Development and comparison of efficient Newton-like methods for voltage stability assessment

Marcos Tostado-Véliz[1], Salah Kamel[2,3], Francisco Jurado[1,*]

[1]Department of Electrical Engineering, University of Jaén, 23700 EPS Linares, Jaén, Spain
[2]Electrical Engineering Department, Faculty of Engineering, Aswan University, 81542 Aswan, Egypt
[3]State Key Laboratory of Power Transmission Equipment and System Security and New Technology, Chongqing University, Chongqing, China

Abstract:

The voltage stability analysis can be carried out using different continuation techniques. The Continuation Power Flow that evaluates all Power-Flow solutions at different loading levels, is typically considered the most standard one. This approach is mainly based on two stages called Predictor and Corrector. The standard Newton-Raphson method is usually used at the corrector stage. In this paper, several corrector techniques based on efficient Newton-like methods are proposed in order to speed up the solution of Continuation Power Flow. Consequently, two high order Newton-like methods as well as a fast corrector technique based on the Dishonest NR method have been proposed. A comprehensive study is addressed in order to check the suitability of the proposed corrector techniques compared with the standard Newton-Raphson (NR). Several small, medium, large and very large-scale test systems are used to achieve the validation. The obtained results prove the effectiveness and superiority of proposed techniques compared with the standard NR which conventionally used in corrector stage of the Continuation Power Flow analysis in term of computational time.

Keywords: *Voltage Stability, Continuation Power Flow, Maximum Loading Point, Newton-like Methods.*

[*] Corresponding author: Tel.: +34 953 648518; Fax: +34 953 648586.

E-mail addresses: mtostado@ujaen.es (M. Tostado-Véliz) fjurado@ujaen.es (F. Jurado), skamel@aswu.edu.eg (S. Kamel)

Acronyms	
MLP	Maximum loadability point
PF	Power Flow
CPF	Continuation Power Flow
NR	Newton-Raphson method
NRPC	Newton-Raphson Predictor-Corrector method
NR3	3rd order Newton-like method
DNR	Dishonest Newton-Raphson method
Parameters and variables	
$V_i \angle \delta_i \in \mathbb{C}$	Complex voltage
$P \in \mathbb{R}$	Active power
$Q \in \mathbb{R}$	Reactive power
$Y_{ij} \angle \theta_{ij} \in \mathbb{C}$	ij^{th} element of admittance matrix
$\lambda \in \mathbb{R}^+$	Loading parameter
$\alpha \in (1, \infty)$	Loading level of the target case
$\sigma \in \mathbb{R}^+$	Step size
$\varepsilon \in \mathbb{R}^+$	Convergence tolerance
$\beta \in \mathbb{R}^+$	Damping factor
$\mu \in \mathbb{R}^+$	Convergence parameter
$\sigma_{\max} \in \mathbb{R}^+$	Maximum step size
$\sigma_{\min} \in \mathbb{R}^+$	Minimum step size
$k_{\max} \in (1, \infty)$	Maximum number of iterations at corrector stage
$\epsilon_* \in \mathbb{R}^+$	Error index using some efficient corrector technique (in this paper, '*' might be 'NRPC', 'NR3' or 'DNR')
Vectors and matrix	
$\boldsymbol{x} \in \mathbb{R}^n$	PF state vector
$\boldsymbol{b} \in \mathbb{R}^n$	Power transfer vector
$\boldsymbol{P_b} \in \mathbb{R}^{n_l+n_g}$	Vector of base case nodal active powers
$\boldsymbol{Q_b} \in \mathbb{R}^{n_g}$	Vector of base case nodal reactive powers
$\boldsymbol{P_t} \in \mathbb{R}^{n_l+n_g}$	Vector of target case nodal active powers
$\boldsymbol{Q_t} \in \mathbb{R}^{n_g}$	Vector of target case nodal reactive powers
$\boldsymbol{z} \in \mathbb{R}^{n+1}$	CPF state vector
$\boldsymbol{z_{NR}} \in \mathbb{R}^{n+1}$	CPF state vector calculated using NR as corrector technique
$\boldsymbol{z_*} \in \mathbb{R}^{n+1}$	CPF state vector calculated using some efficient corrector technique (in this paper, '*' might be 'NRPC', 'NR3' or 'DNR')
$\boldsymbol{u} \in \mathbb{R}^{n+1}$	Tangent vector
$\boldsymbol{\delta_{PV}} \in \mathbb{R}^{n_g}$	Voltage angles at PV buses
$\boldsymbol{\delta_{PQ}} \in \mathbb{R}^{n_g}$	Voltage angles at PQ buses
$\boldsymbol{V_{PQ}} \in \mathbb{R}^{n_l}$	Voltage magnitudes at PQ buses
$\boldsymbol{A_z} \in \mathbb{R}^{(n+1)\times(n+1)}$	CPF Jacobian matrix
Functions and equations	
$\mathbf{g}: \mathbb{R}^n \longmapsto \mathbb{R}^n$	PF equations
$\mathbf{f}: \mathbb{R}^n \longmapsto \mathbb{R}^n$	PF equations including the loading parameter
$p: \mathbb{R} \longmapsto \mathbb{R}$	Parameterized equation
$[\cdot]^T: \mathbb{R}^{m\times n} \longmapsto \mathbb{R}^{n\times m}$	Transpose operator
$[\cdot]^{-1}: \mathbb{R}^{n\times n} \longmapsto \mathbb{R}^{n\times n}$	Inverse operator
$\nabla_{\boldsymbol{x}}\mathbf{f}: \mathbb{R}^n \longmapsto \mathbb{R}^{n\times n}$	First derivatives of $\mathbf{f}$ with respect $\boldsymbol{x}$
$\boldsymbol{\mathcal{F}}: \mathbb{R}^{n+1} \longmapsto \mathbb{R}^{n+1}$	CPF equations
$\lVert\cdot\rVert_\infty: \mathbb{R}^n \longmapsto \mathbb{R}$	Infinity norm
$\lVert\cdot\rVert_2: \mathbb{R}^n \longmapsto \mathbb{R}$	2-norm
Indexes and scripts	
$j \in \mathbb{N} \cup \{0\}$	Step of the CPF procedure
$k \in \mathbb{N} \cup \{0\}$	Iteration of the iterative solution of the corrector stage
$n_l \in \mathbb{N} \cup \{0\}$	Number of PQ buses
$n_g \in \mathbb{N} \cup \{0\}$	Number of PV buses
$n \in \mathbb{N}$	Number of total variables of the PF ($n = 2n_l + n_g$)
$\widehat{(\cdot)}$	Guessed (predicted) value

$(\cdot)^{sch}$	Scheduled	

1. Introduction.

A. Motivation

The voltage stability analysis is a very useful tool for determining the security steady state margin of electrical power systems [1, 2]. The MLP is typically considered as the farthest operational point that used to determine the exact situation of the power system. However, other singular operational points should be considered like the bifurcation or limit-inducted points [3].

Several techniques have been developed for solving the voltage stability problem. Following the classification presented in [4, 5], these techniques might be broadly categorized as follows:

- Direct methods: these methods determine the MLP by solving an optimization problem.
- Continuation or homotopy methods: MLP is found by calculating a set of intermediate points corresponding to different PF solutions for different loading levels. Moreover, the PV curves are obtained as a byproduct. These techniques are normally carried put in two stages called predictor and corrector.

Due to its importance in optimal power system operation, computational performance of voltage stability tools is vital. Consequently, this paper tackles this issue.

B. Literature Review

Although the direct methods have been tackled in some works (see e.g. [6, 7]), the continuation techniques have been by far the most widely studied for voltage stability analysis. In this paper, we focus on continuation techniques, which have been attracted a tremendous interest in numerous technical papers. The standard CPF was introduced in [8], as a tool for voltage stability analysis. It uses a tangent predictor and the standard NR as corrector.

CPF was originally envisaged for voltage stability analysis in transmission systems. Nevertheless, distribution systems are currently facing several challenges mainly related with the massive integration of distributed generators. Consequently, several efforts have been recently carried out to adapt the traditional CPF for distribution systems [9-13]. This tool has also found applications in AC/DC [14] and interconnected systems [15, 16] or for network reconfiguration problems [17].

Several recent references have tackled the computational performance of continuation methods for voltage stability analysis. The interested readers may refer to [18, 19], where different numerical techniques were considered. Different geometric parameterization approaches have been deeply studied in [20-23]. A parameterization technique that is developed from the analysis of PF curves was analyzed in [24]. An alternative CPF model was developed in [25]. An adaptive step size mechanism was proposed in [26].

C. Contributions & Paper Organization

Aforementioned references aim to improve the computational performance of standard CPF, as modifications of the predictor stage, parameterization scheme and step size control. Nevertheless, the computational cost of CPF can be also reduced by improving the PF methods used in the corrector stage. Typically, NR is used for solving the parameterized PF equations in the corrector stage. Nevertheless, any other nonlinear techniques may be used with the aim of improving the computational performance of corrector stage. On the basis of the results reported on different papers [27, 28], we honestly believe that the High Order Newton-like methods may offer very good results at corrector stage of CPF. In this paper, several corrector techniques based on two High Order Newton-like methods with superquadratic and cubic convergence rates are proposed. In addition, an efficient corrector technique based on the Dishonest Newton's method [5] is explored.

Several simulations are carried out using various small, medium, large and very large-scale test systems with the aim of providing a comprehensive analysis of the proposed corrector techniques. In order to check the effectiveness of the proposed techniques, their results are compared with those obtained when the standard NR method used in the corrector stage.

Remainder of this paper is organized as follows. In Section 2, the standard CPF using the tangent predictor and pseudo arc length parameterization is outlined. Section 3 describes the proposed corrector techniques based on Newton-like methods. The simulations carried out and the main results obtained are reported in Section 4. Finally, the main conclusions are presented in Section 5.

2. Continuation Power-Flow

CPF is normally carried out in two stages called Predictor and Corrector. They use an augmented PF formulation which includes the loading parameter. In both stages, a parameterized formulation of the PF problem is included in order to avoid the singularity of Jacobian matrix. Continuation's procedure is thoroughly described in Fig. 1. Let us assume that the PV curve is traced until an active loading level $P^{(j)}$. At this point, the operational steady state of the system for a loading level $P^{(j+1)}$ is aimed to be found, thus, $\hat{V}^{j+1}$ is firstly guessed at the predictor stage. In order to reduce the error introduced at linear predictor stage, the parameterized PF equations are solved for finding $V^{(j+1)}$ using some available iterative techniques. This step is typically called corrector stage. This process is repeated until the full P-V curve is traced. It is worth mentioning that CPF allows to obtain both high and low voltage values.

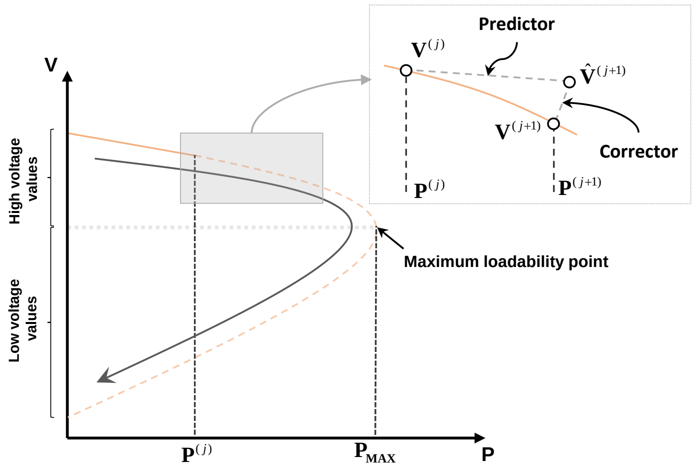

FIGURE 1. Sketch of CPF for voltage stability assessment

In this paper, the standard CPF with tangent predictor and pseudo arc length parameterization is considered. The pseudo arc length parameterization forces the next solution point to lie on the hyperplane orthogonal to the tangent described by the previous solution, this idea is sketched in Fig. 2.

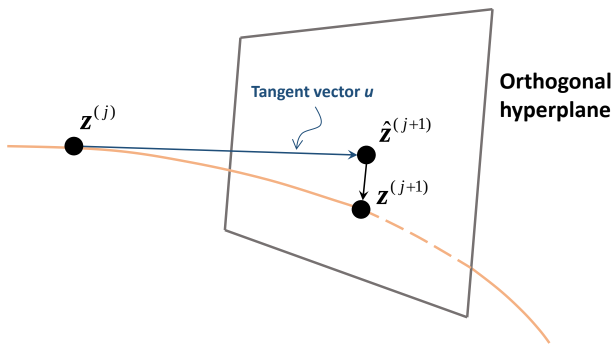

FIGURE 2. Sketch of the pseudo arc length parameterization

Firstly, let us consider the PF equations in polar coordinates as a set of n nonlinear algebraic equations as follows:

$$\mathbf{g}(\boldsymbol{x}) = 0 \tag{1}$$

The CPF starts modifying the problem by introducing the loading parameter as follows [29]:

$$\mathbf{g}(\boldsymbol{x}) \Longrightarrow \mathbf{f}(\boldsymbol{x}, \lambda) = \mathbf{g}(\boldsymbol{x}) - \lambda \boldsymbol{b} \tag{2}$$

$$\boldsymbol{b} = [\boldsymbol{P_t} - \boldsymbol{P_b}, \boldsymbol{Q_t} - \boldsymbol{Q_b}]^T \tag{3}$$

The state vector of CPF problem is given by:

$$\boldsymbol{z} = [\boldsymbol{x}, \lambda]^T \in \mathbb{R}^{n+1} \tag{4}$$

One should note that (4) has a larger dimension with respect $\boldsymbol{x}$, consequently, it is compulsory to add one more equation to the system in order to avoid its singularity. CPF addresses this issue by incorporating a parameterization equation. In this paper, we use the pseudo arc length parameterization, which can be written (for generic k^{th} iteration and j^{th} step of the corrector stage of CPF procedure) as follows:

$$\mathrm{p}\left(\boldsymbol{z}^{(j,k)}\right) = \left[\boldsymbol{z} - \boldsymbol{z}^{(j,k)}\right]^{T} \boldsymbol{u}^{(j)} - \sigma^{(j)} = 0 \tag{5}$$

The tangent vector is calculated by solving the following linear system:

$$\begin{bmatrix} \nabla_{\boldsymbol{z}}^{T} \mathbf{f}\left(\boldsymbol{z}^{(j,k)}\right) & \nabla_{\lambda}^{T} \mathbf{f}\left(\lambda^{(j,k)}\right) \\ \nabla_{\boldsymbol{z}}^{T} \mathrm{p}\left(\boldsymbol{z}^{(j,k)}\right) & \nabla_{\lambda} \mathrm{p}\left(\lambda^{(j,k)}\right) \end{bmatrix} \boldsymbol{u}^{(j)} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow \boldsymbol{A}_{\boldsymbol{z}}^{(j,k)} \boldsymbol{u}^{(j)} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{6}$$

At the predictor stage, the information of tangent vector is used to guess the value of CPF state vector at the following step as:

$$\hat{\boldsymbol{z}}^{(j+1)} = \boldsymbol{z}^{(j)} + \frac{\boldsymbol{u}^{(j)}}{\left\|\boldsymbol{u}^{(j)}\right\|_{2}} \sigma^{(j)} \tag{7}$$

At the corrector stage, the guess value of the state vector obtained at the predictor stage is corrected by iteratively solving the following set of nonlinear equations:

$$\left[\mathbf{f}^{(j,k)}, \quad \mathrm{p}^{(j,k)}\right]^{T} = 0 \Rightarrow \boldsymbol{\mathcal{F}}\left(\boldsymbol{z}^{(j,k)}\right) = 0 \tag{8}$$

Conventionally, NR is employed for solving (8). A generic k^{th} iteration of the standard NR at the j^{th} CPF step for solving (8) is carried out as follows:

$$\Delta \boldsymbol{z}^{(j+1,k)} = -\left[\boldsymbol{A}_{\boldsymbol{z}}^{(j+1,k)}\right]^{-1} \boldsymbol{\mathcal{F}}\left(\boldsymbol{z}^{(j+1,k)}\right)$$

$$\boldsymbol{z}^{(j+1,k+1)} = \boldsymbol{z}^{(j+1,k)} + \Delta \boldsymbol{z}^{(j+1,k)} \tag{9}$$

The convergence of this iterative algorithm can be checked according to the following criteria:

$$\|\boldsymbol{\mathcal{F}}\|_{\infty} \leq \varepsilon \tag{10}$$

The CPF procedure is considered properly finished when the corrector does not converge in a predetermined number of iterations ($k_{\max}$) or when the whole PV curve is traced. This condition can be verified as follows:

$$\lambda^{(j)} = \lambda^{(0)} \tag{11}$$

An adaptive mechanism for the step size based on the difference between the predicted and the corrected values can be employed as:

$$\gamma = \left\| \boldsymbol{z}^{(j+1)} - \hat{\boldsymbol{z}}^{(j+1)} \right\|_{\infty} \tag{12}$$

Then, the step size is adapted according to the following equation:

$$\sigma = \min\left[\sigma_{\max}, \max[1 + \beta(\mu/\gamma - 1), \sigma_{\min}]\right] \tag{13}$$

For the sake of clarity, Fig. 3 shows the flowchart of the CPF with tangent predictor, pseudo arc length parameterization and standard NR corrector.

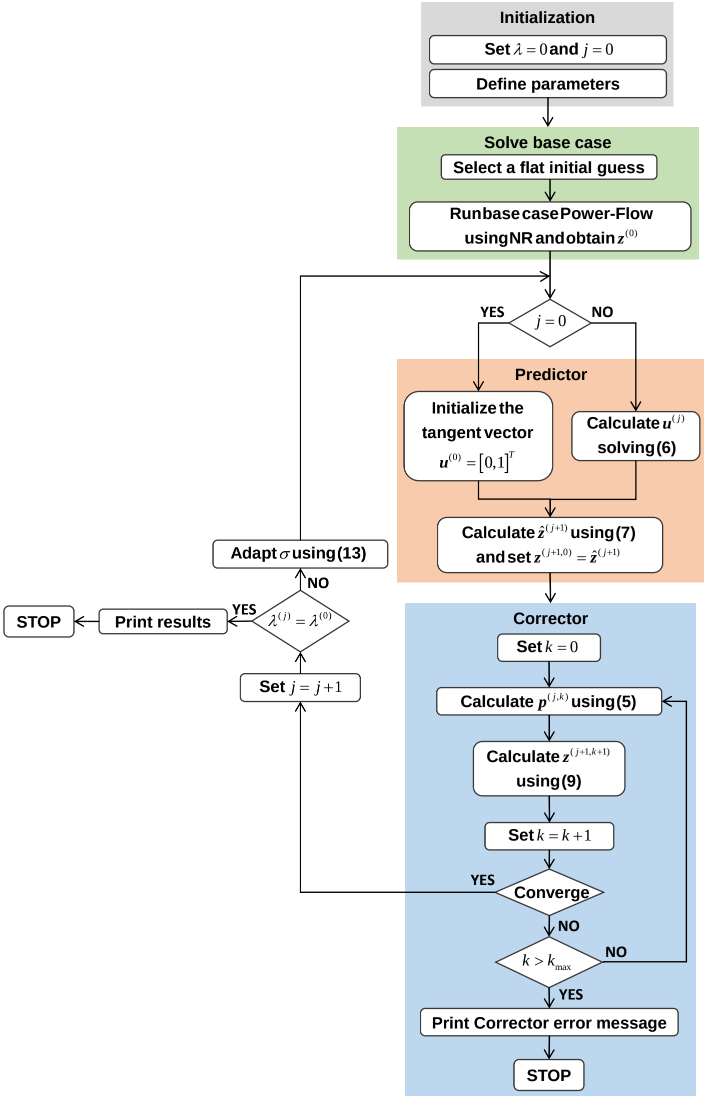

FIGURE 3. Flowchart of CPF with tangent predictor, pseudo arc length parameterization and standard NR corrector

3. Efficient Corrector techniques

The main aim of this paper is studying the suitability of several efficient Newton-like methods at corrector stage of CPF. For this purpose, several corrector techniques based on efficient Newton-like methods have been proposed.

A. King-Werner methods

King-Werner methods [30], are a family of techniques which achieve an order of convergence of $1 + \sqrt{2}$. They are normally carried out at two stages frequently called predictor and corrector, however, we have avoided these terms in this paper in order to avoid confusion with the two stages involved in the CPF. In [31], a King-Werner technique has been proposed for solving nonlinear equations. A PF solution technique based on this method called Newton-Raphson Predictor-Corrector (NRPC) has been studied in [27]. At first iteration, NRPC for the CPF is carried out in the same way of NR (9), while it is defined for following iterations as follows:

For $k \geq 1$

$$\Delta\tilde{\boldsymbol{z}}^{(j+1,k)} = -\left[\boldsymbol{A}^{(j+1,k-1)}_{\left(\frac{1}{2}[\boldsymbol{z}+\tilde{\boldsymbol{z}}]\right)}\right]^{-1} \boldsymbol{\mathcal{F}}\left(\boldsymbol{z}^{(j+1,k)}\right)$$

$$\tilde{\boldsymbol{z}}^{(j+1,k)} = \boldsymbol{z}^{(j+1,k)} + \Delta\tilde{\boldsymbol{z}}^{(j,k)}$$

$$\Delta\boldsymbol{z}^{(j+1,k)} = -\left[\boldsymbol{A}^{(j+1,k)}_{\left(\frac{1}{2}[\boldsymbol{z}+\tilde{\boldsymbol{z}}]\right)}\right]^{-1} \boldsymbol{\mathcal{F}}\left(\boldsymbol{z}^{(j+1,k)}\right)$$

$$\boldsymbol{z}^{(j+1,k+1)} = \boldsymbol{z}^{(j+1,k)} + \Delta\boldsymbol{z}^{(j+1,k)} \tag{14}$$

It is worth noticing that, as NR, only a matrix factorization is required each iteration.

B. High order Newton-like methods

Unlike NR, which has quadratic convergence, a huge variety of high order Newton-like methods have been proposed over decades. Most of them achieve higher order of convergence by evaluating the Jacobian matrix in several points (see e.g. [32]). However, it supposes an extra computational burden. Oppositely, a 3rd order method (NR3) has been proposed in [33], which only requires one LU decomposition each iteration (as NR). A generic k^{th} iteration of NR3 for solving the corrector stage at j^{th} step is carried out as follows:

$$\bar{\boldsymbol{z}}^{(j+1,k)} = \boldsymbol{z}^{(j+1,k)} - \left[\boldsymbol{A}_{\boldsymbol{z}}^{(j+1,k)}\right]^{-1} \boldsymbol{\mathcal{F}}\left(\boldsymbol{z}^{(j+1,k)}\right)$$

$$\boldsymbol{z}^{(j+1,k+1)} = \bar{\boldsymbol{z}}^{(j+1,k)} - \left[\boldsymbol{A}_{\boldsymbol{z}}^{(j+1,k)}\right]^{-1} \boldsymbol{\mathcal{F}}\left(\bar{\boldsymbol{z}}^{(j+1,k)}\right) \tag{15}$$

As it can be seen, NR3 achieves 3rd order of convergence by evaluating (8) twice instead of the Jacobian matrix.

C. Dishonest NR method

The dishonest NR (DNR) method attracted huge interest for PF analysis especially at 70s, when the NR was considered inefficient due to the high cost of factorizing the Jacobian matrix. DNR only evaluates the Jacobian matrix at first iteration, reusing this information for the remainder iterative procedure. It normally requires more iterations than NR, however, iterative procedure can be speed up by avoiding some calculations (e.g. factorizations). A generic k^{th} iteration of the DNR for solving the corrector stage at j^{th} step is carried out as follows:

$$\boldsymbol{z}^{(j+1,k+1)} = \boldsymbol{z}^{(j+1,k)} - \left[\boldsymbol{A}_{\boldsymbol{z}}^{(j+1,0)}\right]^{-1} \boldsymbol{\mathcal{F}}\left(\boldsymbol{z}^{(j+1,k)}\right) \tag{16}$$

It is worth mentioning that reference [34], already studied the application of DNR in CPF, however, this reference uses a parameterization strategy. Moreover, DNR scheme is based on the significant changes in the system for updating the Jacobian matrix, oppositely, the proposed DNR-based corrector technique always factorizes the Jacobian matrix once each corrector step.

The studied corrector techniques mainly differ in the calculations involved each iteration. A preliminary analysis of these techniques may be made in order to determine the features of each technique. Table 1 shows the main calculations involved each iteration of the studied corrector techniques. It is worth mentioning that the heaviest part of any corrector calculation is the factorization of the Jacobian matrix. In that sense, it is expected that DNR shows the lowest execution time per iteration since it only factorizes the Jacobian matrix once (at the beginning of the iterative procedure). Nevertheless, DNR normally shows linear convergence which means this technique normally employs more iterations than remainder techniques, thus, it is expected that NR3 employ less iterations than remainder methods. However, NR3 involves other calculations mainly related with linear systems solution, anyway, this kind of computations may be normally carried out efficiently and they do not have a significant impact in the overall computational performance.

Corrector Method	Order of convergence	Vectors sum	Linear systems solved	LU decompositions
NR	2	1	1	1
NRPC	$1+\sqrt{2}$	2	2	1
NR3	3	2	2	1
DNR	1	1	1	1*

* In the whole iterative procedure

TABLE 1: Main calculations involved in an iteration of proposed corrector techniques

For the sake of completeness, Fig. 4 shows the CPF flowchart using the proposed corrector techniques. It is worth noting that the base case is always solved using the NR. Furthermore, the flowcharts showed in Fig. 4 can be incorporated to that showed in Fig. 3 by just changing the corrector stage, thus, the whole CPF procedure would be described.

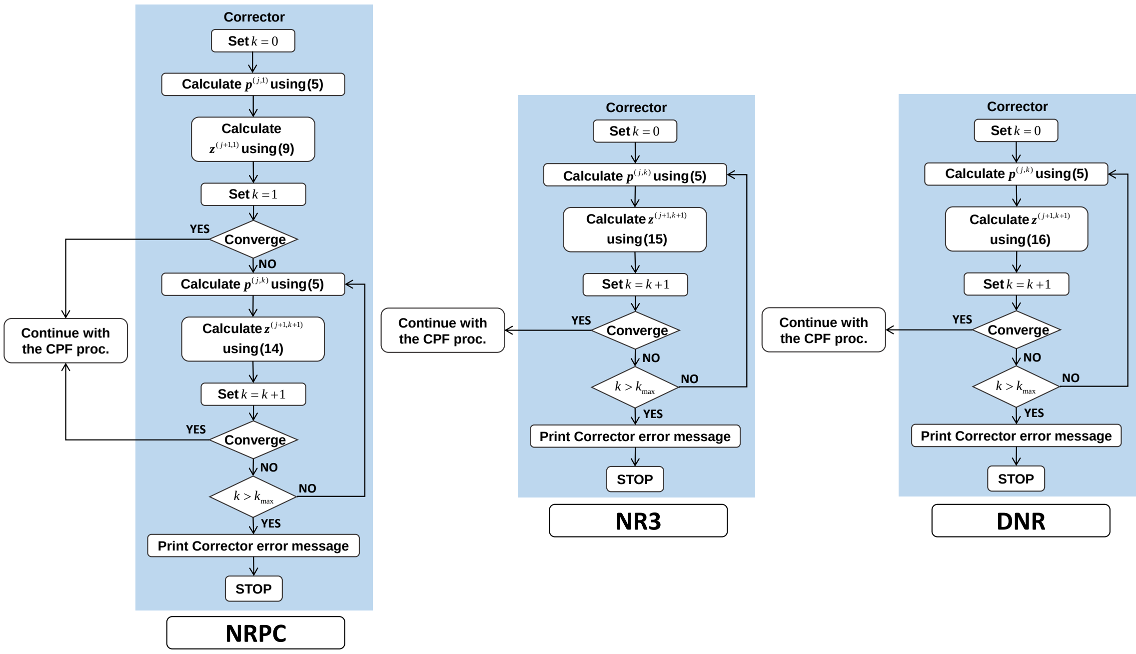

FIGURE 4. Flowchart of proposed corrector techniques

4. Results and discussion

With the aim of checking the suitability of proposed corrector techniques, several numerical experiments have been carried out. All methodologies have been coded using Matpower v6.0 [29] and run under Windows 10 using a 3.4 GHz Intel Core i7-8750H CPU 2.2 GHz personal laptop (16.00 GB RAM). The execution times reported have been computed as the average value of 100 simulations, in order to reduce the influence of parallel computational activities.

For all scenarios considered, NR method with flat start ($|V|$=1.0 p.u. and δ=0) has been used for solving the base case (it is worth mentioning that the flat start is often a good starting point [35]). In all cases, the default parameters $\beta = 0.7$ and $\mu = 10^{-3}$ have been taken. Remainder parameters (i.e. σ, $\sigma_{\max}$, $\sigma_{\min}$ and α) have been considered case dependent and tuned with the aim of achieving reliable results.

A. Description of test systems

The proposed corrector techniques have been tested using the following systems:

- Small-scale system: the 69-bus radial test system [36].
- Medium-scale system: the IEEE 300-bus test system [37].
- Large-scale system: the 2869-bus portion of the European transmission system from the EU PEGASE project [38, 39].
- Very large-scale system: the 9241-bus portion of the European transmission system from the EU PEGASE project [38, 39].

For the sake of completeness, Table 2 presents the main description of studied systems.

Case	Buses	Branches	Generators	Load		No. of
				MW	MVar	variables (n)
69-bus	69	68	--	3.8	2.7	136
300-bus	300	411	69	23525.8	7788	530
2869-bus	2869	4582	510	132437.3	29007.8	5227
9241-bus	9241	16049	1445	312354.1	73581.6	17036

TABLE 2: Main description of studied systems

B. Numerical results

For each test system, two scenarios are considered. Firstly, all studied systems are solved considering ideal generators, it means, they are able to inject as much reactive power are required to keep the voltage magnitude at its connecting bus. Secondly, the generators' reactive limits are considered by following the same strategy implemented in Matpower. Basically, it is based converting those PV buses which are violated to PQ buses with keeping the injected reactive powers at the limits. It is well-known that generators' reactive limits have a direct impact on the voltage stability and, consequently, on the overall performance of CPF.

The total number of iterations, the execution time in seconds and the average number of iterations of a complete CPF procedure are reported in Tables 3-6 for the 30-bus, 300-bus, 2869-bus and 9241-bus systems, respectively. On the other hand, Tables 7-9 are similar to the previous tables but for the scenario of enforcing generators' reactive limits in the 300-bus, 2869-bus and 9241-bus systems, respectively. One should note that the 69-bus radial system does not have any generator bus, therefore, scenarios with and without reactive limits enforcement are essentially the same case.

Corrector technique		Corrector iterations	Time [s]	Average no. of iterations
NR	$\varepsilon = 10^{-5}$	64	0.91	1.0667
	$\varepsilon = 10^{-7}$	104	0.93	1.7333
	$\varepsilon = 10^{-9}$	123	0.95	2.0500
NRPC	$\varepsilon = 10^{-5}$	64	0.91	1.0667
	$\varepsilon = 10^{-7}$	104	0.94	1.7333
	$\varepsilon = 10^{-9}$	122	0.95	2.0333
NR3	$\varepsilon = 10^{-5}$	**62**	0.90	**1.0333**
	$\varepsilon = 10^{-7}$	**65**	0.91	**1.0833**
	$\varepsilon = 10^{-9}$	**86**	**0.93**	**1.4333**
DNR	$\varepsilon = 10^{-5}$	64	**0.89**	1.0667
	$\varepsilon = 10^{-7}$	108	**0.90**	1.8000
	$\varepsilon = 10^{-9}$	153	0.94	2.5500

$\alpha = 2$
$\sigma^{(0)} = 0.2$
$\sigma_{\max} = 0.2$
$\sigma_{\min} = 0.01$

TABLE 3: Obtained results of 69-bus system

Corrector technique		Corrector iterations	Time [s]	Average no. of iterations
NR	$\varepsilon = 10^{-5}$	108	1.79	1.2706
	$\varepsilon = 10^{-7}$	155	2.16	1.8235
	$\varepsilon = 10^{-9}$	188	2.35	2.2118
NRPC	$\varepsilon = 10^{-5}$	107	1.79	1.2588
	$\varepsilon = 10^{-7}$	151	2.12	1.7765
	$\varepsilon = 10^{-9}$	181	2.29	2.1294
NR3	$\varepsilon = 10^{-5}$	**94**	1.77	**1.1059**
	$\varepsilon = 10^{-7}$	**106**	1.83	**1.2471**
	$\varepsilon = 10^{-9}$	**123**	1.94	**1.4471**
DNR	$\varepsilon = 10^{-5}$	115	**1.59**	1.3529
	$\varepsilon = 10^{-7}$	182	**1.64**	2.1412
	$\varepsilon = 10^{-9}$	249	**1.74**	2.9294

$\alpha = 3$
$\sigma^{(0)} = 0.1$
$\sigma_{\max} = 0.2$
$\sigma_{\min} = 0.01$

TABLE 4: Obtained results of 300-bus system

Corrector technique		Corrector iterations	Time [s]	Average no. of iterations
NR	$\varepsilon = 10^{-5}$	183	12.07	1.2200
	$\varepsilon = 10^{-7}$	281	16.46	1.8733
	$\varepsilon = 10^{-9}$	309	18.85	2.0600
NRPC	$\varepsilon = 10^{-5}$	183	12.15	1.2200
	$\varepsilon = 10^{-7}$	281	16.59	1.8733
	$\varepsilon = 10^{-9}$	305	18.71	2.0333
NR3	$\varepsilon = 10^{-5}$	**155**	11.95	**1.0333**
	$\varepsilon = 10^{-7}$	**167**	13.25	**1.1133**
	$\varepsilon = 10^{-9}$	**255**	16.89	**1.7000**
DNR	$\varepsilon = 10^{-5}$	183	**10.28**	1.2200
	$\varepsilon = 10^{-7}$	293	**11.78**	1.9533
	$\varepsilon = 10^{-9}$	416	**13.15**	2.7733

$\alpha = 2$
$\sigma^{(0)} = 1$
$\sigma_{\max} = 1$
$\sigma_{\min} = 0.1$

TABLE 5: Obtained results of 2869-bus system

Corrector technique		Corrector iterations	Time [s]	Average no. of iterations
NR	$\varepsilon = 10^{-5}$	170	45.00	1.0692
	$\varepsilon = 10^{-7}$	196	48.20	1.2327
	$\varepsilon = 10^{-9}$	304	61.13	1.9119
NRPC	$\varepsilon = 10^{-5}$	170	45.96	1.0692
	$\varepsilon = 10^{-7}$	195	48.19	1.2264
	$\varepsilon = 10^{-9}$	300	61.01	1.8868
NR3	$\varepsilon = 10^{-5}$	**166**	44.86	**1.0440**
	$\varepsilon = 10^{-7}$	**172**	46.68	**1.0818**
	$\varepsilon = 10^{-9}$	**188**	48.74	**1.1824**
DNR	$\varepsilon = 10^{-5}$	171	**42.57**	1.0755
	$\varepsilon = 10^{-7}$	207	**44.12**	1.3019
	$\varepsilon = 10^{-9}$	338	**45.83**	2.1258

$\alpha = 2.5$
$\sigma^{(0)} = 0.5$
$\sigma_{\max} = 0.5$
$\sigma_{\min} = 0.1$

TABLE 6: Obtained results of 9241-bus system

Corrector technique		Corrector iterations	Time [s]	Average no. of iterations
NR	$\varepsilon = 10^{-5}$	89	1.98	1.0595
	$\varepsilon = 10^{-7}$	127	2.31	1.5119
	$\varepsilon = 10^{-9}$	160	2.56	1.9048
NRPC	$\varepsilon = 10^{-5}$	**89**	1.89	**1.0595**
	$\varepsilon = 10^{-7}$	127	2.22	1.5119
	$\varepsilon = 10^{-9}$	160	2.37	1.9048
NR3	$\varepsilon = 10^{-5}$	**89**	1.92	**1.0595**
	$\varepsilon = 10^{-7}$	**103**	2.10	**1.2262**
	$\varepsilon = 10^{-9}$	**118**	**2.21**	**1.4048**
DNR	$\varepsilon = 10^{-5}$	**89**	**1.68**	**1.0595**
	$\varepsilon = 10^{-7}$	127	**1.77**	1.5119
	$\varepsilon = 10^{-9}$	174	2.31	2.0714

$\alpha = 3$
$\sigma^{(0)} = 0.02$
$\sigma_{\max} = 0.03$
$\sigma_{\min} = 0.005$

TABLE 7: Obtained results of 300-bus system considering reactive limits enforcement

Corrector technique		Corrector iterations	Time [s]	Average no. of iterations
NR	$\varepsilon = 10^{-5}$	325	32.27	1.3830
	$\varepsilon = 10^{-7}$	491	35.95	2.0894
	$\varepsilon = 10^{-9}$	551	37.15	2.3447
NRPC	$\varepsilon = 10^{-5}$	325	31.16	1.3830
	$\varepsilon = 10^{-7}$	482	33.84	2.0511
	$\varepsilon = 10^{-9}$	539	34.97	2.2936
NR3	$\varepsilon = 10^{-5}$	**280**	29.90	**1.1915**
	$\varepsilon = 10^{-7}$	**386**	31.28	**1.6426**
	$\varepsilon = 10^{-9}$	**451**	33.21	**1.9191**
DNR	$\varepsilon = 10^{-5}$	336	**28.56**	1.4298
	$\varepsilon = 10^{-7}$	532	**29.78**	2.2638
	$\varepsilon = 10^{-9}$	657	**32.57**	2.7957

$\alpha = 1.5$
$\sigma^{(0)} = 1$
$\sigma_{\max} = 1$
$\sigma_{\min} = 0.1$

TABLE 8: Obtained results of 2869-bus system considering reactive limits enforcement

Corrector technique		Corrector iterations	Time [s]	Average no. of iterations
NR	$\varepsilon = 10^{-5}$	530	288.13	0.8521
	$\varepsilon = 10^{-7}$	827	324.88	1.3296
	$\varepsilon = 10^{-9}$	926	341.58	1.4887
NRPC	$\varepsilon = 10^{-5}$	530	286.94	0.8521
	$\varepsilon = 10^{-7}$	826	324.15	1.3280
	$\varepsilon = 10^{-9}$	924	340.02	1.4855
NR3	$\varepsilon = 10^{-5}$	**529**	287.52	**0.8505**
	$\varepsilon = 10^{-7}$	**816**	323.58	**1.3119**
	$\varepsilon = 10^{-9}$	**856**	**332.45**	**1.3762**
DNR	$\varepsilon = 10^{-5}$	530	**275.47**	0.8521
	$\varepsilon = 10^{-7}$	831	**318.74**	1.3360
	$\varepsilon = 10^{-9}$	941	338.54	1.5129

$\alpha = 1.2$
$\sigma^{(0)} = 0.1$
$\sigma_{\max} = 0.1$
$\sigma_{\min} = 0.01$

TABLE 9: Obtained results of 9241-bus system considering reactive limits enforcement

From these tables, it can be observed that NR3 typically required less iterations than the remainder methods in all studied cases. DNR was normally the fastest, except for high convergence tolerances which NR3 is occasionally faster than DNR due to the latter tends to employ too many iterations owing to its linear convergence. Overall performance of NR and NRPC is quite similar in all systems when reactive limits are not taken into account. Oppositely, NRPC significantly outperformed NR in case of considering the generators' reactive limits enforcement, especially for high convergence tolerances.

Finally, the ability of the proposed corrector techniques in addressing distribution systems with high R/X ratio has been checked. To do that, the branch inductances of the radial 69-bus system have been multiplied by 0.5. Thus, the overall R/X ratio of system is doubled. The obtained results for this scenario are reported in Table 10. Similar conclusions from previous experiments can be extrapolated to this scenario. As observed, the proposed techniques have conveniently managed the large R/X ratio of the system. The total number of iterations increased with respect to those results reported in Table 3, which is logic if one takes into account that condition of the system is, in this case, poorer.

Corrector technique		Corrector iterations	Time [s]	Average no. of iterations
NR	$\varepsilon = 10^{-5}$	92	1.4324	1.0455
	$\varepsilon = 10^{-7}$	153	1.5048	1.7386
	$\varepsilon = 10^{-9}$	181	1.5449	2.0568
NRPC	$\varepsilon = 10^{-5}$	92	1.4469	1.0455
	$\varepsilon = 10^{-7}$	153	1.5208	1.7386
	$\varepsilon = 10^{-9}$	180	1.5395	2.0455
NR3	$\varepsilon = 10^{-5}$	**91**	1.4539	**1.0341**
	$\varepsilon = 10^{-7}$	**92**	1.4629	**1.0455**
	$\varepsilon = 10^{-9}$	**131**	1.5091	**1.4886**
DNR	$\varepsilon = 10^{-5}$	92	**1.4216**	1.0455
	$\varepsilon = 10^{-7}$	154	**1.4530**	1.7500
	$\varepsilon = 10^{-9}$	220	**1.4957**	2.5000

$\alpha = 2$
$\sigma^{(0)} = 0.2$
$\sigma_{\max} = 0.2$
$\sigma_{\min} = 0.01$

TABLE 10: Obtained results of 69-bus system with branch inductances reduced to the half

On the other hand, Fig. 5 plots the total number of iterations per step in studied systems. For the sake of brevity, only results for $\varepsilon = 10^{-7}$ are depicted.

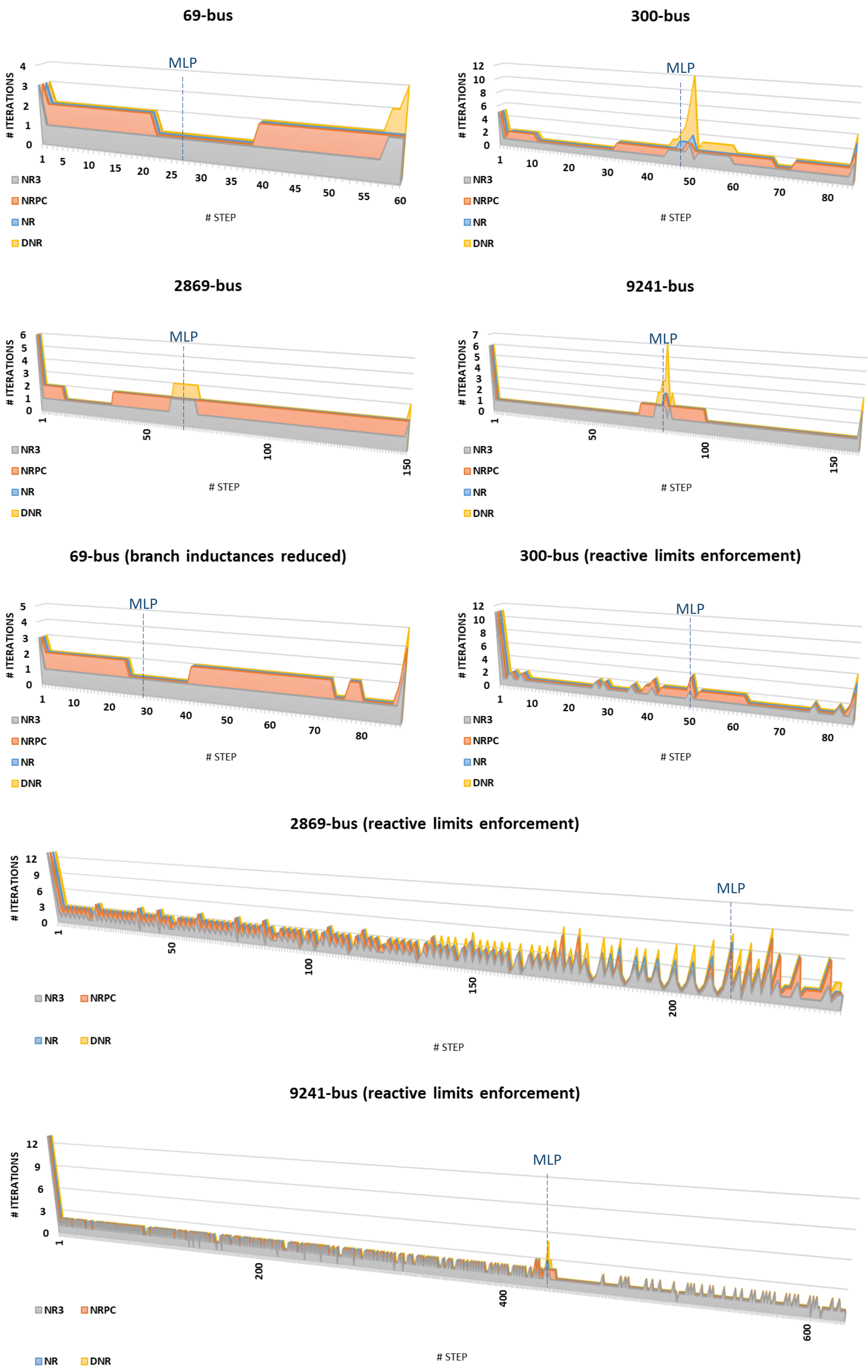

FIGURE 5. Total number of iterations for each corrector step in the studied systems ($\varepsilon = 10^{-7}$)

From Fig. 5, it can be noted that all considered corrector techniques typically tended to employ more iterations in the vicinity of the MLP. In 69-bus test system, oppositely, all studied techniques employed only an iteration for converging at MLP. This is due to the adaptive mechanism forces the step size to be small at these points (see Fig. 9 and 11), which provokes that initial guess produced by the predictor stage is quite good and the corrector methodology is able to quickly converge to the solution. On the other hand, many iterations were required for solving the base case in all studied systems, due to a flat start was used. When the generators' reactive limits are enforced, all studied techniques showed an irregular behavior, employing many iterations in several steps. This is due to the existence of limit induced bifurcation points, which appear when some reactive limit has been violated.

In order to compare the accuracy of the results obtained by the considered corrector techniques, the following index has been defined:

$$\epsilon_*^j = \left\| \boldsymbol{z}_{NR}^j - \boldsymbol{z}_*^j \right\|_\infty \tag{17}$$

where, * is the considered corrector technique.

Expression (17) aims to determine the accuracy of the proposed efficient corrector techniques. To do that, they are compared step by step with those solutions obtained when the standard NR, which considered as a benchmark, is used as corrector. Fig. 6-8 plot the values of (17) of different studied systems. On the other hand, Fig. 9-11 plot the values of the step size of different studied systems. For the sake of brevity, only results for $\varepsilon = 10^{-7}$ are plotted.

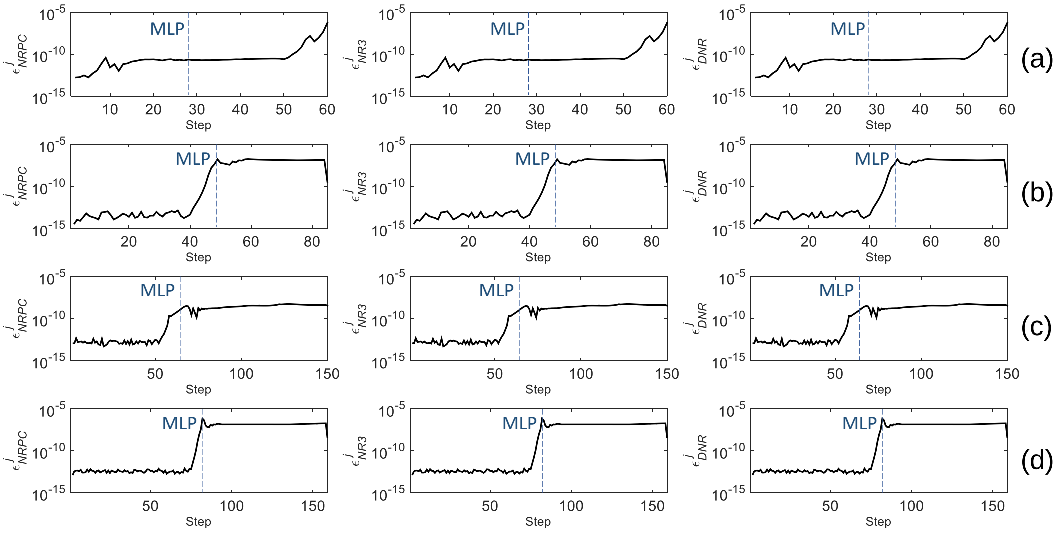

FIGURE 6. Error index (17) for each corrector step of 69-bus (a), 300-bus (b), 2869-bus (c) and 9241-bus (d) systems at base case scenario ($\varepsilon = 10^{-7}$)

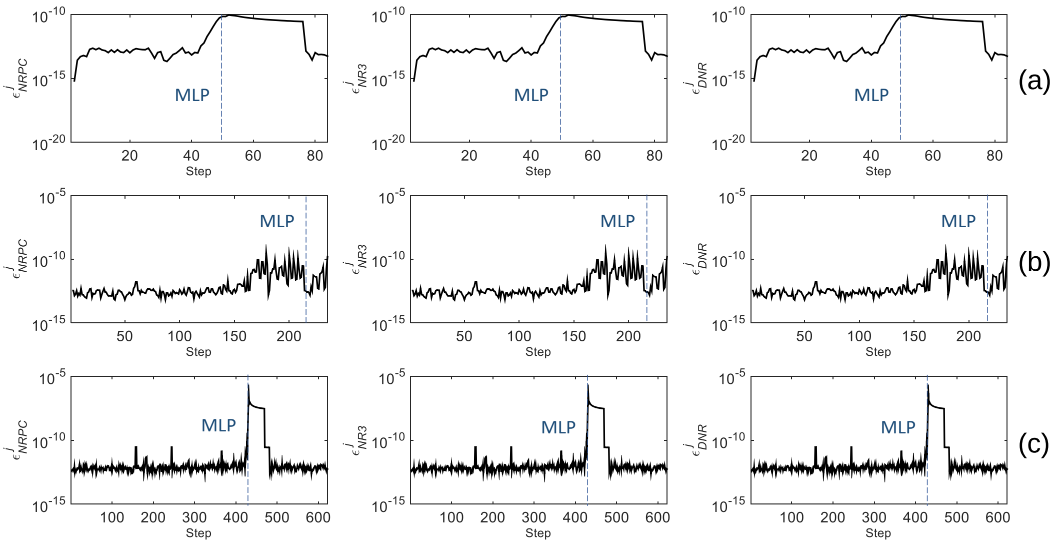

FIGURE 7. Error index (17) for each corrector step of 300-bus (a), 2869-bus (b) and 9241-bus (c) systems with reactive limits enforcement ($\varepsilon = 10^{-7}$)

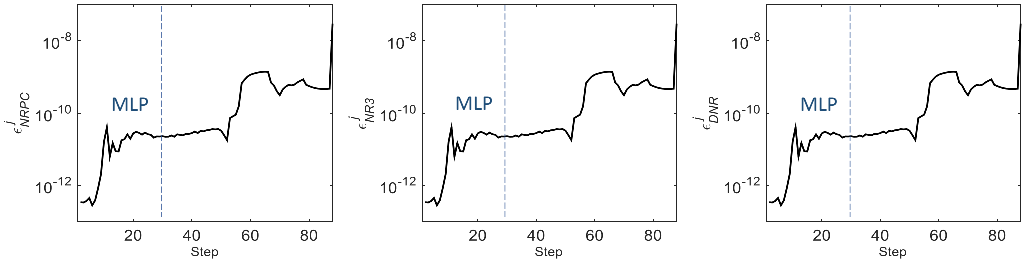

FIGURE 8. Error index (17) for each corrector step of 69-bus system with branch inductances reduced to the half ($\varepsilon = 10^{-7}$)

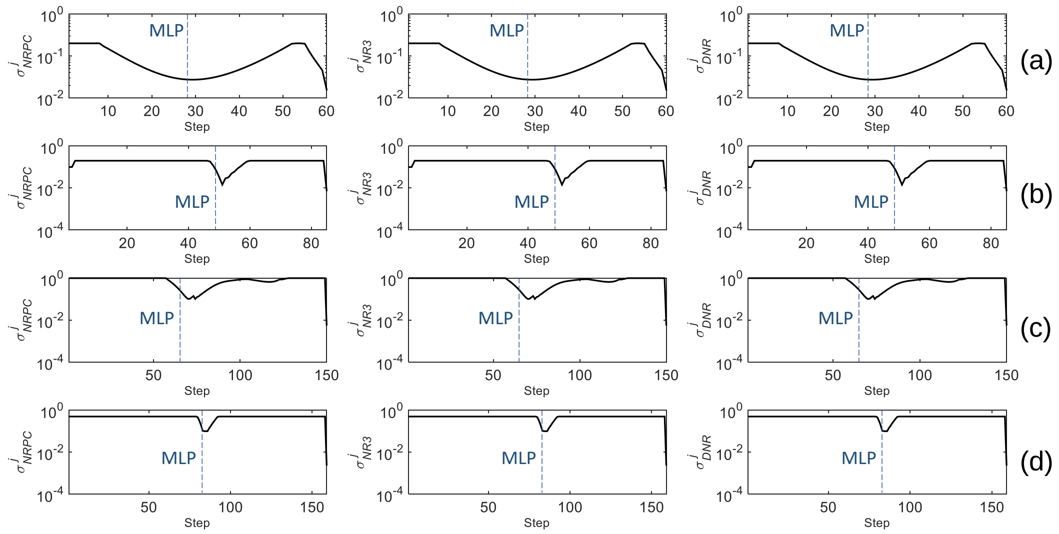

FIGURE 9. Step size for each corrector step of 69-bus (a), 300-bus (b), 2869-bus (c) and 9241-bus (d) systems at base case scenario ($\varepsilon = 10^{-7}$)

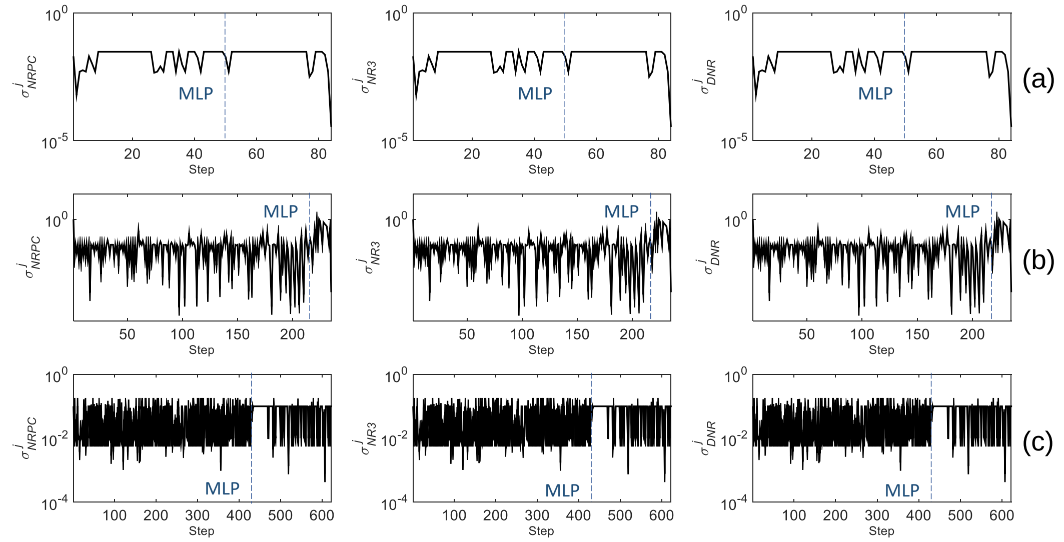

FIGURE 10. Step size for each corrector step of 300-bus (a), 2869-bus (b) and 9241-bus (c) systems with reactive limits enforcement ($\varepsilon = 10^{-7}$)

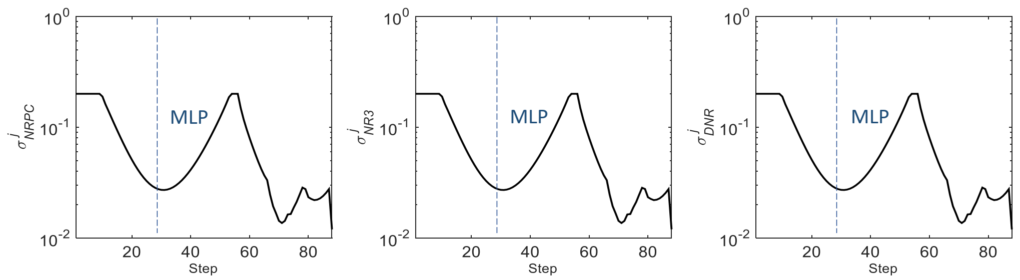

FIGURE 11. Step size for each corrector step of 69-bus system with branch inductances reduced to the half ($\varepsilon = 10^{-7}$)

From Fig. 6-8, it can be observed that all studied techniques essentially exhibited the same accuracy level. It is remarkable that the highest error magnitudes are found either at MLP or at the lower side of the P-V curve (low voltage solutions). On the other hand, it can be observed from Fig. 9-11 that the step size becomes a critic aspect of the CPF calculation when the reactive limits are enforced. In this case, the value of the step size might be even shorter than the minimum allowed (this strategy is set by default in Matpower). For remainder considered scenarios, behavior of the step size is similar in all systems, furthermore, it turns shorter at the vicinity of the MLP and lower side of the P-V curve.

C. Discussion

Results showed that the performance of the proposed corrector techniques was as expected. That is, NR3 saved a notably number of iterations, logically, smaller the convergence tolerance, higher computational saving. Despite that DNR often employed more iterations than NR, it was often faster. On the other hand, NRPC does not suppose a significant advantage with respect to NR when reactive limits are not taken into account. However, it typically outperformed NR when the generators' reactive limits enforcement is considered. This proves the ability of NRPC for saving up two iterations with respect to NR in ill-conditioned systems [27]. Authors believe that NRPC may offer even better results in large-scale ill-conditioned systems. This topic should be covered in future works.

All proposed technique successfully carried out CPF in the radial system, which showed their ability for managing distribution systems even with high R/X ratio.

Following the iterative procedure (15), higher order of convergence can be achieved raising a recursive formula, by evaluating (8) in several intermediate points. Nevertheless, authors believe that employing methodologies with order of convergence higher than three in CPF analysis may be counterproductive. One should note that Region of Attraction tends to be narrower when the order of the method increases [40], which may provoke instability of the corrector stage. Moreover, from Fig. 5, one can observe that NR3 frequently employed one iteration for converging in most steps. Consequently, higher order methods would not get better performance since faster convergence cannot be achieved.

For high convergence tolerances, both DNR and NR frequently performed similar, this is due to, in most steps, both techniques employed only one iteration for converging. DNR only allows saving in computational effort with respect to NR when more than an iteration is required.

Occasionally, computational saving in terms of execution time is observed when NR3 or DNR are used (several seconds). It is worth noting that some optimization problems require the repetitive solution of different CPF procedures. In this case, reducing the computation time is cumulative and outperforming of the proposed techniques would be more noticeable. Authors strongly believe this issue is an interesting topic for future works.

Based on the results shown in Fig. 6-8, we can affirm that all proposed corrector techniques are accuracy enough and, essentially, they found the same solution of the standard NR. This interesting point enables the usage of proposed efficient corrector techniques in industry tools.

5. Conclusion and future works

In this paper, several corrector techniques based on efficient Newton-like methods have been proposed to improve the computational performance of conventional CPF analysis, which uses the standard NR as corrector method. Instead, two high order Newton-like methods (NRPC and NR3) besides a fast methodology (DNR) have been proposed to achieve this target.

The proposed corrector techniques have been validated using four test systems ranging from 69-, to 9241-buses. Handling the generators' reactive power limits has been considered in the proposed corrector techniques. The results obtained by the proposed techniques proved their high computational feature compared with the conventional corrector technique based on NR.

Future works should be focused on exploring the performance of other techniques as corrector methods, for example the powerful techniques recently proposed in [41-44].

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