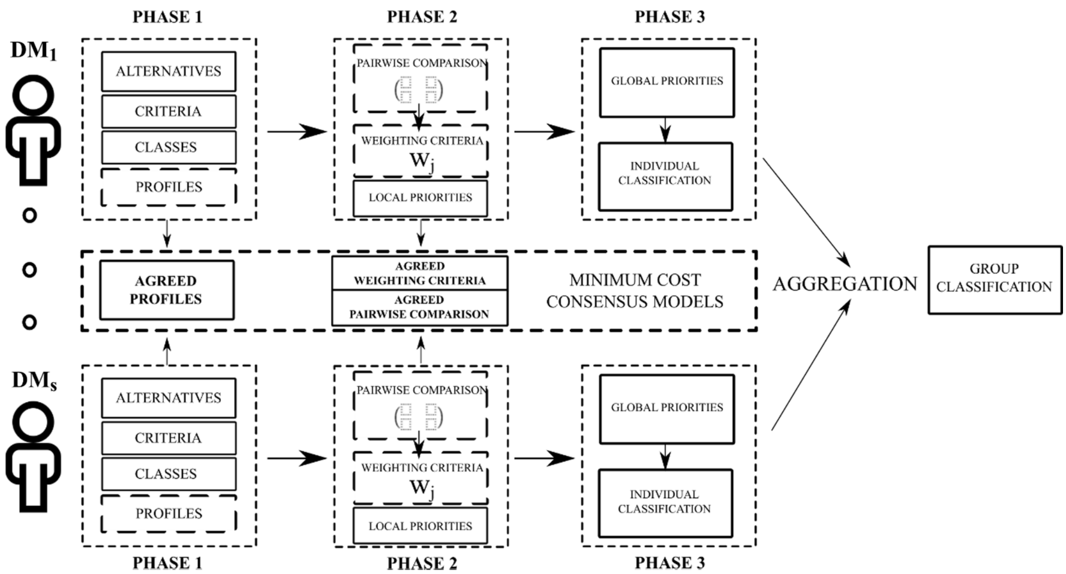

Fig. 5. Consensual GAHPSort approach.

3.1. Agreeing profiles

In Section 2.1, the step 3) was revised. The decision makers taking part in the GAHPSort process define a profile for each different class in which the alternatives will be sorted by means of central/limiting profiles. Without loss of generality, in this section it is assumed the use of central profiles by the decision makers, cp^s_{ij} (but the process is analogous with limiting profiles). Therefore, each decision maker dm_s, provides her/his central profileP^s to belong class C_i for the different criteriac_j.

$$dm_s \Rightarrow P^s = \{ cp^s_{11} \cdots cp^s_{1m} \cdots cp^s_{n1} \cdots cp^s_{nm} \} \tag{6}$$

Hence, the consensual GAHPSort aims at agreeing the central profilescp^s_{ij} provided by the different decision makersdm_s involved in the group decision sorting problem. The idea of achieving a consensus on the decision makers' profiles consists of looking for those values that minimize the change of the original central profilescp^s_{ij}. It obtains adjusted central profiles$\overline{cp}^s_{ij}$, which have a high level of agreement to generate the optimal collective value$\overline{cp}^*_{ij}$. A minimum cost consensus optimization process is applied to obtain such adjusted and collective values (Zhang, Dong et al. 2011). In details, the following steps are:

1. *Values to agree:* each central profile cp^s_{ij} for each criterion c_j and each class C_i is adjusted by means of a non-linear programming model to obtain new profiles with a higher level of agreement.

2. *Collective value*: For each central profile cp_{ij}, its collective value cp^*_{ij}, is computed, by aggregating the individual values provided by the h decision makers dm_s with an aggregation operator, F:

$$cp^*_{ij} = F(cp^1_{ij}, \cdots, cp^h_{ij}) \tag{7}$$

Remark 2.. *For sake of simplicity, without loss of generality, this paper considers Weighted Sum as the aggregation operator F.*

3. This step aims at reaching a consensus among the decision makers regarding profiles by changing the initial decision makers' opinions as little as possible. To do that, a linear consensus cost function is used (Zhang, Dong et al. 2011):

$$f_s\left(cp^s_{ij}, \overline{cp}^s_{ij}\right) = c_s \left| \overline{cp}^s_{ij} - cp^s_{ij} \right|, \tag{8}$$

where c_s is the cost of moving decision maker's dm_s opinion 1 unit and $\overline{cp}^s_{ij}$ are the adjusted central profiles with a higher level of agreement (Zhang, Dong et al. 2011).

Remark 3.. *Note that, the cost value c_s assigned to the decision maker dm_s depends on the attitude of such decision maker towards the suggested changes or his/her expertise degree. Decision makers receptive to accept the suggestions or with a low expertise degree will have a low cost change and vice versa (see* Ben-Arieh & Easton, 2007; Zhang, Dong et al. 2011 *for further details).*

In order to reach a consensus in the decision makers' group by changing as little as possible the initial decision makers' preferences, cp^s_{ij}, a minimization function is defined (Zhang, Dong et al. 2011):

$$\min \sum_{s=1}^{h} c_s \left| \overline{cp}^s_{ij} - cp^s_{ij} \right| \tag{9}$$

4. The minimum-cost consensus model with linear cost that will be used in this proposal is based on (Zhang, Dong et al. 2011) and it is formulated as:

$$\begin{cases} \min_{\overline{cp}^s_{ij}} \sum_{s=1}^{h} c_s \left| \overline{cp}^s_{ij} - cp^s_{ij} \right| \\ s.t. \overline{cp}^*_{ij} = F(\overline{cp}^1_{ij} \cdots, \overline{cp}^h_{ij}) \\ \left| \overline{cp}^s_{ij} - \overline{cp}^*_{ij} \right| \leq \varepsilon, s = 1, \cdots h \end{cases} \tag{10}$$

where ε is the established threshold of acceptable consensus level. From the model in Eq. (10), let $\Gamma = \left\{ \overline{cp}^{1*}_{ij}, \cdots, \overline{cp}^{h*}_{ij}, \overline{cp}^*_{ij} \right\}$ be its optimal solution in which $\overline{cp}^{s*}_{ij} (s = 1 \cdots h)$ are the adjusted individual opinions and $\overline{cp}^*_{ij}$ the optimal collective opinion.

For our proposal, all decision makers use the same consensual profiles, fixed by the optimal collective opinion $\overline{cp}^*_{ij}$. In this way, decision makers are prevented from classifying the alternatives into different classes and obtaining results that do not satisfy the whole group.

Remark 4.. *Note that, the value of the threshold of acceptable consensus ε depends on the level of agreement that wants to be reached in the group of decision makers* (Zhang, Dong, Xu, & Li, 2011)*. The more restrictive its value, the more changes in the initial decision makers' preferences. Then, such modified preferences are aggregated to obtain a new collective opinion to* *compute lately the distance among such modified decision maker's preference and the collective opinion in order to check if distance among both is lower or equal than ε.*

Remark 5.. *Notice for any value of ε, the feasible region defined by the constraints of the minimum cost model are non-empty since, by transforming the values present in the models into linear ones, the feasible region is convex and the objective function is linear, that guarantee the existence of the optimal solution of the minimum cost model for any values of ε* (Vanderbei, 2015).

4.1. Agreed criteria weights

Once the profiles that will be used by the decision makers during the GAHPSort approach have been fixed and agreed, the next step in this sorting approach is to compute the weights, w_j of each criterion c_j. Therefore, each decision maker dm_s, provides her/his pairwise comparison matrix between criteria G^s:

$$G^s = \begin{pmatrix} - & \cdots & p^s_{1m} \\ \vdots & \ddots & \vdots \\ p^s_{m1} & \cdots & - \end{pmatrix} \tag{11}$$

Before computing the weights vector by using Eq. (1), it should be considered the necessity of achieving a minimum level of agreement among the different decision makers regarding the priorities of the different criteria, otherwise the sorting process may result again in an unsatisfactory solution from group's point of view because of conflicts.

The description of the minimum cost consensus process model (Eq. (8)) for agreeing the criteria weights is below:

1. Values to agree: In this case, the values that should be agreed belong to the pairwise comparison matrix of the AHP method. It is necessary to agree only the $\frac{m^2-m}{2}$ values of the triangular superior area of the matrix because the matrix is reciprocal.

5. *Collective value:* For each pairwise value is the collective value is computed as:

$$p^*_{ij} = F(p^1_{ij}, \cdots, p^h_{ij}) \tag{12}$$

Remark 6.. *Aggregation operator F is similar to* Remark 2.

6. The minimum cost consensus process is formulated as:

$$\begin{cases} \min_{\overline{p}^s_{ij}} \sum_{s=1}^{h} c_s \left| \overline{p}^s_{ij} - p^s_{ij} \right| \\ s.t. \overline{p}^*_{ij} = F(\overline{p}^1_{ij}, \cdots, \overline{p}^h_{ij}) \\ \left| \overline{p}^s_{ij} - \overline{p}^*_{ij} \right| \leq \varepsilon_w, s = 1, \cdots h \end{cases} \tag{13}$$

where ε_w is the established threshold of acceptable consensus level for the criteria priorities (see Remark 4). From the model in Eq. (13), let $\Gamma_w = \left\{ \overline{p}^{1^*}_{ij}, \cdots, \overline{p}^{h^*}_{ij}, \overline{p}^*_{ij} \right\}$ be its optimal solution in which$\overline{p}^{s^*}_{ij}, (s = 1, \cdots, h)$, are the adjusted individual opinions and $\overline{p}^*_{ij}$, the adjusted collective opinion.

For our proposal, all decision makers use the same criteria weights: $\overline{p}^*_{ij}$.

6.1. Agreed pairwise comparisons

Finally, before assigning alternatives to classes, each individual provides their pairwise comparisons between alternatives and profiles and then assigns the alternatives to classes. The group sorting process is carried out according to these individual assignments. Therefore, important disagreements among decision makers in the pairwise comparisons may result again in results that may not satisfy several decision makers. Hence, it is considered that although opinions among decision makers eliciting pairwise comparisons could be different it should not present high disagreements. In order to avoid such situations this third step added to the GAHPSort aims at looking for adjusted individual values that shows the individual view of each decision maker but kept within a level of agreement.

Consequently, it is applied the minimum cost consensus model:

1. Values to agree: In this case the values to bring closer to each other are the pairwise comparison values, pw^{sk}_{ij}, comparing each alternative a_k with the central profiles cp_{ij} for each criterion c_j. In this case, it is only necessary the use the upper part of the pairwise comparison matrix because the lower part is reciprocal.

7. *Collective value:* For each pairwise comparison value, pw^{sk}_{ij}, a collective value is computed:

$$pw^{*k}_{ij} = F(pw^{1k}_{ij}, \cdots, pw^{hk}_{ij}) \tag{14}$$

Remark 7.. *Similar to* Remark 2, 5.

8. The minimum cost consensus process is formulated as:

$$\begin{cases} \min_{\overline{pw}^s_{ij}} \sum_{s=1}^{h} c_s \left| \overline{pw}^{sk}_{ij} - \quad pw^{sk}_{ij} \right| \\ s.t. \overline{pw}^{*k}_{ij} = F(\overline{pw}^{1k}_{ij}, \cdots, \overline{pw}^{hk}_{ij}) \\ \left| \overline{pw}^{sk}_{ij} - \overline{pw}^{*k}_{ij} \right| \leq \varepsilon_c, s = 1, \cdots h \end{cases} \tag{15}$$

where ε_c is the established threshold of acceptable consensus level for the pairwise comparisons (see Remark 4). From the model in Eq. (15), let$\Gamma_c = \left\{ \overline{pw}^{1k^*}_{ij}, \cdots, \overline{pw}^{hk^*}_{ij} \overline{pw}^{*k}_{ij} \right\}$ be its optimal solution in which$\overline{pw}^{sk^*}_{ij}, (s = 1, \cdots, h)$, are the adjusted individual pairwise comparisons and $\overline{pw}^{*k}_{ij}$, the adjusted collective opinion.

In this case the adjusted individual pairwise comparisons $\overline{pw}^{hk^*}_{ij}$ are used to carry out the individual assignment to class of each alternative.

Remark 8.. *Even though the cost values c_s for the decision maker dm_s may be different for the different elements to be aggregated, profiles, criteria and pairwise comparisons, for sake of simplicity we consider all the costs have the same value.*

8.1. Consensual GAHPSort process

The GAHPSort algorithm is modified by including the minimum cost consensus processes introduced in the previous section, resulting in the Consensual GAHPSort algorithm composed by the following phases and steps:

A) *Phase 1:* **Problem definition**
1) Similar to 1) in GAHPSort.
2) Similar to 2) in GAHPSort.

3) Similar to 3) in GAHPSort.
1) A non-linear programming minimum cost model (see Eq. (10)) is carried out to achieve an agreement on the decision makers' profiles. Finally, all decision makers use the same profiles represented by the optimal collective opinion $\overline{lp}_{ij}^*$ or $\overline{cp}_{ij}^*$ depending on the type of the profile.

B) *Phase 2:* **Evaluations**

2) A non-linear programming minimum cost consensus model (see Eq. (13)) is carried out to achieve an agreement among the different decision makers regarding the priorities of the different criteria by obtaining the adjusted collective opinion $\overline{p}_{ij}^*$.
3) All the decision maker dm_s use the same weight vector $\overline{w^*}$ for the criteria by using the Eq. (1) and the optimal collective values $\overline{p}_{ij}^*$ previously computed.
4) Each decision maker dm_s, makes a pairwise comparison of each alternative a_k with the limiting profiles $\overline{lp}_{ij}^*$ or central profiles $\overline{cp}_{ij}^*$ for each criterion c_j.
5) A non-linear programming minimum cost consensus model (see Eq. (15)) is carried out to achieve an agreement among the decision makers regarding the pairwise comparison among alternatives and profiles by obtaining the consensual individual pairwise comparisons $\overline{pw}_{ij}^{hk^*}$.
6) Similar to 6) in GAHPSort.

C) *Phase 3:* **Aggregation**

7) Similar to step 7) in GAHPSort but using the global priority $\overline{p}_k^*$ of the alternative a_k and the global priorities $\overline{lp}_i^*$ or $\overline{cp}_i^*$ of the limiting or central profiles.

D) *Phase 4:* **Assignment to classes**

8) By using $\overline{p}_k^*$ and $\overline{lp}_i^*$ or$\overline{cp}_i^*$, the same process 8) that in GAHPSort is carried out.

9. Case Study: Sorting countries according their sustainable development based on consensual GAHPSort

In order to show the validity and advantages of the proposed method, this section presents a case study in which five European countries, Spain, United Kingdom, Croatia, Norway and Romany, have been selected to be sorted according to their sustainable development achievements with respect to social issues. Such a selection has been based on the diversity that exist among the countries in several aspects and their localization in different areas of the Europe. The problem resolution is based on the eleven steps described in section 3.4.

9.1. Problem definition

1) The goal for this problem is to classify five EU countries, which are Spain, United Kingdom, Croatia, Norway and Romany, according to their sustainable development achievements with respect to social issues. Sustainable development (Brundtland, 1987) has been defined as the "*development that meets the needs of the present without compromising the ability of future generations to meet their own needs*". In 2015, United Nations Member States adopted the *2030 Agenda for Sustainable Development* (Resolution, 2015), a global agreement to erase poverty and fight inequality and injustice by 2030, and its 17 Sustainable Development Goals (SDGs). There are 169 targets associate to the 17 SDGs and each one has between 1 and 3 indicators used to measure progress toward reaching the targets. In total, 304 indicators will measure compliance. Considering all the SDGs and indicators might complicate the understanding of the case study and, consequently, the demonstration of the proposed model. For this reason, this case study is only focused on evaluating the sustainable performance of the selected EU countries regarding social policies. Therefore, based on this decision, a preselection of all the SGDs related with this area has been carried out, discarding others related to different areas. The chosen SGDs and their corresponding indicators (criteria) are introduced below:

2) No poverty

3) People at risk of poverty or social exclusion (c_1): This indicator corresponds to the sum of persons who are at risk of poverty after social transfers, severely materially deprived or living in households with very low work intensity.

- Zero hunger
  - *Obesity rate by body mass index (BMI) (% of population aged 18 or over* (c_2): The indicator measures the share of obese people based on their index BMI, BMI is defined as the weight in kilos divided by the square of the height in meters.
- Good health and well-being
  - *Life expectancy at birth* (c_3): Life expectancy at birth is defined as the mean number of years that a new-born child can expect to live if subjected throughout his life to the current mortality conditions (age specific probabilities of dying).
- Quality education
  - *Young people neither in employment nor in education and training* (c_4): The indicator measures the share of the population aged between 15 and 29 who is not employed and not involved in education or training.
- Gender equality
  - *Gender employment gap* (c_5): The indicator measures the difference between the employment rates of men and women aged between 20 and 64. The employment rate is calculated by dividing the number of persons aged between 20 and 64 in employment by the total population of the same age group.
- Peace, justice and strong institutions
  - *Death rate due to homicide (number per 100 000 persons)* (c_6): The indicator tracks deaths due to homicide and injuries inflicted by another person with the intent to injure or kill by any means, by including 'late effects' from assault. It does not include deaths due to legal interventions or war.

The evaluations over the different countries were carried out by a set of three decision makers, $DM = (dm_1, dm_2, dm_3)$, in multiple areas such as Cartography, Geography and Economics through a questionnaire[1].

4) The countries will be classified into three classes, $C = (C_1$*: Good,* C_2*: Moderate,* C_3*: Bad)* according to their sustainable development performance regarding social policies.

5) Each decision maker defined the profiles of each class by using *limiting profiles*. To obtain the limiting profiles values, a survey was conducted through the distribution of a questionnaire among the decision makers. The respondents were asked to assign the limiting profiles of each class for each criterion. An excerpt of the questionnaire for evaluating the limiting profiles is shown in Fig. 6.

6) Once the limiting profiles are provided by the decision makers, the evaluations are introduced in the software application AFRYCA (Labella, Estrella et al. 2017) and a non-linear programming minimum cost consensus model (see Eq. (10)) is carried out to achieve an agreement on the decision makers' profiles. Note that, Consensual GAHPSort has been incorporated into AFRYCA, as there is currently no commercial software tool to carry out the processes that make up model. Finally, all decision makers use the same consensual limiting profiles represented by the collective optimal value $\overline{lp}_{ij}^*$ (see Table 3).

[1] Initially four decision makers provided their preferences over the alternatives through the questionnaire but one of whom was discarded for providing inconsistent valuations.Questionnaire available in https://sinbad2.ujaen.es/afryca/sites/default/files/dataset/Sustainable%20Development-Questionnaire.pdf

1. Could you indicate the limiting profiles** (classes of performance) for each of the criteria described above?

Indicator	**Limiting profile 1** (**good performance**)	**Limiting profile 2** (**moderate performance**)
People at risk of poverty or social exclusion		
Obesity rate by body mass index		
Life expectancy at birth		
Young people neither in employment nor in education and training		
Gender employment gap		
Death rate due to homicide		

Fig. 6. An excerpt of the questionnaire for evaluating limiting profiles.

Remark 9.. *Note that the value of the threshold of acceptable consensus levelε is a key aspect in our approach. Depending on the needs of consensus of the problem, this value can be adjusted, the smaller the value, the higher the consensus to be reached and vice-versa. To show the influence of this parameter, this case study is carried out by using two different values of ε, one more restrictive* 0.15, *and another more moderate,* 0.5. *For costs and weights, the assignments are $c_s = 1$, $s = 1, 2, 3$ and $w_s = 1/h$, $h = 3$ respectively*.

According to Remark 9, we have assumed that the cost value for all the decision makers is the same. We do not focus on its estimation because it is not the aim of the proposal. However from our point of view, in this type of problem, it would be adequate to fix the cost according to the degree of expertise that can be determined by the number of years working on these problems or the number of problems in which the decision maker has been involved. Let us suppose an example in which the three decision makers involved in the resolution of this sorting MCDM problem have distinct degrees of expertise, determined by the number of years working on this kind of problems, respectively 6, 15, 3. The cost for each decision maker may be assigned directly according to the number of years, $c_1 = 6, c_2 = 15, c_3 = 3$.

Example 1:. *To compute the consensual profile$\overline{lp}^{*}_{11}$ with $\varepsilon = 0.15$, the minimum cost model (see Eq. (10)) would be defined as follows (other consensual profiles are computed in a similar way)*:

$$\min\left|\overline{lp}^{1}_{11} - 15\right| + \left|\overline{lp}^{2}_{11} - 10\right| + \left|\overline{lp}^{3}_{11} - 10\right|$$

$$\overline{lp}^{*}_{11} = \frac{\overline{lp}^{1}_{11} + \overline{lp}^{2}_{11} + \overline{lp}^{3}_{11}}{3}$$

$$\left|\overline{lp}^{1}_{11} - \overline{lp}^{*}_{11}\right| \leq 0.15$$

$$\left|\overline{lp}^{2}_{11} - \overline{lp}^{*}_{11}\right| \leq 0.15$$

$$\left|\overline{lp}^{3}_{11} - \overline{lp}^{*}_{11}\right| \leq 0.15$$

The optimization model searches the consensual profiles $\overline{lp}^{1}_{11}, \overline{lp}^{2}_{11}, \overline{lp}^{3}_{11}$ that satisfy the constraints, by returning the following values: $\overline{lp}^{1}_{11} = 10.21696, \overline{lp}^{2}_{11} = 10.26877, \overline{lp}^{3}_{11} = 10.26877$ and thus, $\overline{lp}^{*}_{11} = 10.2515$.

9.2. Evaluations

7) The criteria have been pairwise compared in a questionnaire (Fig. 7) by the decision makers. The evaluations are then entered in the software application AFRYCA (Labella, Estrella et al. 2017) for calculating the criteria weights by using Eq. (13) to achieve an agreement among the different decision makers regarding the priorities of the criteria by obtaining the adjusted collective opinion $\overline{p}^{*}_{ij}$.

8) Then, decision makers make use of the same weighting vector $\overline{w^{*}}$ based on the values $\overline{p}^{*}_{ij}$ (see Table 4).

Example 2:. *The first three rows in Table 4 show the non-consensual weights of the decision makers, computed from the criteria pairwise comparisons provided by them in the questionnaire by using Eq. (1). To compute the consensual weighs, first we extract one by one the comparisons between pair of criteria and use Eq. (13) to obtain a consensual pairwise comparison for such pair. For instance, the pairwise comparisons between the criteria c_1 and c_2 provided by the three decision makers are*: $a^{1}_{12} = 8, a^{2}_{12} = 9, a^{3}_{12} = 8$. *Then, by using Eq. (13), we obtain a collective consensual pairwise comparison for the criteria c_1-c_2 as follows* ($\varepsilon = 0.15$):

$$\min\left|\overline{p}^{1}_{12} - 8\right| + \left|\overline{p}^{2}_{12} - 9\right| + \left|\overline{p}^{3}_{12} - 8\right|$$

$$\overline{p}^{*}_{12} = \frac{\overline{p}^{1}_{12} + \overline{p}^{2}_{12} + \overline{p}^{3}_{12}}{3}$$

$$\left|\overline{p}^{1}_{12} - \overline{p}^{*}_{12}\right| \leq 0.15$$

$$\left|\overline{p}^{2}_{12} - \overline{p}^{*}_{12}\right| \leq 0.15$$

$$\left|\overline{p}^{3}_{12} - \overline{p}^{*}_{12}\right| \leq 0.15$$

The optimization model searches the consensual comparisons $\overline{p}^{1}_{12}, \overline{p}^{2}_{12}, \overline{p}^{3}_{12}$ that satisfy the constraints, by returning the following values: $\overline{p}^{1}_{12} = 7.99857, \overline{p}^{2}_{12} = 8.16652, \overline{p}^{3}_{12} = 7.99857$ and thus, $\overline{p}^{*}_{12} = 8.05455$.

Table 3
Limiting profiles.

Limiting profiles	dm_1		dm_2		dm_3		Consensus			
							$\varepsilon = 0.15$		$\varepsilon = 0.5$	
	lp_{1j}	lp_{2j}	lp_{1j}	lp_{2j}	lp_{1j}	lp_{2j}	$\overline{lp}^{*}_{1j}$	$\overline{lp}^{*}_{2j}$	$\overline{lp}^{*}_{1j}$	$\overline{lp}^{*}_{2j}$
c_1	15	25	10	15	10	20	10.2515	16.4815	10.8937	19.9754
c_2	50	55	50	55	30	50	32.9374	50.6919	39.9317	52.4404
c_3	82	78	82	78	80	75	80.2167	75.3701	80.9162	76.4193
c_4	8	15	8	12	10	15	8.0966	12.4361	8.3534	8.3535
c_5	5	10	5	10	5	10	5.0	10.0	5.0	10.0
c_6	0.5	0.8	0.25	0.75	1	1.5	0.3021	0.8072	0.4534	0.9097

The 1-9 double-scale provided represents how many times a criterion is more important than the one on the opposite side of the row, while 1 represents equal importance.

Circle one number per row below using the scale:

1 = Equal 3 = Moderate 5 = Strong 7 = Very strong 9 = Extreme

2, 4, 6, 8 are intermediate values

People at risk of poverty or social exclusion	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	Obesity rate by body mass index

Fig. 7. An excerpt of the questionnaire for evaluating criteria pairwise comparison.

Table 4
Criteria weights.

Criteria weights	c_1	c_2	c_3	c_4	c_5	c_6
dm_1	0.325	0.031	0.125	0.392	0.1	0.027
dm_2	0.315	0.037	0.234	0.243	0.06	0.111
dm_3	0.142	0.035	0.429	0.297	0.017	0.08
$\overline{p}^*_{ij}(\varepsilon = 0.15)$	0.198	0.031	0.21	0.408	0.064	0.089
$\overline{p}^*_{ij}(\varepsilon = 0.5)$	0.275	0.042	0.252	0.322	0.05	0.059

We apply the same process for the rest of comparisons, c_1-c_3, c_1-c_4, c_1-c_5, c_1-c_6, c_2-c_3, c_3-c_4… by obtaining a consensual value for each one. In this way, we compose a new consensual pairwise comparison matrix between criteria:

$$G^* = \begin{pmatrix} - & \cdots & \overline{p}^*_{1m} \\ \vdots & \ddots & \vdots \\ \overline{p}^*_{m1} & \cdots & - \end{pmatrix}$$

Finally, by using Eq. (1) we obtain the final consensual weights represented in the penultimate row of Table 4 (similar with $\varepsilon = 0.5$).

9) The sorting of the five countries was obtained by comparing pairwise each alternative to the limiting profile for each criterion. An excerpt of the questionnaire for evaluating the limiting profiles is shown in Fig. 8.

10) Before computing the local priorities, possible disagreements among decision makers in pairwise comparison alternatives-limiting profiles are smoothed. The initial pairwise comparisons are introduced in AFRYCA (Labella, Estrella et al. 2017) for calculating the consensual individual pairwise comparison $\overline{pw}^{hk*}_{ij}$ by using Eq. (15).

Example 3:. *Similarly to the process for computing consensual weights, in step* 8, *we extract each pairwise comparison between the alternatives and profiles provided by the decision makers through the survey (see* Fig. 8*) and compute a consensual value for each one by using Eq.* (15). *For instance, the decision makers provided the following comparisons between the alternative* a_1 *and the limiting profile* lp_{11}: $pw^{11}_{11} = 1/8, pw^{21}_{11} = 1/8, pw^{31}_{11} = 1/6$. *Then, by using Eq.* (15), *we obtain a collective consensual pairwise comparison for the alternative-profile* a_1-lp_{11} *as follows* ($\varepsilon = 0.15$):

$$\min\left|\overline{pw}^{11}_{11} - 1/8\right| + \left|\overline{pw}^{21}_{11} - 1/8\right| + \left|\overline{pw}^{31}_{11} - 1/6\right|$$

$$\overline{pw}^{*1}_{11} = \frac{\overline{pw}^{11}_{11} + \overline{pw}^{21}_{11} + \overline{pw}^{31}_{11}}{3}$$

$$\left|\overline{pw}^{11}_{11} - \overline{pw}^{*1}_{11}\right| \leq 0.15$$

$$\left|\overline{pw}^{21}_{11} - \overline{pw}^{*1}_{11}\right| \leq 0.15$$

$$\left|\overline{pw}^{31}_{11} - \overline{pw}^{*1}_{11}\right| \leq 0.15$$

The optimization model searches the consensual comparisons $\overline{pw}^{11}_{11}$, $\overline{pw}^{21}_{11}, \overline{pw}^{31}_{11}$ that satisfy the constraints, by returning the following values: $\overline{pw}^{11}_{11} = 0.12410, \overline{pw}^{21}_{11} = 0.12410, \overline{pw}^{31}_{11} = 0.16576$ and thus, $\overline{pw}^{*1}_{11} = 0.13799$. We apply the same process for the rest of comparisons, a_1-lp_{11}, a_1-lp_{12}, a_1-lp_{13}, a_1-lp_{14}, a_1-lp_{15}, a_1-lp_{16}… by obtaining a consensual value for each comparison. Then, we compose a consensual pairwise comparison matrix between alternatives and profiles with the computed consensual values.

3. Compare the relative performance of the EU countries against the **people at risk of poverty or social exclusion** against the limiting profiles

Spain	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	Limiting profile 1
Spain	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	Limiting profile 2
UK	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	Limiting profile 1
UK	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	Limiting profile 2
Croatia	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	Limiting profile 1
Croatia	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	Limiting profile 2
Norway	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	Limiting profile 1
Norway	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	Limiting profile 2
Romania	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	Limiting profile 1
Romania	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	Limiting profile 2

Fig. 8. An excerpt of the questionnaire for evaluating alternatives-limiting profiles pairwise comparison.

11) From previous consensual matrices, it is computed for each alternative a_k and decision maker dm_s its local priority p_{kj}^s and the local priority lp_{ij} of the limiting profiles $\overline{lp}_{ij}^*$, with Eq. (1).

9.3. Aggregation

12) The global priority $\overline{p}_k^*$ of the alternative a_k and the global priorities $\overline{lp}_i^*$ of the limiting profiles are computed by Eqs. (4) and (5) (see Table 5).

9.4. Assignment to classes

1) By using $\overline{p}_k^*$ and $\overline{lp}_i^*$ the same process that in GAHPSort is carried out.

According to Table 6, depending on the minimal consensus to achieve within the group represented by the parameter ε, different classifications are obtained. The alternatives United Kingdom, Croatia, Norway and Romania are globally classified into the classes *Medium, Bad, Good* and *Bad* respectively with both values ofε, although there are slightly changes in the individual classification of dm_3 for the alternative Romania. This latter case shows clearly the influence of the parameter ε throughout the sorting process. When the level of consensus to achieve within the group is low, as in the case of $\varepsilon = 0.5$, decision makers largely keep their opinions, and possible conflicts among them are ignored (dm_3 disagrees with the rest of the group). On the other hand, with a higher level of consensus,$\varepsilon = 0.15$, decision makers bring positions closer together and achieve more consensual classifications (all decision makers classify Romania into *Bad*). Apart from Romania, the most significant differences take place in the classification of the alternative Spain, concretely, dm_1 classifies Spain into *Medium* with $\varepsilon = 0.5$ and into *Bad* with $\varepsilon = 0.15$, by resulting a different global classification for such alternative. In the same way as above, with a low value of consensus to achieve, each decision maker carries out his individual classification without considering the opinion of the rest of the group. As consequence, each decision makers classifies Spain into a different class, which is globally classified into the class *Good* by applying the tiebreaker process defined in (López & Ishizaka, 2017). On the contrary, with a considerable consensus to reach by the decision makers, their preferences are further modified in order to achieve such consensus level, smoothing possible disagreements. In this way, dm_1 and dm_2 reach an agreement among them, by resulting a classification by majority of Spain into the *Bad* class.

Therefore, it is evident that the level of agreement to reach among decision makers is a determining factor in the classification process. With a more restrictive value of ε, it is intended that the decision makers smooth their disagreements to a greater extent and try to achieve a more consensual solution. With a low value of ε, disagreements among decision makers are not sufficiently considered, which implies that the final classification of the alternatives might not satisfy to the all members of the group or misclassifications.

Table 6
Consensual GAHPSort classification.

Classification	Spain	U.K	Croatia	Norway	Romania	
$\varepsilon = 0.15$	dm_1	Bad	Medium	Bad	Good	Bad
	dm_2	Bad	Medium	Bad	Good	Bad
	dm_3	Good	Good	Good	Good	Bad
	Global	Bad	Medium	Bad	Good	Bad
$\varepsilon = 0.5$	dm_1	Medium	Medium	Bad	Good	Bad
	dm_2	Bad	Medium	Bad	Good	Bad
	dm_3	Good	Good	Good	Good	Good
	Global	Good	Medium	Bad	Good	Bad

Remark 10.. *Note that decision makers sometimes classify alternatives in different categories that are not consecutive even when consensus processes are applied (see Table 6, Spain and Croatia with $\varepsilon = 0.15$). Therefore, in this situation simple majority may be doubtful. However, these cases can be avoided by reducing the value of the parameter ε and thus, increasing the consensus among decision makers. For instance, if $\varepsilon = 0.05$, all decision makers classify Spain and Croatia in the category Bad.*

9.5. Comparison with GAHPSort

The disagreement among decision makers in a sorting process might lead to misclassification of the alternatives. To show the latter and expose the importance and utility of detecting disagreements and smoothing them, the results provided by Consensual GAHPSort and GAHPSort are compared. Table 7 shows the classification of the alternatives provided by GAHPSort.

Table 7
GAHPSort classification.

Classification	Spain	United Kingdom	Croatia	Norway	Romania
dm_1	Medium	Medium	Bad	Good	Bad
dm_2	Bad	Medium	Bad	Good	Bad
dm_3	Good	Good	Good	Good	Good
Global	Good	Medium	Bad	Good	Bad

Table 5
Global priorities.

Global Priorities		$\varepsilon = 0.15$				$\varepsilon = 0.5$			
		Good		Medium		Good		Medium	
		$\overline{lp}_1^*$	$\overline{p}_k^*$	$\overline{lp}_2^*$	$\overline{p}_k^*$	$\overline{lp}_1^*$	$\overline{p}_k^*$	$\overline{lp}_2^*$	$\overline{p}_k^*$
dm_1	Spain	0.692	0.307	0.525	0.474	0.656	0.343	0.408	0.591
	U.K	0.54	0.459	0.285	0.714	0.547	0.452	0.212	0.787
	Croatia	0.876	0.123	0.64	0.359	0.887	0.112	0.539	0.46
	Norway	0.318	0.681	0.15	0.849	0.243	0.756	0.126	0.873
	Romania	0.888	0.111	0.839	0.16	0.887	0.112	0.854	0.145
dm_2	Spain	0.72	0.279	0.64	0.359	0.693	0.306	0.628	0.371
	U.K	0.686	0.313	0.41	0.589	0.669	0.33	0.461	0.538
	Croatia	0.874	0.125	0.762	0.237	0.873	0.126	0.771	0.228
	Norway	0.376	0.623	0.195	0.804	0.375	0.624	0.253	0.746
	Romania	0.895	0.104	0.854	0.145	0.892	0.107	0.851	0.148
dm_3	Spain	0.391	0.608	0.319	0.68	0.213	0.786	0.174	0.825
	U.K	0.352	0.647	0.245	0.754	0.188	0.811	0.156	0.843
	Croatia	0.455	0.544	0.37	0.629	0.233	0.766	0.19	0.809
	Norway	0.245	0.754	0.146	0.853	0.148	0.851	0.111	0.888
	Romania	0.581	0.418	0.5	0.499	0.424	0.575	0.372	0.627

Note that GAHPSort provides a different classification that Consensual GAHPSort when $\varepsilon = 0.15$ but it is the same with $\varepsilon = 0.5$. When the level of agreement ε is more restrictive, disagreements are smoothed to a greater extent and consequently the initial valuations from decision makers regarding profiles, criteria weights and pairwise comparison matrices are modified into more consensual values, which implies at the same time a more consensual and adequate classification. With a value of ε too high, as is the case of $\varepsilon = 0.5$, consensus among decision makers is practically not taken into consideration and disagreement among them, ignored. Therefore, the changes applied to the decision makers' opinions are not so significant and the agreement among decision makers not so high, by obtaining similar results to the one obtained if consensus is not considered. Once again, the relevance of the level of agreement within the group of decision makers is evident.

10. Conclusions

MCDM approaches are mostly focused on ranking and choice problems. However, nowadays the complexity of problems and needs of organizations require not only multiple points of view that lead to group decision making problems but also the sorting of the alternatives instead of its ranking or choice. Among the different MCDM sorting approaches, our interest is focused on a recent proposal as the AHPSort and its extension for group decisions, so-called GAHPSort, in which multiple decision makers provide their preferences for sorting the alternatives. Multiple decision makers involve in a sorting decision making problem with their own attitudes may imply either conflicts in the resolution process or the solution obtained. In order to smooth such conflicts CRPs have been applied to achieve satisfactory solutions for the group of decision makers, looking for an agreed solution.

This paper has introduced Consensual GAHPSort, a new sorting method based on GAHPSort, which aims at smoothing the possible existing disagreements among decision makers and achieving a minimum degree of consensus before the aggregation process. Different minimum cost consensus models based on non-linear programming are applied in different steps of the algorithm to achieve a consensual solution accepted by the most members of the sorting MCGDM problem. In order to validate the method, a case study focuses on the sorting of several EU countries according to their sustainable performance in social policies has been carried out and from the results it is clear that consensus process can not only modify results but also avoid conflicts among decision makers views leading to better accepted solutions.

As future researches, Consensual GAHPSort is not suitable for dealing with large-scale GDM problems, which are quite common today, since the participation of a large number of decision makers might imply that the minimum cost consensus models based on non-linear programming not achieve feasible solution or time consuming problems. Therefore, an extension of Consensual GAHPSort for dealing with large-scale GDM problems seems necessary and a compelling reason for further research. Additionally, Consensual GAHPSort makes use of minimum cost models to obtain consensus values. However, there are a large number of consensus models proposed in the specialized literature, so it would be interesting to carry out a comparative analysis of different consensus models in order to evaluate their performance in sorting MCDM problems.

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