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TESIS DOCTORAL

**ANÁLISIS MODAL EXPERIMENTAL EN
CAMPO COMPLETO EMPLEANDO
CORRELACIÓN DIGITAL DE IMÁGENES EN
ALTA VELOCIDAD/
FULL-FIELD EXPERIMENTAL MODAL
ANALYSIS USING HIGH SPEED DIGITAL
IMAGE CORRELATION**

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RESUMEN

El comportamiento estructural bajo solicitaciones dinámicas presenta, a menudo, mayor criticidad para la integridad estructural y el funcionamiento de un sistema mecánico que las solicitaciones estáticas. El análisis modal experimental es una disciplina encargada de la caracterización de la respuesta a vibración atendiendo al fenómeno de la resonancia. Esta disciplina caracteriza los modos propios de vibración del sistema analizado por medio de los tres parámetros modales: frecuencia natural, forma de modo y factor de amortiguamiento. Con este objetivo, múltiples sensores puntuales, principalmente acelerómetros, se adhieren en diferentes localizaciones de la superficie del elemento analizado conformando una malla de medida. Pese a la alta sensibilidad de medida de estos sensores, sus limitaciones afectan en ocasiones a la fiabilidad y representatividad de los resultados. La propia masa de los sensores y del cableado aporta inercia y disipación de energía adicional, pudiendo ocasionar una variación significativa del comportamiento modal del sistema, con especial incidencia en elementos ligeros. Por otro lado, la resolución espacial de la malla de medida es muy pequeña en comparación con la información obtenida de una simulación por elementos finitos.

Recientemente, algunas técnicas ópticas han cobrado importancia en la medida de vibraciones y en la identificación modal. Estas técnicas aportan mejoras a las citadas limitaciones de las instrumentaciones tradicionales. No existe contacto con el espécimen y, además, realizan medidas en campo completo con una resolución espacial sin precedentes respecto a las instrumentaciones clásicas. En este sentido, cabe destacar por su capacidad y versatilidad las técnicas Correlación Digital de Imágenes en alta velocidad (HS DIC), en sus variantes de 2D y 3D empleando una o dos cámaras en cada caso, y vibrometría laser por escaneo. Actualmente, el uso de vibrometría laser está ampliamente extendido y aceptado en el análisis modal experimental con equipos específicos y software comercial. A pesar de ello y de su menor sensibilidad, HS DIC ha atraído la atención ya que es capaz de aportar una mayor resolución espacial y realizar medidas tridimensionales con menores costes de equipos. Varios estudios han puesto en valor esta técnica para la caracterización de formas modales gracias a la información detallada que aportan las medidas en campo completo. Otros estudios han avanzado en la realización de caracterizaciones modales completas a partir de funciones de respuesta a la frecuencia para fuerzas puntuales. Sin embargo, no ha sido propuesta una metodología para aquellos casos habituales en el que la excitación es aportada como un movimiento de la base. Por tanto, en esta tesis se propone y evalúa una metodología para la identificación modal en campo completo empleando HS DIC en ensayos por movimiento de la

base. Para ello, se propone una equivalencia entre las respuestas a una fuerza puntual y aquella correspondiente a un movimiento de la base partiendo del análisis teórico de un sistema de un grado de libertad. El método de ajuste del círculo se propone para el procesado eficiente de la información proporcionada por HS DIC durante la identificación modal. La metodología fue ensayada en una viga en voladizo y validada experimental, numérica y teóricamente.

La magnificación de movimiento basada en la fase es una rutina recientemente desarrollada que también ha demostrado gran potencial para las caracterizaciones modales. El algoritmo descompone las imágenes obtenidas durante el ensayo y, mediante un filtro pasabanda, amplifica la señal de intensidad en ese ancho de banda. De esta manera, se obtiene una versión magnificada del video que revela movimientos inicialmente imperceptibles. Esto ha sido empleado para generar videos del movimiento magnificado producido durante la resonancia empleando una única cámara. Así, es posible observar formas de modo bidimensionales a simple vista. En esta tesis se evalúa también la combinación de HS DIC y la magnificación de movimiento llevando a cabo medidas sobre imágenes magnificadas. Esto fue realizado tanto para la versión 2D empleando una única cámara como para 3D con un sistema estereoscópico. En primera instancia, la combinación fue empleada para determinar los modos propios de la viga en voladizo y validar las medidas con un modelo de elementos finitos. Gracias a la mayor amplitud de movimiento, se obtuvieron resultados menos ruidosos en las medidas de HS DIC. Así pues, se consigue mejorar las medidas en aquellos casos en los que se emplea baja excitación y en la caracterización de formas de modos en frecuencias más altas. Estos resultados guardaron una alta correlación con los resultados de la simulación, rechazando cualquier deformación introducida en la imagen durante el procesamiento de las mismas. Además de ello, los campos de desplazamientos aportan información muy relevante a la visualización para comprender la deformación sufrida. Por último, la combinación fue empleada para obtener formas de modo tridimensionales en un panel industrial curvo de grandes dimensiones. Esto supone un incalculable avance respecto a la información extraída mediante algoritmos de detección de aristas o la simple visualización.

Con las metodologías abordadas, esta tesis contribuye a extender el uso de HS DIC en el análisis modal experimental aportando un claro potencial industrial.

ABSTRACT

Structural behaviour under dynamic events frequently influences the structural integrity and performance in a more severe way than static loads. Experimental modal analysis studies the vibration response focusing on the resonance phenomenon. This discipline defines the resonant modes of the analysed system using the three modal parameters: natural frequency, mode shape and damping ratio. For this purpose, multiple pointwise sensors, mainly accelerometers, are attached to the studied element at different locations describing a measurement grid. In spite of the high sensitivity of the sensors, some limitations may make the results non-reliable or representative. The mass of the sensors and cabling introduces additional inertial and dissipation effects that could significantly modify the modal behaviour of the system, especially in lightweight elements. Furthermore, the spatial resolution is quite lower than in a finite element method model.

Recently, some optical techniques have become popular for vibration measurements and modal identification. These techniques overcome the traditional instrumentation's drawbacks. On one hand, there is no contact with the specimen and, on the other hand, they are able to perform full-field measurements what notably increases the spatial resolution with respect to classical instrumentation. In this sense, the most remarkable and versatile techniques are high speed Digital Image Correlation (HS DIC), which performs 2D and 3D measurements by respectively employing one or two high speed cameras, and scanning laser Doppler vibrometry. Laser vibrometry is currently a reality in experimental modal analysis with specific equipment and commercial software. Despite that and the lower sensitivity, HS DIC is attracting attention since it provides higher spatial resolution and is able to perform 3D measurements with cheaper equipment. Some studies have demonstrated that the full-field information of HS DIC is quite interesting to obtain detailed mode shapes. Further studies performed complete modal characterisations from the frequency response functions to a pointwise force. However, it has not been proposed any methodology for the common base motion excitation instances. Thus, in this thesis it has been proposed and evaluated a methodology for full-field modal identification using HS DIC under base motion excitation. Theoretical equivalences were found between response to pointwise force and base motion excitation under single-degree-of-freedom assumptions. The circle-fit approach was proposed for efficient modal identification. The methodology was tested in a cantilever beam and validated experimentally, numerically and theoretically.

Phase-based motion magnification is a recently developed algorithm that has also shown potential for modal characterisation purposes. By decomposing the images recorded during an event, a bandpass algorithm filters the detected signal in the intensity field and amplifies this band to reveal subtle motion. As a result, it is obtained a magnified version of the event. Some methodologies have been proposed to generate magnified videos corresponding to a resonance using a single camera. That makes it possible to observe 2D mode shapes deformation motion by the naked eye. In this thesis it is also evaluated the combination of HS DIC and motion magnification by measuring on magnified images. It was tested for 2D measurement using a single camera and then for 3D using stereovision. Initially, the combination was employed to determine the cantilever beam mode shapes and validate them with the finite element model. Clearer results were obtained as a result of the higher amplitudes. Thus, it improves measurements when using lower excitation and characterising higher frequency mode shapes. Results prove to be in agreement with finite element results and reject shape distortions during the magnification processing. Additionally, the quantitative measurements supply the naked eye visualisations with substantial information. Finally, the combination was employed to obtain remarkable 3D mode shapes of an industrial large curved panel. It allowed to assimilate the deformations where edge detection algorithms or simple visualisation fail.

With these approaches, this thesis has contributed to expand the use of HS DIC in experimental modal analysis with industrial potential.

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NOMENCLATURE

a	Denominator polynomial coefficients for Rational Fraction Polynomial method
b	Numerator polynomial coefficients for Rational Fraction Polynomial method
A	Modal constant
B	Residual term employed to define the effect of close modes in single-degree-of-freedom approaches
C	Parameter for the calculation of theoretical beam mode shapes that depends on the boundary conditions
C_{CC}	Cross-Correlation coefficient
C_s	Fourier coefficients in image decomposition
C_{ZNCC}	Zero-Normalised Cross-Correlation coefficient
D	Diameter of the frequency response function circle in the complex plane
E	Young's Modulus
f	Light intensity of the pixel facet in the unloaded state
f_n	Natural frequency (Hz)
F	Light intensity of a 1D image
g	Light intensity of the pixel facet in the loaded state
h	Impulse response function
H	Frequency response function
$\hat{H}$	Frequency response function obtained experimentally
H_0	Frequency response function initially obtained from the conversion with a proposed estimation of the natural frequency
I	Second moment of area
K^R	Residual stiffness of the modes out of the analysed spectrum
l	length of a cantilever beam
m	Mass
M	Integer number to define the facet size
M^R	Residual mass of the modes out of the analysed spectrum
N	Number of modes in the spectrum

P	Central pixel of a facet in the unloaded state
P'	Central pixel of a facet in the loaded state
s	Complex exponent of impulses response function term
T	Non-dimensional transmissibility function in terms of relative motion
T_{exp}	Transmissibility function experimentally obtained with this setup
w	Theoretical mode shape of a beam
x	X coordinate
(x_0, y_0, z_0)	Coordinates of the central pixel of a facet in the unloaded state
(x_0', y_0', z_0')	Coordinates of the central pixel of a facet in the loaded state
y	Y coordinate
z	Z coordinate
α	Parameter for the theoretical estimation of beam mode shapes
β	Mode parameter for the theoretical estimation of modal properties of a beam
$\delta(t)$	Time-domain displacement in a 1D image.
η	Structural damping
θ	Angular position of a point of the frequency response function circle in the complex plane
ξ	Viscous damping ratio
ω	Vector of frequencies in the analysed spectrum (rad)
ω_s	Spatial frequency (rad)
ω_n	Natural frequency (rad)
$\omega_{n,0}$	Natural frequency initially proposed for the conversion to frequency response functions (rad)

SUBSCRIPTS

a	Point of the frequency response function corresponding to a frequency above the natural frequency
b	Point of the frequency response function corresponding to a frequency below the natural frequency
j	Output coordinate (Response)
k	Input coordinate (Excitation)
r	Mode indicator

SUPERSCRIPTS

(1)	Half-power low cut
(2)	Half-power high cut

Capítulo 1

Introducción

1.1 Motivación

La resonancia es uno de los fenómenos dinámicos de mayor importancia puesto que amplifica la vibración producida por una fuente externa en frecuencias concretas. La distribución de masa y rigidez del componente definen las características de este fenómeno mientras que la disipación de energía define la amplitud de la respuesta. Desde un punto de vista mecánico, este comportamiento amenaza el funcionamiento y la integridad estructural tanto de máquinas como de estructuras. Por ejemplo, en la industria del transporte, tal como la automoción o la aerodinámica, existen importantes fuentes de vibración producidas por los motores y turbinas, la fricción con la calzada, flujos turbulentos, etc., que deben ser tenidos en cuenta en la evaluación de los diferentes sistemas que componen el vehículo. El ruido generado por la vibración también afecta negativamente al confort de pasajeros y miembros de la tripulación. En las últimas décadas, se han realizado múltiples esfuerzos en materia de aislamiento y control de las vibraciones. Con este propósito, la selección de los materiales y el diseño mecánico condicionan de manera determinante la masa, el amortiguamiento y la rigidez de una estructura. Así pues, las simulaciones numéricas basadas en el método de los elementos finitos (FEM) constituyen una potente herramienta en fases de diseño para la evaluación de la influencia de las propiedades del material y el diseño en el comportamiento dinámico de un montaje.

Pese al considerable avance de los equipos y técnicas de computación, los ensayos experimentales proporcionan información fundamental para la mejora del diseño y el ajuste de los modelos numéricos. De acuerdo con el tema tratado, el análisis modal experimental es una metodología extendida cuyo objetivo es alcanzar un profundo entendimiento de la naturaleza de la vibración así como la caracterización dinámica incluyendo la disipación de energía y la resistencia a fatiga (McConnell 1995; Maia and Silva 1997; Ewins 2000). Esta información tiene una implicación directa en la verificación de las predicciones numéricas.

Sin embargo, los sensores habitualmente empleados, mayoritariamente acelerómetros, poseen ciertas limitaciones que afecta a la fiabilidad de los resultados. Respecto a la instrumentación, es preciso alcanzar un compromiso entre el tiempo de preparación y la resolución espacial obtenida. En el caso de estructuras de grandes dimensiones, la resolución es generalmente baja e incluso puede desembocar en la aparición de aliasing espacial en la determinación de la forma de los modos. Además, el carácter invasivo de los sensores y el cableado aporta masa y amortiguamiento lo que conduce a que en elementos muy ligeros las medidas sean tan invasivas que los resultados no sean representativos del comportamiento real.

Solventando estas limitaciones, diferentes autores han estudiado recientemente la aplicación de técnicas ópticas de campo completo en el análisis de vibraciones (Sharpe 2008; Baqersad et al. 2017). Correlación Digital de Imágenes (DIC) (Schreier, Orteu, and Sutton 2009) es una de las más comúnmente empleadas. Con el uso de cámaras de alta velocidad (HS DIC), esta técnica ha experimentado un notable avance en los últimos tiempos para caracterizaciones modales. El interés de esta técnica se encuentra avalado por su carácter no invasivo y la simplicidad de equipamiento y ejecución. Frente a las costosas instrumentaciones, esta técnica únicamente requiere la instalación de las cámaras digitales y un ordenador con el software de procesado. Además, proporciona información en campo completo de toda la superficie, desechando cualquier posibilidad de aliasing espacial.

Recientemente ha surgido una interesante aplicación denominada magnificación del movimiento basada en la fase (Wadhwa et al. 2013) empleada para revelar movimientos imperceptibles en videos incrementando la amplitud del movimiento correspondiente a un determinado ancho de banda. Este algoritmo ha sido empleado en metodologías para la monitorización de estructuras y análisis modal (J. G. Chen et al. 2015; Y. Yang, Dorn, Mancini, Talken, Kenyon, et al. 2017). La ejecución de medidas con DIC sobre imágenes magnificadas mejoraría los resultados de la forma de los modos, en especial en medidas en alta frecuencia con bajos desplazamientos.

1.2 Objetivos

Considerando las ventajas ofrecidas por HS DIC, la técnica es aceptada como una alternativa viable y fiable para análisis modal experimental. Este estudio pretende continuar extendiendo el uso de la técnica con el objetivo particular de proponer metodologías de potencial aplicación industrial. Este estudio recoge dos cuestiones identificadas en la bibliografía proponiendo tres metodologías que han sido recientemente publicadas.

El primer objetivo de esta tesis la elaboración de una metodología para la realización de caracterizaciones modales utilizando funciones de transmisibilidad obtenidas con HS 3D-DIC en ensayos por movimiento de la base. Para ello, es preciso establecer equivalencias entre las respuestas de los sistemas a este tipo de excitación y a la excitación por fuerza puntual y definir un algoritmo de identificación modal que sea eficiente manejando la gran cantidad de información generada por DIC.

En segundo lugar se explora la combinación de DIC y la magnificación del movimiento basado en la fase propuesta por Wadhwa et al. (Wadhwa et al. 2013) tanto en 2D empleando una única cámara como en 3D usando un sistema estereoscópico. El objetivo en ambos estudios fue evaluar los beneficios de los gráficos de contornos de DIC junto con la visualización del movimiento magnificado para la interpretación de las formas de deflexión operacionales ODSs empleando diferentes factores de magnificación. Se presta especial atención a ensayos con bajo nivel de excitación y en la caracterización de frecuencias del orden de miles de Hercios. Se busca evaluar la calidad de los resultados comparando con un modelo de elementos finitos, observando la evolución del error respecto al factor de magnificación, así como la posible deformación del objeto en la imagen como consecuencia del proceso de magnificación.

Estos estudios fueron ejecutados sobre una viga en voladizo diseñada como material de referencia para estudios de validación y fabricada con gran precisión (“Validation of Numerical Engineering Simulations: Standardisation Actions (VANESSA). European FP7 Project Grant Agreement No. NMP3-SA-2012-319116” 2014). Este elemento presenta una alta correspondencia con la predicciones de modelos teóricos (Blevins 2001) y numéricos, tanto en estático como en dinámico (Hack 2016), lo que lo hace ideal para la validación de las metodologías. Adicionalmente, la combinación de magnificación y 3D-DIC se lleva a la práctica en un panel industrial de material compuesto de grandes dimensiones.

1.3 Alcance

El trabajo desarrollado durante esta tesis para la consecución los citados objetivos ha sido reunido en el presente documento. Este ha sido construido sobre la base de las tres publicaciones científicas adjuntadas en el Apéndice A.

El Capítulo 2 describe los fundamentos de las técnicas y las metodologías experimentales que componen este trabajo. Los diferentes métodos de análisis modal experimental son descritos con especial atención en aquellos empleados en este trabajo. Se incluye también la teoría de DIC tanto en sus versiones 2D y 3D. Finalmente se exponen los conceptos básicos de la magnificación del movimiento basada en la fase.

El Capítulo 3 evalúa con detalle la literatura relacionada con estos temas. Se presenta una revisión histórica de la evolución de DIC desde los años 80 incluyendo aquellas disciplinas donde la técnica ha adquirido mayor popularidad. Posteriormente, se exponen con profundidad la aplicación de DIC en análisis modal experimental. Por último, se destaca el potencial de la magnificación para el análisis modal a través de los trabajos desarrollados por diferentes estudios. En este punto se destacan los puntos sobre los que se apoya este estudio y que no han sido cubiertos por la literatura.

El Capítulo 4 está dedicado a la descripción general de los equipos y los métodos empleados durante la tesis. En él, se describen las características de la viga en voladizo. Se enumera también todo el equipamiento empleado para HS DIC incluyendo los procedimientos de procesamiento atendiendo el software disponible y los resultados requeridos. En este capítulo se describen los dispositivos típicos de ensayos de vibraciones. Se exponen aquí las herramientas de posprocesado de los resultados de DIC en Matlab para prepararlos para el análisis modal. Se incluye también la descripción de los modelos teóricos y numéricos empleados. Concluyendo este capítulo, se expone el procedimiento para la magnificación del movimiento.

El Capítulo 5 expone de manera particular el procedimiento llevado a cabo para la evaluación de la metodología propuesta de análisis modal en ensayos por movimiento de la base usando DIC. Aquí se describe completamente el proceso y parámetros experimentales así como la conversión de las funciones de transmisibilidad a FRFs y la implementación del método del círculo para la identificación modal. Se especifican también todos los detalles para la validación experimental, numérica y teórica.

Del mismo modo, el Capítulo 6 está dedicado a la combinación de HS DIC y magnificación tanto en la versión 2D y 3D. Se enumera el conjunto de ensayos y procesados de magnificación llevados a cabo. Se describe también el proceso de evaluación de los resultados por medio del modelo de elementos finitos.

El Capítulo 7 presenta un resumen de los resultados incluidos en las tres publicaciones prestando atención a los detalles más relevantes para las conclusiones de esta tesis, recogidas en el Capítulo 8. Basándose en el resultado de este trabajo, en el Capítulo 9 se define la línea de trabajos futuros.

Chapter 1

Introduction

1.1 Motivation

Resonance is one of the most important dynamic phenomenon since it produces the amplification at particular frequencies when external vibration sources are applied. It is conditioned by the mass and the stiffness distribution and the response amplitude is defined by the energy dissipation. From the mechanical point of view, this behaviour threatens the performance and the structural integrity of machines and structures. For instance, in the transport industry (such as the automotive or the aerospace sectors) there are significant vibration sources due to engines and turbines, road friction, airflow turbulence, etc., which must be taken into account during the evaluation of the different systems used in the vehicle. Moreover, the comfort and the occupational performance of the crewmembers and passengers are affected by the presence of noise. Along the last decades, many efforts have been made to provide insulation and vibration control. In this sense, the selection of materials and the mechanical design are critical issues regarding the mass, damping and stiffness of the structure. Thus, in the design stages powerful numerical simulations, conventionally based on the Finite Element Method (FEM), evaluate the influence of the material properties and the mechanical design in the dynamic behaviour of the ensemble.

Despite the exponential advance of the computational techniques and the hardware, experimental testing provides a fundamental feedback to improve design and numerical models. Considering the topic

covered here, experimental modal analysis is an extended methodology that aims to provide a full comprehension of the vibration nature as well as dynamic material characterisation involving energy dissipation and fatigue endurance (McConnell 1995; Maia and Silva 1997; Ewins 2000). This information has direct implication in the verification of numerical predictions.

Nonetheless, traditional sensors, mostly accelerometers, present limitations that affect the reliability of the results. A compromise is always required between time and cost for instrumentation and the resulting spatial resolution. In large structures, this resolution is usually low and it may lead to spatial aliasing in mode shape characterisation. Moreover, some sources of errors are present in the measurements as a consequence of their invasive nature. Mainly, perturbations introduced by mass and damping of transducers and cabling. Especially, the instrumentation of lightweight systems is complex and typically is too invasive so that the results are not representative of the actual behaviour.

In order to overcome those drawbacks, some authors have recently studied the application of full-field optical techniques on vibration analysis (Sharpe 2008; Baqersad et al. 2017). Digital Image Correlation (DIC) (Schreier, Orteu, and Sutton 2009) is one of the most extensively employed. In combination with High Speed cameras (HS DIC), this technique has experienced a notable advance in the implementation for experimental modal analysis in the last decade. The main advantages are the non-contact nature of the technique and its basic setup. There are no requirements of time-consuming instrumentation except from digital cameras and a computer with the image processing software. Additionally, they provide full-field information on a whole surface simultaneously, rejecting any possibility of spatial aliasing.

An interesting tool known as phase-based motion magnification (Wadhwa et al. 2013) has been recently developed. It intends to reveal subtle motion in videos by increasing the amplitude of the motion corresponding to a defined bandwidth. This algorithm has been employed to develop methodologies for structure monitoring and experimental modal analysis (J. G. Chen et al. 2015; Y. Yang, Dorn, Mancini, Talken, Kenyon, et al. 2017). The application of DIC in magnified images would improve mode shape results especially for high frequency measurements.

1.2 Objectives

Considering the exposed benefits that HS DIC offers, the technique is considered as a feasible and reliable alternative for experimental modal analysis. This study means to keep on expanding the use of the technique, particularly aiming for the potential implementation in industrial applications. The present work covers two issues encountered in the literature review. In particular, three related methodologies were established and published.

The first objective of this thesis is to elaborate a methodology to perform modal identification in transmissibility functions obtained by HS 3D-DIC during base motion excitation test. In that work, it is necessary to establish the equivalence between the response system to base motion and to the pointwise force excitation. Moreover, an efficient modal identification algorithm must be chosen to deal with the great amount of data generated by DIC.

Additionally, it is explored the combination of DIC and phase-based motion magnification proposed by Wadhwa et al. (Wadhwa et al. 2013). A single-camera setup was employed for 2D measurement and also a stereovision system was implemented for 3D measurements. In both studies, the objective was to evaluate the benefits of DIC contour plots along with visualisation of magnified motion in the interpretation of video ODSs using different magnification factors, especially for low excitation level tests and frequencies of the order of thousands of Hertz. It is also intended to evaluate the quality of the results using a Finite Element model. The evolution of the error versus the magnification factor was evaluated as well as possible shape distortion in the image introduced by the magnification procedure.

The studies have been developed and tested in a cantilever beam accurately manufactured that was designed as a reference material for validation analysis (“Validation of Numerical Engineering Simulations: Standardisation Actions (VANESSA). European FP7 Project Grant Agreement No. NMP3-SA-2012-319116” 2014). Thus, experimental results are in a good agreement with its well-known theoretical behaviour (Blevins 2001) and numerical models, both in static and dynamic conditions (Hack 2016), what makes it ideal for the validation of the methodologies. Additionally, the combination of motion magnification and 3D-DIC was put into practice in an industrial large composite panel.

1.3 Scope of the thesis

The work developed during this thesis to fulfil the mentioned objectives has been gathered in this document. This document has been built up on the base of the three publications which are attached in Appendix A.

Chapter 2 describes the fundamentals of the techniques and the experimental methodologies that this work consists in. The different approach to perform experimental modal analysis is presented mainly focusing in those approaches employed in this work. Theoretical aspects of DIC are included, describing the 2D and 3D version. The fundamentals of phase-based motion magnification are also briefly explained.

Chapter 3 evaluates in detail the literature works related to these topics. An historical review of DIC shows the evolution of the technique since the 1980s including those fields where the technique is more

popular. Then, the application of DIC in experimental modal analysis is deeply exposed. Finally, the potential of motion magnification in experimental modal analysis is exposed through different works. Here it is highlighted the points which this work is based on and not covered by the literature.

Chapter 4 is devoted to the description of the general equipment and methods employed during this thesis. The cantilever beam employed as test specimen is fully described. The full equipment to perform HS DIC is enumerated, including the processing procedure considering the available software and the required results. Then, vibration devices are described. The post-processing of DIC results in Matlab is exposed, and the preparation of experimental data for modal analysis. The theoretical and numerical model employed for validation purposes are also developed in this section. Finally, the motion magnification procedure is exposed.

Chapter 5 exposes the particular procedure to evaluate the proposed methodology for experimental modal analysis in base motion tests using DIC. The experimental setup is fully described. The proposed conversion of transmissibility functions into FRFs is here deduced together with the implementation of the circle-fit approach for modal identification. Details for the experimental, numerical and theoretical validation are also specified.

In the same way, Chapter 6 is devoted to the combination of HS DIC and motion magnification both in 2D and 3D version. The package of tests and magnification processing is presented. The procedure to evaluate the improvement of the results using the FEM model as a reference is defined.

Chapter 7 is a summary of the results included in the three publications included in Appendix A. Focus is on the most relevant details that lead to the conclusions of the thesis, indicated in Chapter 8. Based on this work, future works can be defined as included in Chapter 9.

Chapter 2

Theoretical background

In this chapter it is presented the basis of the disciplines employed in this thesis. As previously exposed, this work intends to explore new possibilities of DIC in vibration and modal analysis. Thus, the dynamic behaviour concepts and parameters underlie throughout the study. These concepts and methodologies are described under the extended designation of experimental modal analysis. A classification of modal identification algorithms is made, emphasising in the SDOF approaches considered in this study. As the main experimental technique for this thesis, an overview of DIC fundamentals is presented. Algorithms and procedures for 2D and 3D measurements are described, indicating their capabilities and limitations. In this thesis, it has been also explored a tool known as phase-based motion magnification what amplifies subtle periodic motion in videos. In combination with DIC, it would contribute to enhance the quality of the results. Hence, the fundamentals of motion magnification are developed.

Finally, some conclusions sum up the relevant aspect and point out the possibilities and combinations between them that are the starting point of the study.

2.1 Experimental modal analysis

Dynamically, a mechanical system can be modelled according to three stages focusing on different characteristics. Experimental modal analysis is an extended discipline both in researching and industry

that tackles the characterisation through these stages assuming that the structure is linear and time invariant (McConnell 1995; Maia and Silva 1997; Ewins 2000). For practical reason, the characterisation stages typically follow the next order:

- 1.- Response model: in this phase it is estimate the level of response of the system to a unitary excitation typically defined by a set of transfer functions in frequency domain. Time-domain data is acquire from input excitation and output structural response. The signals are transformed into frequency domain using Fourier analysis (K. R. Rao, Kim, and Hwang 2010) and Welch averaging method (Welch 1967) so as to obtain auto and cross Power Spectral Densities (PSD). Further processing of these PSDs allows to obtain the transfer functions and coherence functions, what indicates the correlation level between signals. According to the type of data sets in terms of number of input and output measurements, modal analyses can be referred to as Multi-Input Multi-Output (MIMO), Multi-Input Single-Output (MISO), Single-Input Multi-Output (SIMO) or Single-Input Single-Output (SISO). In this phase, an initial assessment of the results is performed, especially to identify the number of modes present in the spectrum. Mode indicator functions are frequently employed mainly based in eigenvalue decomposition of single value decomposition such as complex mode indicator function (Ewins 2000; R. J. Allemang and Phillips 2004). This function shows local maximum or minimum at the natural frequencies as shown in Fig. 1.
- 2.- Modal model: in this phase, the dynamic behaviour is decomposed in a certain number of independent components known as modes. A mode involves a particular deformation, known as mode shape, vibrating at a particular frequency, known as natural frequency. Additionally, the effect of the different energy dissipation sources is defined by the damping ratio. These modal parameters associated to a set of vibration modes are obtained from response functions. Thus, the way the system tends to vibrate is defined. Different modal identification algorithms have been developed along years seeking more realistic characterisations. A description is presented in the subsequent subsections. The modal model is frequently the last stage in experimental modal analysis. These results are typically compared with those from theoretical models or numerical models based on FEM. This comparison is straight in the case of natural frequencies and damping ratios as single values. For a quantitative comparison of the mode shapes, it is remarkable the Modal Assurance Criterion (MAC) (R J Allemang and Brown 1982; Randall J Allemang 2003) which is a correlation coefficient that indicates the consistency of the different modes identified by all the models.
- 3.- Spatial model: this stage entails the definition of the mathematical model based on mass, stiffness and damping properties of the different degrees of freedom experimentally defined. The natural frequencies and the modes shapes are the eigenvalues and eigenvectors, respectively, of the matrix that defines the system.

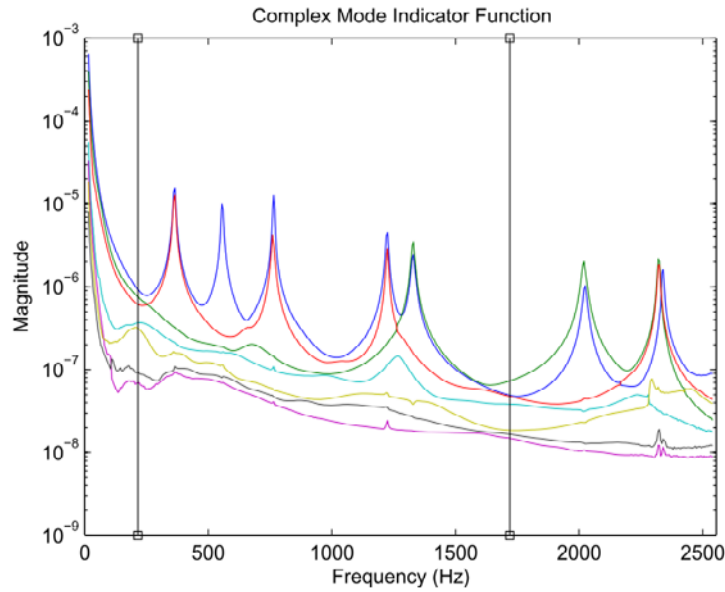


Fig. 1. Example of Complex Mode Indicator Functions (R. J. Allemang and Phillips 2004).

In this thesis, the two first stages are covered as the most common in experimental modal analysis. In the next subsection, further details to define the modal model from the response model are provided and the most significant approaches are explained.

Additionally, although it is not strictly considered as modal analysis, it is common the evaluation of the deformation under certain operating condition known as operational deflection shapes (ODS). This parameter is interesting since, when the excitation is sinusoidal at one natural frequency, the deformation is dominated by the shape of the excited mode. In fact, if natural frequencies are separated enough, the ODS tends to reveal the mode shape. This allows to estimate mode shapes using a simple test and avoiding further processing.

2.1.1 Modal identification methods

Modal identification methods perform the modal parameter identification based on the response formulas of the theoretical dynamic model. They are typically designed to manage frequency response functions (FRFs) or their equivalent in the time domain Impulse Response Functions (IRFs), transfer functions which relate the displacement response of the structure and the force excitation in the frequency and time domain, respectively.

Considering a general multi-degree-of-freedom (MDOF) system, the FRF and the IRF corresponding to the response at the point j exciting at the point k are defined by equation (1) and (2), respectively. As can be deduced, the general response is described by the superposition of the responses of n modes of the structure where ω_r is the natural frequency, $A_{jk,r}$ is the modal constant and η_r the structural damping. Mode shapes are directly related to modal constant. Among the different ways to define energy

dissipation, structural damping is the most common to characterise it since it represents the deformation internal friction. However, IRF are defined by viscous damping ratio, ξ , that can be related with structural damping as $\eta = 2\xi$ in the resonance.

$$H_{jk}(\omega) = \sum_{r=1}^N \frac{A_{jk,r}}{\omega_{n,r}^2 - \omega^2 + i\eta_r \omega_{n,r}^2}, \quad (1)$$

$$h_{jk}(t) = \sum_{r=1}^N A_{jk,r} e^{s_r t} \quad ; \quad s_r = \left(-\xi + \sqrt{\xi^2 - 1}\right) \omega_r \quad (2)$$

Experimentally, the spectrum of analysis is always finite and covers the response of a specific number of modes, N . The effect of the non-registered modes from the spectrum above and below is approximately defined by terms known as residuals.

As the FRF inspection provides more explicit information of the structure, frequency domains approaches are more frequently employed. Thus, emphasis is on this methods hereafter. Despite these equation defines a MDOF, sometimes a MDOF characterisation approach is not efficient and SDOF point of view may be required. Under this classification, it is now exposed the most popular methods.

2.1.1.1 SDOF methods

These methods evaluate a single resonant peaks under the assumption that previous and subsequent modes do not have any influence. Natural frequencies must be separated enough to fulfil this assumption. Thus, this method are based in a single term of the equation (1)

$$H_{jk}(\omega) = \frac{A_{jk,r}}{\omega_{n,r}^2 - \omega^2 + i\eta_r \omega_{n,r}^2} \quad (3)$$

Under this designation, two methods stand out. The peak-picking method is useful because of its simplicity to obtain a first estimation for further operations. The circle-fit approach is a more sophisticated method that provides a more accurate vision of the system.

2.1.1.1.1 Peak-picking method

The peak-picking method is the simplest method for modal identification. In this method, the natural frequency, ω_r , is determined as that where maximum response occurs, what can be identified as peaks in the FRFs. For damping estimation, this method is based on the determination of the bandwidth defined by those points whose response involves half the power of the peak. In terms of amplitude of response, this points are defined by a level of $|H(\omega_{n,r})|/\sqrt{2}$, where $|H(\omega_{n,r})|$ is the maximum response, as observed in Fig. 2. From the equation, it can be demonstrated that the structural damping ratio for the mode r is approximately:

$$\eta_r \cong \frac{\omega_{n,r}^{(1)} - \omega_{n,r}^{(2)}}{\omega_{n,r}} \quad (4)$$

where $\omega_r^{(1)}$ and $\omega_r^{(2)}$ represents the high and low cut of the half-power frequency band. Thus, this method is also known as half-power method. Finally, modal constant, A_r , can be obtained from the absolute response at the natural frequency as follows:

$$|H(\omega_r)| = \frac{A_r}{\omega_{n,r}^2 \eta_r} \quad (5)$$

As can be deduced, in this method the damping and the modal constant depend on the maximum FRF level. This level is identified by peak-picking, what is not very accurate since the position of the maximum response depends on the characteristics of the Fourier analysis performed. Moreover, it is always assumed that the modal constant has no imaginary part, thus complex modes where different phase lags are encountered at the different spatial points cannot be properly characterised.

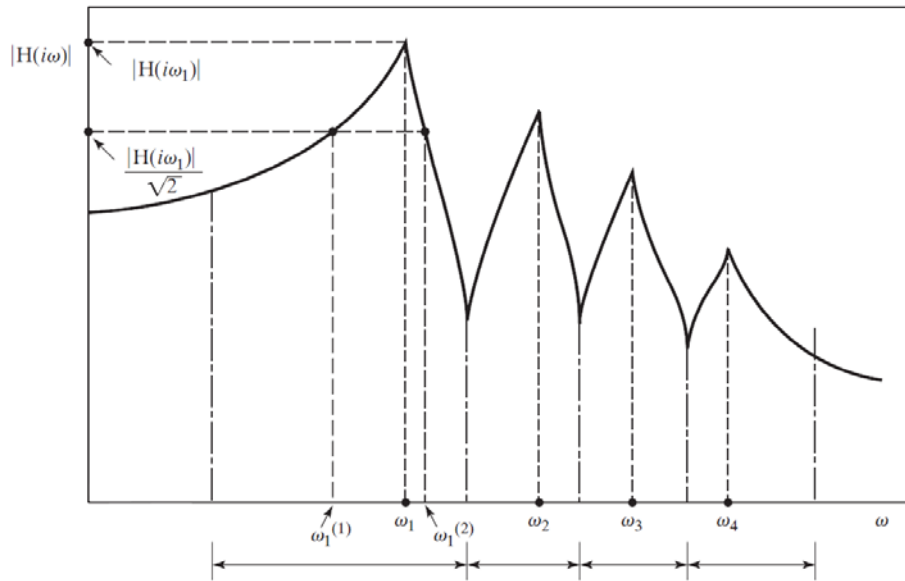


Fig. 2. Half-power points of a resonance (S. S. Rao 2011).

2.1.1.1.2 Circle-fit method

This is an evolution of the half-power method. In a SDOF system with structural damping, the Nyquist plot of a FRF describe a perfect circle. This is illustrated in Fig. 3 corresponding to a system with a structural damping of 0.06.

This method is based in the relations that exist between the modal and the circumference geometrical parameters shown in Fig. 4. The natural frequency, ω_n , is determined as that with maximum response, what implies the furthest point from the origin. The structural damping, η , can be obtained from the frequency, ω , and the geometrical position defined by the angle, θ , of any point above, a , and any below

the resonance, b , as shown in equation (6). Both parameters together with the modal constant, A , are related with the circumference diameter, D , as observed in equation (7).

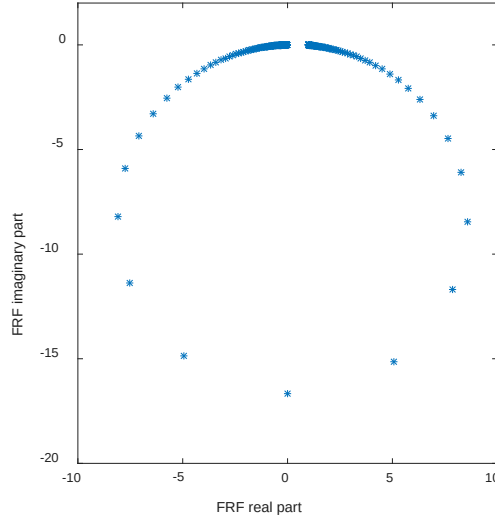


Fig. 3. Nyquist plot of a FRF of a SDOF system with a structural damping of 0.06.

Experimentally, a set of data points are selected at the vicinity of the peak to fit them to a circle using the least-squares method. Unlike peak-picking method, the natural frequency is here obtained as the maximum from the resulting curve which is based on the theoretical equations. Thus, the influence of the experimental conditions is reduced. As the actual experimental curve is not expected to be a perfect circumference, the damping ratio is calculated as the mean value of the damping ratios obtained from the combination of every point above and below the resonance in the selected region using equation (6). Finally, the modal constant is obtained from equation (7). This method can be improved by considering the influence of surrounding modes in the analysed frequency band as a complex constant, $B_{jk,r}$, to be included in the SDOF FRF equation as shown in equation (8).

$$\eta = \frac{\omega_a^2 - \omega_b^2}{\omega_n^2 \left(\tan \frac{\theta_a}{2} + \tan \frac{\theta_b}{2} \right)} \quad (6)$$

$$D = \frac{|A|}{\omega_n^2 \eta} \quad (7)$$

$$H_{jk}(\omega) = \frac{A_{jk,r}}{\omega_{n,r}^2 - \omega^2 + i\eta_r \omega_{n,r}^2} + B_{jk,r}, \quad (8)$$

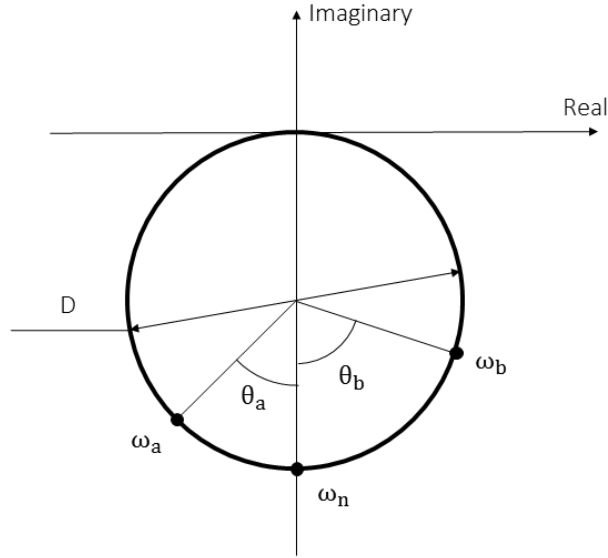


Fig. 4. FRF plotted in the complex plane to show the relation between modal and geometrical parameters.

2.1.1.2 MDOF methods

The MDOF approaches differs from the previous in the simultaneous recognition of the modal parameters of several modes at a time. The influence of every mode is considered. Thus, these methods overcome the closely-coupled modes issue. The MDOF approaches can be split into two subgroups depending on the way they manage the FRF data set.

2.1.1.2.1 Single-FRF analysis

Under this designation, it is found the methods that determine the modal parameters of multiple modes simultaneously by analysing each FRF curve individually. Two principal approaches can be highlighted that are described below.

The Non-Linear Least-Squares method (NLLS) consists in a curve-fit method based on the theoretical expression of FRF (equation (9)). This equation includes the residual effects of the modes out of the analysed spectrum, both above and below. This residual can be approximated as an inertial term for the modes below and a stiffness term for modes above. With this equation, this method performs a fitting of the experimental results by minimizing the total error. The coefficients to be determine are the modal constants, $A_{jk,r}$, the natural frequencies, $\omega_{n,r}^2$, the structural damping ratios, η_r , the residual mass, M_{jk}^R , and the residual stiffness, K_{jk}^R .

$$H_{jk}(\omega) = -\frac{1}{\omega^2 M_{jk}^R} + \left(\sum_{r=m_1}^{m_2} \frac{A_{jk,r}}{\omega_{n,r}^2 - \omega^2 + i\eta_r \omega_{n,r}^2} \right) + \frac{1}{K_{jk}^R} \quad (9)$$

Rational Fraction Polynomial method (RFP) is also a curve-fitting based method that formulate the problem as a polynomial ration formula as shown in equation (10) which implies two linear equations. In the equation, N represents the number of modes in the spectrum. As advantage, the error equation can be expressed as a combination of polynomial as shown in equation (11). Here, $\hat{H}_{jk}$ is the experimental FRF. After determining the coefficients, natural frequencies and damping ratios are firstly obtained from the denominator expression in equation (10). Modal constant are then obtained from the numerator. As the number mode, N , is also a parameter to estimate, it is necessary to assess the detected modes and discern actual modes from computational ones. Typically, a repeating process is performed employing different orders and evaluating the results.

$$H_{jk}(\omega) = \frac{b_0 + b_1(i\omega) + b_2(i\omega)^2 + b_{2N-1}(i\omega)^{2N-1}}{a_0 + a_1(i\omega) + a_2(i\omega)^2 + a_{2N}(i\omega)^{2N}} \quad (10)$$

$$e_{jk} = (b_0 + b_1(i\omega) + b_2(i\omega)^2 + b_{2N-1}(i\omega)^{2N-1}) - \hat{H}_{jk}(a_0 + a_1(i\omega) + a_2(i\omega)^2 + a_{2N}(i\omega)^{2N}) \quad (11)$$

2.1.1.2.2 Multi-FRF analysis

With this methods, beside simultaneous identification of several modes at a time, several curves are fitted at the same time. Hence, a unique set of modal parameters is obtained by processing the whole response data set at once. This has some advantages over the single curve analysis. These are more sophisticated methods that practically do not need any user supervision. Regarding mode shapes, they are obtained explicitly, conversely to previous methods where they are obtained by processing the modal constant matrix. Moreover, they are able to work out very close natural frequencies or even identical, i.e., double modes. Some of the most common methods (R. J. Allemang and Phillips 2004) in frequency domain are polyreference least-squares complex frequency (Van der Auweraer et al. 2001; Guillaume et al. 2003; Verboven et al. 2004), rational fraction polynomial (H. Richardson and L. Formenti 1982), complex mode indicator function (Shih et al. 1988). On the other hand, popular time domain approaches are least-squares complex exponential (Brown et al. 1979) and Ibrahim time domain (Ibrahim and Mikulcik 1977; Pappa 1982).

However, the greater complexity of these methods implies the necessity for more computation power. Furthermore, they present difficulties to identify variations in the characteristics of FRF and would produce unsatisfactory results.

2.1.2 Conclusions

Here it has been summarised how the knowledge of experimental modal analysis fundamentals and modal identification algorithms is essential to perform efficient processing and accurate modal characterisations. Some points can be remarked:

- SDOF approaches are computationally simpler. Although the peak-picking approach provides a good initial estimation of the modal parameters, the circle-fit approach is not much more computationally consuming and is more accurate instead.
- MDOF are able to evaluate the effect of all the modes in the analysed spectrum what provide reliable information with closely spaced modes. They require more computationally powerful devices than SDOF methods, what might be a limitation for large data sets.
- Under MDOF approaches, multi-FRF has remarkable advantages such as:
 - Practically no need for user supervision.
 - Mode shapes are obtained explicitly, without further processing
 - Double modes are discerned.
- The main disadvantage of multi-FRF MDOF methods is the high computational requirements for large data sets.

2.2 Digital Image Correlation

Digital Image Correlation (DIC) is an optical technique used for measuring strain and displacements in mechanical elements (Schreier, Orteu, and Sutton 2009). DIC correlates a sequence of digital images captured during the test based on intensity and compares them with an image from an initial state, generally unloaded. The area of interest is virtually divided into squared groups of pixels known as facets or subsets. The size of the facets is $(2M+1) \times (2M+1)$ pixels, where M an integer number and the central pixel, P , is located at (x_0, y_0, z_0) (Bing Pan et al. 2009). A facet is the smallest unit and on which the algorithm performs a tracking. In a loaded state, the structure undergoes deformation and the initial facet is moved from its original location, P , as shown in Fig. 5. In the loaded state, the DIC algorithm seeks a pixel, P' , located at (x_0', y_0', z_0') whose surrounding subset area has the most similar intensity. From this analysis, it is possible to infer the displacements the specimen experienced and the associated strain field. In order to perform the tracking, every facet must be unique and hence a random speckle pattern, is needed in the area of interest as observed in Fig. 6. The speckle must be composed by various pixels to improve the accuracy of the results (a diffuse grey pattern would be observed otherwise) (Lecompte et al. 2006).

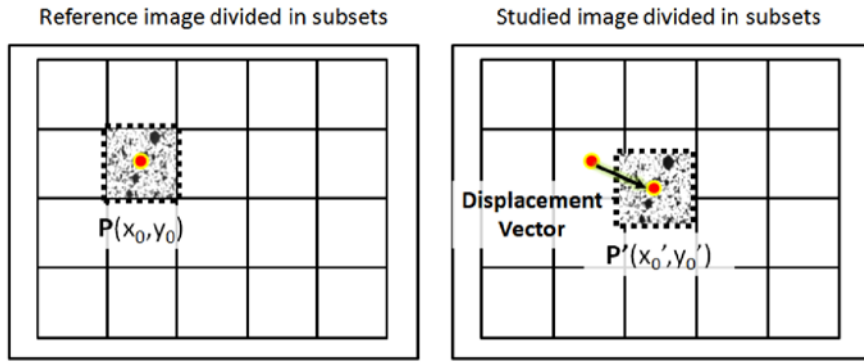


Fig. 5 Displacement of a facet from a reference image to a loaded image using Digital Image Correlation (L. A. Felipe-Sesé 2014).



Fig. 6. An example of a random speckle pattern.

The way DIC performs the facets matching is by means of a correlation criterion. Many of them can be found in the literature but two methods have the greatest popularity: cross-correlation criteria (CC) and sum-squared differences (SSD). Two variation of this methods are obtained employing normalisation and zero-normalisation. Pan et al. (Bing Pan et al. 2009) evaluated the efficiency of this methods and it was found out that Zero-Normalised Cross Correlation (ZNCC) and Zero-Normalised Sum-Squared Differences (ZNSSD) have the highest noise robustness. Moreover, the effect of the intensity offset and linear scale due to illumination light sources is avoided, conversely to simple CC or SSD methods.

$$C_{CC} = \sum_{i=-M}^M \sum_{j=-M}^M [f\{x_i, y_j\} \cdot g\{x'_i, y'_j\}] \quad (12)$$

The correlation methodology experienced improvements with the methodology called reliability-guided DIC (B. Pan 2009; Bing Pan, Wang, and Lu 2010). This method propose a seed point to start the correlation which is guided by firstly analysing the points in the neighbourhood using the ZNCC coefficient. In this way, the correlation calculation is performed in the most reliable direction. This methodology can be applied despite different issues in the images such as shadows or discontinuous

areas and reduces the error propagation. In equation (13), ZNCC formula is shown where $f\{x_i, y_j\}$ and $g\{x'_i, y'_j\}$ are the grey intensity level at the point $\{x_i, y_j\}$ in the reference state and the loaded stated, respectively. The remaining terms are the mean intensity values in the subset, f_m and g_m , calculated as shown in equation (14) and the normalising factors, Δf and Δg , defined by equation (15). After evaluating the ZNCC in the search window, the position of the facet in the loaded stated is found as that what maximise the correlation coefficient.

$$C_{ZNCC} = \sum_{i=-M}^M \sum_{j=-M}^M \left[\frac{[f\{x_i, y_j\} - f_m] \cdot [g\{x'_i, y'_j\} - g_m]}{\Delta f \Delta g} \right] \quad (13)$$

$$f_m = \frac{1}{(2M+1)^2} \sum_{i=-M}^M \sum_{j=-M}^M f\{x_i, y_j\} \quad ; \quad g_m = \frac{1}{(2M+1)^2} \sum_{i=-M}^M \sum_{j=-M}^M g\{x'_i, y'_j\} \quad (14)$$

$$\Delta f = \sqrt{\left(\sum_{i=-M}^M \sum_{j=-M}^M f\{x_i, y_j\} - f_m \right)^2} \quad ; \quad \Delta g = \sqrt{\left(\sum_{i=-M}^M \sum_{j=-M}^M g\{x'_i, y'_j\} - g_m \right)^2} \quad (15)$$

With this procedure, the maximum resolution of the technique would be a pixel. However, different enhancements allow to achieve subpixel resolution. One approach consists in adjusting a 2D-quadratic surface to the correlation values around the displaced pixel position (Hung and Voloshin 2003). This method evaluates the ZNCC in the nine pixels around the initial match and fits the values to a 6 parameter 2D quadratic function. The resulting function is inspected to obtain the position that maximise the ZNCC, providing a subpixel displacement. Another approach takes into account the heterogeneous deformation of the subset to provide more accurate results (Bing Pan 2007). This method employs a second-order displacement mapping function to approximate complex subset deformations.

This concepts and method in combination with different technologies allows DIC to be employed in dynamic events, such as vibration, using high speed cameras (HS DIC), for microscopic measurements on the material microstructure or the volumetric version employing X-ray scanning known as Digital Volumetric Correlation (DVC).

2.2.1 2D Digital Image Correlation

Digital Image Correlation arose as a full-field technique to measure in-plane displacements in flat surfaces by employing a single camera, what is currently known as 2D-DIC. In Fig. 7 it is shown a generic setup for 2D-DIC. As can be observed, the camera is placed in such a way that the optical axis is perpendicular to the specimen surface. Thus, this technique is intended for tests under in-plane loading such as for the quantification of the crack tip plastic zone during fatigue crack growth (J. M. Vasco-

Olmo et al. 2016). Light sources are frequently employed to improve the speckle contrast and increase the accuracy.

Here, the pixel displacement of every subset in the area of interest is determined as exposed above. The lateral magnification of the lens relates the displacements in terms of pixel and in length units. This parameter can be obtained by determining the pixel distance between two points whose actual distance is known. Likewise, the sensitivity of 2D-DIC in length units is also related to this parameter. Considering that the subpixel resolution is about 0.1 ± 0.01 pixels (Schreier, Orteu, and Sutton 2009), it would depend on the lens lateral magnification and the resolution of the camera.

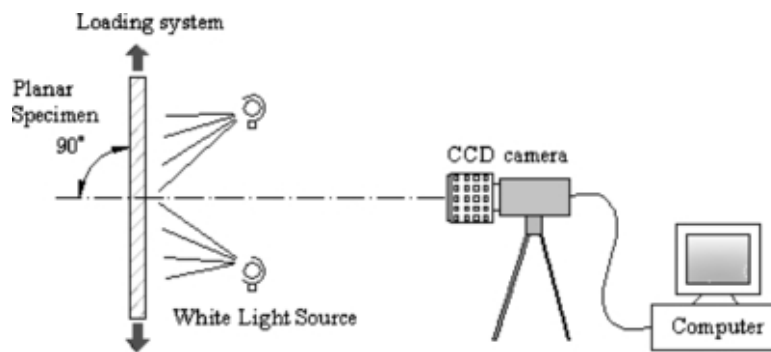


Fig. 7. Typical setup for 2D-DIC (Bing Pan et al. 2009).

A limitation of this technique is that it is sensitive to possible out-of-plane displacements that introduce fake in-plane deformation as long as telecentric lenses are not employed (M. A. Sutton et al. 2008). However, these lenses are expensive, heavy and voluminous.

2.2.2 3D Digital Image Correlation

This is a particular performance of DIC where a system of two synchronised cameras is employed to provide a stereoscopic vision, shown in Fig. 8 (a), and hence three-dimensional measurements and digitalisations. Thus, out-of-plane displacements are now registered. The calibration of the system is a differential element with respect to 2D version. For a complete calibration, it is necessary to establish certain parameters that can be sorted in two groups. The six extrinsic parameters define the relative position of the cameras and establish the relation of the coordinate systems as seen in Fig. 8 (b). Moreover, five additional parameters, known as intrinsic parameters, provides the position of the sensor which depends on the camera, the lenses and the light. A standard calibration procedure consists in employing a grid with a known pattern which is recorded by the cameras in different positions and orientations. Later on, the shape recognition algorithm obtains the intrinsic and extrinsic parameters according to the grid properties. The whole process can be approached by calibrating the cameras individually or simultaneously (Michael A. Sutton 2008).

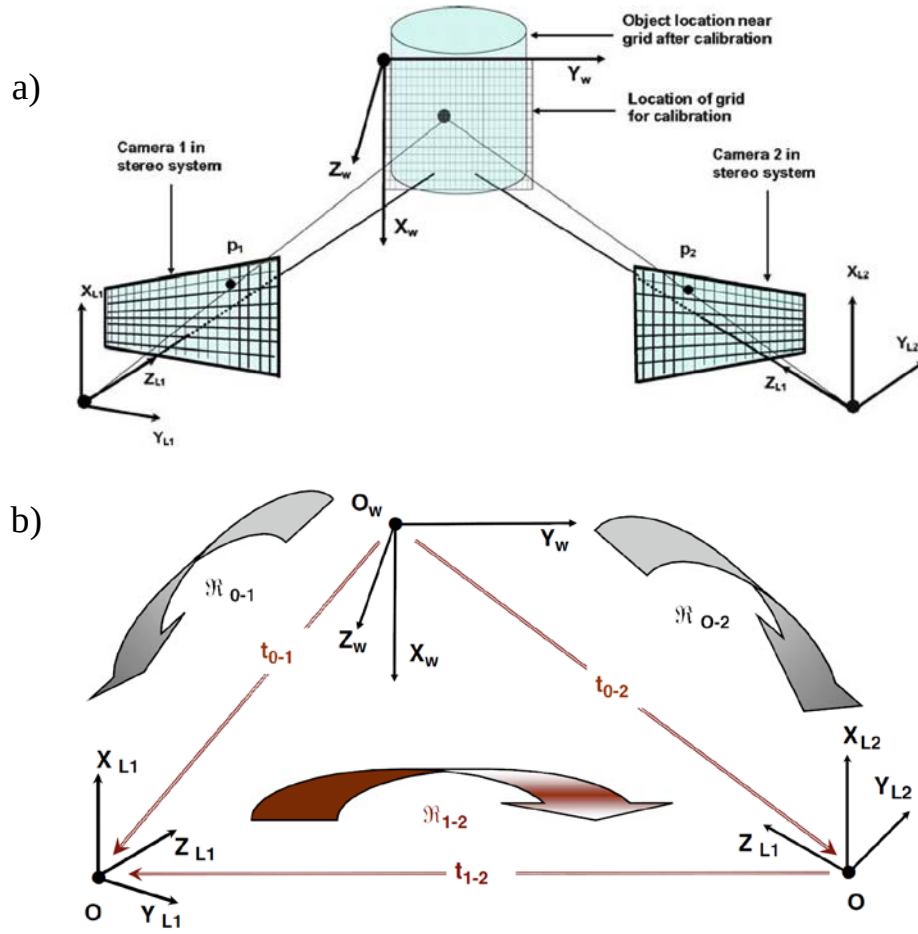


Fig. 8. (a) Generic stereoscopic system diagram. (b) Cameras coordinates transformation defined by extrinsic calibration parameters (Schreier, Orteu, and Sutton 2009).

In the correlation procedure, the central pixel of each facet is initially identified in both cameras as it corresponds to two different states in a 2D system. According to the calibration parameters, the three-dimensional position of the whole set of facets is established by stereo-triangulation and a 3D digitalisation of the surface is obtained. In the loaded state, the same process is performed. After that, an additional correlation procedure is carried out to match the positions of every facet from the non-loaded and the loaded state. Hence, three-dimensional displacements are determined in a full-field manner. The synchronisation of the cameras is crucial to obtain reliable results.

This system is more sensitive than 2D system to calibration parameters and thus the experimental setup must be very stable. Moreover, the technique is less sensitive to out-of-plane displacements which are defined as a projection in in-plane pixel motion. The out-of-plane sensitivity is of the order of $d/50000$, where d is the distance of the cameras to the specimen (Michael A. Sutton 2008).

2.2.3 Conclusions

It can be deduced from above that DIC is versatile, non-contact and full-field which is attractive for different disciplines. It does not required complex instrumentation. Conversely, the setup is simple even for 3D-DIC where two cameras are required. Moreover, the specimen preparation is reduced to applying paint.

The main limitation of 2D-DIC regarding out-of-plane displacements is solved by a stereoscopic system. This is a more complex setup where the calibration is sensitive to small changes. Nevertheless, the calibration method has experienced significant advances and the present-day commercial software performs accurate calibration with simple procedures. Additionally, the 3D version requires the synchronisation of both cameras, especially during dynamic events, to guarantee that each pair of images represents the same time instant.

The drawbacks of DIC with respect to traditional sensors is that the correlation procedure to produce full-field measurements requires longer processing time and generate large data sets. However, it is a worthy cost for more detailed characterisations.

2.3 Phase-based motion magnification

Phase-based motion magnification is a new Eulerian approach to motion processing developed by Wadhwa et al. (Wadhwa et al. 2013) to reveal subtle motion in videos by obtaining a magnified version. The video signal are decomposed using complex steerable pyramid filters (Simoncelli et al. 1992; Simoncelli and Freeman 1995; Portilla and Simoncelli 2000). This is an overcomplete transform based on complex sinusoids modulated by a Gaussian window function as shown in Fig. 9. Local amplitudes and phase were obtained according to spatial scale, orientation, and position.

The algorithm employs the phase signals of the complex steerable pyramids to identify the local motions in spatial sub-bands of an image. Previous studies (Fleet and Jepson 1990; Gautama and Van Hulle 2002) demonstrate that phase is more robust than amplitude defining the motion field reducing contrast and scale changes. The concept is analogous to Fourier shift theorem for sine functions transforms. A generic 1D spatial-domain intensity profile that is moved according to a time domain function, $\delta\{t\}$, can be described by a Fourier series decomposition as follows:

$$F\{x + \delta\{t\}\} = \sum_{\omega_s = -\infty}^{\infty} C_s e^{i\omega_s(x + \delta\{t\})} \quad (16)$$

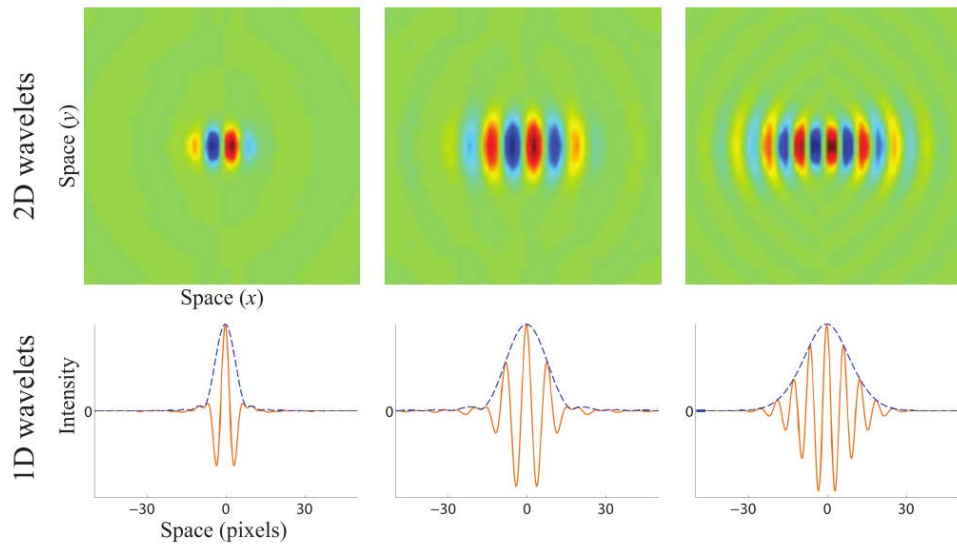


Fig. 9. Sinusoid wavelets employed for image decomposition in motion magnification (Wadhwa et al. 2013).

As can be observed, the displacement term is encoded in the phase of each spatial frequency component, s . Thus, the time-domain local phase at every spatial scale and orientation of a steerable pyramid can be temporally bandpass filtered to isolate specific temporal frequencies relevant to a given application and remove any temporal DC component. This frequency band is then manipulated by multiplying by an amplification factor. Analogously to Fourier shift theorem, the original phase can be modified by the amplified bandpassed phases which involves a magnification of the harmonic motion in the sequence corresponding to this frequency band after reconstructing the images. The complete process is exposed in Fig. 10.

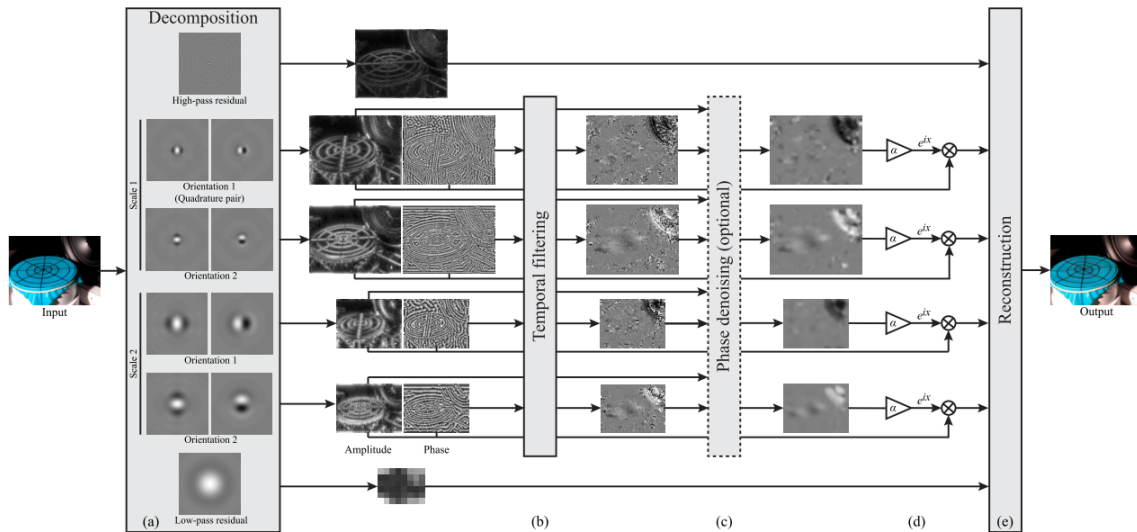


Fig. 10. Schematic of the general procedure of phase-based motion magnification. (a) Complex steerable pyramid decomposition, (b) bandpass filtering of phase time-domain signal, (c) optional phase denoising to improve signal-to-noise ratio, (d) amplification of the frequency band and (e) reconstruction of the video (Wadhwa et al. 2013).

This method is advantageous over previous linear amplitude-based Eulerian magnification method (Wu et al. 2012) since it is possible to achieve larger amplitudes. Moreover, it provides substantially better noise performance since amplitudes are not modified and hence there is no increment of spatial noise magnitude.

2.4 General conclusions

In this section, it has been exposed the methodologies that have been employed in this work, i.e., experimental modal analysis, DIC and phase-based motion magnification.

Regarding the use of DIC results to perform experimental modal analysis, DIC is attractive to perform modal identification since it is a contactless and full-field technique, conversely to traditional sensors. The main issue concerns the large data set to be analysed. The order of points for a typical DIC measurement is 10^4 according to the current state of technology whereas traditional tests instrumentation consist of tens or hundreds points, at the most. Although MDOF are less time consuming, the size of the data sets has no precedent in traditional sensor instrumentation. Moreover, multi-FRF require unaffordable computational cost in these cases. Conversely, SDOF approaches are easily implementable with DIC despite that SDOF assumption is not always valid.

Phase-based motion magnification is an image processing procedure that consists in time-frequency bandpass filtering and amplifying the signal present in the image intensity to afterwards reconstruct it back into a video with larger motion amplitude in the selected frequency band. The resulting higher amplitudes would allow to obtain clearer results when using DIC considering its lower sensitivity with respect to other conventional sensors.

Chapter 3

Literature review

In the previous chapter, the different techniques and methodologies employed in this thesis have been described. In this chapter, it is exposed the relevance and popularity they had in different scientific disciplines. For this purpose, a deep analysis of the related research is performed. Initially, different application of DIC are included to highlight the capabilities of the technique. According to the objectives of this thesis, the current state-of-art of DIC on vibration and experimental modal analysis is detailed. The significant aspect for this thesis are clearly remarked in this point. In the same way, the research on phase-based motion magnification in modal analysis is presented, showing the main advantages that would provide the combination of DIC and motion-magnification.

3.1 Digital Image Correlation

This epigraph is a general review of DIC. Since it was born, the technique has experienced remarkable improvements that make it attractive and reliable in experimental mechanics. Thus, the main evolution of the technique is outlined and later on relevant applications are exposed.

3.1.1 Historical review

The germen of DIC appeared in the early 1980s based on electronic speckle photography. Peters and Ransom (Peters and Ranson 1982) performed a computer-based image acquisition and deformation measurement system by comparing small areas in the specimen surface under different loading states. The location of these areas was found through digital image analysis. With this base, Sutton et al. (M. Sutton et al. 1983) developed the algorithms of the methodology currently known as 2D-DIC, and the experiments demonstrated that the methodology was feasible. Thus, the first development was experienced in 2D-DIC with a single camera to measure in-plane displacements and strain. In-plane rigid body motions, both translation and rotation, were measured by Peters et al. with satisfactory results (Peters et al. 1983). Chu et al. (Chu, Ranson, and Sutton 1985) validated it also for deformations in solids.

DIC demonstrated to be more flexible and versatile than similar techniques due to its simpler surface preparation and the wide measuring range. As a consequence of the limitation and the effect that out-of-plane displacements produce in the lens magnification (M. A. Sutton et al. 2008), in the 1990s it was developed a complex optical calibration for stereo-vision. It was based on the stereo-vision principles employed in robotics and allows DIC to measure 3D displacements (3D-DIC) (Kahn-Jetter and Chu 1990; Luo et al. 1993). From then on, multiple applications started arising what increased the popularity of DIC in experimental mechanics. Several commercial 3D-DIC equipment and software packages were developed to optimise the calibration and the measuring processes.

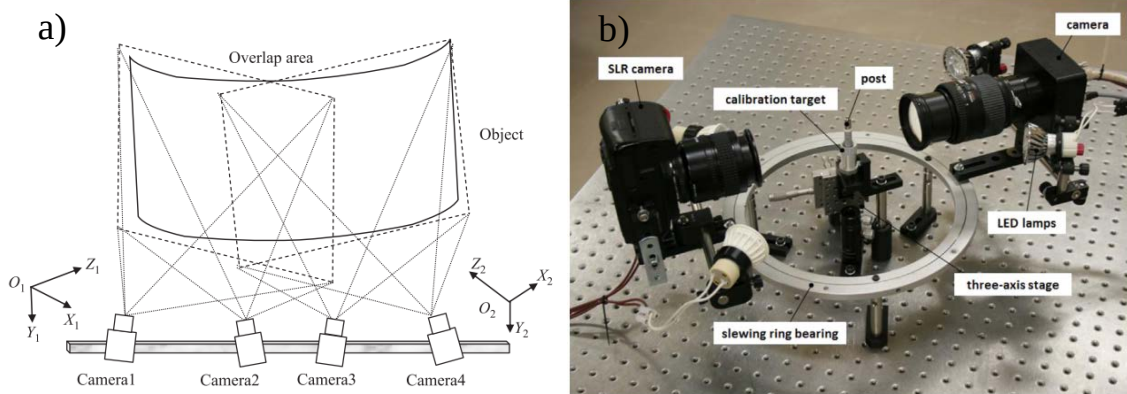


Fig. 11. Methodologies to increase the field of view. (a) Multi-camera system. (b) 360° 3D-DIC system.

The technique is still evolving in order to overcome software and hardware limitations. The decorrelation due to large deformations is an issue that has been tackled by different approaches (Tang et al. 2012; Bai et al. 2017). Focus has been also placed on the speckle pattern. Different speckle distributions have been studied and optimised to increase the accuracy of the measurements (Lecompte et al. 2006; Bomarito et al. 2017). Different studies have also tried to extend the field of view increasing so the measuring area. Chen et al. (F. Chen et al. 2013) implemented a multi-camera 3D system as shown

in Fig. 11 (a). The cameras are grouped into pairs which are calibrated and measured as single stereovision systems. Then, information from different pairs of cameras are mapped into a universal coordinate system based on the calibration data. An alternative method has been proposed by Genovese et al. (K. Genovese et al. 2016) to perform 360° 3D-DIC by mounting a single SLR digital camera in multiple positions around the specimen by means of a large rotation stage as shown in Fig. 11.

3.1.2 Applications

As a consequence of the benefits of the technique, it has been established in many experimental mechanics fields as a reliable technique with high potential for industrial applications.

Fracture mechanics take the most advantage of the setup simplicity of 2D-DIC. In this field, specimens are typically plates undergoing in-plane loading and the influence of out-of-plane displacements is negligible. For instance, Vasco-Olmo et al. (J. Vasco-Olmo et al. 2012) evaluated the stress intensity factor and T-stress in polycarbonate Middle Tension specimens using 2D-DIC. A good agreement was achieved in comparison with photoelasticity. The full-field information was employed by Sun and Pang (Sun and Pang 2008) to validate the numerical model of the near-crack-tip deformation of solder alloy under monotonic tensile loading. An industrial application was performed by Du et al. (Du et al. 2011). They monitored the evolution of a crack in a large aircraft panel with a measuring uncertainty of 5%. The effect of the reinforcements in the crack path and the propagation speed were analysed. They were able to analyse the mixed mode (I+II) stress intensity factors. Beside fracture mechanics, civil engineering is another application of 2D-DIC as demonstrated by Yoneyama et al. (Yoneyama et al. 2007). In this study, the authors carried out measurements of the deflexion of a bridge when a truck was crossing. They achieved an error of 0.31 mm in an area of interest of 15 m.

However, the mentioned limitations of 2D-DIC avoid its widespread use as 3D-DIC. Among the applications, it is remarkable its popularity in the automotive industry due to its versatility. Schmidt et al. (Schmidt, Tyson, and Galanulis 2003) investigated the strain fields experience by the tires during spinning. Burguete et al. (Burguete et al. 2014) evaluated the effect of low energy impacts in an inner bonnet liners using 3D-DIC, showing a good agreement with FEM predictions. Additionally, 3D-DIC has been also employed in biomechanics to evaluate the strain distribution of a loaded bone (Sztefek et al. 2010) or in fracture mechanics (Lopez-Pedrosa et al. 2012).

In the last decade, additional applications have been encountered such as microscale measurements (Katia Genovese, Leeflang, and Zadpoor 2017) or the development of Digital Volume Correlation which is able to obtain the inner strain field in porous materials using X-Ray scanning. Nevertheless, the greatest advance is due to the development of high speed cameras which has significantly contributed to propel DIC in dynamic characterisations (P L Reu and Miller 2008). In this study, special attention is

paid in this thesis to vibration measurements and modal analysis (Baqersad et al. 2017). This topic is deeply discussed in the next subsection.

3.2 Experimental modal analysis and HS DIC

The main limitations of traditional instrumentation in experimental modal analysis are the mass-loading effect as a consequence of the invasive nature of the transducers and the pointwise measurement that entails low spatial resolution and even spatial aliasing. In this sense, optical techniques provides full-field non-invasive measurements. Different studies have attempted to apply different optical techniques in the analysis of Operational Deflection Shapes (ODSs) during sinusoidal excitation at its natural frequencies such as Digital Shearography (L. Yang et al. 1998), Electronic Speckle Pattern Interferometry (Caponero et al. 2000), Digital Holographic Interferometry (Caponero et al. 2000; Stetson 2014) and Fringe Projection (Schirripa Spagnolo, Ambrosini, and Paoletti 1998). Besides the mentioned techniques, Laser Doppler Vibrometry (LDV) and Digital Image Correlation (DIC) appeared as real alternatives to invasive transducers. LDV is a pointwise interferometric technique that performs velocity measurements in the laser beam direction. It can provide full-field information by sequential or continuous scanning (A.B. Stanbridge and Ewins 1999; Anthony B. Stanbridge, Martarelli, and Ewins 2004). This is typically carried out by an automated mirror that modifies the direction of the laser beam towards different points. Anthropomorphic robot arms are also employed to perform sequential measurements of objects with three-dimensional geometry by modifying the position and the orientation of the laser (O'Malley et al. 2009; Bertini et al. 2014; Bertini et al. 2017).

High speed cameras are able to capture thousands images per second making it possible to employ DIC in experimental modal analysis. However, it is not possible to employ filters to avoid temporal aliasing (Beborniss and Ehrhardt 2017), which is an intrinsic limitation of the use of digital video cameras. Although LDV is more sensitive and extensively developed for experimental modal analysis, High Speed 3D Digital Image Correlation (HS 3D-DIC) provides more spatial resolution and 3D measurements with a considerably cost reduction in the equipment (Helfrick et al. 2011; Warren et al. 2011; Phillip L. Reu, Rohe, and Jacobs 2017). Unlike LDV, which is insensitive to rigid solid motion in the specimen, HS DIC provides an accurate tracking of every point. This is especially important in low frequency modes, where large displacements make this effect significant.

These capabilities of DIC caught the attention of several researchers (Baqersad et al. 2017). The characterization of full-field mode shapes or Operational Deflection Shapes (ODSs) is one of the most interesting features using HS 3D-DIC. Previous studies have highlighted the potential by employing a sinusoidal signal forcing a single resonance, what is known as Forced Normal Mode tests (FNM). Siebert et al. (Thorsten Siebert et al. 2007) performed this kind of excitation to evaluate the errors

employing different kind of specimens. Helfrick et al. (Helfrick et al. 2011) compared DIC with traditional accelerometers and a scanning laser vibrometry in a dryer cabinet. Results indicate that all three approaches correlated well with a finite element model and provided validation for the DIC approach for full-field vibration measurement. Baqersad et al. (Baqersad et al. 2012) employed HS 3D-DIC to extract mode shapes of a turbine blade. They compared the results with those from marker tracking photogrammetry, accelerometers and a FEM model. Ehrhardt et al. (Ehrhardt et al. 2017) evaluated the linear and non-linear behaviour of a beam and a plate in FNM tests using HS 3D-DIC and Continuous Scanning Laser Doppler Vibrometry (CSLDV). Both techniques provided similar results regarding natural frequencies, mode shapes (Fig. 12) and nonlinear harmonics deformation. The authors found that complex deformation is better characterised by DIC whereas CSLDV demonstrated higher sensitivity. Barone et al. (Barone et al. 2017) explored the possibility of employing a projected speckle pattern to measure mode shapes using a single sinusoidal component. Thus, it is possible to optimise the speckle pattern and avoid surface preparation.

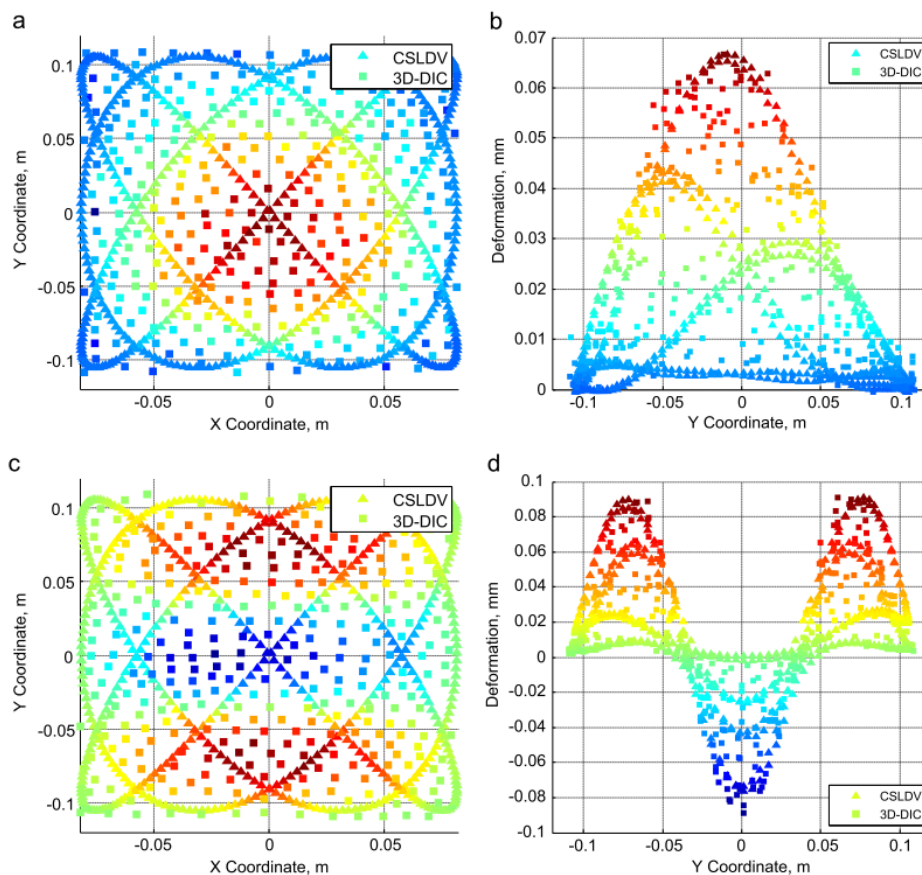


Fig. 12. (a)-(b) First and (c)-(d) fourth mode shape of a plate using 3D-DIC and CSLDV (Ehrhardt et al. 2017).

In these studies, the natural frequencies are usually determined by using other techniques such as accelerometers or laser vibrometry. In most of the cases, the performed studies were limited to a direct comparison between experimental modal vectors or mode shapes with those obtained by numerical

simulations. This comparison was performed by using MAC (R J Allemang and Brown 1982; Randall J Allemang 2003).

However, by performing only a FNM test it is not possible achieve a complete modal characterization. Employing a broadband excitation instead of a fixed sine, the full-field response of the specimen is registered, and frequency response functions or other frequency-domain transfer function are easily obtained. By analysing the peaks that maximize the response it is possible to estimate the natural frequencies and the damping ratios.

First advances were performed by some authors, such as Siebert et al. (T Siebert, Wood, and Splithhof 2009), who attempted to determine natural frequencies with 3D-DIC by employing different excitation types (such as through an explosion or white noise signal). However, the lack of excitation measurements did not make it possible to perform the calculation of the frequency response function to obtain the modal characterization of the component. Similarly, a HS DIC approach was proposed to estimate natural frequencies and mode shapes of helicopter rotor blades under operating conditions (Rizo-Patron and Sirohi 2017).

Warren et al. (Warren et al. 2011) implemented for first time the FRFs of a system consisting of two flat plates joined in a "T" shape, using the 3D point-tracking technique (Schreier, Orteu, and Sutton 2009), as shown in Fig. 13. This technique is not considered as full-field technique, and the authors showed that the large amount of data obtained by 3D-DIC processing could be difficult to analyse. Furthermore, only natural frequencies were identified from these FRFs and no modal identification algorithm was employed.

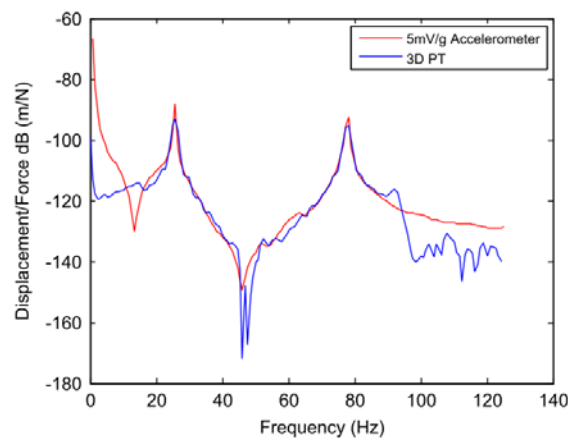


Fig. 13. Frequency response function corresponding to a point of a T-shape element using 3D PT and an accelerometer (Warren et al. 2011).

Trebuña et al. (Trebuña and Hagara 2014) were able to perform a complete modal analysis of flat plates using 3D-DIC by performing an impact test and synchronizing the capture of the cameras with the analogue signal of an instrumented hammer. Results were correlated with those obtained in a similar

test using a laser vibrometer, although the mode shape comparison was only performed qualitatively. Damping ratios were obtained with the peak-picking method, which is one of the simplest method. Afterwards, a computationally efficient method was proposed by (Huňady and Hagara 2017) with a more accurate modal parameter estimation method tested in a composite plate also excited by an impact hammer. Yu and Pan (Yu and Pan 2017) employed a low-cost HS 3D-DIC composed by a single camera and a mirrors system as observed in Fig. 14. The peak-picking method was also employed for modal identification.

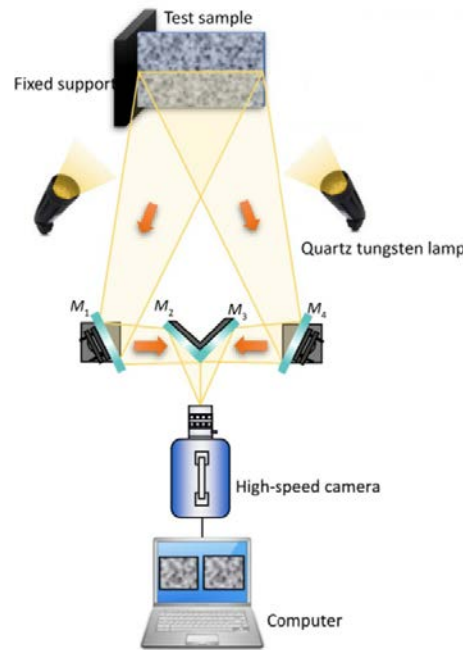


Fig. 14. Schematic of the low-cost HS 3D-DIC system composed by a single camera and a mirrors system (Yu and Pan 2017).

An interesting research field was discovered by the use of image decomposition approaches to suppress redundancies or reduce the amount data obtained to just a few shape descriptors (Lampeas et al. 2015; Wang, Mottershead, Sebastian, et al. 2011; Sebastian, Hack, and Patterson 2013; Wang, Mottershead, and Mares 2009a; Wang, Mottershead, and Mares 2009b). In a first approximation, Tchebichef shape descriptors (Fig. 15) were employed to simplify the mode shapes obtained by 3D-DIC in a FNM test of a panel (Wang, Mottershead, Ihle, et al. 2011). In this way, it was possible to reduce the processed information by one hundredth part with an accuracy of 99.8%. Berke et al. (Berke et al. 2015) employed this approach to evaluate changes in ODSs due to high temperature conditions. Felipe-Sesé and Díaz (L. Felipe-Sesé and Díaz 2018) developed a methodology for damage detection and location based in the variation in the shape descriptors of ODSs under different damage severities. In a later work a complete modal analysis was performed (Wang et al. 2012), using a random signal excitation in a car bonnet. In this work, the concept of shape-descriptor frequency response function, SD-FRF, was

introduced. This SD-FRF defined the response of the component response in terms of a descriptor rather than in terms of displacement.

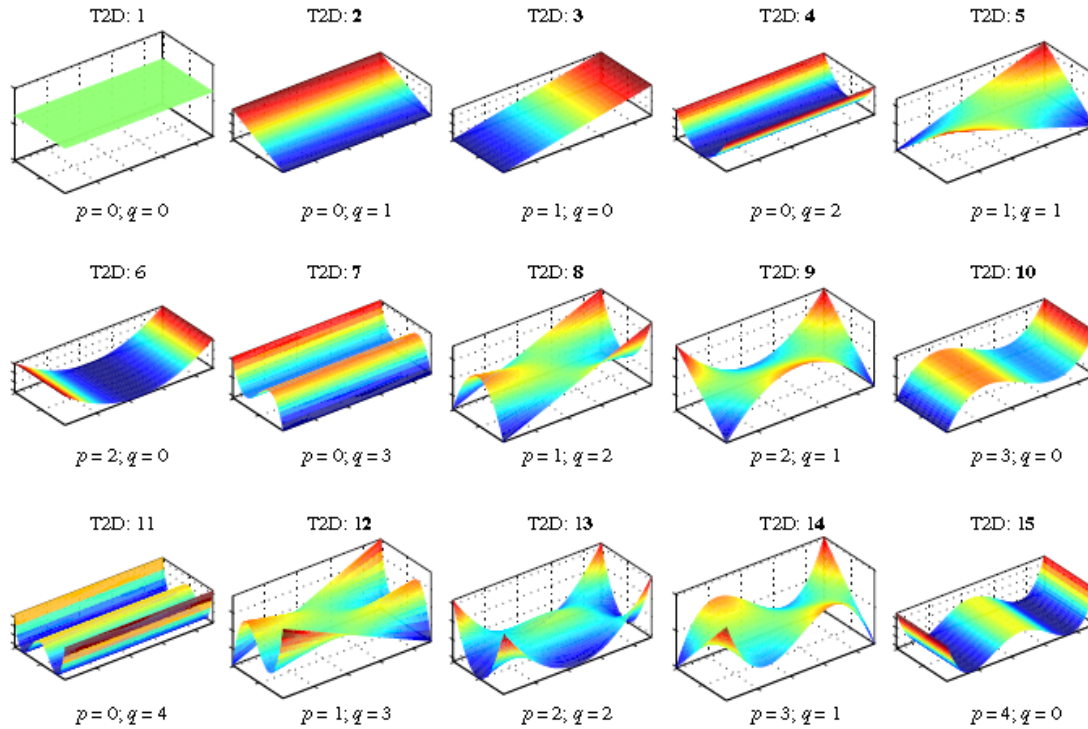


Fig. 15. First 15 Tchebichef shape descriptors (Wang, Mottershead, Ihle, et al. 2011).

Cited work up to this point collect research in which pointwise forces are applied to the system, obtaining full-field FRF relating the force input and the displacements response. However, modal testing is not restricted to the evaluation of this configuration and some practical situations require that excitation is mainly transmitted to the structure through the attaching points. This is typically characterized as a base motion excitation problem. Experimentally, the excitation is registered as a motion instead of force in such cases. The transfer function relates the transmission of base motion to structure response motion and are hence named as transmissibility functions. Nevertheless, modal identification algorithms are mostly developed to manage FRFs. Despite the similarities between FRFs and transmissibility functions, straight modal identification on transmissibility functions must be avoided if, as usual, FRF-based algorithms are considered (Béliveau et al. 1986).

In this line, base motion excitation was performed by Ha et al. (Ngoc San Ha, Jin, and Goo 2013) to obtain the mode shapes of an artificial beetle's wing in FNM tests employing HS DIC. Previously, the natural frequencies were identified in a swept-sine test using a laser vibrometer. No transmissibility function was determined from DIC data. In later work, Ha et al. (N. S. Ha, Vang, and Goo 2015) obtained transmissibility functions of the wing using HS DIC employing a random signal. Modal identification was performed using the peak-picking method from the smoothed transmissibility functions, employing them as if they were FRFs. However, the full transmissibility functions matrix was not employed for the

determination of the natural frequency and the damping factor. Just some of them corresponding to selected points were employed. That procedure does not justify the use of DIC versus other techniques. Additionally and as a consequence, they resorted to an FNM test to obtain mode shapes, shown in Fig. 16. Thus, no previous work on HS DIC and base motion excitation for a complete modal analysis has been found in the literature.

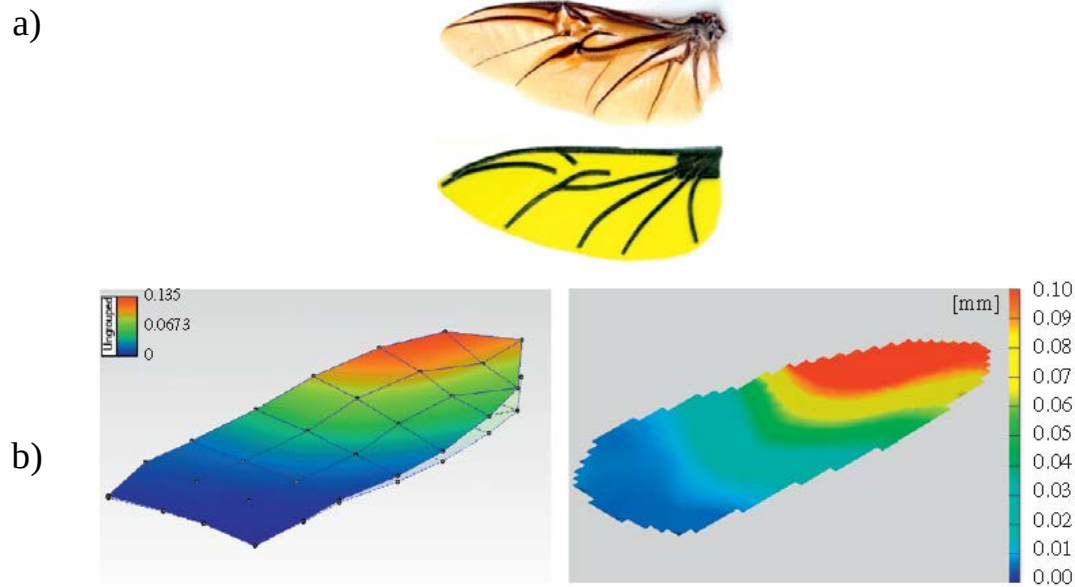


Fig. 16. Real beetle wing (a) and artificial beetle wing (b). First vibration mode at 104 Hz obtained in FNMT tests. (A) Using laser vibrometry (b) Using DIC (N. S. Ha, Vang, and Goo 2015).

In all cases, relevant information of the behaviour of the whole structure is provided using HS DIC, and hence constituting a powerful tool to validate numerical models (Wang, Mottershead, Sebastian, et al. 2011; Wang, Mottershead, Ihle, et al. 2011; Wang, Mottershead, and Mares 2009a). However, DIC is a displacement measuring technique and low level of displacements related to high frequency excitation represents a challenging task. Cited work presented high quality results regarding ODSs and mode shapes covering a spectrum of hundreds of Hertz and just a few of them obtained acceptable results up to 1500 Hz. High frequency characterisation is also conditioned by the resolution of high speed cameras. Due to current technical limitations of the cameras in the managing of the data memory, the image resolution has to be reduced when higher frame ratios are required. The pixel-millimetre ratio, on which DIC bases its accuracy (Schreier, Orteu, and Sutton 2009), becomes major limitation. Thus, DIC has been better positioned for low frequency events.

3.3 Experimental modal analysis and phase-based motion magnification

The recent development of methodologies of magnification of subtle periodic motion in digital videos presents notable advantages in the characterisation of modal behaviour for non-destructive testing and structural health monitoring using a single camera. Based on complex steerable pyramid filters for image decomposition (Wadhwa et al. 2013; Rubinstein et al. 2013), these methodologies make a magnified version of the motion by increasing the amplitude of a narrow frequency band. In their work, Chen et al (J. G. Chen et al. 2015) employed motion magnification for visualisation of magnified Operational Deflection Shapes (ODSs). By properly choosing the frequency band, they obtained a video of the structure's ODSs at one of its resonant frequencies, assuming that modes are well spaced. In other studies (Y. Yang, Dorn, Mancini, Talken, Kenyon, et al. 2017; Y. Yang, Dorn, Mancini, Talken, Nagarajaiah, et al. 2017) natural frequencies, mode shapes and damping ratios were blindly provided by means of an autonomous phase-based methodology. This methodology is proposed by the authors to monitor structures using just a video-camera with neither surface preparation, like for the case of DIC and laser vibrometry, nor a laborious, inefficient instrumentation. Fig. 17 shows two video stills from a three-story structure corresponding to the non-magnified state and the magnification of the third mode. Although magnified motion visualisation is invaluable for the interpretation of the mode shape, there are cases in which magnification is not enough for a satisfactory visualisation and too much magnification blurs the images, as shown in Fig. 18, so the perception of the shape of the motion is affected (Wadhwa et al. 2017). More difficulties are found in higher order modes as a result of the increasing complexity of the shape.



Fig. 17. Video stills of a three-story structure. (a) Non-magnified state. (b) Third mode magnified (Y. Yang, Dorn, Mancini, Talken, Kenyon, et al. 2017).

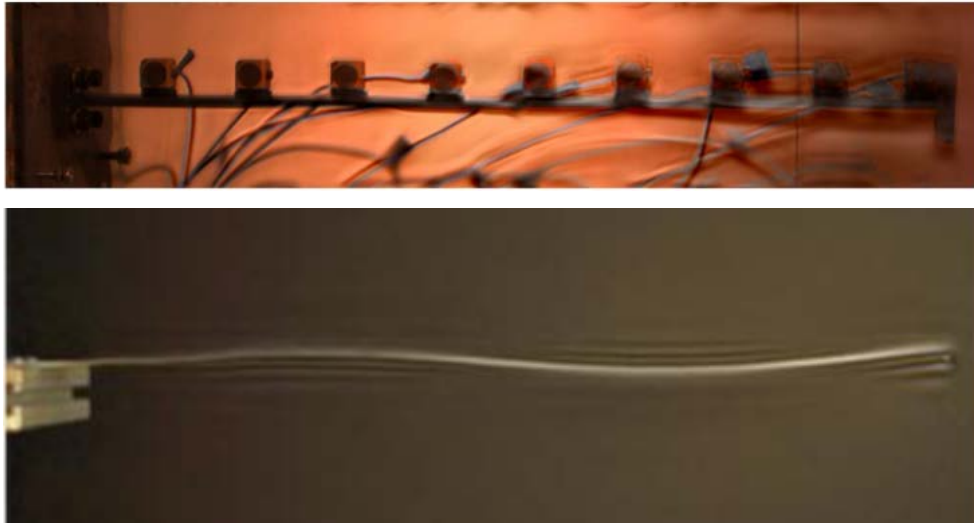


Fig. 18. Blurring in images due to aggressive motion magnification processing (J. G. Chen et al. 2015; Y. Yang, Dorn, Mancini, Talken, Kenyon, et al. 2017).

The previously mentioned works have been performed under the assumption that only in-plane motion occurs, dealing with images from a single camera. However, most scenarios in real-life involve shape and motion in the space. Hence, although the naked eye visualisation is useful in monitoring structures, three dimensional interpretation is biased.

Considering this, the combination of HS DIC and motion magnification algorithms arises as a possibility to increase signal-to-noise in HS DIC measurements in low displacement tests and provide quantitative information to magnified video since DIC technique is able to quantify motion that is not perceived by the naked eye. Moreover, magnification in stereovision images would allow to obtain 3D information of the motion. Thus, it would be built up a consistent methodology to interpret mode shapes.

3.4 Conclusions

In this chapter, it has been demonstrated that DIC has experienced extraordinary improvements in the last decades as a consequence of the possibilities it offers. As has been shown, it is currently a real alternative to traditional sensors in many experimental disciplines based on its non-invasive nature and the full-field information.

In particular, these advantages and the development of high speed cameras made DIC gain in popularity in experimental modal analysis. Many researchers have focused on exploring it with excellent results. ODSs and mode shapes are the most outstanding results since they contain explicitly the full-field information provided by DIC and in a more effective way. Some authors have proved that it is possible to obtain full-field FRFs using DIC and perform modal identification on them. However, it has not been

provided an alternative to employ the technique for modal characterisation from base motion excitation tests that take into account the particularities of these tests.

In these studies, the analysed modes are usually in a spectrum of hundreds of Hertz. For higher frequencies, the acceleration amplitude becomes more relevant in detriment of displacements. Consequently, these frequencies are difficultly characterised by DIC. Phase-based motion magnification have demonstrated to amplify the subtle displacements corresponding in a frequency band in digital videos. Thus, performing measurements in magnified images would contribute to reduce the noise in ODSs at higher frequencies. Additionally, this can be extended to tests that employ low power exciters. However, not only DIC would be benefited but blurring would be avoided since DIC is able to characterise deformations that are not detected by naked eye. Thus, large magnification factors are not necessary. The particular combination with HS 3D-DIC would be a step ahead, providing information where HS 2D-DIC is not applicable (M. A. Sutton et al. 2008).

Chapter 4

Apparatus and Methods

To perform the studies related to experimental modal analysis and measuring in magnified images using HS DIC different equipment and software have been employed. They are described in this chapter. Firstly, the specimens are commented. In a second point, the optical equipment for HS DIC measurements is presented. The acquisition system employed to synchronise signals from additional sensors is indicated here. The general procedure for DIC processing is also exposed. The third classification involves all the equipment and methodologies related to vibration. Thus, exciters, sensors, and acquisition hardware is enumerated. In this point it is also described the post-processing procedure for DIC results to perform frequency analysis and modal identification. The validation modal models, both numerical and theoretical, are also exposed. Finally, the general procedure for motion magnification is described. In later chapters particular procedures and setup are defined.

4.1 Specimens

4.1.1 Cantilever beam specimen

The methodologies developed in this thesis have been tested using a 2024 aluminium cantilever beam, shown in Fig. 19. The specimen is in a stepped shape whose thinner part was 160 mm long, 40 mm wide

and 4 mm thick. The thicker part is used to clamp the specimen, keeping the thinner part fully released. Thus, the thinner part behaves as a cantilever beam. It has been designed as reference material (RM) in previous validation analysis (“Validation of Numerical Engineering Simulations: Standardisation Actions (VANESSA). European FP7 Project Grant Agreement No. NMP3-SA-2012-319116” 2014). This design shows high conformity with analytical approaches due to its stable and reproducible behaviour. Hence, it is conventionally employed for calibration of full-field optical techniques of displacement and strain measurements comparing full-field measurement with analytical results, both static and dynamic (Hack 2016). Overall, modes are separated and uncoupled and the behaviour of this geometry is well-known, easily reproducible by FEM models. Besides the manufacturing high standard, that makes it ideal for validation purposes.



Fig. 19. Cantilever beam, employed for validation purposes of the methodology.

In the tests, the cantilever beam was fixed to the shaker armature through the thicker part, as observed in Fig. 20. In this way, the excitation vibration is transmitted to the thinner part and induces the cantilever modes. A clamping device was designed, shown in Fig. 21, for that purpose. It consists of two aluminium parts to tight the beam by pressure. One is a plate that is attached to the shaker armature through eight screws. The second one is a thicker cylinder piece with a milled slot that cover the thicker part of the cantilever. Both parts are joined by four screws.

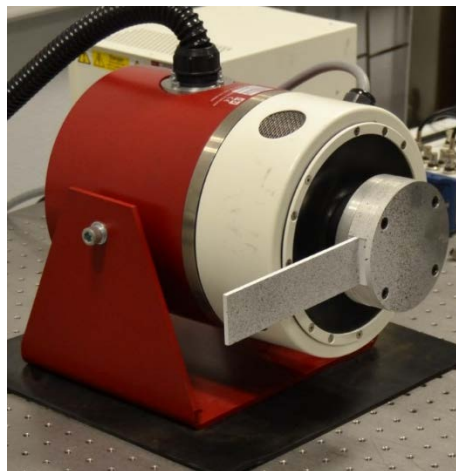


Fig. 20. Cantilever beam attached to the shaker armature through a clamping device.

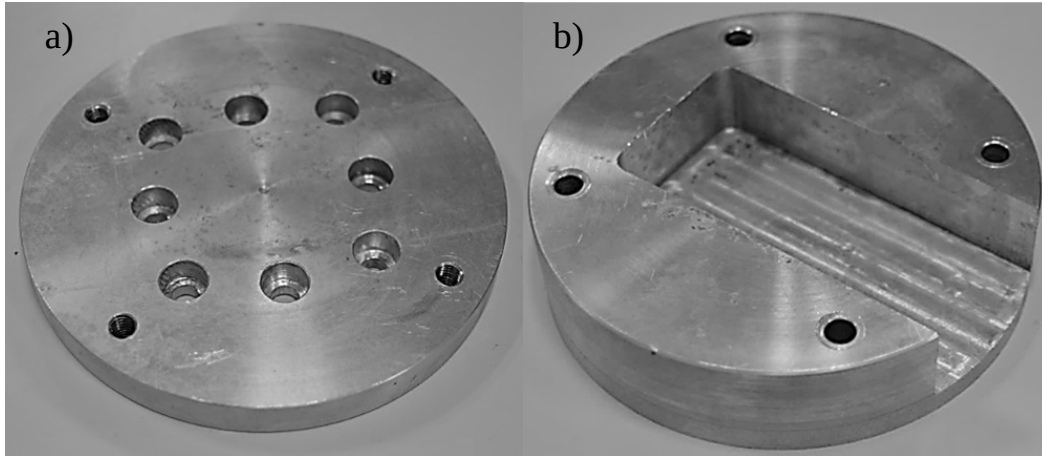


Fig. 21. Clamping device to attach the beam to the shaker. It consists of two parts: (a) the base that is fixed to the shaker armature and (b) the upper part that is fixed to the base through four screws.

4.1.2 Composite panel

The panel shown in Fig. 22 has been employed to test the combination of motion magnification and HS 3D-DIC in a 3D industrial piece. It is a large glass-fibre composite panel of 1.45 m high, 2.0 m wide and 0.6 m deep, which presents stiffening regions on the structure. With these dimensions, this panel was intended to evaluate the capabilities of the methodology using a little shaker. As can be seen, it is curved and therefore in a 3D-shape what would allow to reveal the advantages of HS 3D-DIC over the 2D version. The panel was tested in free-free boundary conditions. Hence, a triangular frame structure was employed to hang it.

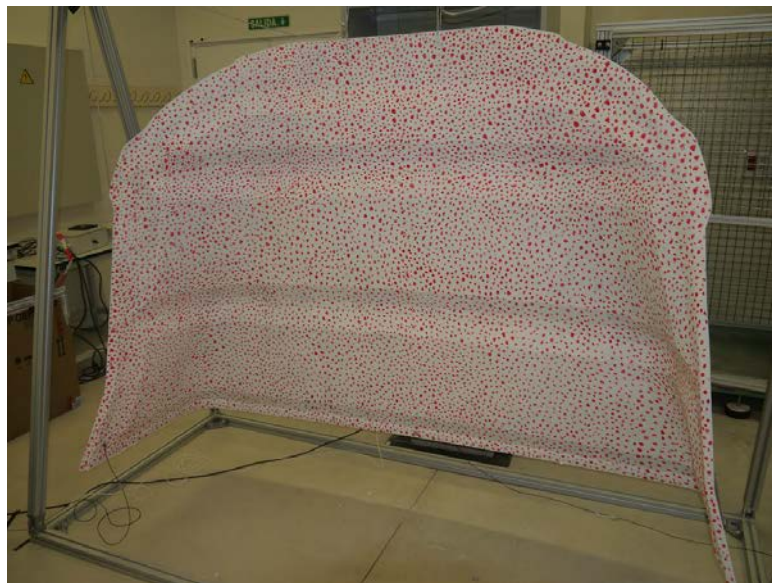


Fig. 22. Large composite curved panel.

4.2 High Speed Digital Image Correlation

4.2.1 Equipment

In this study, Digital Image Correlation has been employed in conjunction with High Speed cameras (HS DIC) both in 2D and 3D version. Hence it was possible to provide high sampling frequencies for the modal identification. Two CMOS high speed cameras were employed, model FASTCAM SA4 by Photron (Fig. 23) with 1024x1024 pixels full resolution. The cameras are able to provide a maximum frame rate of 3600 fps at full resolution with storage for 5457 images (5.33 Gb). The frame rate can be increased up to 500000 fps by reducing the resolution of the images. As the image resolution defines the accurateness of DIC, a trade-off must be achieved between image resolution and speed to obtain accurate results in a reasonable spectrum. The cameras are connected to the controller computer via Ethernet connexion.



Fig. 23. Photron FastCam SA4 high speed camera.

The cameras were provided with two lenses Nikon model AF Nikkor 50 mm f/1.4D. Each camera was mounted on a Manfrotto New Geared Head 405 and fixed to a tripod Manfrotto model 475B. These systems are adjustable and allow to provide the desired position and orientation of the cameras.

The cameras can be synchronised between them to build a stereoscopic system for 3D measurements. The internal clock signal of one camera (master) is assigned to the other one (slave). Simultaneous trigger is then employed to start capturing. The pertinent configuration is defined via software. In the same way, it was possible the synchronization of the cameras with external sensors such as a load cell or accelerometers. A data acquisition board NI USB-6251 DAQ (Fig. 24) is used for simultaneous data acquisition from accelerometers and images recorded during the tests. It has eight analogue input channels (± 10 V, 16 bit). Eight additional digital channels are also available which perform the timing and synchronisation of the signals.

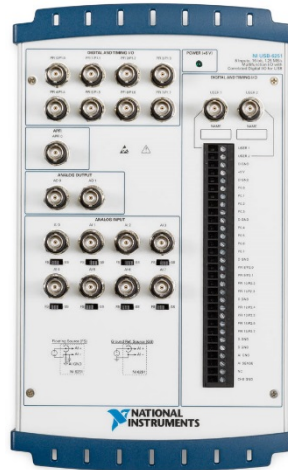


Fig. 24. Data acquisition board NI USB-6251 DAQ.

Photron FASTCAM Viewer (PFV) (Photron 2018) is the software that controls the images and external signal acquisition. Besides the necessary connections, the synchronisation parameters are set with this software. The test parameters are also selected such as frame rate, resolution, shutter exposure time and the trigger mode according to the test requirement. This information is exported for further processing with DIC software or Matlab.

In high speed recording, it is necessary to provide additional illumination due to the short exposure time of the sensor. In particular, for accurate DIC measurements, it is necessary a good speckle contrast as well as to avoid shadows and blurring of the images. Different light sources were available. One is a 150 W cold halogen illumination system DX 15 flood Light, brand Hedler. Additionally, two 1000W light sources model Videolight 6 were also employed.

4.2.2 DIC processing

DIC processing has been performed using the commercial software Vic-2D and Vic-3D for 2D-DIC and 3D-DIC, respectively, from Correlated Solutions Inc. (Correlated Solutions 2017).

The general procedure using this technique starts with the specimen preparation. A random speckle pattern is typically made by applying black paint over a white paint layer. Special attention must be paid to the mean size, about some pixels, and the homogenous distribution of the speckle to improve the correlation.

The calibration procedure depends on the system. For in-plane measurements using 2D-DIC, a known reference in the image is normally employed to establish the lens magnification in terms of pixel-millimetre ratio. The stereoscopic system calibration is conceptually more complex but modern software provides a very simple procedure. For Vic-3D, the calibration is performed by placing a calibration plate with a printed grid on it at the test space where the specimens is located during the experiment as

observed in Fig. 25. The grid is recorded in different positions and orientations. The software recognises the pattern and determines the intrinsic and extrinsic calibration parameters considering the grid parameters.



Fig. 25. Calibration grid from the point of view of the stereo-vision system for 3D-DIC measuring.

Later on, an image of the static state of the specimen is recorded which is taken as the reference image to process the deformation fields. In this image, the region of interest is defined including the subset size and the overlap. Fig. 26 shows the region of interest with a grid to evaluate the subset size in relation with the speckle. Finally, the images recorded during the test are imported to the software to perform the correlation. The analysis algorithm is efficiently designed and is compatible with multi-core processor, reducing the processing time. As a result, the coordinates and the displacements in the three spatial direction are obtained. The capabilities of the software includes, among other tools, the modification of the global coordinate system, a rigid body motion removing tool and different post-processing and mathematical operation tools. The results can be exported for further processing in different formats such as Matlab variable format. Images and videos of the event including the full-field map can be also exported.

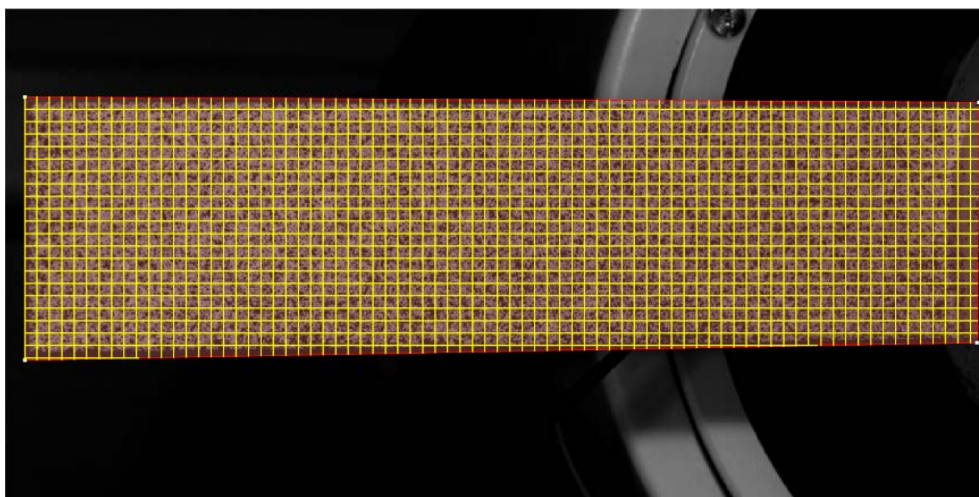


Fig. 26. Area of interest and facets size visualisation.

4.3 Vibration and Modal Analysis

Complementary to HS DIC, in this study it has been employed different equipment and utilities that are extensively employed in mechanical vibration testing and, specially, in experimental modal analysis.

4.3.1 Equipment

As exciters, two electrodynamic shakers were employed, model GW-V55/PA-300E and GW-V20/PA30E from Data Physics (Fig. 27). The first one was employed to excite the cantilever specimen. This shaker has a frequency working range from DC till 8000 Hz. It is able to provide a maximum sinusoidal force of 310 N and 110 N for random signals. The second one is less powerful and was utilised to excite the large composite panel. It covers a frequency range from DC till 14000 Hz and the maximum excitation force is 53 N and 17.6 N for sinusoidal and random signals, respectively. In the armature heads, there is a plate in both cases with a screw matrix to fix stingers or any other systems to transmit the vibration to the specimen.

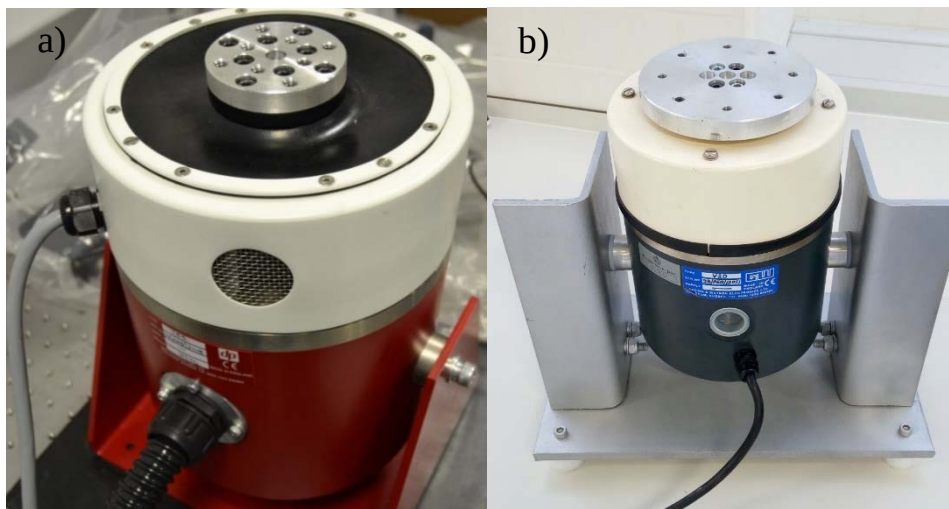


Fig. 27. Electrodynamic shakers model GW-V55/PA-300E (a) and GW-V20/PA30E (b) from Data Physics.

Besides DIC, different accelerometers were employed. The acceleration range of the accelerometers goes from 50 g to 500 g. They were devoted to register the excitation signal or for validation purposes of DIC measurements. Although the signals of the accelerometers employed in the test were recorded using the NI DAQ system, a real-time analyser, model Photon+ Real Time analyser from Brüel & Kjaer (Fig. 28), was also employed to deal with the signals in a faster way when no synchronisation with HS DIC was required. It consists of four inputs and one output channels. Real time data acquisition and signal processing is available. This system was also employed as signal generator using the output channel. RT Pro software is the controller of the analyser that allows different functionalities and utilities for data recording and frequency analysis. Typical vibration analysis functions can be calculated such

as Fast Fourier Transforms, Auto and Cross Power Spectral Densities (PSD), FRF and Coherence functions. Both time-domain and frequency-domain signals and functions can be exported to be manipulated.



Fig. 28. Brüel & Kjær Photon+ Real Time analyser.

4.3.2 DIC postprocessing

Matlab has been extensively employed in this study to post-process DIC measurements. Different scripts have been developed to manipulate the data and obtain modal information. The first task is reading the exported DIC files and collecting the displacement fields. Vic software exports the variables (coordinates, displacements, strains, etc.) of each image to an individual Matlab variable file. Each variable is provided as a matrix where each element represents a DIC measurement point. In Matlab, the displacements matrices are put together to build a time-domain vector in every measurement point. Then, frequency analysis is performed using Welch's method family functions such as `pwelch` for auto-PSD, `cpsd` for cross-PSD and `mscohere` for Coherence functions. Excitation signal from external sensors is also taken to calculate transfer functions (FRF and transmissibility functions) using the mentioned PSD Matlab functions. Hence, the results is a matrix composed by the transfer functions for every single measurement point what entails the full-field frequency response behaviour of the specimen. Modal parameters can be hence estimated. It exists different open-source modal identification algorithm for Matlab such as EasyMod (Kouroussis et al. 2012). In this Matlab toolbox, circle-fit and line-fit SDOF approaches and LSCE MDOF approach are available.

4.3.3 Numerical and theoretical models

In experimental analysis, it is important to have good reference tools to validate the results. In this thesis, traditional sensors are often employed for this purpose. However, experimental issues can alter the real behaviour of the specimen. In this sense, FEM and theoretical models which represents the ideal behaviour are very valuable.

In this study, both FEM and theoretical models have been employed to estimate the natural frequencies and the mode shapes of the cantilever beam to be compared with HS DIC results. For the numerical analysis the commercial software Abaqus was employed (Dassault Systèmes Simulia Corp. 2018). The cantilever beam was modelled as a 2D plate considering only the thinner part and using S4R linear shell elements and simulation experimental boundary conditions. Standard aluminium material properties were employed: density 2700 kg/m³ and Young modulus 70 GPa. The focus was only on the bending modes of the beam, as shown in Fig. 29.

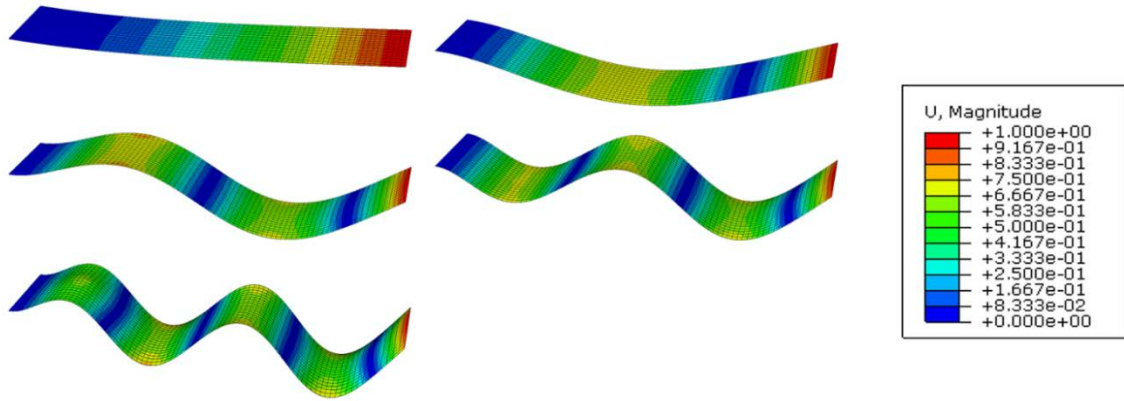


Fig. 29. First five bending mode shapes of the cantilever beam using Abaqus software.

On the other hand, the modal behaviour of a cantilever beam is well described by theory and it can be commonly found in the bibliography as a benchmark example of continuous system vibration analysis (Blevins 2001). Natural frequencies are obtained as:

$$\cos \beta_r l \cosh \beta_r l = -1 \quad (17)$$

where l represents the length of the beam and β_r is a parameter that relates natural frequencies and material properties, such as Young's Modulus, E , the mass, m , and the second moment of area, I , as follows:

$$\omega_{n,r} = \beta_r^2 \sqrt{\frac{EI}{m}} \quad (18)$$

Equation (17) provides infinite solutions, which represent the infinite modes of the continuous system. Table 1 shows the five first solutions of that. Equation (19) defines the r -mode shape along the beam of all the modes depending on the β_r mode parameter and the length of the beam.

$$w_r\{x\} = C_r (\sin \beta_r x - \sinh \beta_r x - \alpha_r (\cos \beta_r x - \cosh \beta_r x)) \quad (19)$$

where

$$\alpha_r = \frac{\sin \beta_r l + \sinh \beta_r l}{\cos \beta_r l + \cosh \beta_r l} \quad (20)$$

The parameter C_r is a constant that depends on the boundary conditions. In the cases presented here, it was calculated to obtain normalised mode shapes with maximum displacement equal to 1.

Table 1. Analytical parameters for natural frequencies and mode shapes estimation of a cantilever beam.

N	$\beta_r l$	β_r	α_r
1	1.875104	11.719400	1.362221
2	4.694091	29.338069	0.981868
3	7.854757	49.092231	1.000776
4	10.995541	68.722131	0.999966
5	14.137168	88.357300	1.000001

4.4 Phase-based motion magnification

To perform DIC measurement in magnified images it has been employed the algorithm developed by (Wadhwa et al. 2013). This is also a Matlab-based algorithm. The input parameters are the original video which will be decomposed using the complex steerable pyramid filters bank, the frequency band in which the phase signal will be bandpass filtered and, finally, the magnification factor to amplify the selected frequency band.

The results is a video which shows the motion corresponding to the modified frequency band with higher amplitude than the original. The video must be previously reconverted in a sequence of images so that the DIC algorithm can perform the correlation.

Chapter 5

Experimental modal analysis using transmissibility functions

In this chapter, it is described the methodology that addresses the proper modal identification in transmissibility functions obtained by HS 3D-DIC. This is the work covered by the first paper included in this document in Appendix A. As previously stated, the huge amount of data generated by DIC is an unfeasible challenge to modal identification in many cases when using MDOF Multi-FRF approaches, which evaluate the whole FRFs data set simultaneously. Thus, the SDOF Single-FRF circle-fit approach is proposed as a more computational efficient method for modal characterization dealing with the FRF data generated by DIC. The issue concerning the differences between transmissibility function and FRF is solved by proposing a theoretical conversion from transmissibility functions to their equivalent FRFs under single-degree-of-freedom (SDOF) assumptions, in agreement with circle-fit approach. The methodology was put into practice in the cantilever beam by analysing the two first bending modes in the spectrum from 0 Hz to 1000 Hz.

This work also tackle the experimental methodology validation using different means. Experimentally, the measurements of an accelerometer were taken as a reference. In particular, the accelerometer transmissibility function were compare with that simultaneously obtained using DIC at the same spatial point. This comparison involved the assessment of the signal quality using of Coherence functions along with the identification of DIC floor noise in Power Spectral Density (PSD), considering the lower

sensitivity of DIC. As temporal aliasing is an intrinsic limitation of DIC measurements (Phillip L. Reu, Rohe, and Jacobs 2017; Beberniss and Ehrhardt 2017), accelerometer signals were also employed to reject aliased frequencies.

Concerning modal parameters, a comparison with FEM and theoretical modal analysis was performed. To provide a quantitative indicator of the agreement for mode shapes, a correlation of mode shapes between these methodologies has been also performed using Modal Assurance Criterion (MAC).

5.1 Experimental setup

For 3D measurements, the setup was composed by both Photron FastCam SA4 high speed cameras. They were focusing on the upper surface of the cantilever beam (RM) during the test, as shown in Fig. 30. The stereoscopic system calibration was satisfactory with just 0.015 pixel deviation. The beam was excited with a white noise random excitation up to 1000 Hz by means of the shaker GW-V55/PA-300E. The signal was provided by the real-time analyser Photon+. In this spectrum, the first two bending modes of the cantilever were encountered. According to this, the frame ratio of the cameras was set at 2000 fps to fulfil the Nyquist frequency criterion. The cameras resolution was adjusted to 1024x512 pixels, fitting the beam shape. Thus, the lower images size allows to record a larger number of images at a time, in particular, 10500 frames during per camera. Two recordings were conducted in order to obtain more time-domain information of the random event and hence reduce the presence of noise when averaging PSD according to Welch's method.

Blurring in the higher frequencies motion was avoided by setting the lens exposure time at 0.1 ms, what implies sampling time ten times shorter than the highest excitation frequency period, 1000 Hz. Due to the short exposure time, the lack of light was overcome by using the 150W lamp.

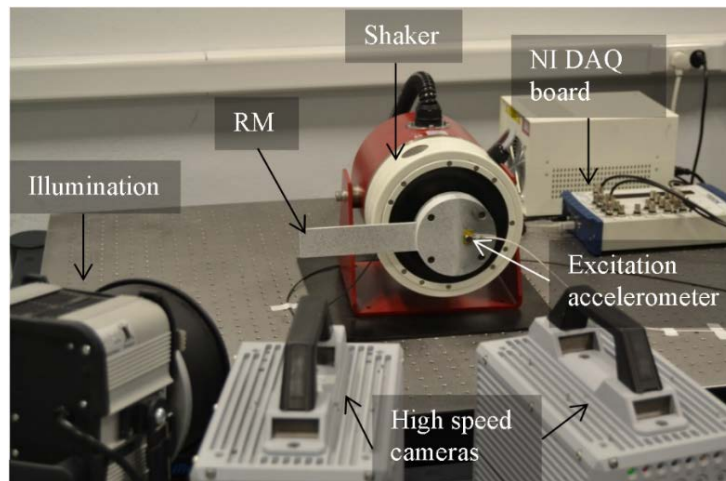


Fig. 30. Experimental setup for full-field transmissibility functions estimation using HS 3D-DIC.

The input of the transmissibility functions, i.e., the excitation, was registered using an accelerometer on the clamping system, as observed in Fig. 30. Hence, it recorded the acceleration of the shaker armature which represents the base motion of the beam. The DAQ system (NI USB-6251 DAQ) was used for registering accelerometer signal and synchronising with high speed cameras. A schematic of the full data recording process is shown in Fig. 31.

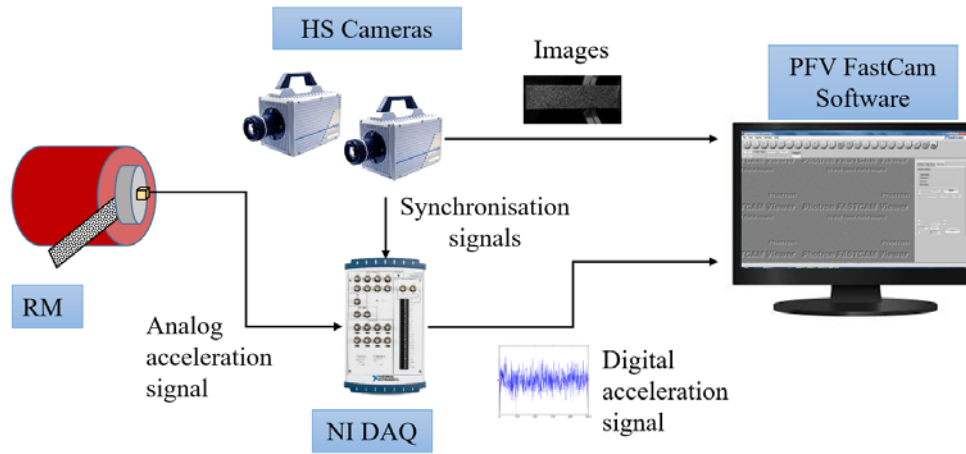


Fig. 31. Data flow diagram for full-field transmissibility functions estimation using HS 3D-DIC.

The speckle was applied covering about 6 pixels as an average size, what provide a good definition for the correlation. Hence, DIC processing was performed using a facet size of 13 pixels and an overlap of 4 pixels. It resulted in a matrix of 66x246 measurement points along the beam's surface. At every point there is a time domain displacement vector. Together with accelerometer signal, Welch averaging method was performed so as to determine the transmissibility functions. Each sampling window was composed by 2000 points what provides a frequency resolution of 1 Hz. Hanning window was adopted to avoid leakage and an overlap of 75% to increase the number of averages. Since there is a discontinuity between the two recordings during the random tests, each one was individually processed. As a result, two transmissibility functions sets were obtained and the final transmissibility function at each point was calculated as the mean value of both data sets.

5.2 SDOF conversion of transmissibility functions

The circle-fit approach evaluates the modal parameters on an individual resonance by fitting the peak considering that the shape of the FRF peak in the imaginary plane is circular, as exposed in Chapter 2. The adopted methodology to adapt the transmissibility functions to the FRF form for modal

identification is based on single degree of freedom assumptions. For a SDOF vibrating system (Ewins 2000), it is possible to obtain a relation between the transmissibility function, T , and the FRF, H :

$$H\{i\omega\} = \frac{T\{i\omega\}}{(\omega/\omega_n)^2}, \quad (21)$$

where ω is the vector of frequencies from the analyzed spectrum and ω_n is the natural frequency of the SDOF system. This relation is valid for the particular form of transmissibility function that fulfils the following conditions: it has to be non-dimensional, i.e., excitation and response motions are defined by the same magnitude; the response motion is relative to the base (excitation) motion.

According to the experimental procedure, previous modifications must be applied to transmissibility functions to employ equation (21). The excitation signal in terms of acceleration must be integrated twice to obtain non-dimensional transmissibility functions, i.e., displacements divided by displacements. Finally, HS 3D-DIC measures absolute displacements and thus the excitation must be subtracted to the response to obtain relative displacements. Both operations can be performed in the frequency domain as follows:

$$T\{i\omega\} = T_{\text{exp}}\{i\omega\} \cdot (i\omega)^2 - 1, \quad (22)$$

where T_{exp} represents the transmissibility functions obtained experimentally with the adopted setup.

As can be deduced from equation (21), the natural frequency, ω_n , is an input parameter in the conversion procedure. However, it is initially unknown since it is also an output parameter of the circle-fit approach. To work this out, an initial value is proposed, $\omega_{n,0}$, for the conversion defined in equation (21) to obtain an initial FRF, H_0 . As a result of performing circle fitting in H_0 , the actual natural frequency is obtained since $\omega_{n,0}$ scales the function but does not modify the shape. Thus, no additional iteration is required and the actual FRF, H , can be obtained as:

$$H\{i\omega\} = H_0\{i\omega\} \cdot \left(\frac{\omega_n}{\omega_{n,0}}\right)^2, \quad (23)$$

Note that the proposed frequency could be 1 to simplify the calculation.

The circle-fit approach was performed for every DIC measurement point and for every detected resonance. As a result, the information of each resonance involves three full-field matrices of natural frequencies, modal constants and structural damping loss factors. The elements of the matrices correspond to the measurement points of HS 3D-DIC. Hence, the mode shapes were obtained by mapping the modal constant matrices. The natural frequencies and damping ratios were obtained as the mean value.

5.3 Validation methodologies

The experimental validation relied on traditional sensors. An accelerometer was fixed to the rear surface of the beam, as seen in Fig. 32, placed at 35 mm from the clamped edge of the beam. Close to the clamp, accelerometer's mass inertia is non-significant, reducing mass-loading effect. The same accelerometer on the clamping device was employed to calculate the transmissibility function. The signal was captured during the same time lapse as DIC measurements. However, different recording parameters were employed for the accelerometer in order to additionally assess the presence of alias frequencies. Hence, the accelerometers signals were recorded and processed using the real-time analyser Photon +. The analysis spectrum was extended up to 3200 Hz, so that potential aliased frequencies in DIC measurements could be detected. In order to achieve the same frequency resolution than in DIC transmissibility functions, 3200 frequency lines were established. Several windows were averaged employing Hanning window with no overlapping because of the real-time analysis.

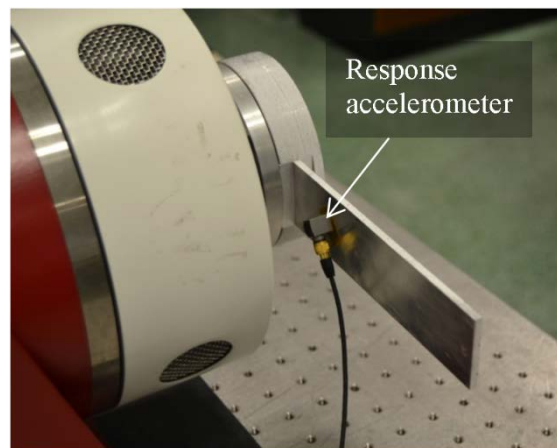


Fig. 32. Validation accelerometer for 3D-DIC results.

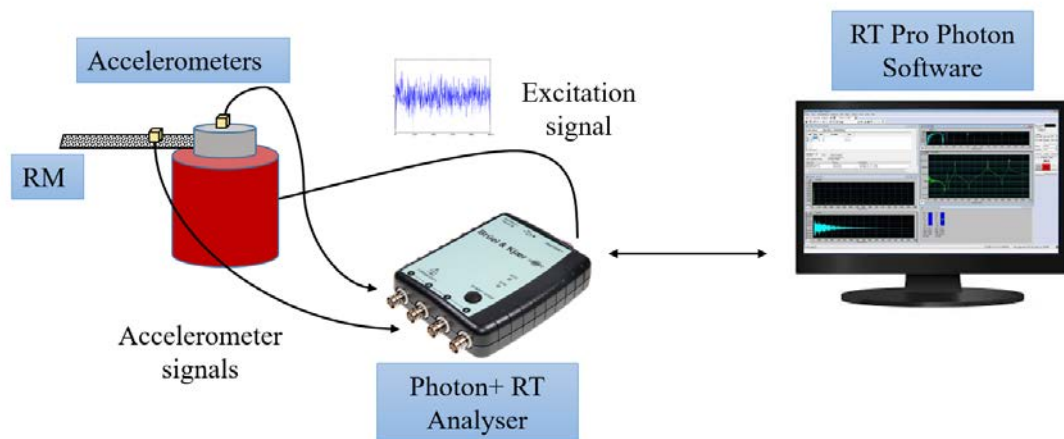


Fig. 33. Accelerometers experimental measuring diagram.

Finally, from the numerical and theoretical models described in section 4.3.3, the natural frequencies and mode shapes of the two first mode shape were determined to be compared with experimental ones. MAC indicator was employed for the comparison of mode shapes.

Chapter 6

Combination of HS DIC and motion magnification

In this section, it is described the experimental procedure to evaluate the combination of the phase-based motion magnification and HS DIC for ODSs characterisations under sinusoidal excitation. Based on the studies presented in section 3.3 that employed a single camera to reproduce magnified videos, initially it was explored the combination for in-plane measurements on the cantilever beam. It is presented a thorough set of tests to evaluate the effects of performing 2D-DIC measurements in magnified images depending on the level of excitation and the magnification factor. The expected signal-to-noise ratio pull the study to analyse higher frequencies modes. The FEM model was considered as the ideal reference to assess the measurements in the magnified images regarding both the noise reduction and possible shape distortion.

However, few applications would benefit from this 2D methodology since in-plane deformation only occur in particular cases. Hence, 3D measurements were performed in pairs of magnified images. This study initially presents a validation analysis using the cantilever beam to demonstrate that no shape distortion was introduced by aberration in the pairs of images. Then, the methodology was employed to characterise different ODSs of the composite panel in the three directions, exhibiting the capabilities of 3D measurements. These tests also allowed to prove the efficiency of the combination when small exciters are employed in the analysis of large and relatively heavy elements.

6.1 Validation on the cantilever beam

6.1.1 Experimental setup

In this occasion, the specimens were excited at single natural frequencies using sinusoidal signal. The shaker model GW-V55/PA-300E was employed to excite the cantilever beam. The results from the cantilever beam intended to validate both 2D and 3D methodologies. On one hand, a single camera setup was designed to register the bending deformation of the beam by focusing on the lateral surface as shown in Fig. 34. The deformation is registered so in the sensor plane. Regarding the shape of the beam, the images were cropped using a resolution of 1024x112, as observed in Fig. 35 (a). Hence, the frame rate and the number of recorded images were notably increased.

The stereoscopic setup was analogous to that designed for the modal identification in base motion identification (Chapter 5) tests shown in Fig. 30. The deformation is registered here as out-of-plane displacements using images of 832x320 pixels as shown in Fig. 35 (b).

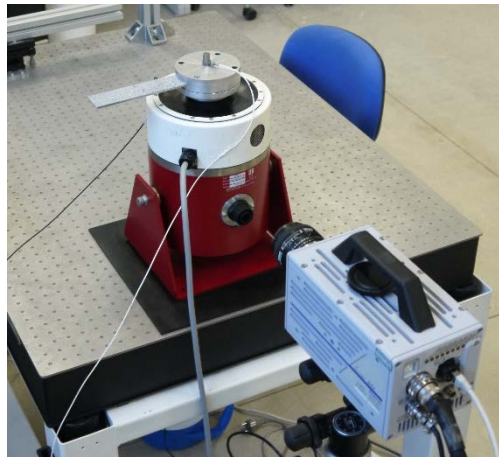


Fig. 34. Optical setup for 2D-DIC measurements in the lateral surface of the cantilever beam.

The natural frequencies were previously identified by performing a random excitation. In an analogous configuration to that exposed in section 5.3 and shown in Fig. 33, two accelerometers were employed, one in the clamping device and the other in the beam at 27 mm of the clamped edge, to obtain a transmissibility function of the cantilever beam. Noise excitation from DC to 7500 Hz was employed considering that the performance frequency range of the shaker reach up to 8000 Hz. The frequency analysis was performed considering a frequency resolution of 1 Hz, Hanning anti-leakage windows and several averaged windows. The response accelerometer was not later removed to maintain the same conditions during sine tests. Five bending modes were selected at the following frequencies: 135 Hz, 793 Hz, 2140 Hz, 4150 Hz and 6710 Hz. The first four modes were considered in the 3D version.

In both versions, the same cameras parameters were employed. The frame rate was defined at 15000 fps, which provides a good discretisation up to the third mode, 2140 Hz, with 7 images per cycle. However, for the 4th and the 5th modes it was not possible to increase the frame rate maintaining the same resolution. Therefore, these modes were subsampled recorded using the camera strobe effect. The frame rate for these modes was chosen so that the aliased frequency was low. Hence, the frame rate was set to 2000 fps for the 4th mode, so this mode appeared in the spectrum as an aliased frequency at 150 Hz. For the 5th mode, it was set at 750 fps being 40 Hz the aliased frequency. The criterion to define the shutter exposure time was about ten times shorter than the period of the mode in order to avoid any blurring. Hence, for the first three modes, it was set to 1/20000 s and 1/60000 s was employed for the last two.

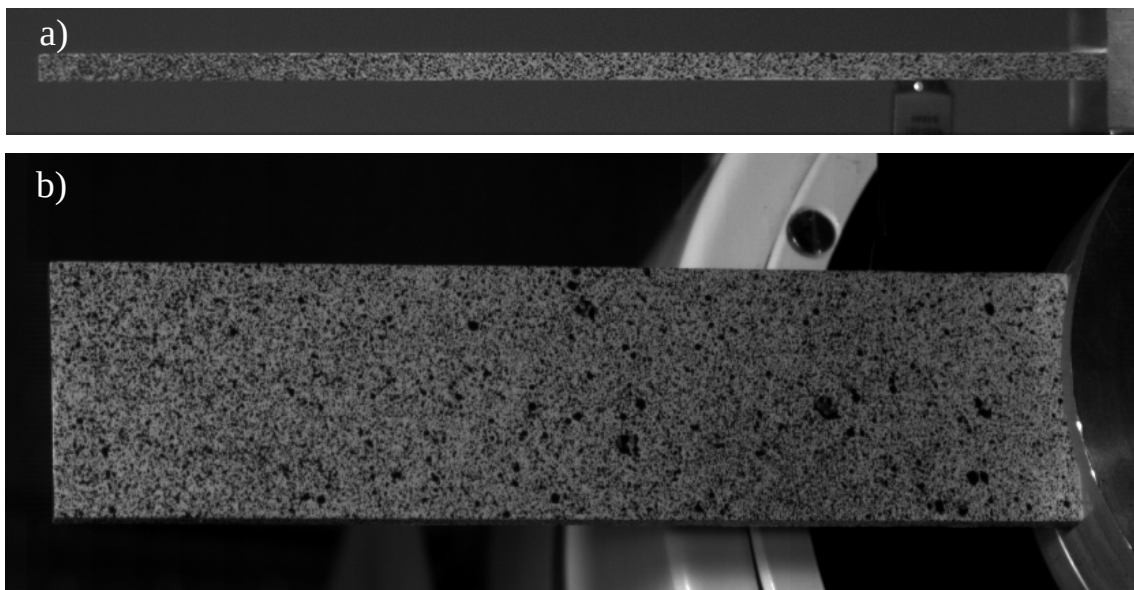


Fig. 35. (a) Lateral surface of the beam recorded by the high speed camera in a 1024x112 pixels image. (b) Upper surface of the beam recorded by one high speed camera of the stereo system in an 832x320 pixels image.

6.1.2 Motion magnification

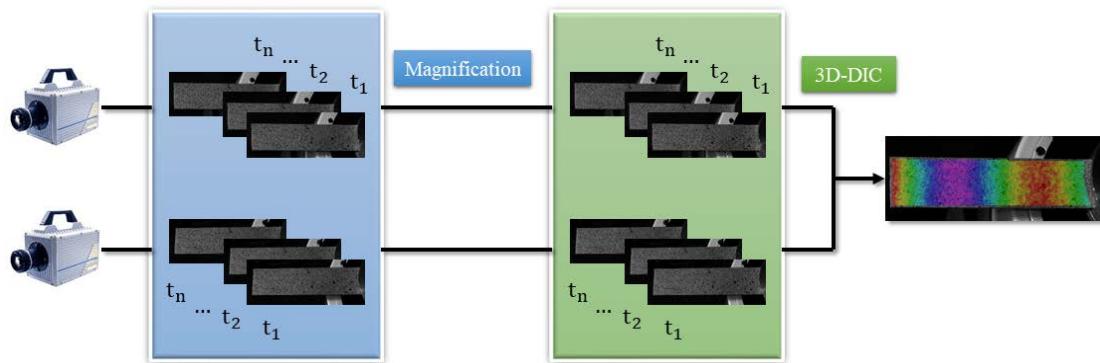
For each of the five identified modes, two opposite level of excitation were applied to evaluate motion magnification. Low level of displacement represents a case of subtle motion to reveal the improvements in DIC results applying magnification. High level of excitation was defined by the shaker's specifications. Different magnification factors were employed for each test in the natural frequencies vicinity to study the relation between magnification and quality of the results. Therefore, the evolution of the contour plot versus the magnification factor was analysed. Therefore, magnification factors were determined for each mode considering the quality of the results. Some large magnification factors have been also employed to reveal the effect of distortion. Table 1 shows the magnification factors employed for each natural frequency.

Table 2. Magnification factors employed in each mode in both configuration for the cantilever beam.

Mode	2D version	3D version
135 Hz	(2, 5, 10)	(2, 5, 10)
793 Hz	(2, 5, 10)	(2, 5, 10)
2140 Hz	(2, 5, 10, 50)*	(2, 5, 10, 50)
4150 Hz	(50, 100)	(10, 50, 100)
6710 Hz	(50, 100)	-

* Magnification factor 50x was only applied to high excitation test

DIC was performed at every magnified state as well as the non-magnified images. The specimen surface was treated with white and black paint to generate the necessary random speckle pattern in the corresponding surface. In the 2D system, images were processed to obtain full-field displacement contour plots using a facet size of 9 pixels and a spacing of 2 pixels. The resulting calibration provided a relation of 0.1657 mm/pixel. As a result of recording the event with two cameras in the stereovision system, motion magnification must be performed individually at each set of images. Then, the two sets of images were gathered together and processed employing Vic-3D software for 3D-DIC to obtain displacements (Fig. 36). It was employed a facet size of 15 pixels and overlap spacing of 5 pixels. Assuming that the surface is flat, it can be estimated a mm/pixel ratio of 0.1961.

**Fig. 36** Measuring procedure in a stereoscopic set of magnified images using 3D-DIC.

6.1.3 Comparison with FEM

The deviation with respect to the ideal shape was assessed using the FEM model. Considering that the modes are well-separated, the comparison between experimental ODSs and FEM mode shapes can be made without incurring any error. The mass of the accelerometer (Brüel & Kjaer 4520-001) was also included in the model. As a result, five bending modes were identified at 130.16 Hz, 807.68 Hz, 2208.2

Hz, 4211.4 Hz and 6902.8 Hz. The maximum difference with respect to the experimental frequencies was 3.58% for the first mode.

For comparison with experimental results, FEM mode shapes were defined as continuous functions which were considered to fit experimental data. The whole process is graphically described in Fig. 37.

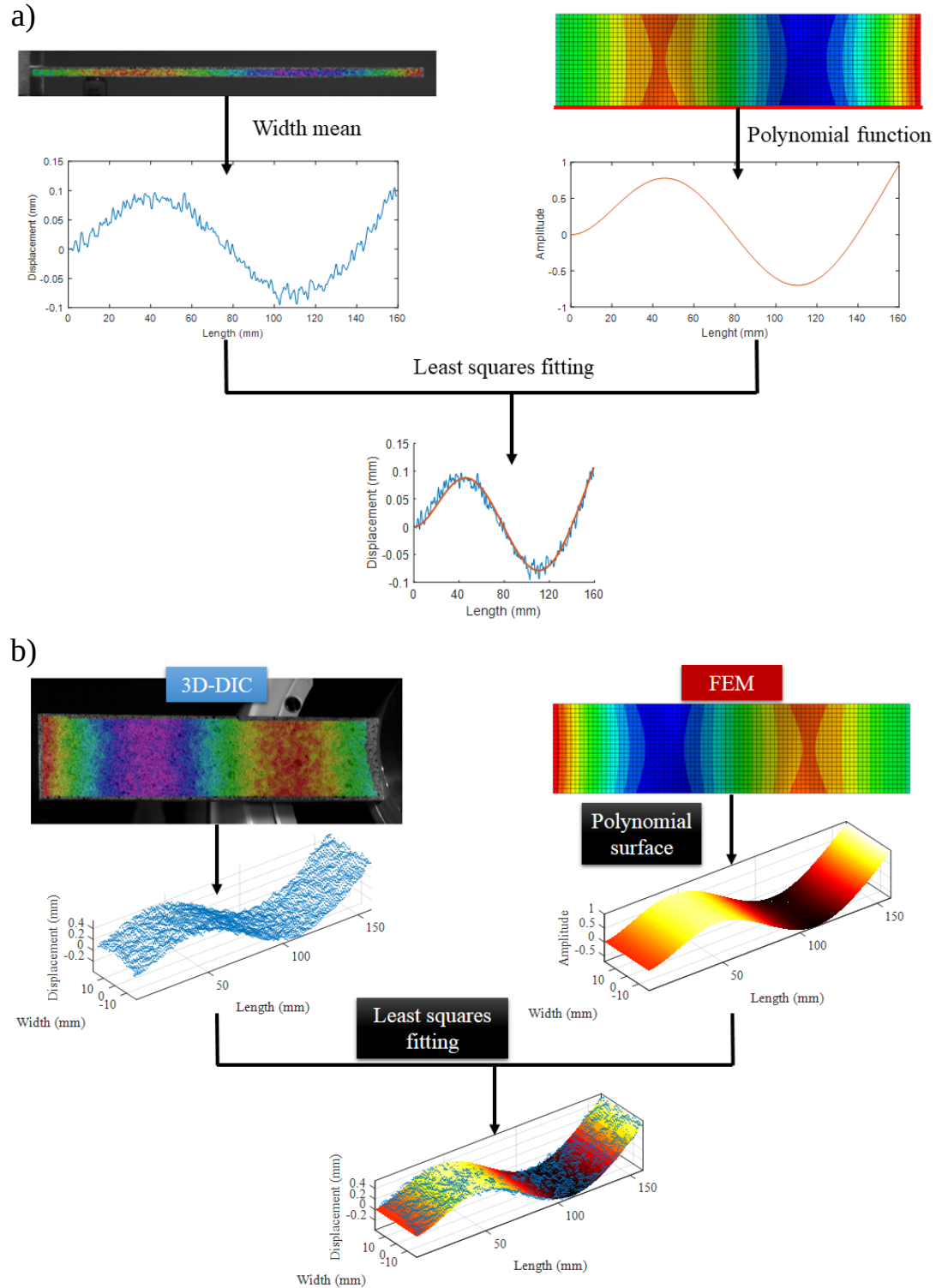


Fig. 37. Fitting process of the DIC results to FEM for (a) HS 2D-DIC and (b) HS 3D-DIC.

In the 2D version, the displacement of the beam can be simplified as a 1D problem where they are only function of length. Thus, experimental measurements on the lateral surface were reduced to a line by calculating the mean along the thickness, neglecting any variation. From FEM shapes, the values of the lateral edge nodes were extracted. A least squares fitting was employed to fit FEM mode shapes using polynomial functions. The polynomial order of the functions varied from 4 to 12.

On the other hand, the 3D measurements provides information in the same surface of the beam and the whole data can be employed. Polynomial surfaces, functions of length and width, are now assigned to fit the FEM results. The order varied from 4 to 12 longitudinally and from 2 to 4 transversally depending on the mode.

The magnification modifies the amplitude of the displacement results for each test. Thus, once FEM results are converted into continuous functions, the scales of the polynomial functions were adjusted by fitting experimental data using the least squares method. Thus, the Normalised Root Mean Squares Error (NRMSE), which is non-dependent on the amplitude of the shape, can be obtained and adopted as an indicator of the improvements in 3D-DIC mode shapes due to magnification. In this evaluation, the correlation coefficient is also interesting and complementary to NRMSE. This is a statistical indicator less sensitive to small changes that provides the degree of consistency between shapes, despite the presence of noise.

6.2 Evaluation on an industrial component

The measurements of the panel meant to try the methodology in an industrial component once the methodology had been validated in the previous analyses on the beam. Nevertheless, additional techniques could be employed, such as 3D scanning laser vibrometry, to validate the results. Particularly, it presents complex shape and modal behaviour what would demonstrate the potential of the combination of 3D-DIC and motion magnification.

The composite panel was excited using the less powerful shaker GW-V20/PA30E, forcing it from the rear face using a stinger fixed at the bottom of the panel at the symmetry axis. With this relative small exciter, it was intended to produce clear magnified results from pure noise fields due to the low level of displacements. As a consequence of the panel shape, it is difficult to make the whole panel visible by the cameras. An intermediate angle of view between the two principal planes of the panel was defined in order to monitor most of the surfaces. Focus was placed on half the panel under the assumptions of symmetry as can be seen in Fig. 38.

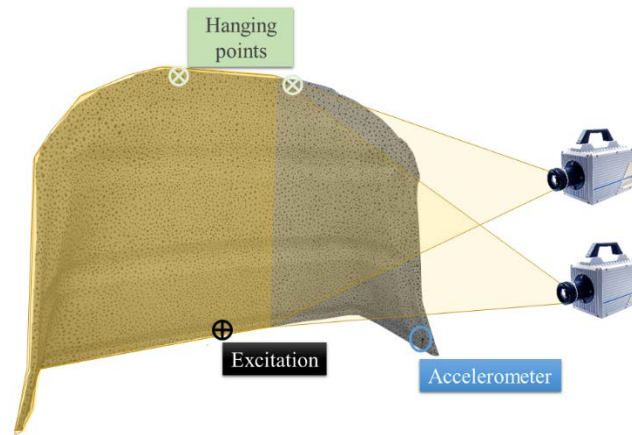


Fig. 38. Optical setup for 3D-DIC measurements in the large composite panel.

For the identification of natural frequencies, the response accelerometer was located on the panel corner, where the highest response is expected for many modes. From the inspection of the response in the spectrum up to 300 Hz, four peaks were selected to evaluate ODSs under low excitation level: 137 Hz, 190 Hz, 215 Hz and 285 Hz.

The same parameters were employed for every mode according to the criteria established for the beam: 3600 fps, 1/3600 s of exposure time. For DIC processing 25 pixels facets were defined, overlapped 5 pixels. In Fig. 39, an image of the panel from one camera and the three-dimensional digitalisation of the surface are shown. The light supply was arranged to minimise the effect of brightness and it did not affect to the global results of this study.

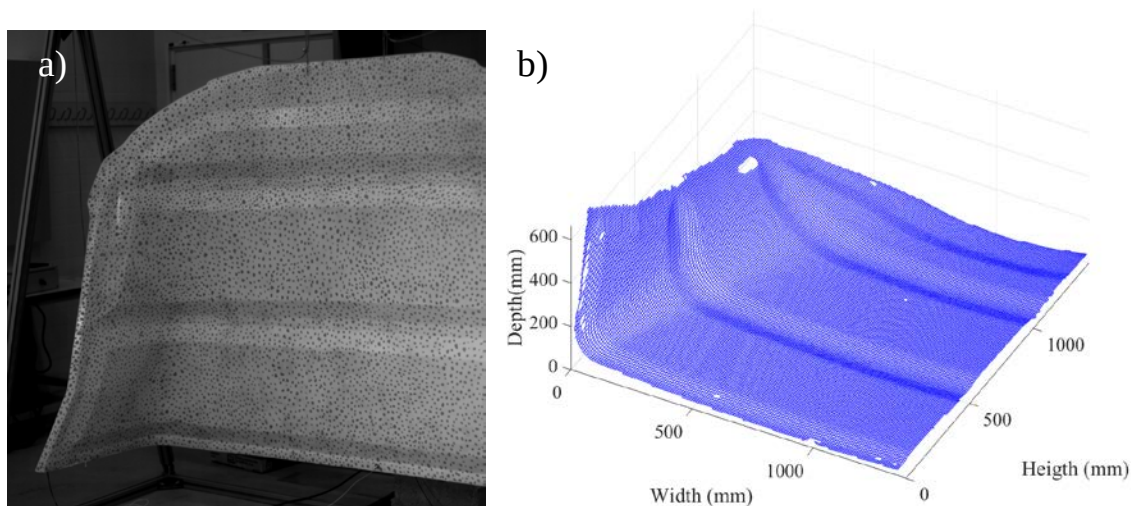


Fig. 39. (a) Image of the panel with 1024x1024 pixel resolution. (b) 3D digitalisation of the panel using DIC.

This part of the study involved just one magnification factor to reveal information from non-discernible ODSs tests, defined in Table 3. The magnification factors provide significant improvements avoiding blurring.

Table 3. Magnification factor employed in every mode of the composite panel.

Mode	Magnification factor
137 Hz	20
190 Hz	50
215 Hz	50
285 Hz	100

Chapter 7

Summary of results

In this section, the most relevant results from the attached papers are briefly exposed and commented to conduct to the conclusions of this thesis. According to the classification throughout the document, this chapter has been divided in two subsections. On one hand, the result from the experimental modal analysis in base motion excitation tests are shown. The second section includes the result from the combination of HS 2D-DIC and HS 3D-DIC with phase-based motion magnification.

7.1 Experimental modal analysis using transmissibility functions

Here it is presented the results corresponding to the full-field experimental modal analysis in base motion excitation tests employing the circle-fit method. The transmissibility function obtained in the cantilever beam using HS 3D-DIC were converted into equivalent FRFs employing the exposed methodology in every resonance peak individually. An example of the conversion corresponding to a point at the tip of the beam is shown in Fig. 40.

Previous to modal identification, measurements were experimentally assessed using the validation accelerometer. Aliasing was rejected with this comparison since, as observed in Fig. 41, the noise floor

of DIC, marked as a red line, is much higher than the amplitude of any line over the Nyquist frequency, 1000 Hz.

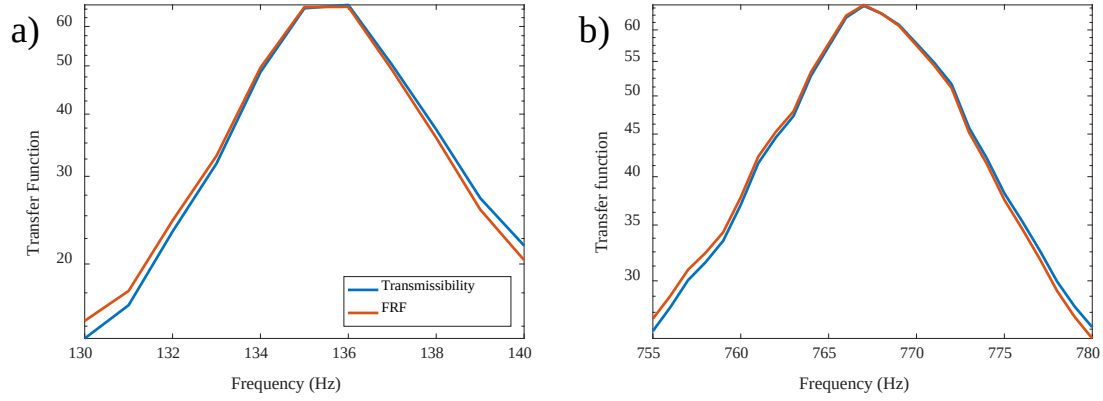


Fig. 40. Comparison between transmissibility function (T) and FRF (H) in a point of the free end of the beam: (a) First mode. (b) Second mode.

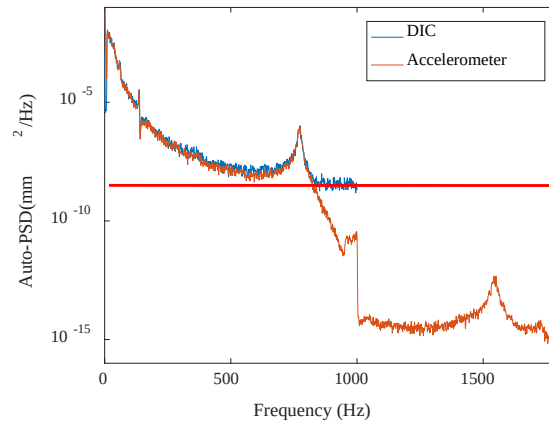


Fig. 41. PSDs of the response of the beam obtained by DIC and the accelerometer at the same point.

Table 4. Statistical indicators of natural frequencies, f_n , and structural damping, η , identified by circle-fit approach in 3D-DIC measurements.

Mode	f_n (Hz)		η (%)	
	Mean	Standard deviation	Mean	Standard deviation
1	135.38	0.08	1.69	0.08
2	767.78	0.74	1.45	0.04

After the validation and conversion, circle-fit approach could be properly performed in every peak of every single measurement point to provide full-field modal characterisation. The analysis of one mode produces three matrices corresponding to the three modal parameters. The size of those matrices is the map of DIC measurements. Natural frequencies and damping ratios were given by a unique value per

mode calculating the mean value of the matrices. Table 4 shows these values together with the standard deviation. The low deviation indicates high homogeneity in the detection of the modal parameters along the beam.

The experimental modal characterisation was assessed with the FE model and theoretical expressions. The natural frequencies of these models are shown in Table 5 and agree with experimental results with an approximate error of 5%. The mode shapes are shown in Fig. 42 and Fig. 43 (Fig. 15 and Fig. 16 in the manuscript, respectively) for the first and second mode, respectively. Evident similarities are found comparing methods. Agreement is definitively proved by the correlation coefficient provided by MAC, which is always over 98%. As it is observed in Fig. 43, the second mode shape is slightly noisier. As deduced in the paper, displacements in this mode are closer to floor noise. Furthermore, there is one more zero-displacement node, increasing the area of low displacements.

Table 5. Natural frequencies of the two first mode identified in FEM and theoretical analyses.

Mode	FEM	Theoretical
1	131.03	129.34
2	817.81	810.41

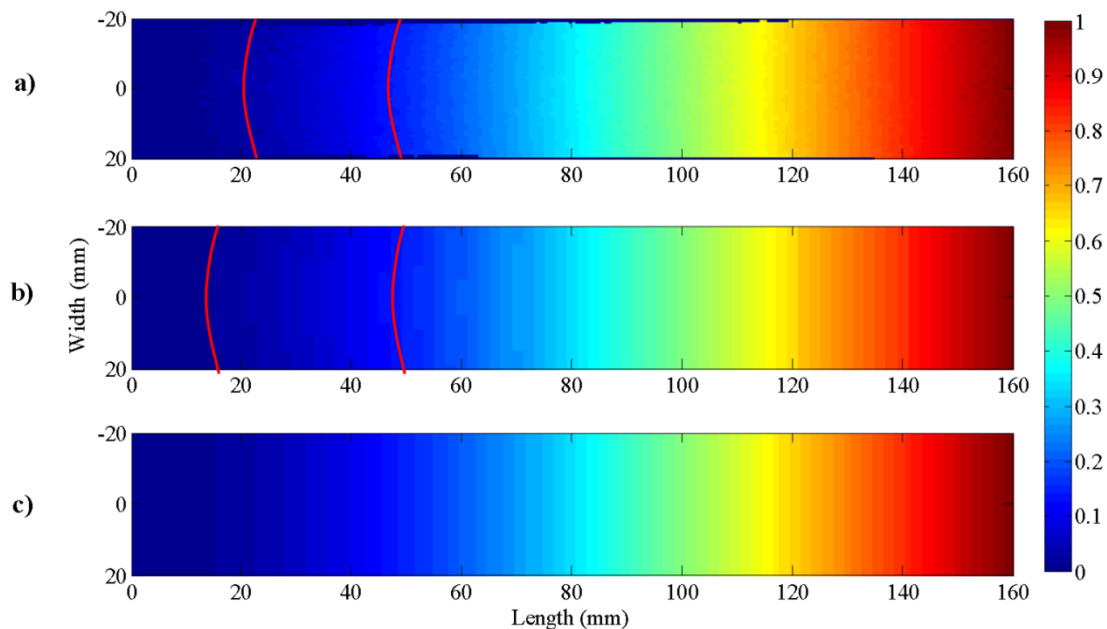


Fig. 42. First resonance's mode shape: (a) circle-fit approach from 3D-DIC; (b) FEM analysis; (c) theoretical analysis. Plate behaviour is highlighted by red lines.

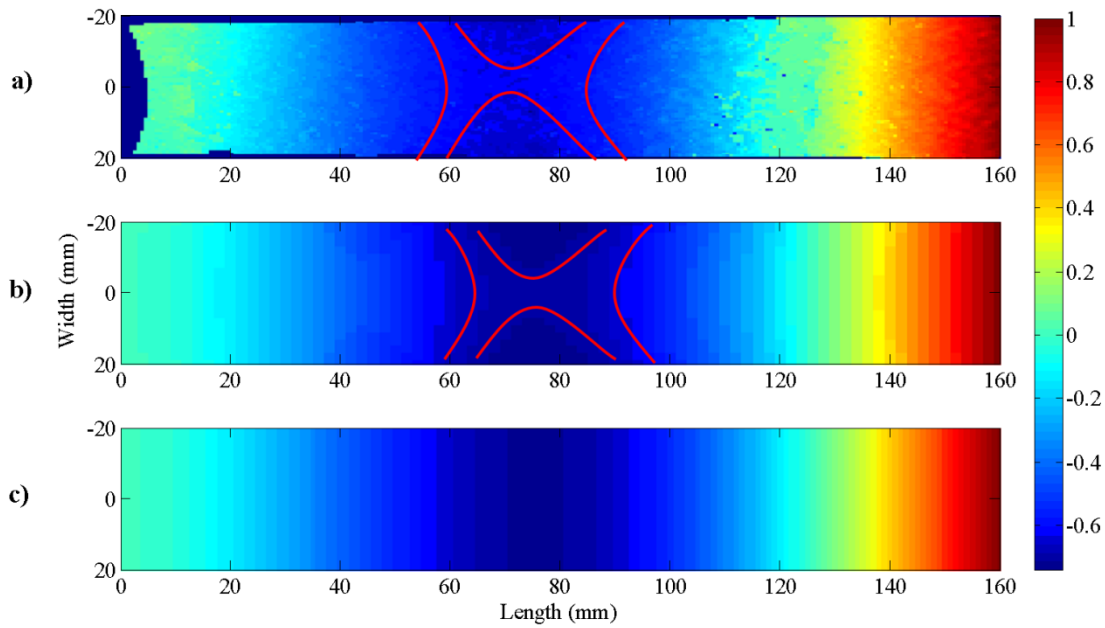


Fig. 43. Second resonance's mode shape: (a) circle-fit approach from 3D-DIC; (b) FEM analysis; (c) theoretical analysis. Plate behaviour is highlighted by red lines.

7.2 Combination of HS DIC and motion magnification

This section sums up the results of the experiments of the combination of motion magnification and HS DIC in 2D and 3D. The first part of the evaluation experiments are devoted to try the methodology in a simple specimen like the cantilever beam. Five bending modes were identified to conduct FNM tests using 2D and 3D configurations: 135 Hz, 793 Hz, 2140 Hz, 4150 Hz and 6710 Hz. The two first modes are in the usual spectrum analysed by HS DIC and correspond with those from the previous work. From contour plots, it was found out that motion magnification contributes to generate higher amplitude displacements that improve ODSs HS DIC measurements. This is especially remarkable for the last three resonances whose measurements in non-magnified images are highly affected by noise, making it possible to visualise ODSs at unprecedented frequencies. Optimal results were found for the forth mode at 4150 Hz according to the frequency value and the quality of the magnified measurement as seen in Fig. 44 and Fig. 45 for 2D and 3D, respectively.

The reduction of noise was evidenced in the comparison with FE results. In Fig. 46 and Fig. 47 the fourth mode is again shown as an example for both configurations. Experimental numerical shapes are compared for different magnification factor. It is possible to observe how the experimental data fits the numerical results. Moreover, the exponential reduction of the NRMSE and the increment of the

correlation coefficient indicate this fact quantitatively. A remarkable enhancement was achieved as demonstrated in Fig. 48-Fig. 51. Furthermore, with this results it is possible to see that the magnified results fitted and agreed with numerical mode shapes.

Some visualisation aspects were also highlighted. Excessive magnification may imply speckle blurring at the regions of higher amplitude and avoids DIC measuring. It is more important if non-magnified motion is already high. In the 2D system, this was the case of the first and second mode, those with higher displacement. Thus, with an intermediate level of excitation and even with a not too high magnification factor it is possible to achieve good results. In the 3D version, this effect was less important since the beam deformation was out-of-plane and the effective motion in pixels was reduced. As a consequence, blurring only appeared for the first mode.

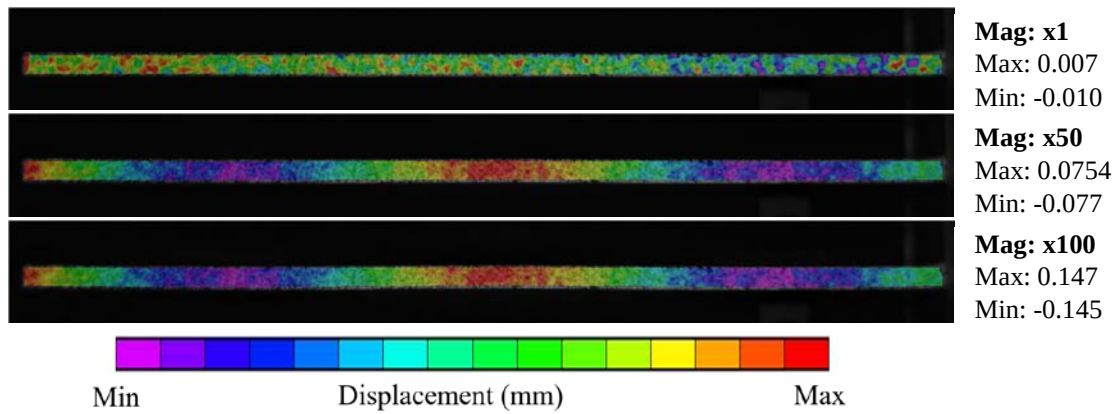


Fig. 44 Visualisation of the fourth mode shape (4150 Hz) with DIC contour plots under low level of excitation and different motion magnification factors

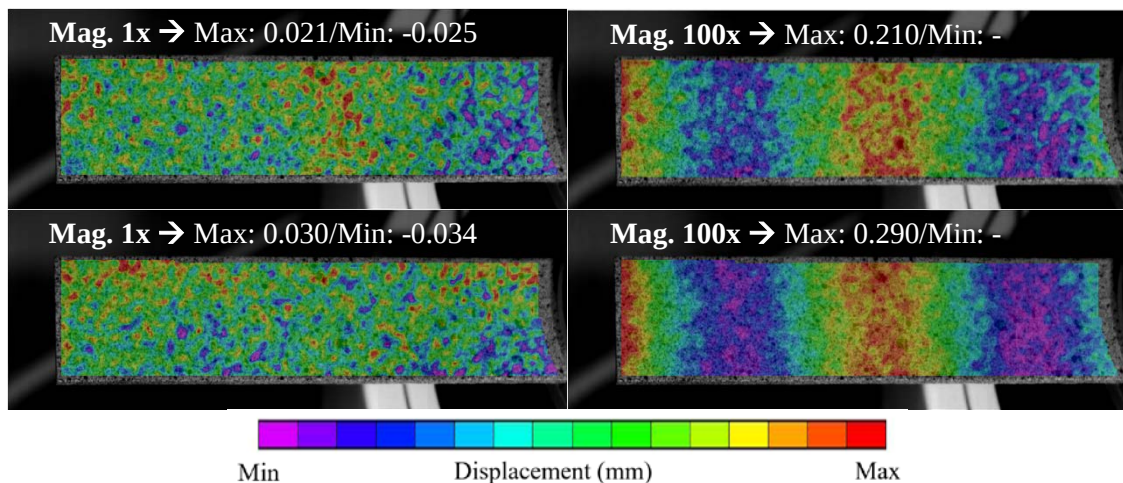


Fig. 45. Visualisation of the out-of-plane displacements of the fourth mode shape (4150 Hz) at non-magnified and maximum magnified states. (Above) Low excitation level, (Below) high excitation level.

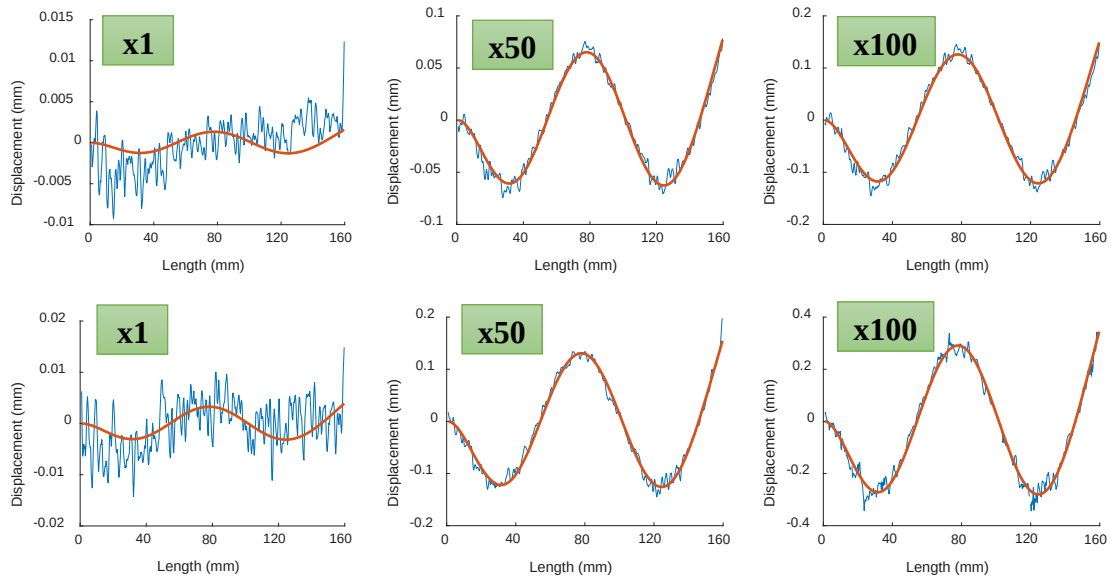


Fig. 46. Comparison of the experimental shape (blue) with the ideal shape from FEM model (orange) of the fourth mode under low excitation (upper row) and high excitation (lower row) with different magnification factors.

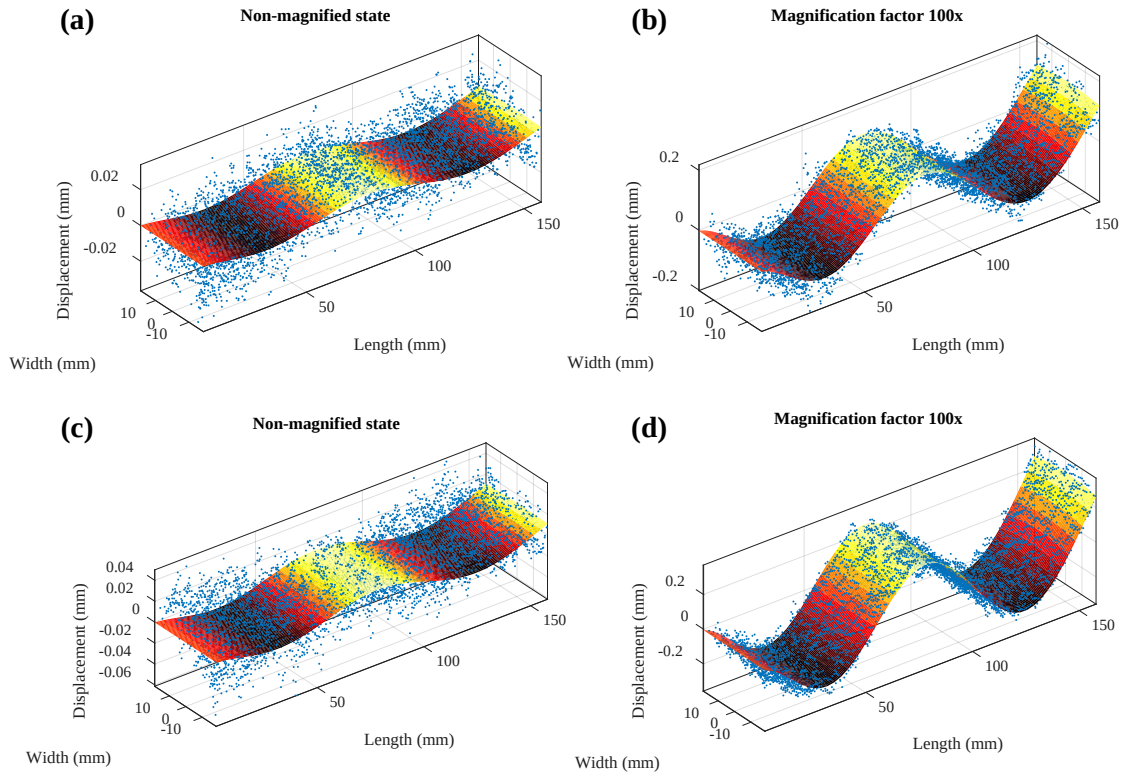


Fig. 47. Comparison of experimental (blue dots) and FEM mode shapes (contour surface) for the fourth mode. (a) Non-magnified state with low excitation. (b) Maximum magnification with low excitation. (c) Non-magnified state with high excitation. (d) Maximum magnification with high excitation.

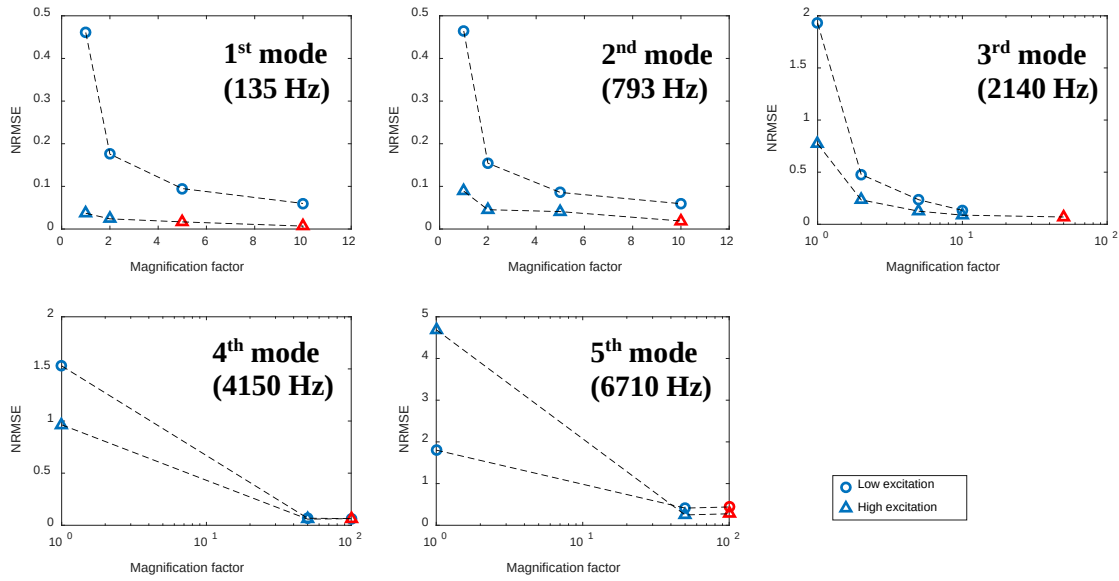


Fig. 48. Evolution of normalized Root Mean Squares Error with the magnification factor. Red markers represent the test where blurring occurred in speckle.

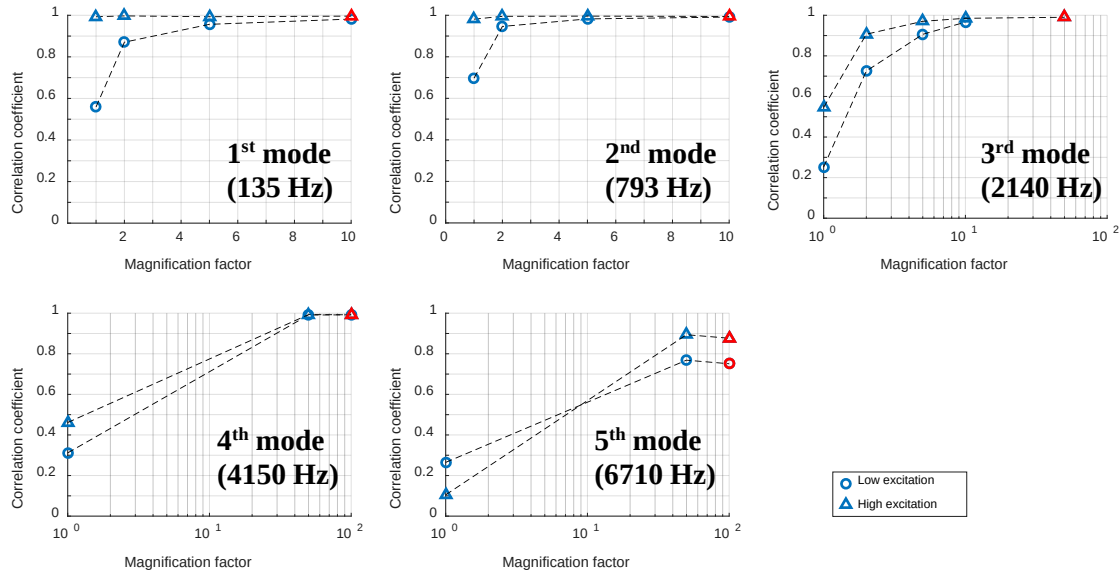


Fig. 49. Evolution of correlation coefficient between experimental and numerical mode shapes with the magnification factor. Red markers represent the test where blurring occurred in speckle.

From the analysis of the relation of amplitudes of different magnified states for the low excitation of the first and second mode shown in Fig. 52, it was shown the proportional trend of the amplitude. However, the relation was not the expected regarding the commanded magnification factor. Further, the presence of an offset in the origin indicated non-linearity.

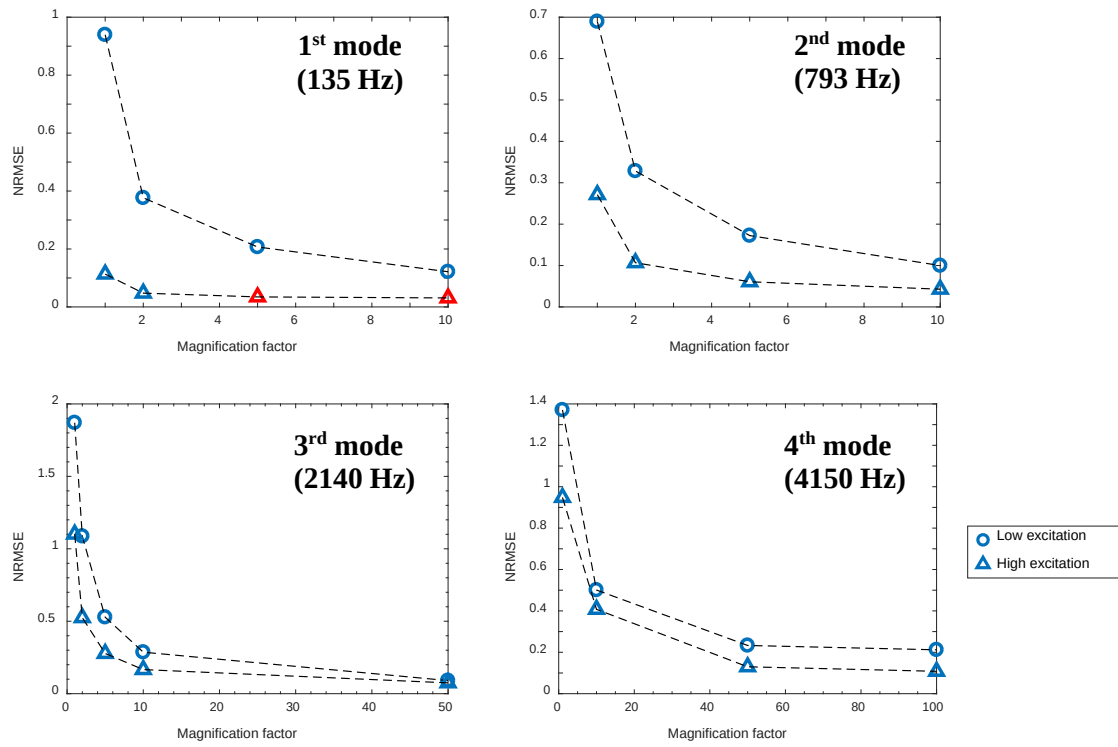


Fig. 50 Normalized Root Mean Squares Error versus magnification factor. Red markers indicate presence of non-correlated areas.

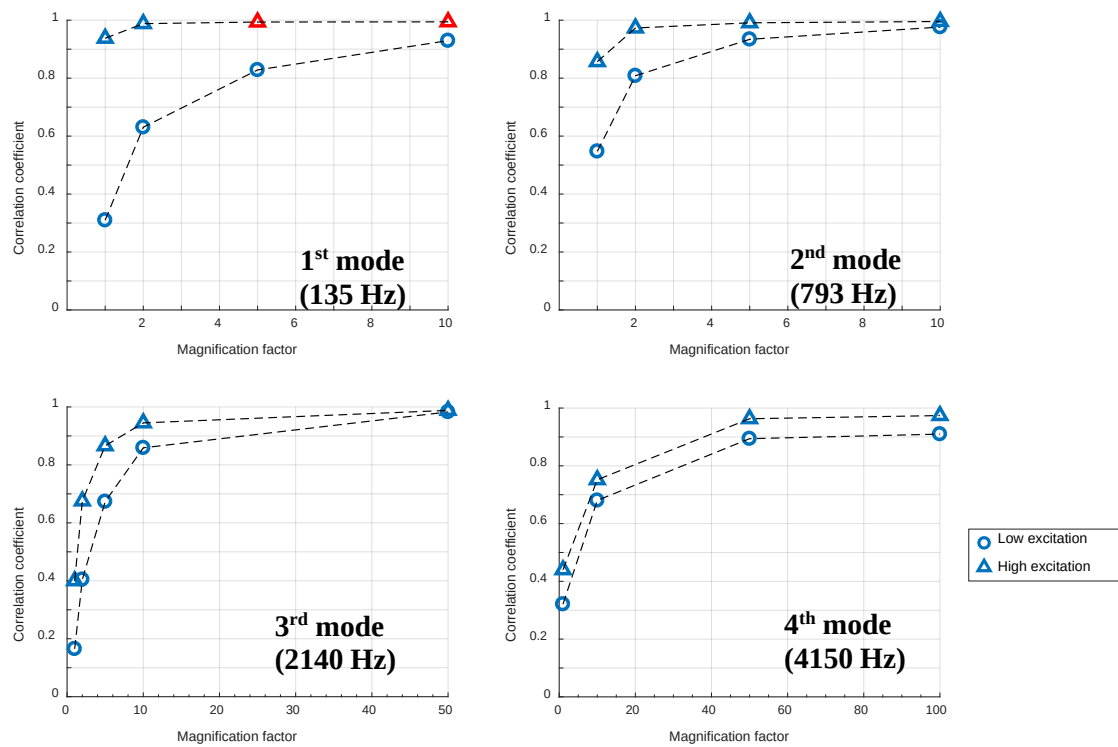


Fig. 51 Correlation coefficient between experimental and numerical mode shapes versus magnification factor. Red markers indicate presence of non-correlated areas.

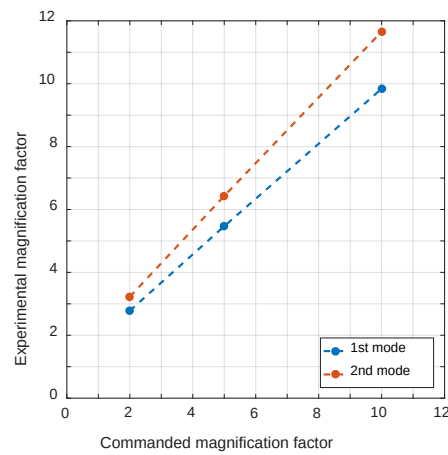


Fig. 52 Evaluation of the resulting magnification factor for the first two modes in low excitation tests.

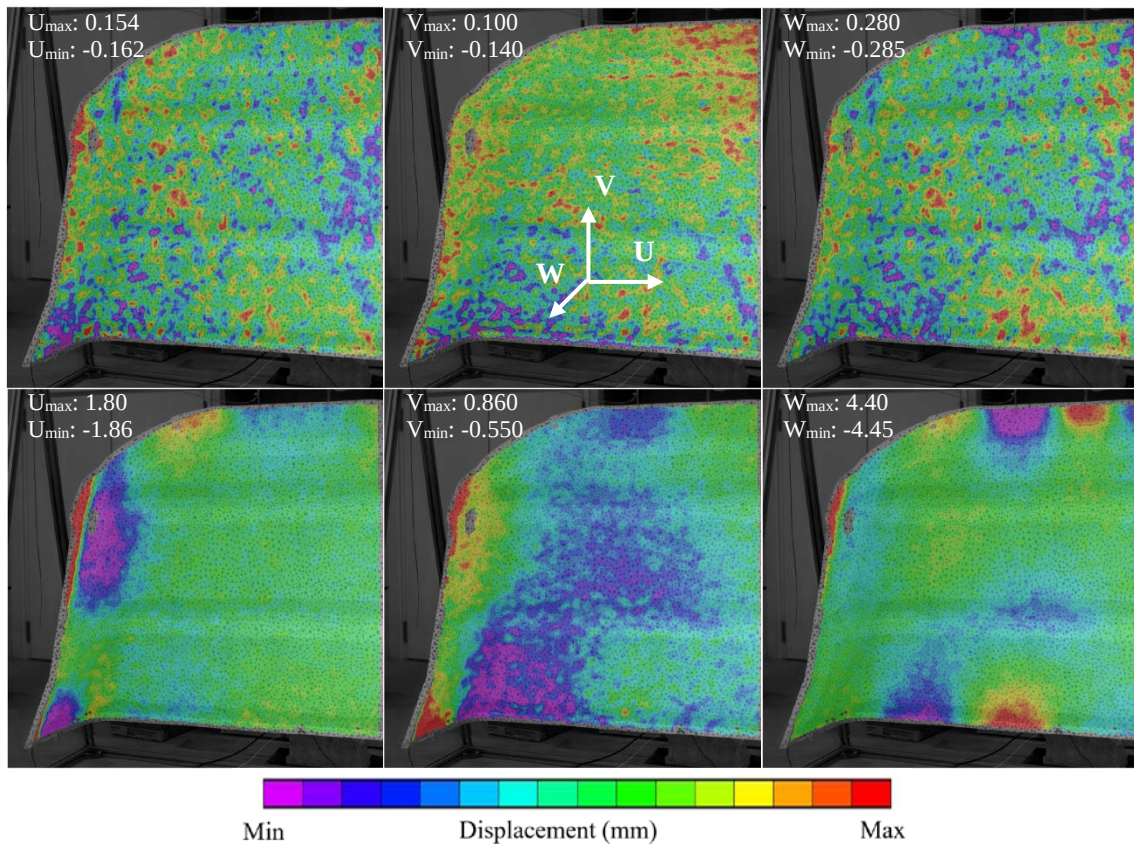


Fig. 53. Operational Deflection Shape of the composite panel at 190 Hz from non-magnified images (upper row) and 50x magnified images (lower row).

Overall, the assessment of cantilever results through the numerical model indicates the absence of distortion during the magnification procedure. This is especially important in the 3D version where the measurement is performed in matched pairs of images that were individually magnified. From this point, the methodology was evaluated in the composite panel, a 3D complex industrial component. Four modes were analysed from what the shape of the 190 Hz and 285 Hz modes are shown in Fig. 53 and Fig. 54, respectively. Each figure shows the ODSs corresponding to the three spatial components for the non-

magnified and magnified states. The methodology reveals the high order mode shapes from nothing initially distinguishable in the non-magnified state.

The results demonstrate the benefits of motion magnification in DIC measurements. However, DIC also enriches simple visualisation of videos. The colour maps allows the user to achieve a full interpretation of the deformation, including secondary motion which is indistinguishable by naked eye. Moreover, in previous works edge detection algorithms were considered instead of DIC to highlight the 2D deflection shape on beam-like structures (J. G. Chen et al. 2015; Y. Yang, Dorn, Mancini, Talken, Nagarajaiah, et al. 2017; Y. Yang, Dorn, Mancini, Talken, Kenyon, et al. 2017). However, this methodologies are not valid for plate-like structures. Three-dimensional displacement measurement employing 3D-DIC is hence demonstrated to be advantageous.

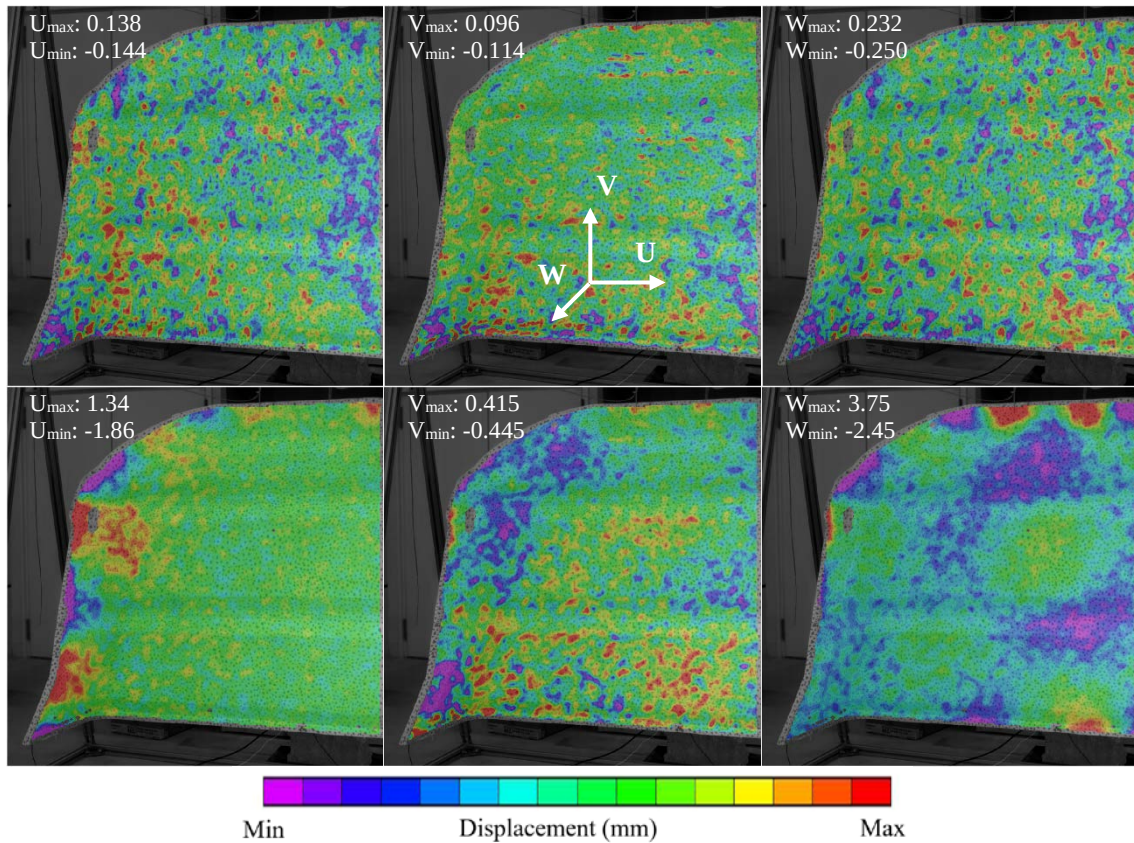


Fig. 54. Operational Deflection Shape of the composite panel at 285 Hz from non-magnified images (upper row) and 100x magnified images (lower row).

Capítulo 8

Conclusiones

En esta tesis, se ha mostrado el potencial de HS DIC para la medida de vibraciones y el análisis modal experimental por medio de las metodologías recogidas en los artículos. Estos trabajos contribuyen a la difusión de HS DIC con contenido original y destacables resultados.

En el primer trabajo se recogen aspectos de interés para la identificación modal en campo completo utilizando HS 3D-DIC para ensayos por movimiento de la base. El método de ajuste del círculo ha sido propuesto como un método no exigente computacionalmente teniendo en cuenta la cantidad de datos procesada. Las funciones de transmisibilidad, obtenidas directamente en los ensayos, han sido transformadas en sus equivalentes FRFs para una aplicación adecuada del método del círculo bajo consideraciones de un grado de libertad. Este tipo de consideraciones no fueron consideradas en ningún trabajo previo sobre funciones de transmisibilidad obtenidas con DIC. Como resultado, se obtuvieron las frecuencias naturales, las formas de los modos y los factores de amortiguamiento sin recurrir a técnicas de medida adicionales ni a la ejecución de ensayos complementarios. Esta metodología, aplicada a la viga en voladizo, ha demostrado ser consistente y precisa. Se obtuvo una gran homogeneidad de las frecuencias naturales y de los factores de amortiguamiento en campo completo en la superficie de la viga. La precisión de la medida fue probada comparando con las medidas tomadas por el acelerómetro de validación. Además, la metodología ha sido validada mediante modelos analíticos y numéricos. Se comprobó que las frecuencias naturales obtenidas experimentalmente eran similares a

las de estos modelos. Por otra parte, el indicador MAC mostró un alto nivel de correlación entre las formas de los modos obtenidas experimentalmente y las de los modelos de validación.

Respecto al método de ajuste por el círculo, fue posible obtener la caracterización modal en la práctica totalidad de la superficie de la viga de una manera eficiente y más precisa que el método de la media potencia, frecuentemente empleado en la bibliografía. No obstante, existen limitaciones en este método derivado de la simplificación del sistema a un grado de libertad. En el caso de modos muy cercanos, este método pierde precisión puesto que es incapaz de diferenciar la influencia de los modos externos.

Tanto en este estudio como en la bibliografía se ha señalado la menor sensibilidad de HS DIC, lo que dificulta realizar medidas de bajos desplazamientos encontrados en ensayos de alta frecuencia o de un espectro de medida extenso. Como segundo trabajo, la metodología para la combinación de HS DIC y la magnificación del movimiento ha sido capaz de proporcionar formas de deflexión significativas de ensayos con bajos desplazamientos donde la medida de DIC inicial estaba fuertemente contaminada de ruido. De este modo, esta metodología se indica para filtrar el ruido presente en la medida de ODSs a altas frecuencias. Para frecuencias bajas, se pueden obtener resultados de alta calidad con esta metodología utilizando bajos niveles de excitación en lugar de niveles elevados. Esto es posible gracias a que con el aumento de la amplitud del movimiento se alcanzan desplazamientos mayores que los obtenidos aumentando físicamente la ganancia de la excitación. Por tanto, el shaker no precisa trabajar a altos niveles de carga y excitadores menos potentes podrían ser empleados.

Además, la combinación con HS 3D-DIC conlleva un avance muy significativo respecto a los trabajos bidimensionales previos que emplean una única cámara. Esta metodología facilita la interpretación del movimiento y de los ODS de elementos complejos cuya dificultad de análisis sería notable mediante la simple visualización desde un punto bidimensional. Por otro lado, los métodos de detección de aristas empleados en trabajos previos de magnificación del movimiento basada en la fase presentan limitaciones obvias en elementos con comportamiento de placa. Por consiguiente, el empleo de la magnificación y su combinación con DIC ha sido así habilitado para la mayoría de aplicaciones prácticas, permitiendo la comparativa en campo completo con simulaciones FEM.

Pese a que no es posible obtener una estimación de los desplazamientos originales a partir de las medidas en imágenes magnificadas, ha sido demostrado que es posible obtener la forma real independientemente de la amplitud. De este modo, la combinación presenta gran utilidad ya que la amplitud en términos de desplazamientos no es el objetivo de la caracterización sino la proporción, es decir, la forma.

En general, la combinación de DIC y magnificación significa un avance importante para la interpretación de formas modales gracias a la visualización cualitativa del movimiento magnificado y a la información cuantitativa proporcionada por los desplazamientos en campo completo.

Chapter 8

Conclusions

In this thesis, it has been shown the potential of HS DIC to measure vibrations and experimental modal analysis through the methodologies exposed in the papers. These works contribute to the expansion of HS DIC with original contents and remarkable results.

In the first work some interesting features are involved for full-field modal identification using HS 3D-DIC when base excitation tests are required. The circle-fit approach has been proposed as a non-demanding method considering the amount of processed data. The transmissibility functions, directly obtained from measurements, have been transformed into their equivalent FRFs for proper circle-fit method application under single-degree-of-freedom assumptions. It has not been considered in any previous work on DIC transmissibility functions. As a result, natural frequencies, mode shapes and damping ratios were determined without the support of any other techniques, nor the execution of additional tests. This methodology, applied to a cantilever beam, has proved to be consistent and accurate. Great homogeneity was achieved along the beam regarding natural frequencies and damping ratios from a full-field estimation. Accuracy was proved by comparing DIC measurement with accelerometer measurements. Furthermore, the methodology was validated by comparing with analytical and numerical models. Experimental natural frequencies were similar with those from these models. In particular, mode shape comparison was quantitatively made by the MAC indicator, which showed good correlation.

Regarding circle-fit method, it was able to provide modal characterisation in almost the whole surface in an efficient manner and more accurate than half-power method, frequently employed in the bibliography. However, there are limitations regarding the circle fit approach mainly from the SDOF assumption. In the case of very close modes, this approach becomes less accurate and sometimes unable to discern the influence of a close mode.

One main issue of HS DIC encountered here and in referenced work is the lower sensitivity, with usual difficulties to measure low displacements involved at high frequencies and in broadband measurement. With the second work, the methodology for the combination of HS DIC and motion magnification was able to provide meaningful deflection shapes from low displacements tests where DIC measurements are dominated by noise. Hence, this methodology is indicated to filter the noise out in high frequency ODSs. For lower frequencies, high quality results can be obtained with this methodology in a low excitation instead of employing high level of excitation. The reason is that with motion magnification it is possible to achieve larger displacements than physically increasing the gain of the exciter. Hence, the shaker does not have to work overloaded, and a smaller exciter may be used.

Moreover, the combination with HS 3D-DIC implies a significant advance regarding 2D previous works using a single camera. This methodology ease the interpretation of the motion and ODSs of a complex element which would be difficult to analyse by visualization from a 2D point of view. Moreover, edge detection, employed in previous phase-based motion magnification studies, presents evident limitations in plate-like structures. Thus, the use of motion magnification and its combination with DIC is extended to most practical real applications, enabling full-field comparison with FEM models.

Despite that it is not possible to estimate the original displacements from measurements on magnified images, it was proved that the actual shape is obtained independently of the amplitude. Thus, the combination is quite useful since the amplitudes in displacements are not the objective but the proportion, i.e., the shape.

As a whole, the combination of DIC and phase-based motion magnification provides an important advance in the interpretation of mode shapes due to the qualitative visualisation of the magnified motion and the quantitative information using the full-field displacements.

Capítulo 9

Trabajos futuros

Con esta tesis, queda establecida una línea de investigación para la continua mejora de las metodologías de HS DIC en análisis modal experimental y sus aplicaciones derivadas. En concreto, algunas conclusiones de este trabajo señalan evidentes campos de trabajo.

En la metodología de análisis modal experimental en funciones de transmisibilidad, las consideraciones de un grado de libertad son válidas para casos como el de la viga en voladizo donde los modos están muy separados. Sin embargo, esta consideración no es siempre válida y sería necesario emplear métodos MDOF. Teniendo en cuenta que los métodos FRF-múltiple tales como el de ajuste exponencial complejo por mínimos cuadrados es incapaz de manejar todo el conjunto de datos de manera simultánea, los métodos MDOF FRF-singular surgen como los métodos probablemente más eficientes y precisos para la identificación modal. En este sentido, el trabajo futuro consistiría en el estudio del método no lineal de mínimos cuadrados y el método de cocientes polinómicos así como la evaluación en comparación con otros métodos. Con el objetivo de abarcar todos los casos, debería ser implementado tanto para el análisis de FRFs como de funciones de transmisibilidad. No obstante, el continuo desarrollo de los algoritmos de identificación modal contribuirá a la difusión de DIC en este campo.

En cuanto a la menor sensibilidad de HS DIC respecto a otras técnicas y sensores, esta es una limitación técnica de la resolución sensor de las cámaras de alta velocidad, lo que se espera que continúe evolucionando en el futuro gracias a la electrónica mejorada. Mientras tanto, el análisis modal experimental está restringido a espectros de baja frecuencia. Las cámaras de video estándar, en cambio,

proporcionan una mayor resolución espacial en detrimento de la velocidad de captura. Estas incrementarían la sensibilidad de DIC limitando el espectro de análisis. Sin embargo, los amortiguamientos y las formas de los modos podrían ser obtenidos con cámaras estándar a mayores frecuencias pese a que apareciesen como frecuencias alias. Mediante simulaciones numéricas o medidas puntuales con otras técnicas se podría determinar el valor real de estas frecuencias. Esta idea podría ser probada en un elemento con pocos modos en un espectro de 0 a 10 kHz con el objetivo de obtener picos alias muy claros en el espectro analizado. Para este propósito, la viga en voladizo empleada en este estudio sería ideal.

En esta tesis, la combinación de magnificación del movimiento y DIC ha sido evaluada y validada para la obtención de ODSs. No obstante, únicamente se ha empleado excitación sinusoidal. En este sentido, se plantea el empleo de esta metodología para eventos con un mayor espectro de excitación. Entre ellos, cabría destacar la excitación de ruido blanco de la que podría extraerse mayor información de un único ensayo. Otro caso de interés concierne a la respuesta de los sistemas frente a eventos de carácter transitorio como puede ser un impacto de baja energía. En ambos casos, se podrían amplificar determinadas componentes de la respuesta para realzarlas sobre el resto.

Por otro lado, considerando las ventajas que ofrece DIC y la creciente popularidad de las técnicas de visión para la monitorización y evaluación (Ye, Dong, and Liu 2016; Poudel, Fu, and Ye 2005; Patsias and Staszewski 2002; Wei Fan and Pizhong Qiao 2011; Feng and Feng 2017; Busca et al. 2014; Kohut et al. 2013; Dworakowski et al. 2016; Bing Pan, Tian, and Song 2016; L. Felipe-Sesé and Díaz 2018), la detección de daño basada en vibración es una aplicación emergente. La información en campo completo solventaría la limitación de los métodos de detección de daño basados en el análisis de formas modales o en la curvatura modal como consecuencia de la baja resolución espacial. Por tanto, un trabajo futuro abordaría la tarea de explorar las capacidades y aplicaciones de DIC en ensayos y evaluación no destructivos, donde la magnificación del movimiento podría jugar un papel relevante. Este trabajo podría constituir la investigación posdoctoral y futuras tesis.

Para finalizar, HS DIC posee un gran potencial para aplicaciones industriales a consecuencia de las ventajas que ofrece frente a instrumentaciones tradicionales. Parte del trabajo desarrollado en esta tesis ha sido realizado empleando un material de referencia con comportamiento de viga en voladizo. Del mismo modo, gran parte de los trabajos bibliográficos analizaron simples elementos tipo viga o placa. En un futuro, se considera necesario explotar estas metodologías en elementos reales e industriales.

Chapter 9

Future works

With this thesis, it has been established a line of research to keep on improving the methodologies of HS DIC in experimental modal analysis and related applications. Some conclusions of this research remarks evident fields of work.

In the methodology for experimental modal analysis in transmissibility functions, the SDOF assumptions are appropriate considering the separated modes of the cantilever beam. However, this is not always the case and a MDOF approach would be required. Taking into account that a multi-FRF approach such as Least Squares Complex Exponential is unable to deal with the whole data set simultaneously, the MDOF single-FRF approaches likely arise as the most efficient, accurate methods for modal identification. In this sense, the future work would consist in studying the Nonlinear Least Squares and Rational Fraction Polynomial methods and evaluating them in comparison with other methods. In order to achieve a wide use, it should be considered both for FRFs and transmissibility functions analysis. Overall, the continuous development of modal identification algorithms and methods will help to spread the use of DIC for experimental modal analysis.

The main issue of HS DIC is the relatively lower sensitivity in comparison with other experimental techniques and sensors. This is a technical limitation from the sensor resolution of the high speed cameras, what is expected to evolve with improved hardware in the future. Meanwhile, the experimental modal analysis is currently restricted to lower frequencies. Standard video cameras provide higher resolution images to the detriment of the frame rate. They would provide more sensitive DIC

measurement but limiting the analysed spectrum. However, information of damping and mode shapes might be extracted with standard cameras at higher frequencies although aliasing occurs. Numerical simulation or selected pointwise measurement with other techniques could provide the actual value of the aliased natural frequencies. This idea could be tested in an element with few modes in a spectrum from 0 to 10 kHz to obtained clear alias peaks in the spectrum to be analysed. The cantilever beam employed in this thesis would be ideal for this purpose.

In this thesis, the combination of motion magnification and DIC to obtain ODSs has been evaluated and validated. Nevertheless, only sinusoidal excitation has been employed. Hence, it is set out employing the methodology for wider excitation spectrum events. For instance, it should be consider the white noise excitation to extract multiple information from a unique test. An additional case is the response to transit forces such as low energy impacts. In both cases, certain components would be amplified to stand them out from the rest.

Considering the advantages of DIC and the increasing popularity of vision-based monitoring and evaluation (Ye, Dong, and Liu 2016; Poudel, Fu, and Ye 2005; Patsias and Staszewskiy 2002; Wei Fan and Pizhong Qiao 2011; Feng and Feng 2017; Busca et al. 2014; Kohut et al. 2013; Dworakowski et al. 2016; Bing Pan, Tian, and Song 2016; L. Felipe-Sesé and Díaz 2018), vibration-based damage detection is an emerging application. Full-field information would solve the limitation of damage detection methods based in the analysis of mode shapes and modal curvature due to sparse sensors. Thus, a future work would address the task of exploring the capabilities and applications of DIC in Non-Destructive Testing and Evaluation. Here, motion magnification might play a relevant role. This work could define the postdoctoral research and future theses.

Finally, HS DIC has a great potential for industrial applications considering the advantages over traditional instrumentations. Part of the work developed in this thesis was performed using a reference material with cantilever behaviour. In the same way, most of the referred literature analysed beam-like or plate-like simple elements. In the future, it is considered to explore this methodologies in real and industrial component.

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APPENDIX A

PUBLISHED PAPERS IN THE JOURNAL CITATION REPORTS

JOURNAL: Measurement Science and Technology

ISSN: 0957-0233

IMPACT FACTOR: 1.585

CATEGORIES: Engineering, Multidisciplinary (29/85), Instruments & Instrumentation (28/58)

TITLE: Full-field modal analysis during base motion excitation using high-speed 3D digital image correlation

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YEAR: 2017

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JOURNAL: Mechanical Systems and Signal Processing

ISSN: 0888-3270

IMPACT FACTOR: 4.116

CATEGORIES: Engineering, Mechanical (4/130)

TITLE: High frequency mode shapes characterisation using Digital Image Correlation and phase-based motion magnification

VOLUME: 102

PAGES: 245-261

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AUTHORS: A.J. Molina-Viedma, L. Felipe-Sesé, E. López-Alba, F. Díaz.

JOURNAL: Mechanical Systems and Signal Processing**ISSN:** 0888-3270**IMPACT FACTOR:** 4.116**CATEGORIES:** Engineering, Mechanical (4/130)**TITLE:** 3D mode shapes characterisation using phase-based motion magnification in large structures using stereoscopic DIC**VOLUME:** 108**PAGES:** 140-155**YEAR:** 2018**DOI:** 10.1016/j.ymssp.2018.02.006**AUTHORS:** A.J. Molina-Viedma, L. Felipe-Sesé, E. López-Alba, F. Díaz.

Full-field modal analysis during base motion excitation using high-speed 3D digital image correlation

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Abstract

In recent years, many efforts have been made to exploit full-field measurement optical techniques for modal identification. Three-dimensional digital image correlation using high-speed cameras has been extensively employed for this purpose. Modal identification algorithms are applied to process the frequency response functions (FRF), which relate the displacement response of the structure to the excitation force. However, one of the most common tests for modal analysis involves the base motion excitation of a structural element instead of force excitation. In this case, the relationship between response and excitation is typically based on displacements, which are known as transmissibility functions. In this study, a methodology for experimental modal analysis using high-speed 3D digital image correlation and base motion excitation tests is proposed. In particular, a cantilever beam was excited from its base with a random signal, using a clamped edge joint. Full-field transmissibility functions were obtained through the beam and converted into FRF for proper identification, considering a single degree-of-freedom theoretical conversion. Subsequently, modal identification was performed using a circle-fit approach. The proposed methodology facilitates the management of the typically large amounts of data points involved in the DIC measurement during modal identification. Moreover, it was possible to determine the natural frequencies, damping ratios and full-field mode shapes without requiring any additional tests. Finally, the results were experimentally validated by comparing them with those obtained by employing traditional accelerometers, analytical models and finite element method analyses. The comparison was performed by using the quantitative indicator modal assurance criterion. The results showed a high level of correspondence, consolidating the proposed experimental methodology.

Keywords: full-field measurement, experimental modal analysis, transmissibility functions, high-speed digital image correlation

(Some figures may appear in colour only in the online journal)

1. Introduction

Experimental modal analysis is an essential part of the characterisation of the dynamic behaviour of structures [1]. Different transducers and different modal identification algorithms have commonly been developed and employed throughout the years, in the search for more realistic characterisation.

Theoretical and numerical models have shown significant improvement by using experimental validation. Modal identification algorithms are designed to manage the frequency response functions (FRFs) and transfer functions, which relate the displacement response of the structure to the force excitation. As a result, natural frequencies, mode shapes and damping ratios are obtained.

In recent years, research has been focussed on exploring the capabilities of digital image correlation (DIC) combined with high-speed cameras (HS DIC) [2–7]. DIC is a consistent and powerful optical technique for obtaining full-field displacement measurements [8], and only requires one image for each deformation state to be analysed. This interest is due to the intrinsic limitations of traditional transducers, like accelerometers, which can be invasive when used on light-weight elements, modifying their natural frequencies. This phenomenon is widely known as the mass-loading effect [1]. Moreover, most of the transducers perform pointwise measurements, and hence only a limited number of points are measured in the whole structure. To overcome these disadvantages, HS DIC offers a non-invasive full-field displacement measurement and a considerable improvement in the performance of experimental modal analysis. On the other hand, DIC is considerably less sensitive than accelerometers and the equipment is also expensive. More detailed comparisons between accelerometers, laser vibrometry and DIC have been made [4, 7, 9].

Complete modal characterisation has already been performed using HS DIC. Different broadband excitation, such as that of impact [10] or random signal [9, 11], was applied to generate full-field FRFs. However, modal testing is not restricted to the evaluation of transfer functions between the force input excitation and response. In fact, some experimental scenarios involve tests in which it is not possible to register the excitation magnitude as a force. One of the most common tests evaluates the response of the structure when it is excited by base motion. The transfer function in this case relates the transmission of the base motion to the structure response motion. Thus, the transfer function is known as the transmissibility function. Despite the similarities between FRFs and transmissibility functions, straight modal identification on transmissibility functions must be avoided if, as is usual, FRF-based algorithms are considered [12].

In this way, base motion excitation was performed by Ha *et al* [13] to obtain the mode shapes of an artificial beetle's wing in forced normal mode (FNM) tests employing HS DIC. The natural frequencies had previously been identified in a swept-sine test using a laser vibrometer. No transmissibility function was determined from the DIC data. In later work, Ha *et al* [14] obtained the transmissibility functions of the wing using HS DIC. However, the mode shapes were measured in FNM tests rather than transmissibility functions. Furthermore, the damping ratios were calculated using the half power method from the smoothed transmissibility functions. Thus, no previous work on HS DIC and base motion excitation for a complete modal analysis is found in the literature.

In the present work, a complete experimental methodology for full-field modal analysis based on transmissibility functions is proposed, in this way avoiding FNM tests for mode shape characterisation. A set of tests on a calibrated cantilever beam [15] has been performed to validate this experimental methodology. The beam was excited using a broadband random signal when clamped to the shaker's armature. The natural frequencies, mode shapes and damping ratios were determined using EasyMod algorithms [16]. Despite the benefits of full-field displacement, managing the huge amount of

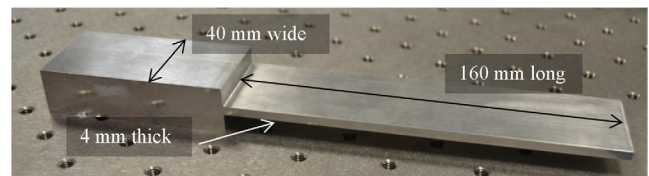


Figure 1. The cantilever reference material, RM, with dimensions.

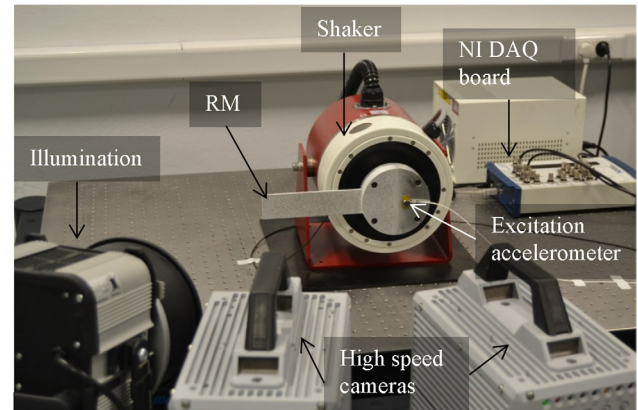


Figure 2. Optical set-up for experimental modal analysis.

data generated by DIC is a challenge to modal identification. A typical DIC measurement involves a huge number of points (of the order of 10^4 points according to the current state of technology) whereas traditional test instrumentation consist of tens or hundreds points at most. Multi-FRF identification methods, which evaluate the whole FRF data set simultaneously, may thus be unfeasible. Due to its simplicity and the lower computational costs, a single-FRF circle-fit approach [1] was applied, based on single degree of freedom (SDOF) assumptions. Considering the differences between the transmissibility functions and FRFs, this study is also a step forward regarding previous work. A theoretical transformation from transmissibility functions to FRFs was performed to increase the accuracy of modal identification, according to the circle-fit assumptions.

To validate the experimental methodology, the theoretical modal behaviour of the cantilever beam was compared with the modal parameters from the experimental modal analysis. The modal behaviour of a cantilever beam is well described by theory and it can be commonly found in the bibliography as one of the examples of continuous system vibration analysis [17]. From an experimental point of view, DIC transmissibility functions were compared with the transmissibility functions simultaneously obtained by accelerometer measurements. Considering the apparently relatively lower sensitivity of DIC, evaluation of the coherence functions is presented here as an interesting indicator of the measurement quality along with the identification of floor noise in the power spectral density (PSD). As temporal aliasing is an intrinsic limitation of DIC measurements [9, 18], accelerometer signals were also employed to detect the possible aliased frequencies in the DIC transmissibility functions. To conclude the validation, a comparison was also performed with finite element method (FEM) modal analysis. A correlation of the mode shapes between the methodologies was also performed using a modal

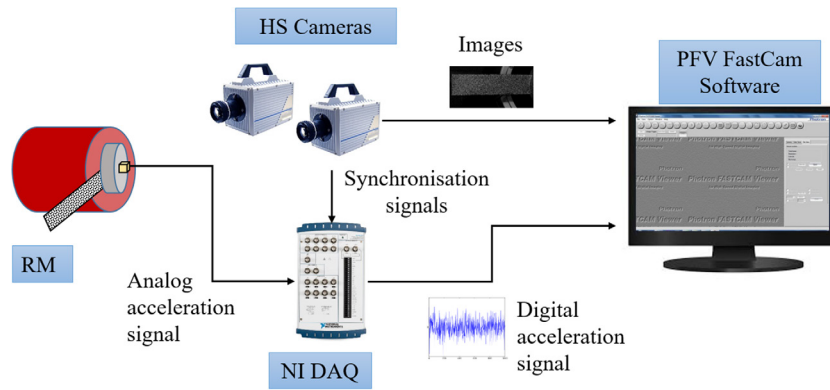


Figure 3. DIC experimental measuring diagram.

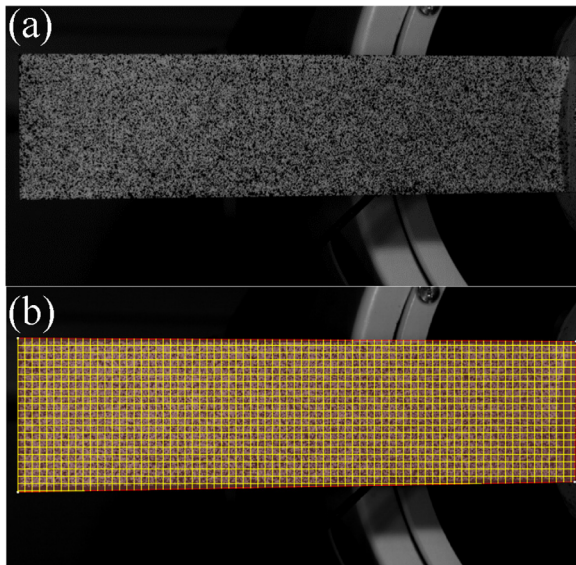


Figure 4. (a) The speckle pattern on the upper face of the beam; (b) the area of interest and the facet size visualisation.

assurance criterion (MAC) [19]. Thus, a quantitative indication of the mode shape agreement was provided.

2. Experimental procedure

2.1. Cantilever reference material

The analysed element was a stepped cantilever bar employed as a reference material (RM) in a previous validation analysis [15]. This design shows high conformity with the analytical approaches due to its stable and reproducible behaviour. Hence, it is conventionally employed for the calibration of the full-field optical techniques of displacement and strain measurements comparing the full-field measurement with the analytical results, both static and dynamic [20].

In this work, the RM was considered suitable for proper validation of the proposed experimental methodology by comparing the results with analytical and numerical solutions.

The RM dimensions are shown in figure 1. It is made of aluminium 2024-T3, the thinner part is 4 mm thick and 160 mm long, the thicker part is 20 mm thick and 80 mm long and the

width is 40 mm throughout the bar. Thus, when the thicker part is clamped, the thinner part behaves as a cantilever beam.

For experimental modal analysis, the RM was clamped to the armature reproducing a perfectly rigid boundary condition. Therefore, vibration was induced in the thinner part of the RM by transmitting the motion of the shaker through the clamp. The symmetrical configuration made it possible to obtain typical cantilever mode shapes in an easy way. To determine the beam transmissibility functions, a broadband spectrum excitation was applied [1]. In this study, a random signal was chosen. This signal consists of a flat profile in the frequency domain from the 10–1000 Hz spectrum. Within this range, two bending modes appeared during the excitation previously mentioned.

2.2. DIC analysis

The full-field response of the beam was captured by a stereoscopic system. Two high-speed camera FastCam SA4 models produced by Photron with 50 mm $f/1.4D$ focal length lenses (Nikon) were employed for recording the upper face of the cantilever, figure 2. The frame rate was set to 2000 fps, according to Nyquist's frequency criterion [21], and the resolution was adjusted to 1024×512 pixels obtaining 10500 frames during each test, limited by the internal memory of the camera. Two different random spectrum tests were conducted in order to increase the number of frames and hence reduce the presence of noise when averaging the PSD according to Welch's method.

The lens exposure time was set to 0.1 ms in order to achieve a sampling period ten times shorter than the highest excitation frequency period. To improve the lack of light due to the short exposure time, a 150 W lamp was adopted to guarantee a uniform light intensity over the specimen, as shown in figure 2.

An accelerometer was set over the clamping system, as shown in figure 2, to register the base motion excitation, and it was synchronised with the stereoscopic system. A DAQ system (NI USB-6251 DAQ) digitalised the accelerometer signal and synchronised it with the camera acquisition. All the devices were synchronised using the same sampling parameters and a perfect correlation was achieved between the response and the excitation measurements for the evaluation of the transmissibility functions. For a better understanding of

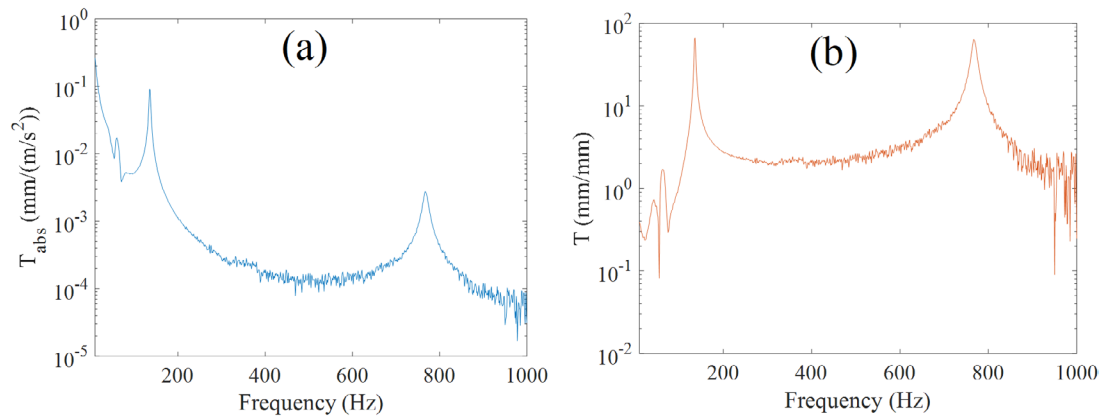


Figure 5. The experimental transmissibility function at the point of the free end of the beam: (a) considering the absolute response and base motion in terms of acceleration; (b) considering the relative response and base motion in terms of displacement.

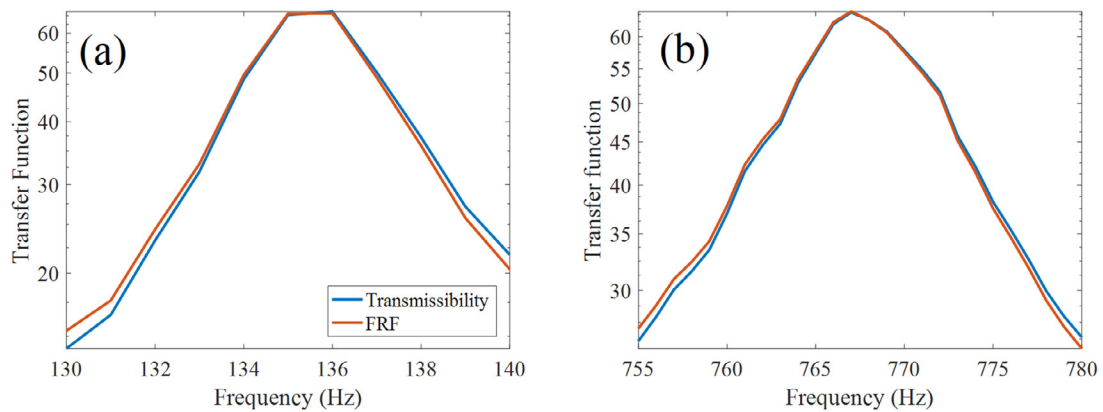


Figure 6. A comparison between the transmissibility function (T) and the FRF (H) at the point of the free end of the beam: (a) first mode; (b) second mode.

the set-up, a complete measurement diagram is represented in figure 3.

The DIC measurement required surface preparation: since the surface colour of the beam was homogeneous, a grey-scale random pattern had to be applied by spraying black paint over a white background. The images captured during the tests were processed using Vic-3D software (produced by Correlated Solutions Inc.) for 3D DIC analysis [8]. Before processing the displacements, Vic-3D was employed to determine the calibration parameters of the stereoscopic system of the camera. Considering the size of the specimen, a grid of 12×9 points with a spacing of 4 mm was chosen. As a result, a pixel deviation of just 0.015 was achieved, which implies excellent calibration.

The area of interest was divided into facets, each of which is a measurement point. The facet size must be chosen according to the speckle size, shown in figure 4(a). The speckle was applied to cover about 6 pixels as an average size [22]. The processing parameters had a facet size of 13 pixels and an overlap of 4 pixels. The grid shown in the area of interest (figure 4(b)) is a representation of the chosen facet size. Thus, DIC processing provided a matrix of 66×246 points, where every element of the matrix contained a vector of time domain displacement.

The proposed methodology for frequency analysis consists of determining the transmissibility function at every

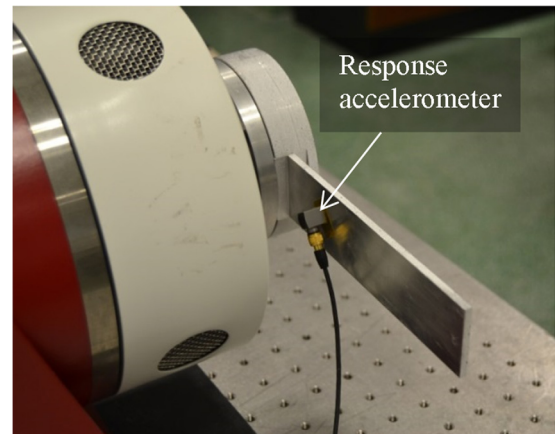


Figure 7. Additional accelerometer for response measurement to be compared with the 3D-DIC results.

data point provided by DIC displacement measurements with respect to the excitation accelerometer signal. In this sense, Welch's averaging method [23] was applied using a script developed in Matlab. Since the image acquisition rate for both cameras was 2000 frames per second, windows of 2000 frequency lines were defined to provide a frequency resolution of 1 Hz. Since there is discontinuity between the two random tests they were independently processed employing Welch's

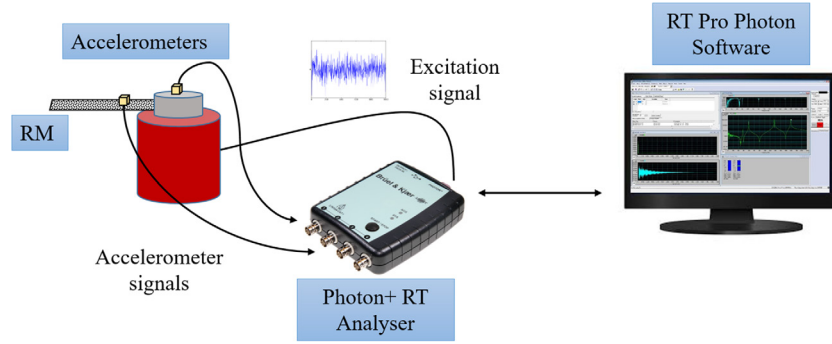


Figure 8. Accelerometer experimental measurement diagram.

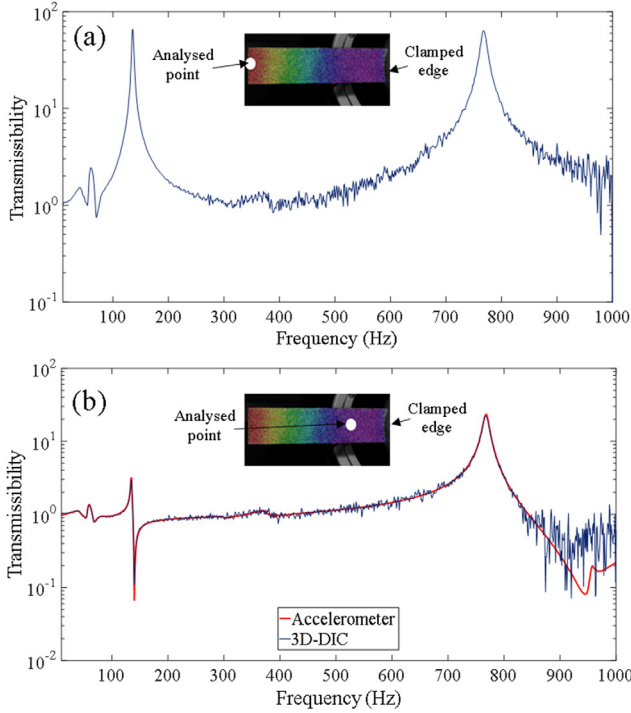


Figure 9. The absolute transmissibility function from 3D-DIC measured at (a) the point of the tip of the RM from the 3D-DIC, (b) the point where the accelerometer was located.

method. A Hanning window was adopted to avoid leakage, and an overlap of 75% was employed to increase the number of averages. As a result, two transmissibility function sets were obtained and the final transmissibility function was calculated at each point as the mean value of these data sets.

2.3. Circle-fit modal identification

In the studied case, the transmission of the base motion to the response experienced in the beam was analysed. Nevertheless, the circle-fit approach [1] was also employed for modal identification from the FRFs, based on the relationship between the displacement response and the force excitation. The circle-fit considers that just one mode influences the behaviour of the structure in the vicinity of the peak. In the same way, conversion is here proposed from transmissibility to its equivalent FRF, considering that in every peak the SDOF theoretical equations govern the dynamic behaviour. Thus, it can be found that [1]

$$H(i\omega) = \frac{T(i\omega)}{(\omega/\omega_n)^2} \quad (1)$$

where H represents the FRF, T is the transmissibility function considering the relative response displacement from the base motion excitation, ω_n is the natural frequency of the system and ω is the frequency vector from the corresponding spectrum.

According to the adopted experimental set-up, this conversion is not straightforward. The transmissibility functions obtained by T_{exp} represent the relationship between the absolute displacement response D_{DIC} , using DIC, and the base motion acceleration A_{acc} , using the accelerometer, both in the frequency domain. The transmissibility in equation (1) is a non-dimensional relation between the response and excitation motions; thus, the base acceleration must be integrated twice to determine the base displacement D_{acc} . Additional transformations are required in order to obtain transmissibility functions in terms of the relative displacement response. These evaluations in the frequency domain are represented as follows:

$$T_{\text{abs}} = \frac{D_{\text{DIC}}}{D_{\text{acc}}} = \frac{D_{\text{DIC}}}{A_{\text{acc}}/(i\omega)^2} = T_{\text{exp}} \cdot (i\omega)^2 \quad (2)$$

$$T = \frac{D_{\text{DIC}} - D_{\text{acc}}}{D_{\text{acc}}} = T_{\text{abs}} - 1. \quad (3)$$

Equation (3) represents the experimental relative transmissibility functions whose theoretical conversion can now be applied. In figure 5 the result of the transformation can be evaluated.

Hence, after detecting the resonance peaks, the experimental transmissibility function transformation in the vicinity of the analysed resonance is composed of the calculation of the absolute displacement transmissibility function by integrating the base acceleration using equation (2), the relative displacement transmissibility functions applying equation (3) and finally, the FRF, equation (1). Since the natural frequency of the analysed resonance must be previously known in equation (1), for this evaluation, it is applied with a proposed natural frequency $\omega_{n,0}$, as shown in equation (4). Then, the circle-fit approach determines the actual natural frequency ω_n , from the resulting FRF, H_0 . Since the natural frequency is constant, it does not distort the actual FRF shape. Once the actual natural frequency is determined, a second circle-fit

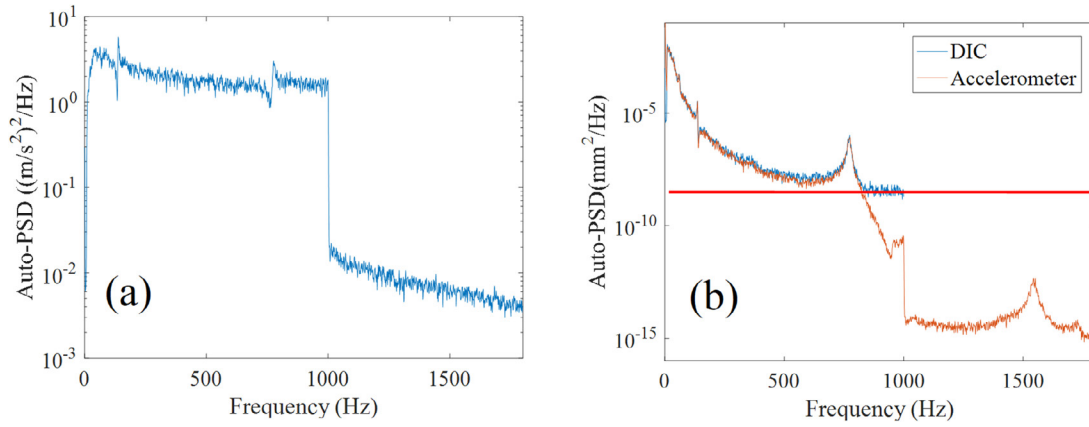


Figure 10. The PSDs of (a) the excitation and (b) the response obtained by DIC and the accelerometer.

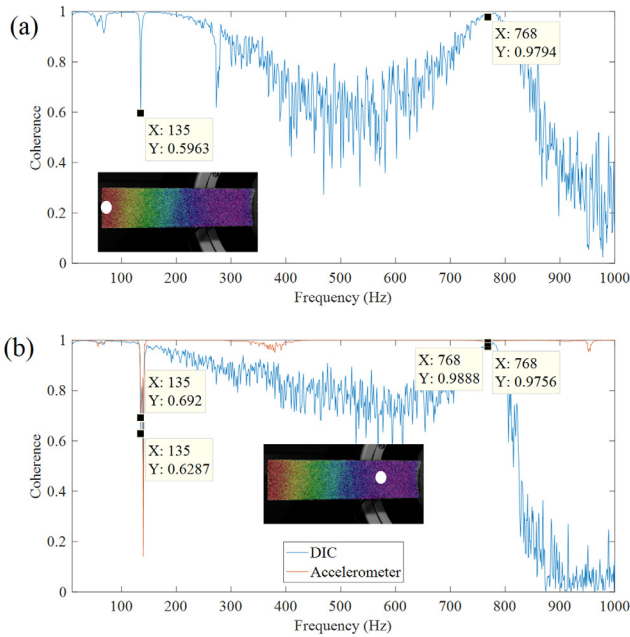


Figure 11. The coherence function at (a) the point of the tip of the RM, (b) the point where the accelerometer was located.

approach identification is necessary after correcting the FRF with the actual natural frequency, as shown in equation (5). The final result of the conversion is shown in figure 6, which shows the divergence of both kinds of transfer functions corresponding to the first two bending modes of the beam:

$$H_0 = \frac{T}{(\omega/\omega_{n,0})^2} \quad (4)$$

$$H = H_0 \cdot \left(\frac{\omega_n}{\omega_{n,0}} \right)^2. \quad (5)$$

Natural frequencies, mode shapes and structural damping loss factors η were identified by performing the circle fitting in every mode. The process was performed in the full-field using every measured point provided by DIC. Hence, the mode shapes were obtained by mapping these results. For the identification of a unique natural frequency and damping ratio, the mean from the mapping results was evaluated.

2.4. Experimental set-up for validation with accelerometer

An accelerometer (figure 7) employed for validation purposes was placed at 35 mm from the clamping grip so that the mass-loading effect was negligible. Its signal was processed together with the excitation accelerometer signal to obtain the transmissibility function at this point during the same random test. This study also allowed the identification of potential aliasing in the DIC measurement, so an alternative acquisition system was used. With a real time vibration analyser (Photon+ produced by Bruel and Kjaer), controlled by the commercial software RT Pro Photon, it was possible to perform this validation. The versatility of this system allowed for a wider spectrum—up to 3200 Hz—so that the potential aliased frequencies in the DIC measurements could be detected. In order to achieve the same frequency resolution as in the DIC transmissibility functions, 3200 frequency lines were established. Several windows were averaged employing a Hanning window with no overlapping because of the real time analysis.

Additionally, the analyser provided the excitation signal to the shaker. A representation of this set-up is shown in figure 8. Since the DIC and accelerometer measurements were performed simultaneously, a combination of figures 7 and 8 illustrates the global experimental set-up.

3. Results and discussions

The methodology is evaluated here and the most significant results are also presented. In section 3.1, the DIC measurements during the random test were validated based on the accelerometer measurements. This validation includes the identification of the noise floor in the DIC measurement and its influence on the coherence function as well as a determination of the possible aliasing. In section 3.2, the results are shown from a modal analysis on the equivalent FRFs using circle fitting, and a comparison is made of the modal parameters with the FEM and theoretical results.

3.1. Experimental validation

The transmissibility results according to the proposed methodology show two resonance peaks. Figure 9(a) shows the

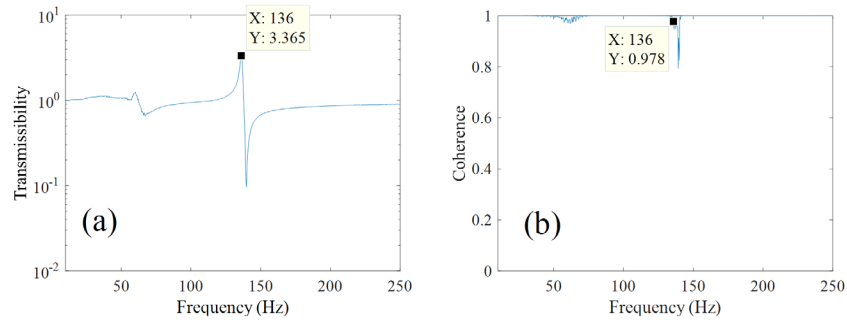


Figure 12. The transmissibility function (a) and coherence (b) provided by the accelerometer measurement in a higher frequency resolution test to analyse the influence of the resolution on coherence.

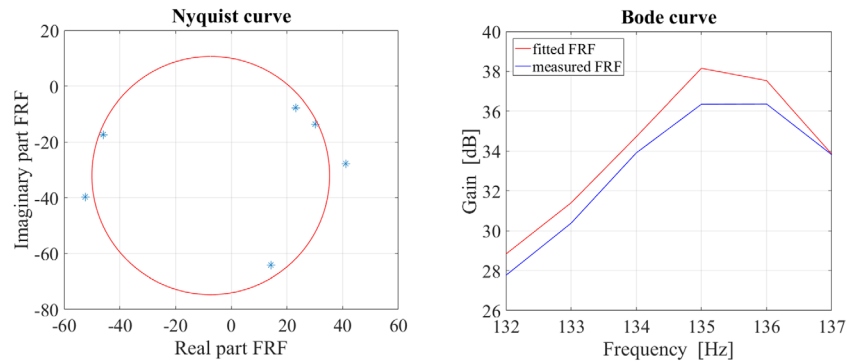


Figure 13. A Nyquist and Bode diagram of the circle-fitting of the first mode at the point of the tip of the beam.

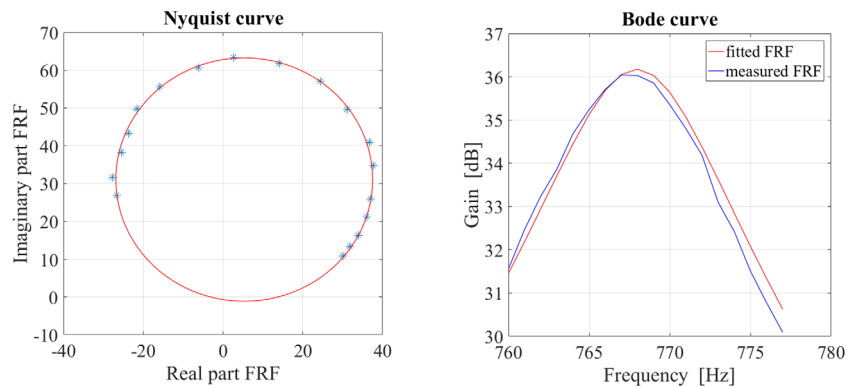


Figure 14. A Nyquist and Bode diagram of the circle-fitting of the second mode at the point of the tip of the beam.

Table 1. Statistical indicators of the natural frequencies, f_n , and structural damping, η , identified by the circle-fit approach in the 3D-DIC measurements.

Mode	f_n (Hz)		η (%)	
	Mean	Standard deviation	Mean	Standard deviation
1	135.38	0.08	1.69	0.08
2	767.78	0.74	1.45	0.04

absolute transmissibility functions versus the frequency. The results correspond to a point in the released tip of the beam, where the maximum response occurs.

Figure 9(b) shows the absolute transmissibility functions from the accelerometer placed on the beam compared with the DIC results at the same point. A high level of agreement is observed. However, the DIC transmissibility function is

Table 2. The natural frequencies of the two first modes identified by FEM and the theoretical analyses.

Mode	FEM	Theoretical
1	131.03	129.34
2	817.81	810.41

noisier than that of the accelerometer due to its lower sensitivity. This noise appears as a consequence of taking the measurement at a point with a low level of displacements, due to its location near the clamping grips. Furthermore, since a higher frequency involves lower displacement, conversely to acceleration, the signal-to-noise ratio also decreases at non-resonant high frequencies.

Aliasing, investigated on the basis of the excitation level, was negligible out of the excitation spectrum. In figure 10(a), the PSD of the excitation confirms this fact. Despite this,

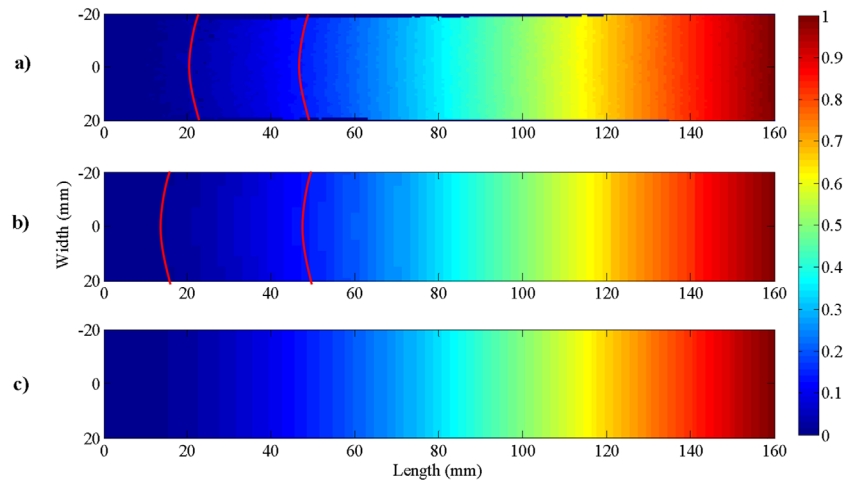


Figure 15. The mode shape of the first resonance: (a) the circle-fit approach from 3D-DIC; (b) FEM analysis; (c) theoretical analysis. The plate behaviour is highlighted by red lines.

figure 10(b) shows that the accelerometer set on the RM detected a peak of resonance around 1500 Hz. The PSD of the same point measured by the DIC is truncated at 1000 Hz, which is the Nyquist frequency. This could thus indicate some aliasing in the DIC measurements. However, it is seen that the DIC is unable to follow the accelerometer PSD above 800 Hz, providing a flat profile. This indicates the level of the noise floor, marked as a red line, and implies that the DIC is unable to identify any components under this level. The displacement signal provided by the accelerometer out of the excitation spectrum is many orders of magnitude lower than this threshold. Thus, it is demonstrated that DIC measurements are free from the aliased frequencies in this test.

The DIC coherence function at the tip (figure 11(a)) shows some interesting aspects as well. First, as was expected, the coherence tends quickly to zero, which means the presence of pure noise above 800 Hz, since it is below the floor noise. The coherence is also expected to be close to one at resonances where the level of noise is negligible due to the high level of response. In the same way, coherence should decrease in the range between both modes. As seen in figures 9 and 10, there is a significant presence of noise in this range, since the beam response is close to the noise floor. However, the first resonance value is defined by a low value of coherence. Figure 11(b) compares the coherence from DIC and the accelerometer; the accelerometer also shows this trend. This happened since the frequency resolution was not high enough to define a peak as sharp as the first one. An additional test was performed using the accelerometer set-up described in epigraph 2.4. In this case, the frequency resolution was increased to 0.25 Hz. In figure 12 it is seen that coherence has congruent values when the peak is better defined; only anti-resonance reduced the coherence.

3.2. Modal identification

The next step was to employ the circle-fit approach for modal characterisation, including the previous conversion of

transmissibility functions. Since this approach is an SDOF method of characterisation, one fitting per mode was performed. Therefore, for the first mode characterisation the interval from 132 Hz to 137 Hz was selected, whereas for the second mode the interval was from 760 Hz up to 777 Hz. Figures 13 and 14 show the Nyquist and Bode diagrams for both modes, respectively, at the point at the tip of the beam. As explained with the coherence function in figure 12, it was possible to improve the results with a higher frequency resolution, which implies either an increase in the number of images or a reduction of the analysed spectrum. Nevertheless, the aim of this study is focused on the methodology.

The adopted experimental methodology employs every single point of the displacement map for its evaluation. It provides three matrices with the natural frequency, the amplitude of the mode shape and the structural damping ratio. A single value for the natural frequency and damping ratio for each mode is obtained by calculating the mean of the natural frequency and the damping ratio matrices. These results are presented in table 1, showing consistent uniformity in both the natural frequencies and damping ratios.

The numerical modal analysis was performed using Abaqus software considering the experimental conditions; it consisted of 1600 S4R linear shell elements. The results of the natural frequencies of both bending modes obtained by performing FEM and theoretical analyses are presented in table 2. Comparing the results with the experimental ones, similar natural frequency values were obtained with a 5% error.

Finally, the results of the mode shapes corresponding to each analysed resonance frequency are presented from the experimental, FEM and theoretical analyses. The normalised results are presented as three contour plots in figures 15 and 16. From a qualitative point of view, the mode shapes are very similar for the three evaluated methods, and they clearly show typical bending modes. The first mode has one node with zero displacement at zero length, where the beam is clamped. The second bending mode implies an additional

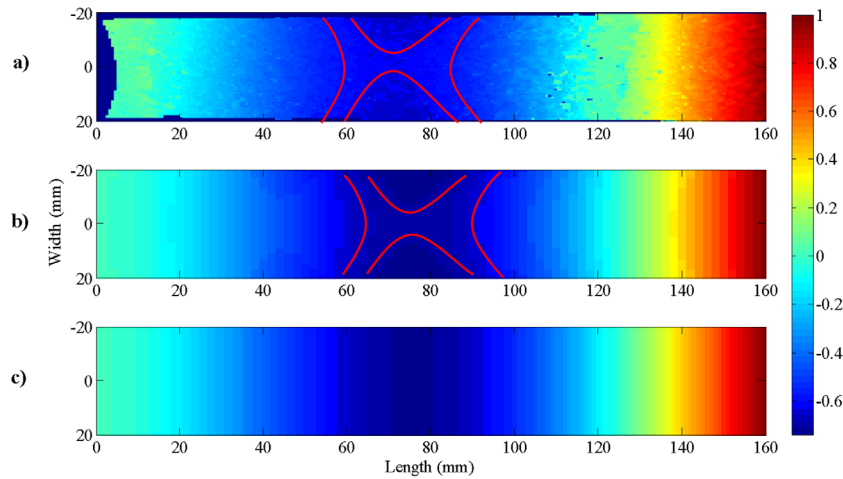


Figure 16. The mode shape of the second resonance: (a) the circle-fit approach from 3D-DIC; (b) FEM analysis; (c) theoretical analysis. The plate behaviour is highlighted by red lines.

Table 3. The MAC matrix from a comparison between the FEM/theoretical analyses and 3D-DIC.

		Experimental modal analysis	
		$f_1 = 135.38$ Hz	$f_2 = 767.49$ Hz
FEM analysis	$f_1 = 130.19$ Hz	1.0000	—
	$f_2 = 812.60$ Hz	—	0.9836
Theoretical analysis	$f_1 = 128.52$ Hz	0.9999	—
	$f_2 = 805.25$ Hz	—	0.9841

mode, at approximately 120 mm as shown in figure 16. Both modes reproduce the maximum displacements at the released tip of the cantilever, i.e. at 160 mm in the plots. As is observed in figure 16, the second mode shape is slightly noisier. As previously seen in figure 10, the displacements in this mode are closer to the floor noise. Furthermore, there is one more zero-displacement node, increasing the area of low displacement.

It is perceptible that only the numerical and the proposed full-field experimental procedures are able to appreciate the slight plate behaviour in the beam. As shown by the red lines in the figures, the contours have a certain curvature which involves deformation along the transverse direction. In the first mode (figure 15), it is observed in the interval from 20 mm to 60 mm, in the second mode (figure 16), this behaviour is detected clearly surrounding the middle of the beam due to its greater shape complexity. If only beam behaviour occurs, the contours should be parallel as the theoretical analysis shows (figures 15(c) and 16(c)).

Despite the agreement shown in the contour plots, for a more accurate comparison providing a quantitative indication, MAC was adopted to compare the experimental modal analysis with both the FEM and theoretical analysis. The application of MAC between FEM and the theoretical results makes no sense, since the objective of this work is the validation of the experimental methodology of the DIC by means of these two methods. The results shown in table 3 highlight that the correlation of the mode shapes is supported by values close to unity.

4. Conclusions

This work presents an experimental methodology for modal identification in base motion tests using HS 3D-DIC, founded on the interest of these kinds of experimental modal analyses and the features of DIC. The methodology takes into account the conversion of DIC transmissibility functions into equivalent FRFs based on SDOF assumptions for validating the use of modal identification algorithms. This has not been considered in previous work on DIC transmissibility functions. No additional technique or test for the identification of any modal parameters was employed. To address the challenge of managing the huge amount of data provided, the circle-fit method was proposed as the modal identification algorithm. This is particularly interesting for full-field modal identification since the method is less computationally demanding than multi-FRF methods and provides a high level of accuracy, although the SDOF method might not be suitable when the modes are very close.

The methodology performed the complete experimental modal characterisation of a cantilever beam. The full-field identification was consistent and congruent considering every single point, making it especially attractive for mode shape visualisation. An experimental validation of the DIC measurements was conducted by simultaneously employing an accelerometer. A comparison between the transmissibility functions from DIC and the accelerometer reveals a high level of correspondence. Analysis of the PSDs reveals DIC floor noise. Considering this threshold and the magnitude of the accelerometer PSD over the Nyquist frequency of the DIC test, it was concluded that no aliasing had occurred. This conclusion is crucial for validation of the results since no anti-aliasing filter can be applied to the DIC measurements.

The influence of floor noise was also revealed in the coherence functions. Regarding the current limitations of DIC regarding the sensitivity in comparison with other transducers, coherence is thus a powerful indicator for detecting poor components in the spectrum, especially when no information about the floor noise is known.

Furthermore, the experimental results were validated with analogous theoretical and numerical analyses. Both the natural frequencies and the mode shapes of the circle-fit approach were very similar to those from the theoretical and numerical methods. In particular, the MAC criterion was applied to the mode shape comparison. It is necessary to point out that this experimental methodology employs no filtering process to enhance the results. These facts have led to proof that the proposed experimental methodology is robust and consistent.

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High frequency mode shapes characterisation using Digital Image Correlation and phase-based motion magnification



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ABSTRACT

High speed video cameras provide valuable information in dynamic events. Mechanical characterisation has been improved by the interpretation of the behaviour in slow-motion visualisations. In modal analysis, videos contribute to the evaluation of mode shapes but, generally, the motion is too subtle to be interpreted. In latest years, image treatment algorithms have been developed to generate a magnified version of the motion that could be interpreted by naked eye. Nevertheless, optical techniques such as Digital Image Correlation (DIC) are able to provide quantitative information of the motion with higher sensitivity than naked eye. For vibration analysis, mode shapes characterisation is one of the most interesting DIC performances. Full-field measurements provide higher spatial density than classical instrumentations or Scanning Laser Doppler Vibrometry. However, the accurateness of DIC is reduced at high frequencies as a consequence of the low displacements and hence it is habitually employed in low frequency spectra. In the current work, the combination of DIC and motion magnification is explored in order to provide numerical information in magnified videos and perform DIC mode shapes characterisation at unprecedented high frequencies through increasing the amplitude of displacements.

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1. Introduction

Digital Image Correlation (DIC) [1] has been presented in the last decade as an alternative to classical transducers in vibration analysis [2]. High speed cameras allow the use of DIC in dynamic events. Hence, DIC offers solutions to overcome typical drawbacks of accelerometer such as mass addition and improves spatial resolution through full-field displacement measurements. The possibility that DIC offers showing high-resolution contour plots is especially interesting when Operational Deflection Shapes (ODSs) are evaluated exciting with sine signal [3–8] or mode shapes are obtained during broadband excitation [9–14]. In all cases, relevant information of the behaviour of the whole structure is provided, and hence constituting a powerful tool to validate numerical models. As DIC is a displacement measuring technique, low level of displacements related to high frequency excitation represents a challenging task. Cited work presented high quality results regarding ODSs and mode shapes covering a spectrum of hundreds of Hertz and just few of them obtained acceptable results up to 1500 Hz. High frequency characterisation is also conditioned by the resolution of high speed cameras. Due to current technical limitations of the cameras in the managing of the data memory, the image resolution has to be reduced when higher frame

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ratios are required. The relation of pixel-millimetre, on which DIC bases its accuracy [1], is compromised. Thus, DIC has been better positioned for low frequency events.

A new tool has been explored recently in the interpretation of subtle periodic motion in digital videos. It consists in phase-based motion magnification using image decomposition through complex steerable pyramid filters [15]. Motion magnification has multiple applications in revealing invisible periodic phenomena in the real world such as changes due to blood flow or even as a visual microphone extracting sounds from objects motion [16]. In particular, in this study the interest is focused on structural vibration analysis. Chen et al. [17] applied for first time phase-based motion magnification to visualise ODSs of different structures. The video of the specimen undergoing broadband excitation was magnified by identifying natural frequencies and magnifying the vicinity of the peak. Then, magnified reconstruction of the image shows clearer motion of the corresponding ODS. Yang et al. [18] proposed an autonomous phased-based methodology for complete modal analysis with practically no need for supervision. This methodology is proposed by the authors to monitor structures using just a video-camera with neither surface preparation, as is for DIC and laser vibrometry, nor a laborious, inefficient instrumentation. Although magnified motion visualisation is invaluable for the interpretation of the mode shape, there are cases in which magnification is not enough for a satisfactory visualisation and too much magnification blurs the images, so the perception of the shape of the motion is affected. More difficulties are found in higher order modes as a result of the increasing complexity of the shape.

With an adequate magnification, DIC technique is able to quantify motion that is not perceived by the naked eye, providing contour plots that offer a straightforward interpretation of the shape. Hence, DIC is a helpful complement in the search of a reliable tool to visualise and assimilate vibration shapes. Furthermore, the presentation of the results in form of displacement fields is suitable to compare with numerical models unlike simple visualisations. Additionally, full-field displacements provide more detailed information for model updating [4,19,20]. Thus, the combination of phase-based motion magnification and Digital Image Correlation would represent an interesting advance which allows the measurement of high quality displacements maps of periodic events even when low displacements (i.e. high order modes) occurs on the studied element.

In this work, a combination of DIC and phase-based motion magnification is explored in order to analyse mode shapes at unprecedented frequencies with DIC. The benefits of contour plots together with visualisation of magnified motion in the interpretation of video mode shapes are also presented. Special attention is paid to frequencies of the order of thousands of Hertz. To evaluate the procedure, a cantilever beam was analysed considering its separated and well-defined modes. Identification of natural frequency was firstly performed and compared with FEM model results. Subsequently, mode shapes were studied by acquiring images of the lateral edge of the beam while resonances are individually forced. Two extreme levels of excitation at each frequency, low and high, were studied to evaluate limitations in the measuring process, in the excitation hardware and in motion magnification. Limitation of resolution in high speed cameras to register high frequency mode shapes was worked out using strobe effect. Maximum resolution was thus available. Phase-based motion magnification method proposed in [15] was employed to evaluate improvements in contour plot video using different magnification factors. Mode shape of the FEM model was considered as reference to study the evolution of the error in the shape versus the magnification factor and analyse if this magnification procedure introduces some aberration in the image. A selection of tests is also presented in video format as online resources in the electronic version to enable the visualisation and interpretation of the motion in conjunction with DIC measurement and evaluate the capabilities of this combination.

2. Materials and methods

The proposed combination of DIC and phase-based magnification was performed in this study on a stepped bar of 2024 aluminium, shown in Fig. 1 which was designed for optical system calibration [21]. The behaviour of the thinner part corresponds to a cantilever beam and it is easy to predict by theoretical analysis or numerical models. The dimension of the thinner part was 160 mm in length, 40 mm in width and 4 mm in thickness.

The experimental procedure consisted in two stages of modal characterisation of the beam: natural frequencies identification and mode shapes characterisation. Excitation was performed by an electrodynamic shaker (Data Physics GW-V55/PA300E) in both cases. A clamping system fixed the beam from the thicker part to the shaker's armature, as seen in Fig. 2.

2.1. Natural frequencies identification

Natural frequencies were determined by inspecting the frequency response. As seen in Fig. 2(b), two accelerometers were employed, monitoring the excitation in the clamping device and the response signal at a point located 27 mm from the



Fig. 1. Specimen of analysis: stepped cantilever bar.

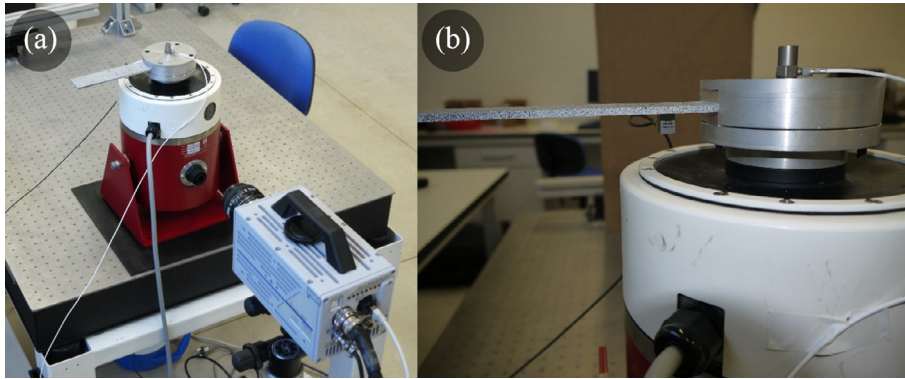


Fig. 2. Experimental setup showing the specimen fixed to the armature of the shaker. (a) High speed camera, focused on the lateral surface of the beam for mode shapes characterisation. (b) Location of the accelerometers to register the excitation on the clamping and the response on the beam for natural frequencies identification.

clamped edge. The response point, close to the clamping device, was chosen since it was a low response point, avoiding the influence in the modal behaviour. Hence, the transmissibility function was obtained comparing the excitation signal and response signal from the accelerometers. The registration of the accelerometer signal and the calculation of transmissibility function was performed by the real-time analyser RT Photon+ and the controller software RT Pro Photon. This system also provided to the shaker the excitation signals employed in this study. As the shaker is limited to 8000 Hz, a random signal was employed to generate broadband response in the beam up to 7500 Hz.

The frequency analysis was performed considering a frequency resolution of 1 Hz, hanning anti-leakage windows and several averaged windows. Fig. 3 shows the transmissibility function where some resonances can be detected. Since measurements were performed at one single point, the shape of these resonances was unknown and the mode shape will be interpreted with the help of FEM.

2.2. FEM model

A modal identification of the beam was performed through a FEM model in Abaqus 6.14-2 software. The beam was simulated as a plate using S4R linear shell elements. The clamping of the thicker part of the specimen was modelled by constraining the edge nodes as encastred. The dynamic influence of the accelerometer was also considered by adding the mass of the Bruel & Kjaer 4520-001 accelerometer (4 g) in the simulation.

Focus was placed on the natural frequencies and mode shapes up to 8000 Hz, which correspond to maximum frequency of the shaker. Modes corresponding to non-cantilever bending modes (torsion, transverse bending and traction) were ignored in this study. As a result, five bending modes were found whose frequencies are presented in Table 1 and their predicted shapes are shown in Fig. 4.

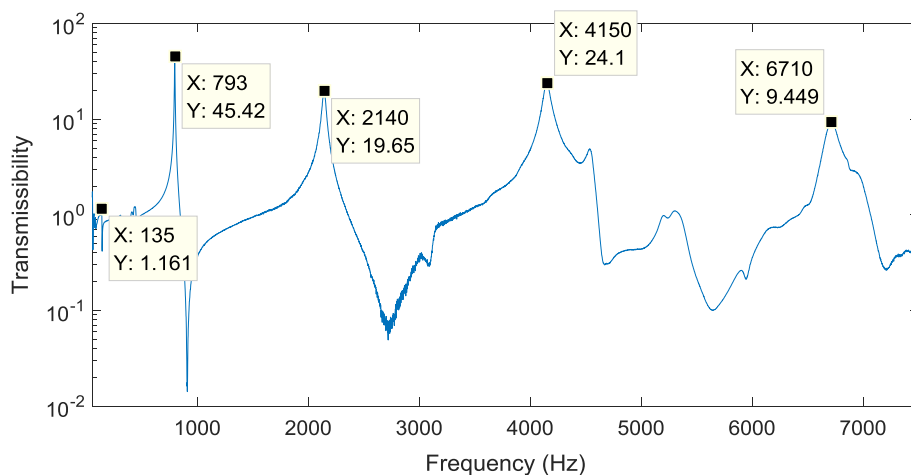


Fig. 3. Natural frequencies of the bending modes of the cantilever obtained in a random test using accelerometers.

Table 1
Natural frequencies of the cantilever bending modes.

Mode	FEM natural frequency (Hz)	Experimental natural frequencies (Hz)
1	130.16	135
2	807.68	793
3	2208.2	2140
4	4211.4	4150
5	6902.8	6710

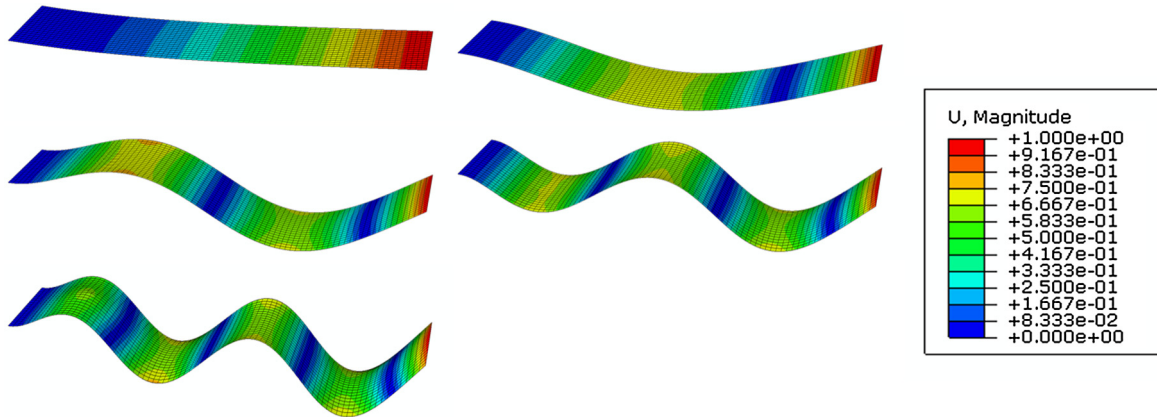


Fig. 4. Bending mode shapes obtained in FEM modal analysis.

As revealed in Table 1, there is a significant conformity in the identification of natural frequencies. FEM analysis allowed to confirm these frequencies according to the shape. Observing Fig. 3, these modes also correspond with the predominant peaks considering that, with this experimental setup, other kind of modes is barely excited. The first mode peak is not predominant, since the measuring point is close to the node of zero-displacement, according to the shape of this mode seen in Fig. 4. Later measuring of full-field mode shapes confirmed that these were the bending modes. Although strictly the experimental shapes are ODSs, in Fig. 3 it is possible to see that the modes are sufficiently separated so that ODSs tend to mode shapes with no influence of close modes. Thus, it can be assumed that experimental ODSs measured by DIC are mode shapes, enabling the comparison with FEM mode shapes.

2.3. DIC mode shapes characterisation

The process of characterising mode shapes with DIC involves the capture of images during sine tests at resonance frequencies and the processing of the images with a DIC algorithm. Experiments were performed using a high speed camera recording the motion on the lateral surface of the beam. The camera (model FastCam SA3 by Photron 1 Mpx full-resolution) was focused perpendicular to the thickness with a Nikon lens with 50 mm of focal length. Images were reduced to 1024×112 pixels because the shape of the beam, much longer than wide, covers all the width of the images but not the height (Fig. 5). This resolution allows a maximum frame rate of 15,000 fps. With this frame rate, it is possible to achieve a good definition of the cycles up to the third mode, 2140 Hz, capturing in this case about 7 images per cycle. Shutter exposure time was even more restrictive and was set to $1/20,000$ s, which is nearly ten times shorter than the period of the third mode.

It is important to highlight that a frame rate of 15,000 fps is not enough to capture the evolution of the periodic motion for 4th and 5th mode. In this study, strobe effect is proposed as an alternative to carry out recordings at higher frequency modes with a good definition of the cycle. Hence, a frame rate lower than the analysed frequency was chosen for these modes. The same resolution, fitted to the shape, was used in order to minimise the size of the images. The frame rate was chosen considering that one multiple was close to the studied frequency, thus many points per cycle were registered. Hence, with the available frame rates at this resolution, 2000 fps were adopted for the fourth mode (4150 Hz) and 750 fps for the fifth (6710 Hz). Maintaining the same criterion as for the first three modes, $1/60,000$ s exposure time was used for these modes.

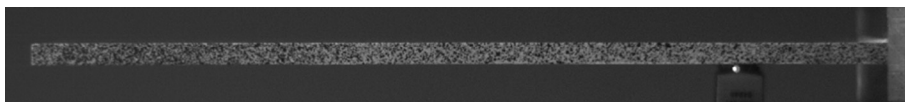


Fig. 5. Beam profile captured by the high speed camera in a 1024×112 pixel image.

The resulting calibration provided a relation of 0.1657 mm/px. The specimen surface was treated with black and white paint to generate a random speckle pattern necessary for DIC. The accelerometer was always present in the beam in order to exactly maintain the same conditions as those registered in the random test. Images were processed employing Vic-2 D software to obtain full-field displacement contour plots using facet size of 9 pixels and spacing of 2 pixels.

Each mode was tested twice under two different level of excitation. It allowed a thorough study of the possibilities of combining DIC and video magnification. On one hand, a low level of displacement represents a case of subtle motion to reveal the improvements in DIC results by magnification. On the other hand, a high level of excitation would show limitations in the definition of the speckle as a consequence of distortion in the images due to an excessive amplitude after magnifying. The high level of excitation was conditioned by the shaker's specifications.

3. Motion magnification

Magnification of the captured images sequence in each test was performed according to the phase-based magnification method proposed by Wadhwa et al. [15]. By this methodology, the sequence is decomposed using complex steerable pyramids. Temporal variation of the local phase is found as an indicator of the motion. By analysing the phase signal in the frequency domain, a magnification factor can be applied to a desired band in the spectrum so that the video reconstruction considering modified phase signal provides a magnified version of the subtle motion in the original video. Thus, the execution of the magnification required three parameters to be known: frame rate, magnification band width and magnification factor. In the particular cases treated in this study, the band covered the vicinity of the excited frequency. It must be considered that the band width of the 4th and 5th mode depends on the stroboscopic sampling. The frequencies of these modes are in both cases clearly over the Nyquist frequency, according to the established frame rate. Defining the cycle using the strobe effect can be considered as an intentioned case of aliasing. Thus, it is necessary to determine the corresponding aliased frequency in the analysed spectrum. For the 4th mode, considering that the natural frequency was 4150 Hz and the frame rate was 2000 fps, this frequency would appear in the spectrum as 150 Hz. The 5th, at 6710 Hz defined with 750 fps, would appear at 40 Hz. As can be deduced, more than 10 points per cycle would be captured in both modes.

Various magnification processes were performed per test to study the relation between magnification and quality of the results. Therefore, the evolution of the contour plot is analysed versus the magnification factor. Magnification implies improvements in DIC results but large magnification factors were eluded to prevent image distortion. Therefore, magnification factors were determined for each mode considering the quality of the results.

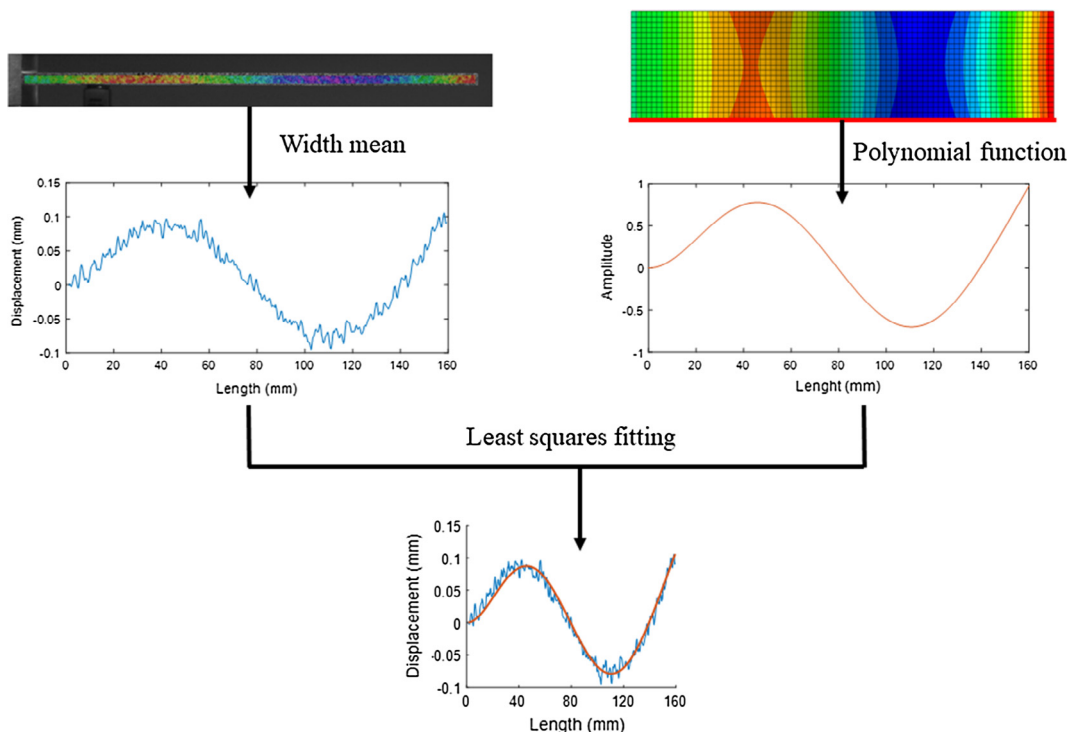


Fig. 6. Scheme of the process of fitting the experimental results to numerical ones.

3.1. Comparison with FEM mode shapes

Besides the identification of natural frequencies, FEM mode shapes are useful for assessing magnified mode shapes. In this epigraph it is exposed the methodology performed to compare FEM mode shapes with DIC ones, outlined in Fig. 6. From FEM shapes, the values of the lateral edge nodes were extracted. Experimental measurements on the surface were reduced to a line by calculating the mean along the thick, neglecting any variation. In order to scale the shapes and establish an estimation of the error with respect to the ideal deformation, FEM mode shapes were approximated to a polynomial function with an order that perfectly fits the shape. Function's order varies from 4 to 12 with the mode order. The polynomial function obtained for each mode represents the continuous ideal shape. By a least squares fitting, experimental mode shapes and the polynomial function were fitted and the deviation from the ideal shape was obtained by means of the Normalised Root Mean Squares Error (NRMSE). NRMSE is not dependent on the amplitude of the shape, so it is an indicator of the improvement achieved in DIC results by means of magnification.

In this evaluation, the correlation coefficient is also interesting and complementary to NRMSE. This is a statistical indicator less sensitive to small changes that provides the degree of consistency between shapes. For this study in which different levels of noise are present in experimental measurements, it represents the agreement of the shapes and is less affected by noise if shapes are coherent enough.

4. Results

In this epigraph, results concerning the visualisation of DIC mode shapes using motion magnification and the deviation with respect to the ideal shape are presented. The first set of results is presented in Figs. 7–16. Contour plots corresponding to vertical displacement at each test and their magnified equivalents are presented. The magnification factor and the amplitude (in millimetres) of the scale are provided for each state of magnification.

Subsequently, Figs. 17–21 show the comparison of experimental mode shapes (blue line) and FEM mode shapes (orange line), performed as previously explained and outlined in Fig. 6. For illustration purposes, some magnification states are omitted from each test considering that do not provide additional information. As observed, displacements were shown scaled to the maximum and minimum registered values to illustrate how the noise is reduced, and experimental results fit to numerical results as the magnification factor increases.

NRMSE represents a quantitative indicator of the deviation of the experimental shapes from the ideal shapes. Fig. 22 shows one plot per mode with the NRMSE from the low excitation level (circular markers) and the high excitation level tests (triangular markers) versus the magnification factor. A complementary analysis is established by determining the correlation factor between experimental and ideal shapes, shown in Fig. 23. Hence, quantitative assessment is presented. Those cases in which magnification is excessive and distorts the images are presented as red markers. Fig. 8 shows that images corresponding to the first mode amplified five and ten times were distorted and DIC algorithm was not able to perform the correlation. In the second mode it occurs when a magnification factor of 10 was applied, as shown in Fig. 10. This is also observed in the comparison with the FEM model (Figs. 17 and 18, second row), where experimental shape is truncated. Therefore, they are no representative of the tendency of error reduction. Nevertheless, this loss of information is very small for the third to the fifth mode (Figs. 19–21) that NRMSE and the correlation coefficient can be considered as representative.

Finally, an analysis of the magnification factor was performed. The experimental magnification factor was determined by comparing the amplitude of the adjusted FEM mode shape of the magnified results with the non-magnified results for the first two modes. Higher modes are not evaluated since the low displacement of the non-magnified shape is not representative of the behaviour. The results are presented in Fig. 24 where the blue and the orange line represent the actual magnification factor obtained in the first and the second mode, respectively.

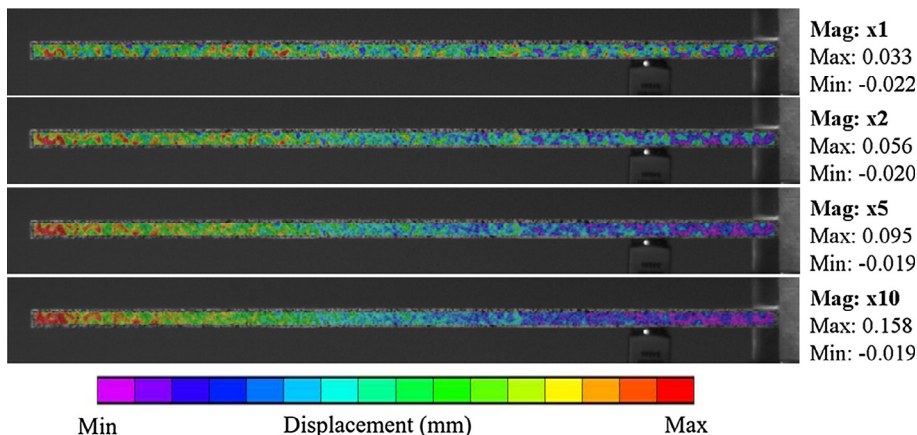


Fig. 7. Visualisation of the first mode shape (135 Hz) with DIC contour plots under low level of excitation and different motion magnification factors.

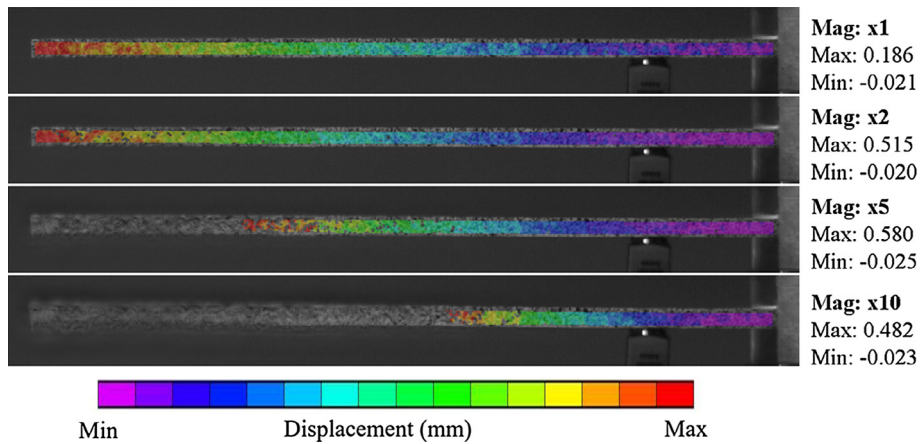


Fig. 8. Visualisation of the first mode shape (135 Hz) with DIC contour plots under high level of excitation and different motion magnification factors.

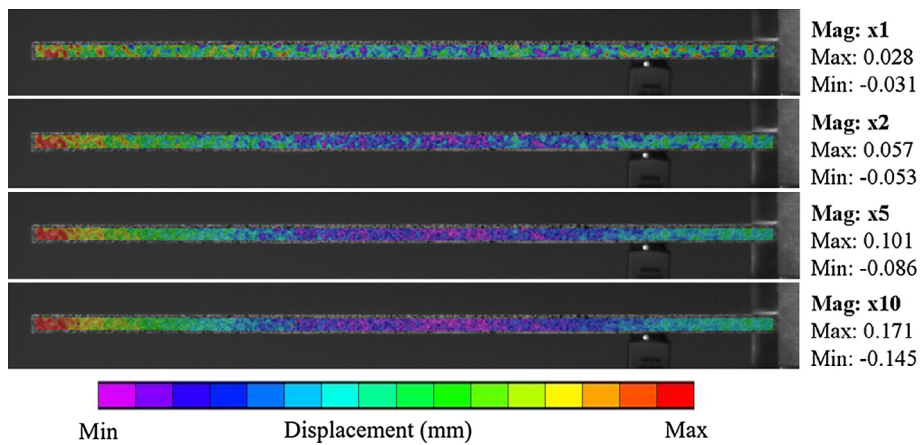


Fig. 9. Visualisation of the second mode shape (793 Hz) with DIC contour plots under low level of excitation and different motion magnification factors.

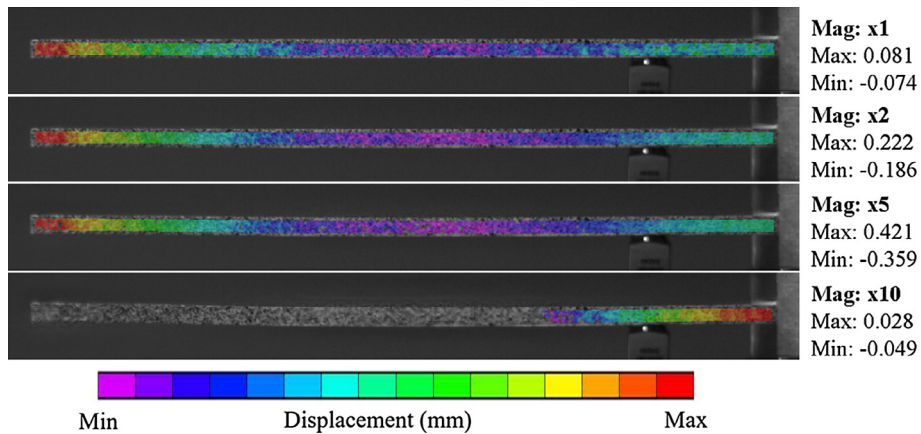


Fig. 10. Visualisation of the second mode shape (793 Hz) with DIC contour plots under high level of excitation and different motion magnification factors.

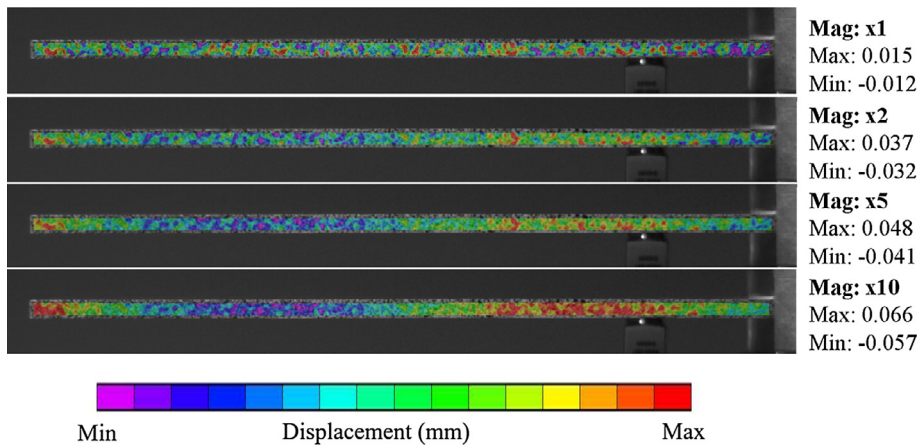


Fig. 11. Visualisation of the third mode shape (2140 Hz) with DIC contour plots under low level of excitation and different motion magnification factors.

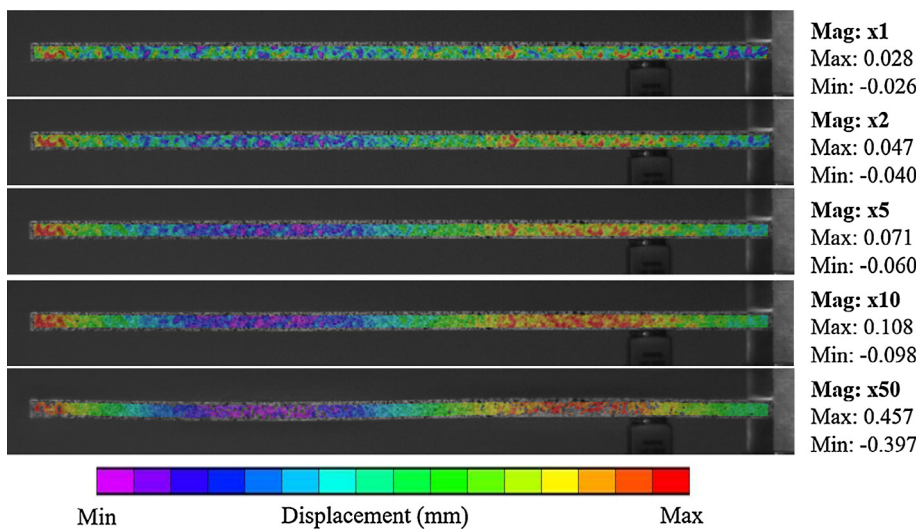


Fig. 12. Visualisation of the third mode shape (2140 Hz) with DIC contour plots under high level of excitation and different motion magnification factors.

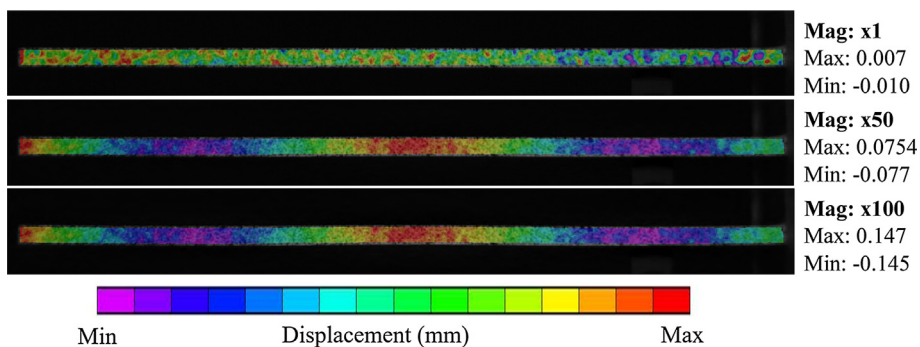


Fig. 13. Visualisation of the fourth mode shape (4150 Hz) with DIC contour plots under low level of excitation and different motion magnification factors.

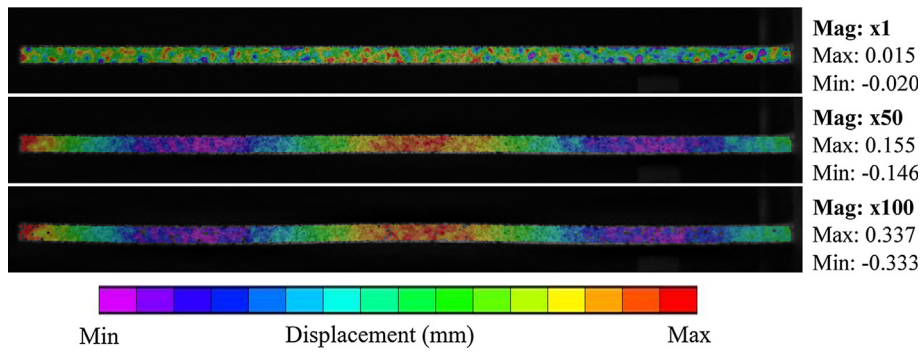


Fig. 14. Visualisation of the fourth mode shape (4150 Hz) with DIC contour plots under high level of excitation and different motion magnification factors.

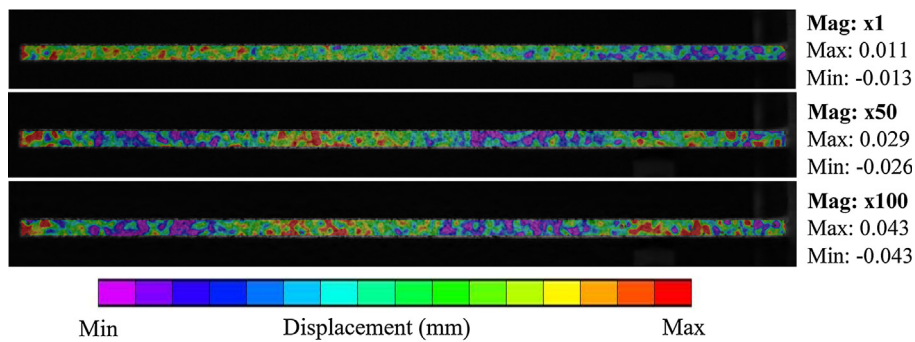


Fig. 15. Visualisation of the fifth mode shape (6710 Hz) with DIC contour plots under low level of excitation and different motion magnification factors.

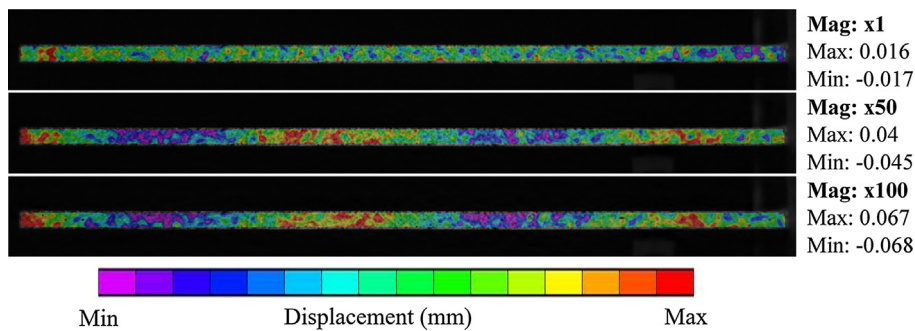


Fig. 16. Visualisation of the fifth mode shape (6710 Hz) with DIC contour plots under high level of excitation and different motion magnification factors.

5. Discussions

Regarding non-magnified state, it is observed that for the first two modes (135 Hz and 793 Hz), with the largest displacement, experimental mode shapes are clearly interpretable from contour plots in Figs. 8 and 10 with the higher level of excitation. Hence, magnification is not determinant. For those frequencies, Fig. 22 reflects that the NRMSE with no magnification is 3.69% and 8.88% for the first and second mode, respectively, at high level of excitation (triangular markers). Correlation is also excellent, over 98%, as seen in Fig. 23. These frequencies are in the usual spectrum that is analysed by DIC due to the largest displacements.

There is a great increment of frequency regarding the next modes. The third mode occurs at 2140 Hz, which is out of the usual spectrum analysed by DIC according to the bibliography. In Figs. 11 and 12, the mode shape is difficult to observe in the

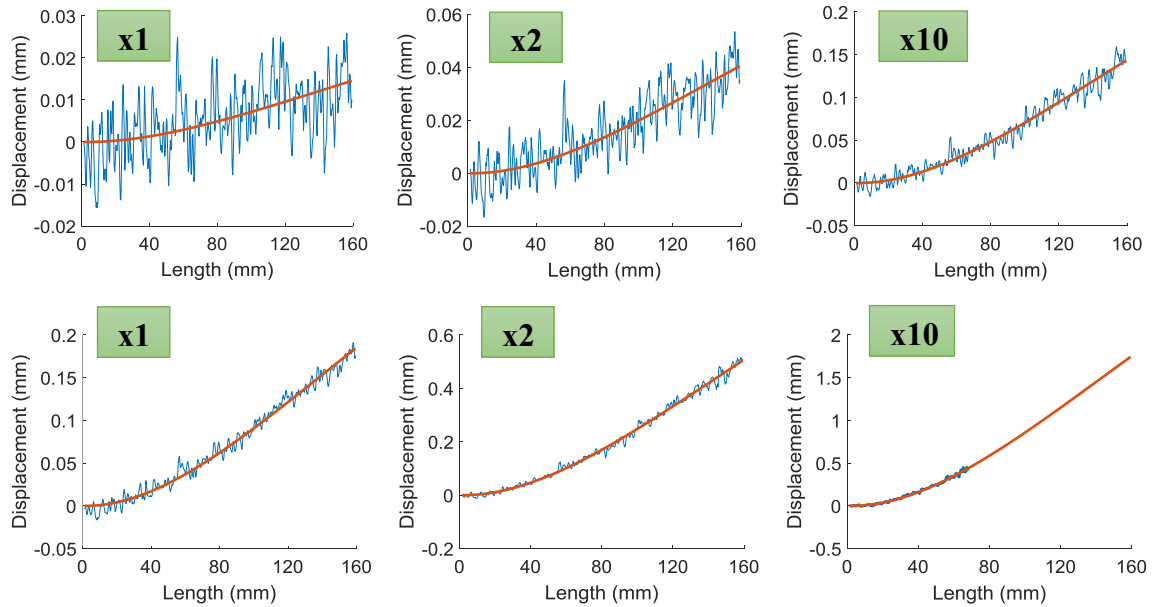


Fig. 17. Comparison of the experimental shape (blue) with the ideal shape from FEM model (orange) of the first mode under low excitation (upper row) and high excitation (lower row) with different magnification factors. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

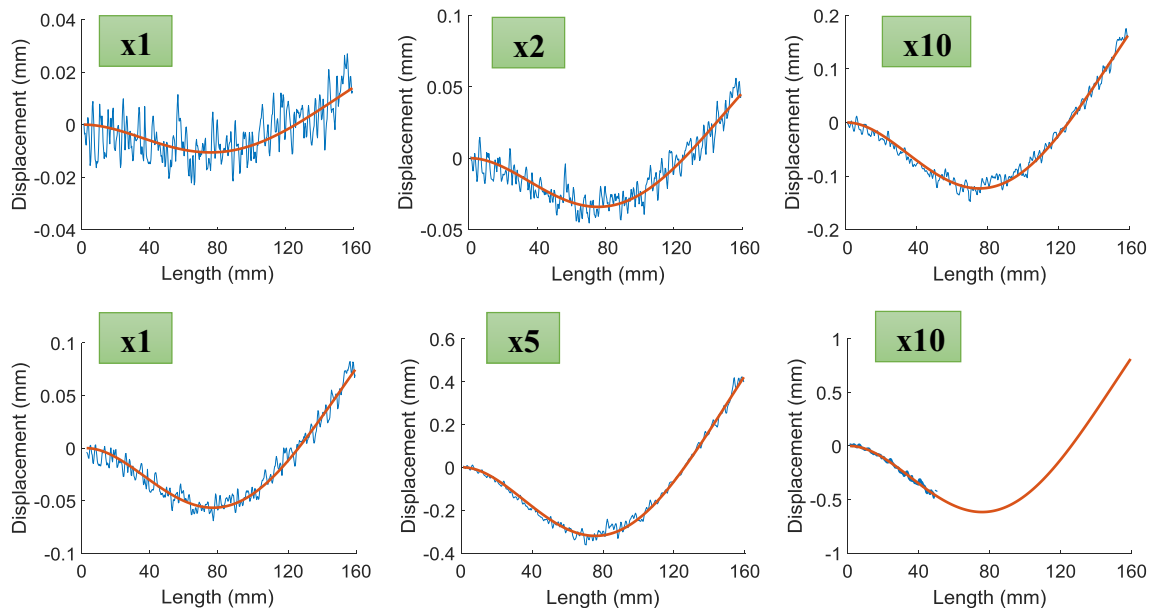


Fig. 18. Comparison of the experimental shape (blue) with the ideal shape from FEM model (orange) of the second mode under low excitation (upper row) and high excitation (lower row) with different magnification factors. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

displacements maps, independently of the excitation level. The visualisation of the motion and contour plot video in the non-magnified state, Video 1, is also unsatisfactory in the case of the highest excitation. This evolution is more notorious as the mode order increases (Figs. 13 and 14 for mode 4 and Figs. 15 and 16 for mode 5). Hence, from the presented results it is found out that phase-based motion magnification contributes to enhance DIC visualisation of the cantilever mode shapes. In particular, third to fifth mode shapes can be now visualised expanding the limits of DIC to unprecedented frequencies, up to 6710 Hz in this study.

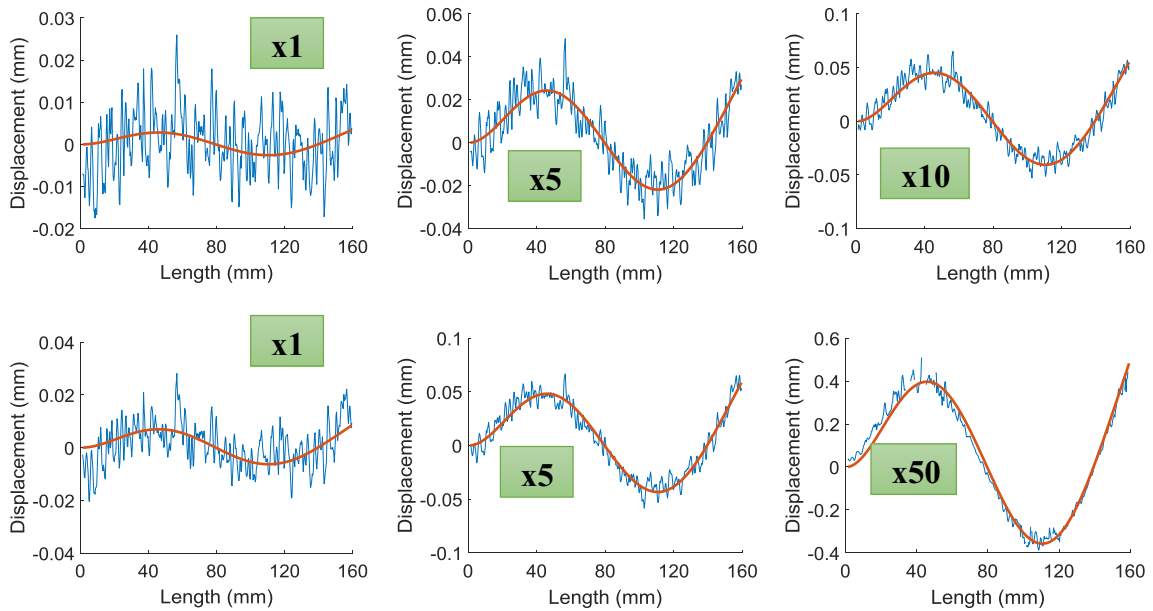


Fig. 19. Comparison of the experimental shape (blue) with the ideal shape from FEM model (orange) of the third mode under low excitation (upper row) and high excitation (lower row) with different magnification factors. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

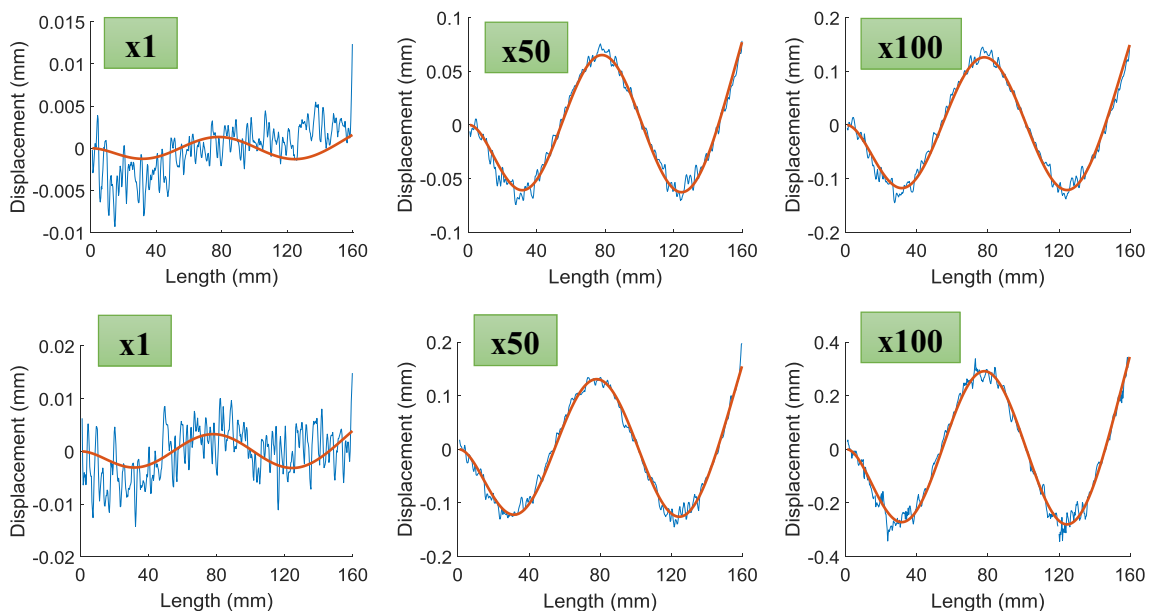


Fig. 20. Comparison of the experimental shape (blue) with the ideal shape from FEM model (orange) of the fourth mode under low excitation (upper row) and high excitation (lower row) with different magnification factors. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

The effect of the magnification is also shown in the evolution of the NRMS and the correlation coefficient with the magnification level in Figs. 22 and 23, respectively. NRMSE experiences a considerable fall with an asymptotic tendency with the magnification factors. Inverse trend but the same meaning is shown in correlation coefficient. For the first three modes, an excellent improvement is achieved in low excitation test magnifying by 2. The most remarkable case corresponds to the third mode where the NRMSE is reduced from 193.1% to 47.8% and the correlation coefficient reaches a value of 0.725 from 0.252. It is relevant that magnification in low excitation tests provides better results in mode shapes visualisation than just employing high level of excitation, except from the first mode. In fact, low excitation tests with maximum evaluated magnification

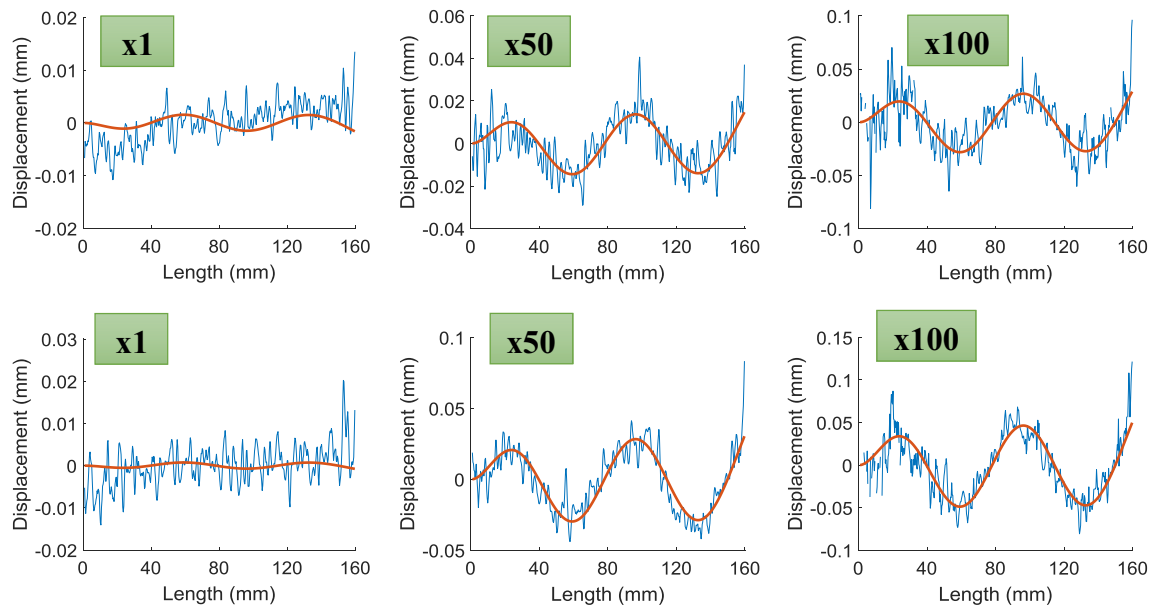


Fig. 21. Comparison of the experimental shape (blue) with the ideal shape from FEM model (orange) of the fifth mode under low excitation (upper row) and high excitation (lower row) with different magnification factors. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

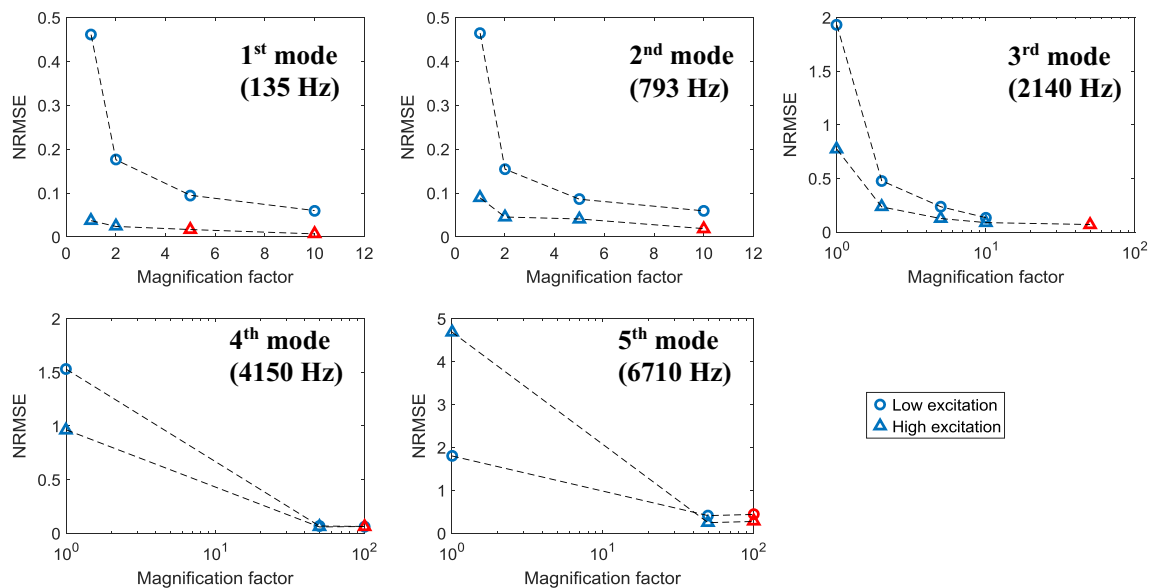


Fig. 22. Evolution of normalised Root Mean Squares Error with the magnification factor. Red markers represent the test where blurring occurred in speckle. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

factor produce lower NRMSE and higher correlation than non-magnified high excitation tests. The reason is that with motion magnification it is possible to achieve larger displacements than physically increasing the gain of the exciter. For instance, the highest excitation in the third mode produced an amplitude of 0.009 mm whereas the low excitation test magnified ten times produced 0.058 mm of amplitude. It is of course reflected in the contour plots. Hence, the correlation factor is improved from 0.546 to 0.966 and, NRMSE from 77.2% to 13.4%. These results can be enhanced by just increasing the excitation level. However, excessive magnification may imply speckle blurring at the regions of higher amplitude. It is more important if non-magnified motion is already high. This is the case of the first and second mode, those with higher displacement. Thus, with an intermediate level of excitation and even with a not too high magnification factor it is possible to achieve good results.

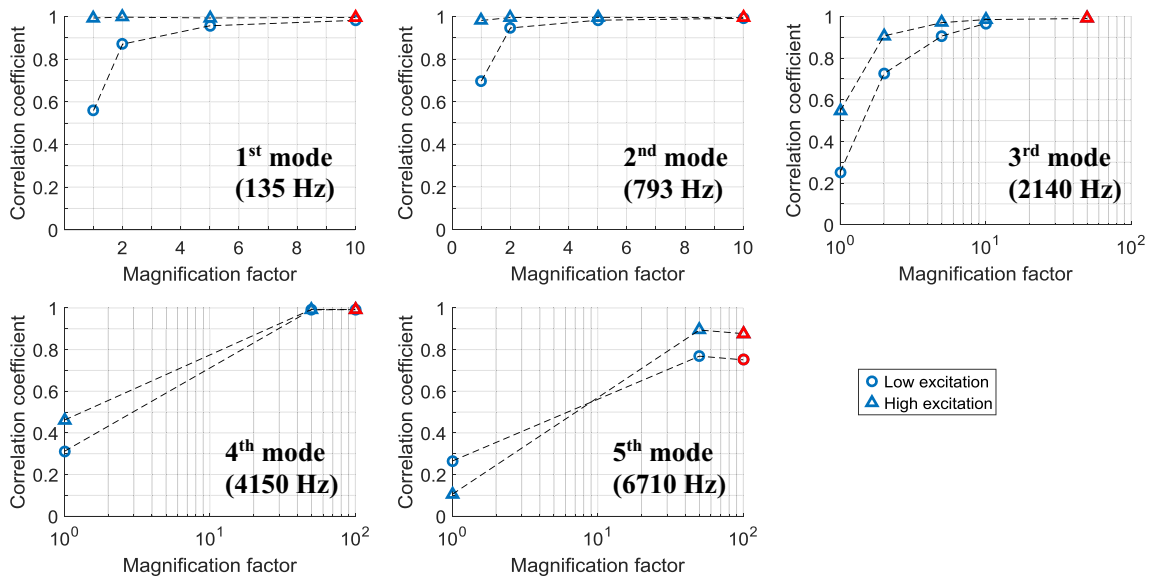


Fig. 23. Evolution of correlation coefficient between experimental and numerical mode shapes with the magnification factor. Red markers represent the test where blurring occurred in speckle. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

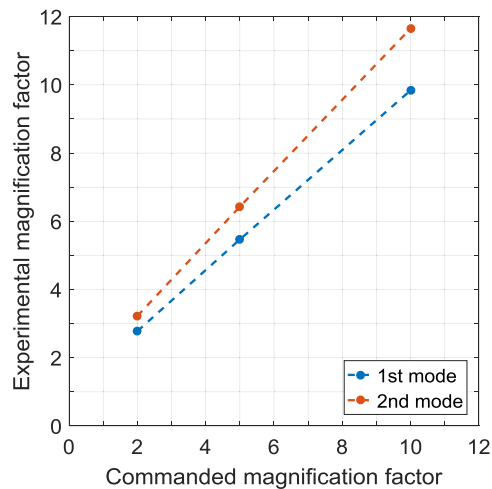
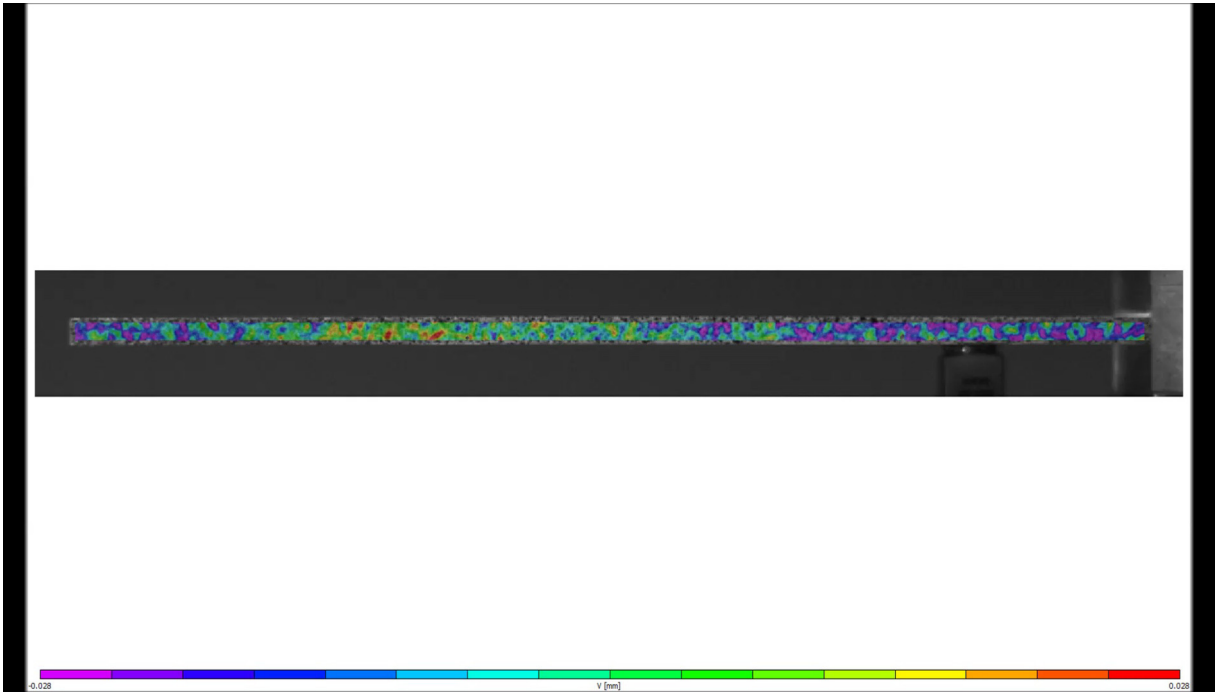


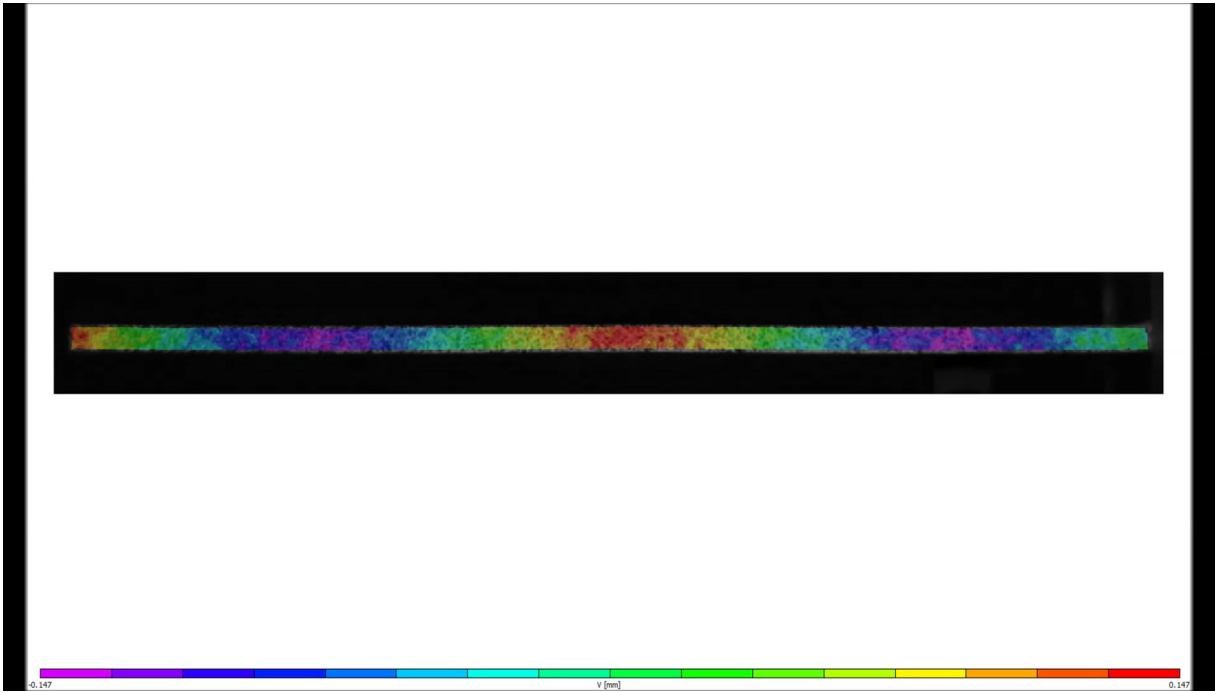
Fig. 24. Evaluation of the resulting magnification factor for the first two modes in low excitation tests.

From Figs. 22 and 23 it is observed that low and high excitation curves converge with the magnification factor. This is observed for all the modes, but the fourth mode shows the most evident case. For this mode, markers from low (circles) and high (triangles) excitation levels are almost coincident with magnification factors of 50 and 100 both in NRMSE and correlation plots. It implies that it is possible to obtain results as good as for high excitation with a low level of excitation. Hence, the shaker does not have to work overloaded, and a little and cheaper exciter may be used. Despite the quality of low excitation test results, a better naked eye visualisation of the motion can be achieved with larger displacements. Although the motion at low excitation test magnified one hundred times (Video 2) can be interpreted, the interpretation is enhanced by the evident motion of the high excitation test (Video 3). Hence, it could be interesting to apply intermediate excitation so that the motion is more evident.

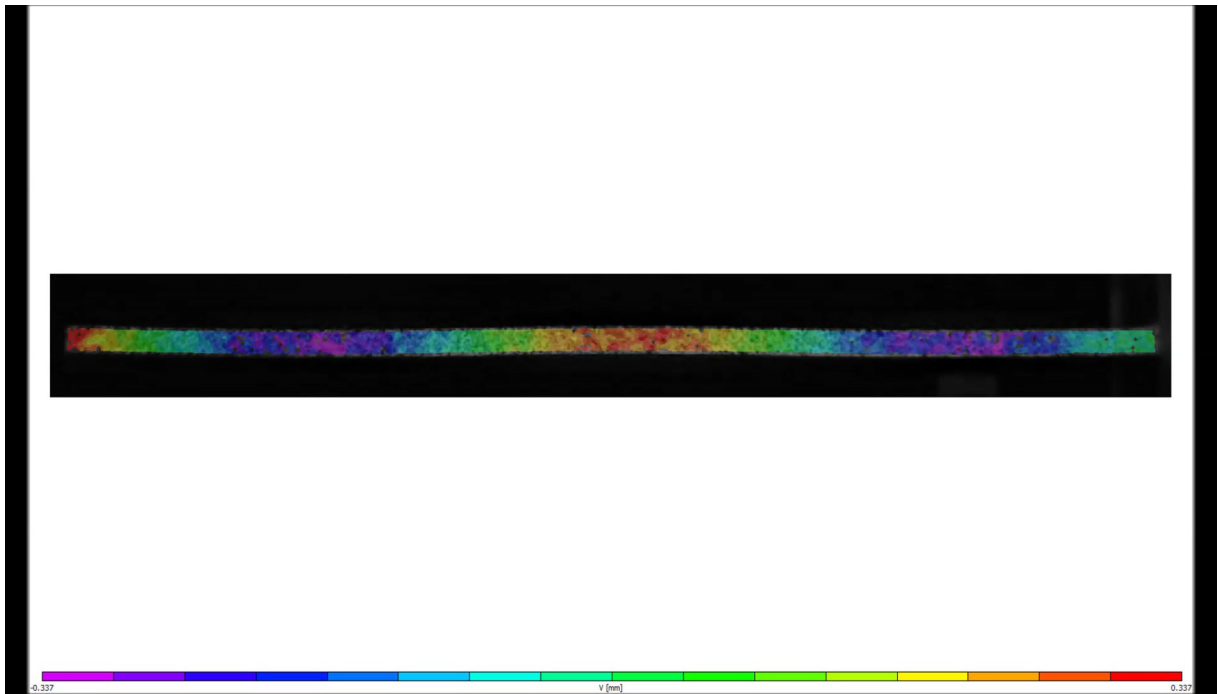
The fifth mode is the extreme case in this study considering the high frequency, 6710 Hz, with respect to usual DIC measurements. A change in the trend of NRMSE and correlation coefficient is observed for a factor of 100 due to the non-correlated areas (see Fig. 21). Thus, the best result is achieved with maximum excitation and 50 magnification factor, where the NRMSE is 24.8%. It means that the level of noise present in the shape is still high due to the original extremely low displacements. In fact, the fitting of the non-magnified state is strongly influenced by the random noise along the beam,



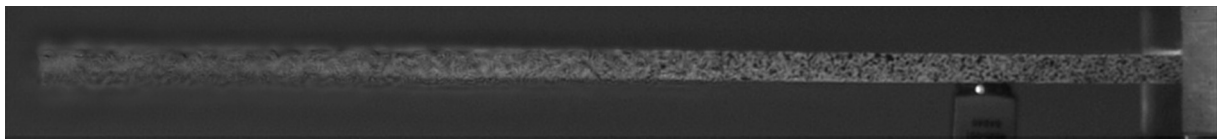
Video 1. Video of the non-magnified motion along with displacement contour plot, provided by DIC, of the third mode (2140 Hz) under high excitation level



Video 2. Video of the motion along with displacement contour plot, provided by DIC, of the fourth mode (4150 Hz) magnified 100 times under low excitation level



Video 3. Video of the motion along with displacement contour plot, provided by DIC, of the fourth mode (4150 Hz) magnified 100 times under high excitation level

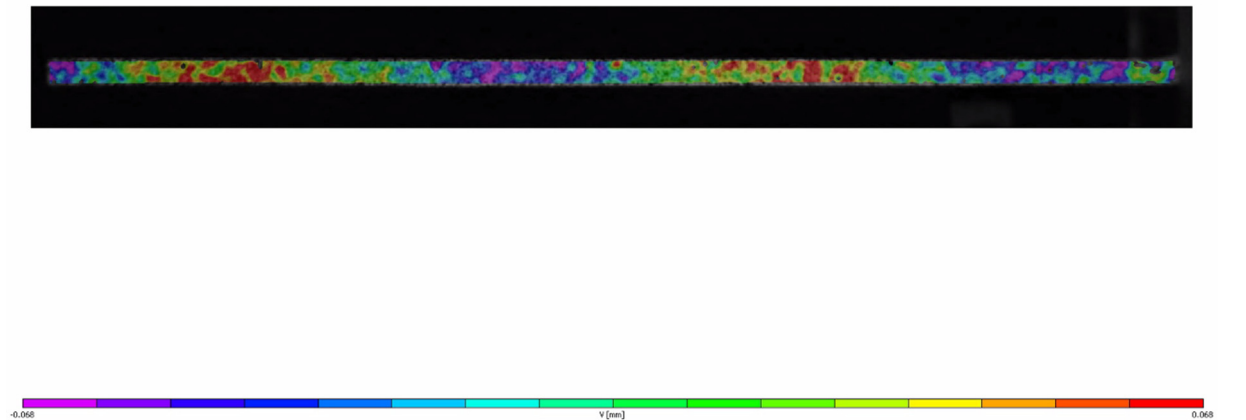


Video 4. Video of the motion of the first mode (135 Hz) magnified 10 times under high excitation level

up to the point that the low excitation test produced better NRMSE and correlation coefficient. The correlation coefficient, 0.894, is satisfactory, though. This evidences that the noise is around a shape that is in good agreement with the ideal one. Hence, the improvement with respect non-magnified state is significant and mode shape can be clearly appreciated, as seen in Figs. 15, 16 and 21.

Nonetheless, it is also necessary to ensure that the magnification is homogeneously performed along the whole image and no aberrations of the mode shapes are performed. For that purpose, the comparison of experimental and numerical shapes (Figs. 17–21) show how the noise is reduced and the experimental mode shapes fit the numerical model. From this plots, it is remarkable that no distortion in the shape is made with any magnification factor. It is proved by the fact that experimental shapes are always well-fitted to the ideal shape from numerical model. As previously highlighted, only for the first two modes using a high level of excitation it is possible to discern shapes with no magnification. In both cases in Figs. 17 and 18, it is seen that the magnified images reproduced the same shape that non-magnified ones, so it is concluded that no alteration of the real motion appears during this processing. All this contributes to achieve correlation factors close to one although the NRMSE may be considered high in some cases. This is an important issue since, by magnifying, the actual shape is obtained independently of the amplitude in a visualisation format (contour plots) that is easily comparable with FEM models, especially in the upper spectrum.

The possibility that DIC offers leads to the quantification of the actual magnification. The estimation of the actual displacements in low amplitude tests would be one application of that. Experimental magnification factors are calculated considering the amplitude of the non-magnified shapes, corresponding to a fitting from a very noisy result. When comparing with the commanded value in Fig. 24, it is observed that there is a substantial error. In the second mode, it is found an actual magnification of $3.22\times$ in the case of $2\times$ magnification, $6.43\times$ in $5\times$ factor and $11.65\times$ in the case of a commanded $10\times$ magnification. That would imply an error in the magnification of 61%, 28.6% and 16.5% for those commanded magnification respectively. Despite these differences, the trend is linear, what means a proportional relation during magnification. However, the proportion is not the supposed 1:1, independently of the actual amplitude estimation (non-magnified). For



Video 5. Video of the motion along with displacement contour plot, provided by DIC, of the fifth mode (6710 Hz) magnified 100 times under high excitation level

instance, the experimental magnification factor when using a commanded one of 10 should be 5 times greater than using 2. Fig. 24 does not reflect so. It implies that experimental magnification will not be the commanded one. Thus, it has been proved that motion magnification is useful and accurate in the characterisation of mode shapes or mode shape, usually normalised modal parameters, but the estimation of actual non-magnified amplitude from magnified images would be based on a false assumption.

In previous work, phase-based motion magnification was presented as a tool for visualisation of mode shapes [17,18]. This study is a step forward. Results evidence some aspects in which DIC provides additional information. In the cited work, visualisation by naked eye usually requires large magnification factors in order to reproduce a motion amplitude large enough to be interpreted. However, specimen on the images suffered a strong transgression and got blurred. It is proved in Fig. 8 for the first mode with high level of excitation. When magnified by 10, the released tip of the beam is completely undefined, as seen in Video 4. Some previous knowledge of the shape of the mode would be required to discern the shape with the naked eye. On the other hand, the increasing complexity of the deformation in higher-order modes makes difficult for the naked eye to discern some shapes. Considering the 5th mode with larger displacements (high excitation magnified by 100) presented in Video 5, DIC measurement is completely necessary to assimilate the shape with almost indiscernible motion. DIC provides numerical information which is complementary for the interpretation of motion on videos and which would be interesting in the validation of numerical models. There are favourable cases in which naked eye visualisation and DIC measurement coexist, as seen in Video 3. Here the benefits of visual interpretation and numerical information are provided.

6. Concluding remarks

In this study, a combination of the capabilities of DIC and phase-based motion magnification have been presented and evaluated. Remarkable results have been obtained regarding the visualisation and measurement of Operational Deflection Shapes of structures. In some situation, naked eye is not sensitive enough to identify motion even after magnification. As a consequence, large magnification factors are required but low improvement is achieved since images suffer from distortion. For those situations, Digital Image Correlation has been employed in this study to provide relevant displacement maps of that imperceptible displacements. Hence, DIC contour plots together with visualisation of magnified motion is a powerful tool as it provides graphical and also quantitative information for intuitive interpretation of the motion corresponding to mode shapes. Contour plots are also useful for comparison with FEM models and their validation. It has been

demonstrated that the magnification procedure presents no aberration of the actual displacements which is crucial to perform a correct measurement.

As DIC is based on measuring displacements, it was evidenced by results that DIC is restricted to low frequency vibrations which involve larger amplitudes. This study has also managed with the intrinsic DIC issues in high frequency vibration analysis when low displacements occur. As a result, by employing motion magnification it has been presented a procedure to visualise and measure mode shapes at unprecedented frequencies, from about 1500 Hz to 6710 Hz with DIC. Furthermore, it has been demonstrated that it is not necessary to employ high levels of excitation to reach an admissible level of displacement for an optimal DIC measurement. It also has implications on the use of little exciter for general spectrum analysis. This contributes to extend the use of DIC in vibration analysis, with the advantage of contactless full-field information.

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3D mode shapes characterisation using phase-based motion magnification in large structures using stereoscopic DIC

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ABSTRACT

Phase-based motion magnification has been proposed as an effective methodology to visualise Operational Deflection Shapes (ODSs) using a single camera. In recent work, magnification allowed the observation of these ODSs by naked eye or more advanced measurements on magnified images by using 2D Digital Image Correlation (DIC). The combination of a full-field optical technique and motion magnification shows a high potential improving DIC measurement and providing qualitative and quantitative information for the interpretation of ODSs. However, both motion magnification and the combination with 2D-DIC is limited to a few cases where plane elements experiences in-plane motion, mainly beam-like structures. In order to provide a more general application, in this study measurements on couples of magnified images using stereoscopic 3D-DIC were performed. A cantilever beam was employed for validation purposes, comparing the results with numerical models. Afterwards, a measurement application on a large curved panel was carried out to show full capabilities of the proposed methodology for 3D characterisation. Visualisation in slow-motion video format enables an intuitive understanding of the deformation. The adopted methodology demonstrates to be highly useful in low amplitude tests, involving the use of little exciters or high frequency applications.

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1. Introduction

Vision-based monitoring and assessment of machines and structures has become popular in last decades [1–10]. The main advantages are the non-contact nature of the techniques and no requirements of time-consuming instrumentation except from digital cameras and a computer with the image processing software. Recently, the development of methodologies of magnification of subtle motion in digital videos presents notable advantages in the characterisation of modal behaviour for non-destructive testing and structural health monitoring using a single camera. Based on complex steerable pyramid filters for image decomposition [11,12], these methodologies make a magnified version of the motion by increasing the amplitude of a narrow frequency band. In their work, Chen et al [13] provides visualisation of magnified Operational Deflection Shapes (ODSs). Natural frequencies, mode shapes and damping ratios are blindly provided in other studies [14,15] by means of an autonomous phased-based methodology.

In a previous work [16] it was performed full-field displacement measurements in magnified images. Two-dimensional Digital Image Correlation (2D-DIC) [17] was performed in magnified and non-magnified images of the resonant vibration of

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a cantilever beam. Although naked eye visualisation of the motion is very valuable, DIC is more sensitive quantifying the motion. Additionally, blurring due to excessive magnification or the increasing complexity of the deformation in higher-order modes make also difficult for the naked eye to discern some shapes. It was hence proved that DIC provided invaluable information to naked eye visualisation. Furthermore, full-field measurements is a valuable and powerful tool as validation or feedback for numerical models [18–20]. On the other hand, motion magnification was demonstrated to be very useful improving the quality of the mode shape characterisation using 2D-DIC with no distortion of the actual shape. Hence, the presented methodology allowed low level of excitation for the measurement of ODSs, reducing the requirement of more powerful (and probably expensive) exciter. Hence, the combination of DIC and phase-based motion magnification provides an important advance in the interpretation of mode shapes due to the qualitative visualisation of the magnified motion using the full-field displacements.

Mentioned works have been performed under the assumption that only in-plane motion occurs, dealing with images from a single camera. However, most scenarios in real-life involve shape and motion in the space where 2D-DIC is not applicable [21]. Hence, although the naked eye visualisation is useful in monitoring structures, three dimensional motion are usually tough to interpret.

Based on the conclusions and limitations, in this work measurements have been explored using 3D-DIC in stereoscopic sets of magnified images for 3D ODSs characterisation. 3D-DIC is the typical configuration in the studies related to the use of DIC in modal characterisation, employing a calibrated system of two cameras [19,22–34]. The proposed combination of 3D-DIC and phase magnification entails the challenge of performing 3D measurements from two sets of individually magnified images, corresponding to each camera, and evaluate the validity of the final result. Specifically, this study presents a validation analysis combining phased-based motion magnification proposed by Wadhwa et al. [11] and 3D-DIC. The validation experiment consisted in the evaluation of the effect of different magnification factors in ODSs of a cantilever beam using both low and high level of excitation. Deviation of magnified shapes with respect to FEM model was evaluated. With the study of the beam, the magnification was validated in stereoscopic images sets, demonstrating that no shape distortion was introduced by aberration in the pairs of images. It was also possible to explore the methodology at high frequencies non-reported in the literature using 3D-DIC.

Bending modes of the cantilever beam involved motion mainly in one direction. In order to exhibit the capabilities of 3D measurement, the proposed methodology was performed for first time in a large curved composite panel. Different ODSs are presented showing displacements in the three directions. These tests also allowed to prove the efficiency of the combination when small exciters are employed in the analysis of large and relatively heavy elements. Thus, it is shown improvements in displacement field obtained by 3D-DIC, providing a full analysis of the motion of the structure in the three spatial directions of the space. This ease the interpretation of the motion and ODSs of a complex element which would be difficult to analyse by visualisation from a 2D point of view. In addition to static ODSs displacement maps, visualisation of videos of displacements measured by 3D-DIC and phase magnification would further facilitate the interpretation of the motion. Therefore the most representative results are also provide in video format, available in the online version.

In Section 2, the outline of the common tasks is presented. Section 3 provides detailed information about the validation analysis, results and discussions. Section 4 is devoted to expose details and results of the analysis in the panel to evaluate the three displacement components. Finally, the main conclusions are presented in Section 5.

2. General procedure

The combination of motion magnification and 3D-DIC was performed in forced normal mode test using a shaker, registering the resonant motion of different modes with a stereoscopic system of two high speed cameras (Photron FastCam SA4) with a resolution of 1024×1024 pixels. Additional illumination was supplied to enhance images quality. Previous to forced normal mode test, natural frequency identification was performed using broadband random excitation measuring with different accelerometers. A real-time analyser RT Photon+ was employed to generate excitation signal, record the accelerometers signals and obtain the transfer functions to identify frequencies. As a result of recording the event with two cameras, motion magnification must be performed individually in each set of images. Then, the two sets of images were gathered together and processed employing Vic-3D software [35] for 3D-DIC to obtain displacements (Fig. 1).

The magnification method here adopted, developed by Wadhwa et al. [11], applies a magnification factor to a user-defined bandwidth of the frequency domain phase signal from the decomposed images. Decomposition is performed using complex steerable pyramids filters. Reconstruction of images after modifying phase shows a higher amplitude motion corresponding to the magnified frequency band. The input parameters for the algorithm are the followings: cameras frame rate, magnification bandwidth, corresponding to the vicinity of the excited frequency, and magnification factor. This information is reported in subsequent sections according to each test.

3. Methodology validation

A 2024 aluminium cantilever beam, shown in Fig. 2, was employed to validate the methodology. The specimen is in a stepped shape whose thinner part was 160 mm long, 40 mm wide and 4 mm thick. The thicker part is used to clamp the

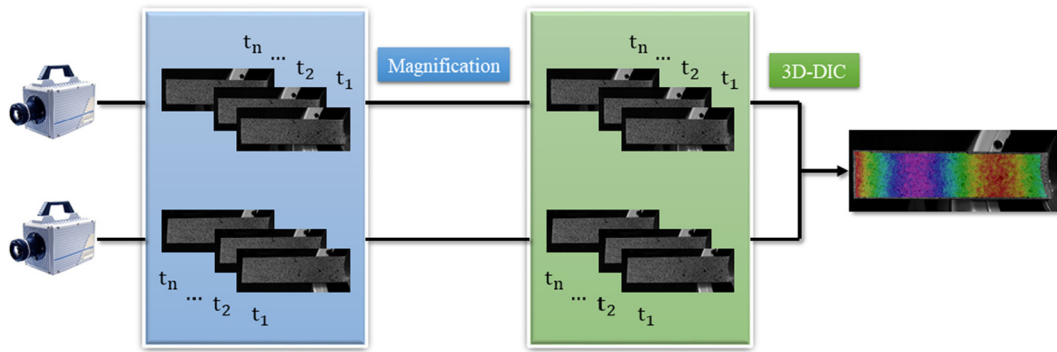


Fig. 1. Procedure of measuring in a stereoscopic set of magnified images using 3D-DIC.

specimen, keeping the thinner released. The behaviour of this geometry is well-known and easily reproducible by FEM models. Besides the manufacturing high standard, this makes it ideal for validation purposes.

The general setup of the beam tests is shown in Fig. 3. An electrodynamic shaker (Data Physics GW-V55/PA300E) was employed. During the tests, the beam was attached to the shaker's armature to transmit the excitation. Natural frequency identification was the previous step to this study. Noise excitation from DC to 6000 Hz was employed. Broadband excitation and response were registered using an accelerometer at the base and another one on the rear surface of the beam, respectively. The location of the last one was close to the clamping device, reducing the influence of the added mass. Nevertheless, this accelerometer was not later removed to maintain the same conditions during sine tests. Four bending modes were selected at the following frequencies: 135 Hz, 793 Hz, 2140 Hz and 4150 Hz.

3.1. Images recording and 3D-DIC processing

For DIC measurements, the high speed cameras were focused on the surface of the beam, as shown in Fig. 3, using a matched pair of 50 mm lenses (Nikkor AF-S 85 mm f/1.8D). Hence, the expected cantilever behaviour would imply out-of-plane displacements. The capability of recording images was limited by the cameras resolution in terms of speed and

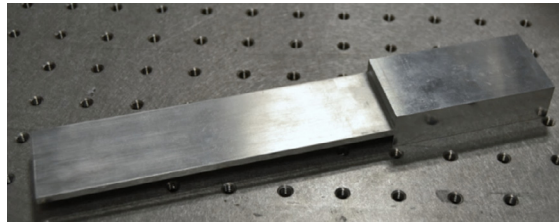


Fig. 2. Cantilever beam, employed for validation purposes of the methodology.

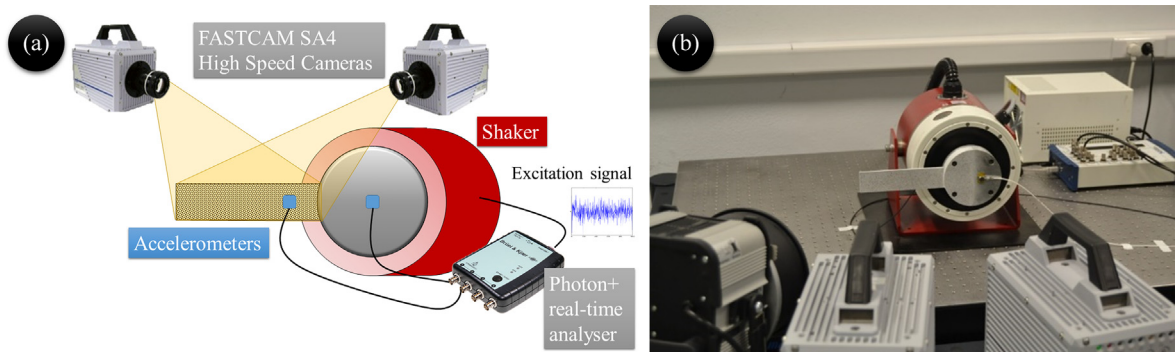


Fig. 3. (a) Sketch of the adopted experimental setup for the cantilever beam with the high speed cameras focused on beam surface for ODSs characterisation and the accelerometers for natural frequencies identification. (b) Actual setup.

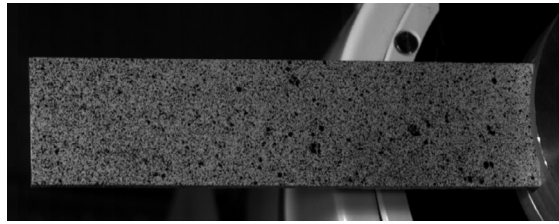


Fig. 4. Image of the beam with 832×320 pixel resolution.

number of images. Consequently, the image resolution was resized to fit the beam, resulting 832×320 pixels images (Fig. 4). The frame rate was defined at 15,000 fps, which provides a good discretisation up to the third mode, 2140 Hz, with 7 images per cycle. However, for the 4th mode it was not possible to increase the frame rate maintaining the same resolution. Therefore, this mode was subsampled recorded using the camera strobe effect. The frame rate for this mode was chosen so that the aliased frequency was low. Hence, the frame rate was set to 2000 fps, so this mode appeared in the spectrum as an aliased frequency at 150 Hz. The criterion to define the shutter exposure time was about ten times shorter than the period of the mode in order to avoid any blurring. Hence, for the first three modes, it was set to $1/20,000$ s and $1/60,000$ s was used for the fourth. The specimen surface was treated with white and black paint to generate a random speckle pattern necessary for DIC. Images were processed to obtain full-field displacement contour plots using a facet size of 15 pixels and spacing of 5 pixels. A pixel/mm ratio of 5.1 was achieved.

3.2. Motion magnification

For each of the four identified modes, two opposite level of excitation were applied to evaluate motion magnification. Low level of displacement represents a case of subtle motion to reveal the improvements in DIC results applying magnification. High level of excitation was defined by the shaker's specifications. Different magnification factors were employed for each test to study the relation between magnification and quality of the results. Therefore, the evolution of the contour plot versus the magnification factor was analysed. The maximum magnification factor was inferred as the magnification level at which image blurring starts occurring. Table 1 shows the magnification factors employed for each natural frequency.

The first results are shown in Figs. 5–8. Each figure represents the out-of-plane displacements results for each mode. Due to illustration purposes, results are presented for non-magnified (left) and maximum magnification (right) image sets at low (up) and high (down) excitation tests. The influence of the magnification factor and the excitation level are thus revealed.

3.3. Comparison with FEM model

The deviation with respect to the ideal shape was assessed using a FEM model in Abaqus 6.14-2 software. The analysis consisted in the modal identification of the beam, which was designed as a plate using S4R linear shell elements (1620 elements of 2×2 mm) considering only the thinner part. Standard aluminium material properties were employed: density 2700 kg/m^3 and Young modulus 70 GPa. The boundary conditions were reproduced by constraining the nodes of one edge. The mass of the accelerometer (Bruel&Kjaer 4520-001) was also included in the model. As a result, four bending modes were identified at 130.16 Hz, 807.68 Hz, 2208.2 Hz and 4211.4 Hz. The maximum difference with respect to experimental frequencies was 3.58% for the first mode.

For comparison with experimental results, FEM mode shapes were defined as continuous functions which were considered to fit experimental data. The whole process is graphically described in Fig. 9. Least squares fitting was employed to fit FEM mode shapes using polynomial surfaces. The order varied from 4 to 12 longitudinally and from 2 to 4 transversally depending on the mode. This order made it possible to obtain a reasonable root mean square error always under 0.0022, i.e., a 0.22% of the shape amplitude. Then, as magnification modifies the amplitude of the displacement results for each test, the scales of the polynomial surfaces were adjusted by fitting experimental data using least squares. Thus, the Normalised Root Mean Squares Error (NRMSE) can be obtained and adopted as an indicator of the improvements in 3D-DIC mode shapes

Table 1
Magnification factors employed in each mode.

Mode	Magnification factors
135 Hz	(2, 5, 10)
793 Hz	(2, 5, 10)
2140 Hz	(2, 5, 10, 50)
4150 Hz	(10, 50, 100)

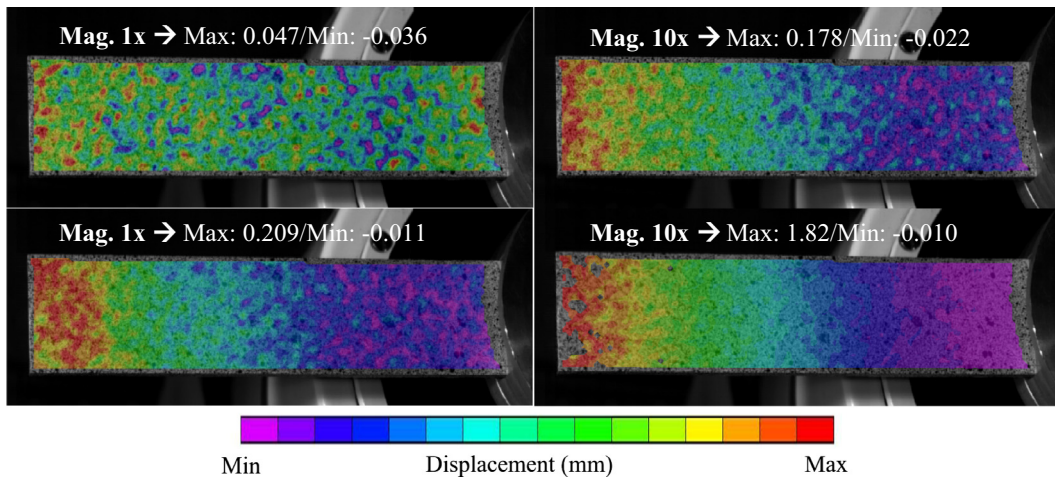


Fig. 5. Visualisation of the out-of-plane displacements for the first mode shape (135 Hz) at non-magnified and maximum magnified states. (Above) Low excitation level, (Below) high excitation level.

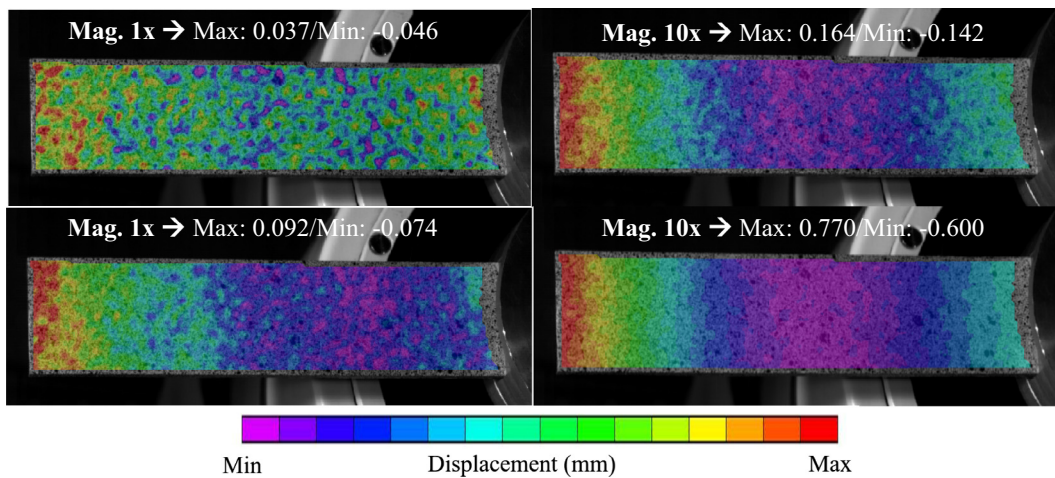


Fig. 6. Visualisation of the out-of-plane displacements for the second mode shape (793 Hz) at non-magnified and maximum magnified states. (Above) Low excitation level, (Below) high excitation level.

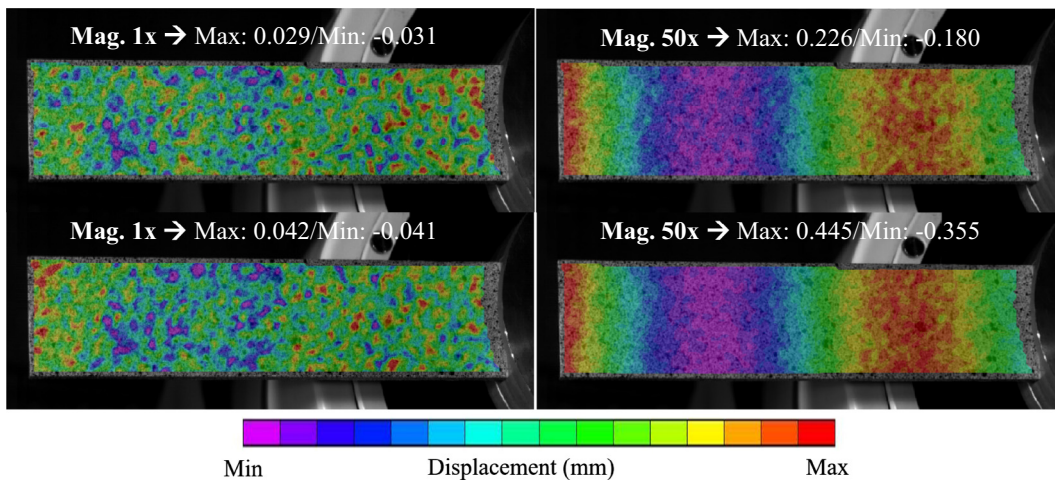


Fig. 7. Visualisation of the out-of-plane displacements for the third mode shape (2140 Hz) at non-magnified and maximum magnified states. (Above) Low excitation level, (Below) high excitation level.

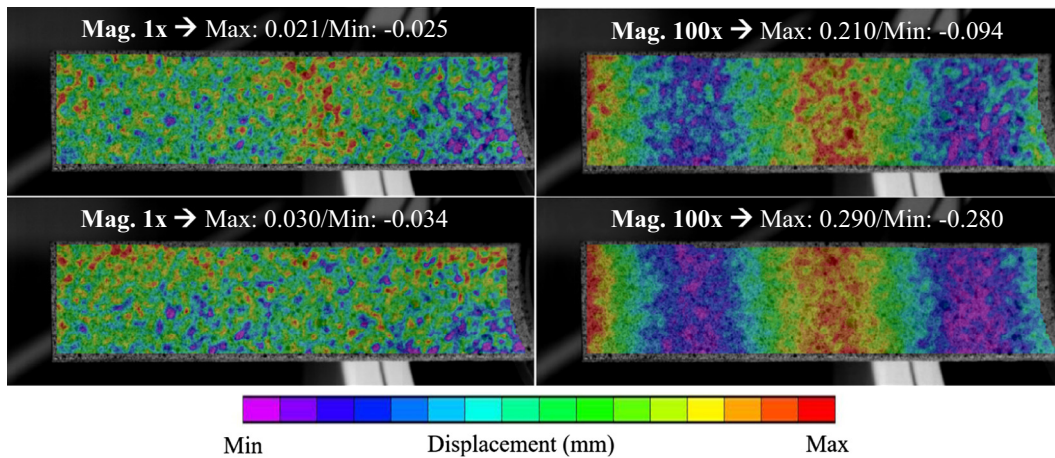


Fig. 8. Visualisation of the out-of-plane displacements of the fourth mode shape (4150 Hz) at non-magnified and maximum magnified states. (Above) Low excitation level, (Below) high excitation level.

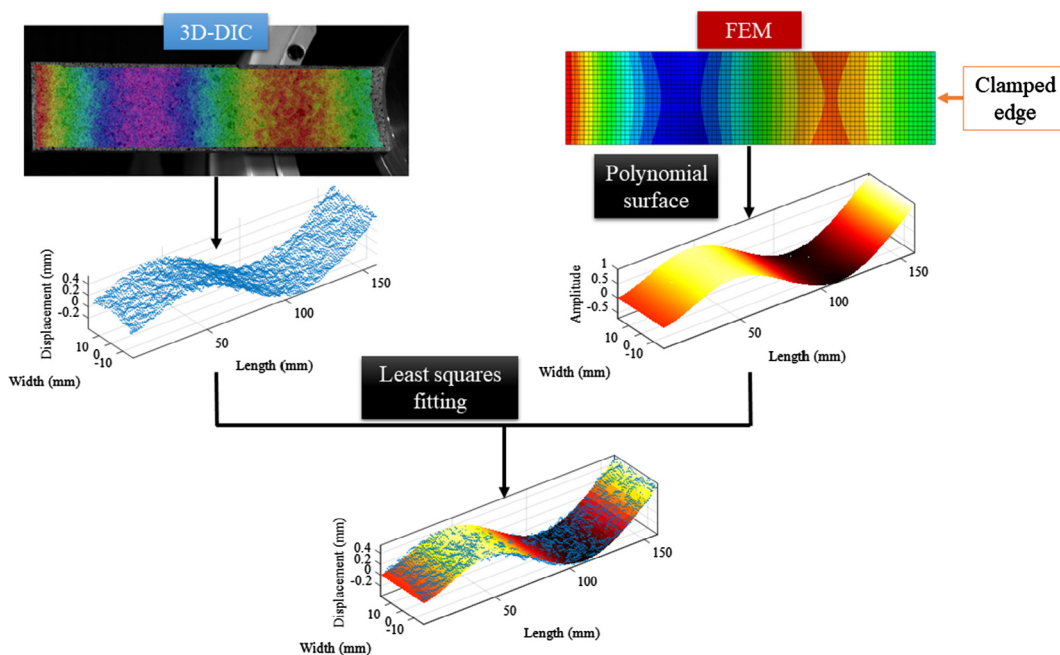


Fig. 9. Scheme of the process of fitting the experimental results to numerical data.

due to magnification. The correlation factor was also analysed since it provides relevant information for comparison, revealing the degree of consistency between shapes despite the presence of noise.

In Figs. 10–13, 3D-DIC and FEM mode shapes are shown together. The experimental shapes are presented as blue points whereas numerical ones are presented as a surface. For illustration purposes, only non-magnified states and maximum magnified states are presented. As it is observed, the displacements are shown scaled to the maximum experimental values to illustrate how the noise is reduced and experimental results improves their fitting to numerical results as the magnification factor increases.

NRMSE and correlation factor are shown in Figs. 14 and 15, respectively, versus magnification factor. Each mode is presented in an individual plot, where low excitation tests are indicated with circular markers and high excitation tests are indicated with triangular markers. Red markers indicate that there are non-correlated areas as a consequence of blurring in the images. As out-of-plane displacements do not involve large changes in the images sequence, blurring only occurs for the first mode using high excitation level, which is the highest amplitude case. However, in the worst scenario with a magnification factor of 10 $\times$, the non-correlated areas were 4.5% of the whole measurement surface (see Fig. 5).

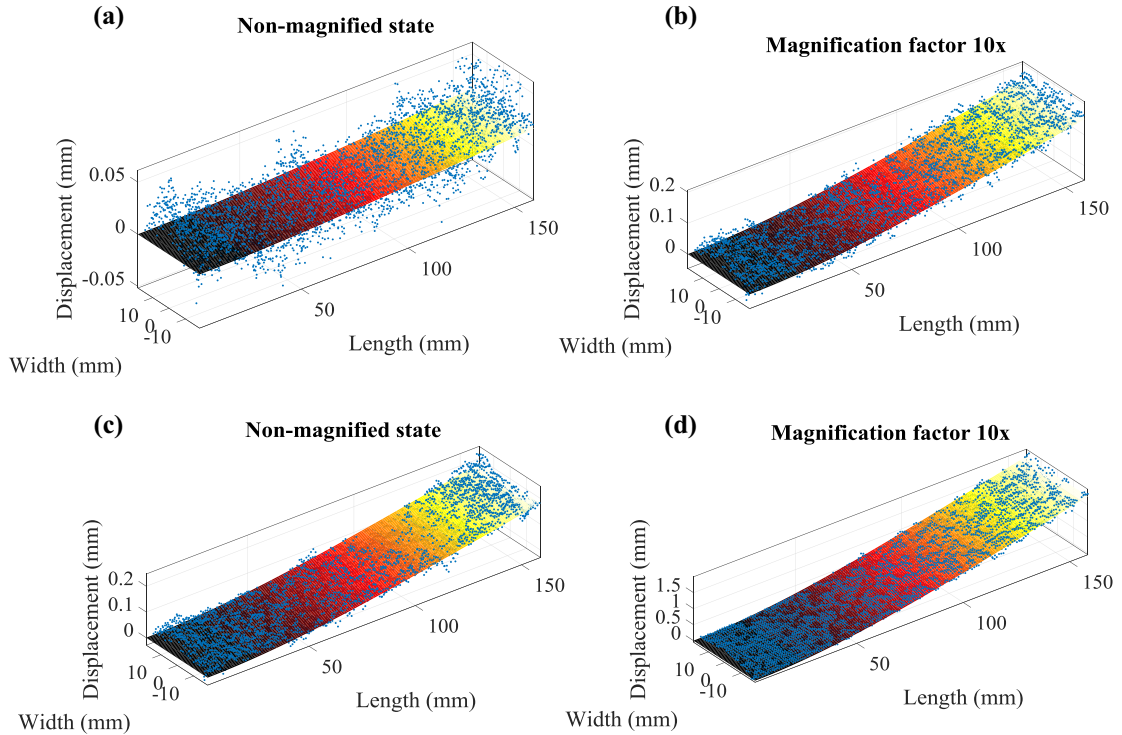


Fig. 10. Comparison of experimental (blue dots) and FEM mode shapes (contour surface) for the first mode. (a) Non-magnified state with low excitation. (b) Maximum magnification with low excitation. (c) Non-magnified state with high excitation. (d) Maximum magnification with high excitation. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

3.4. Discussion

Figs. 5–8 shows improvements in every mode and how magnification reveals shapes in contour maps from mainly noisy results. The initial appearance of the results is satisfactory but, considering the challenge of performing measurements combining two sets of magnified images, it was completely necessary to reject the presence of aberrations due to magnification. For that purpose, FEM results play an essential role as a reference. The different plots shown in Figs. 10–13 demonstrate in a graphical way how experimental results fit to FEM mode shapes using magnification. It is validated observing the decreasing exponential trend in NRMSE, shown in Fig. 14, and the exponential trend to 1 in the correlation coefficient in Fig. 15. Correlation factors are always above 0.9 for maximum magnification factor, indicating a strong correspondence with the reference numerical shape. Observing a low frequency like the first mode, large displacements without magnification are achieved when high excitation level is employed (Figs. 5 and 10). Although a slight presence of noise is noticed, NRMSE is 11.34% and the correlation between shapes is 93.77%. This example is taken as a reference to evaluate shape distortions during magnification. When magnification is applied, both graphical results and indicators improve, reducing the error to 3.11% and increasing the correlation coefficient up to 99.44%. This fact leads to conclude that no distortion has been occurred when independently magnifying the images from the two cameras and then processed together. The improvements made by magnification are consequence of revealing the actual shape without any distortion in the couple of images.

The first two modes, 135 Hz and 793 Hz, are in the usual spectrum analysed by 3D-DIC in the literature. However, the combination with motion magnification offers a new possibility. Regarding these two modes, using very low excitation level it is almost impossible to discern the ODSs, as seen in Figs. 5 and 6 or Figs. 10 and 11. However, the magnification allows to extract relevant information from these tests. Consequently, achieving an appropriate level of displacement can be relied on motion magnification instead of excessively high levels of excitation. Smaller exciters can be employed in combination with this methodology. For higher frequencies, such as the third (2140 Hz) and the fourth (4150 Hz) modes, typical setups are not able to achieve enough level of displacements, as can be deduced from the literature. In the present work, it is not possible to observe the shape in direct 3D-DIC results even at high level of excitation, defined by technical limitation of the shaker. Consequently, the proposed methodology become relevant for ODS visualisation. As the highest order mode, the fourth mode quality of results are very remarkable. The maximum amplitude resulting from the fitting of the fourth mode at high level of excitation and magnified 100 times was 0.3368 mm, what physically means an acceleration of 23,343 g at the tip of the beam, what is completely over any reasonable value for a real test. An additional remarkable point is the trend of low and high excitation NRMSE and correlation coefficient curves to converge, as it is clearly evidenced in the third mode. Almost identical results were obtained when using 50 $\times$ magnification factor. This yields that graphical results in Figs. 7 and 12

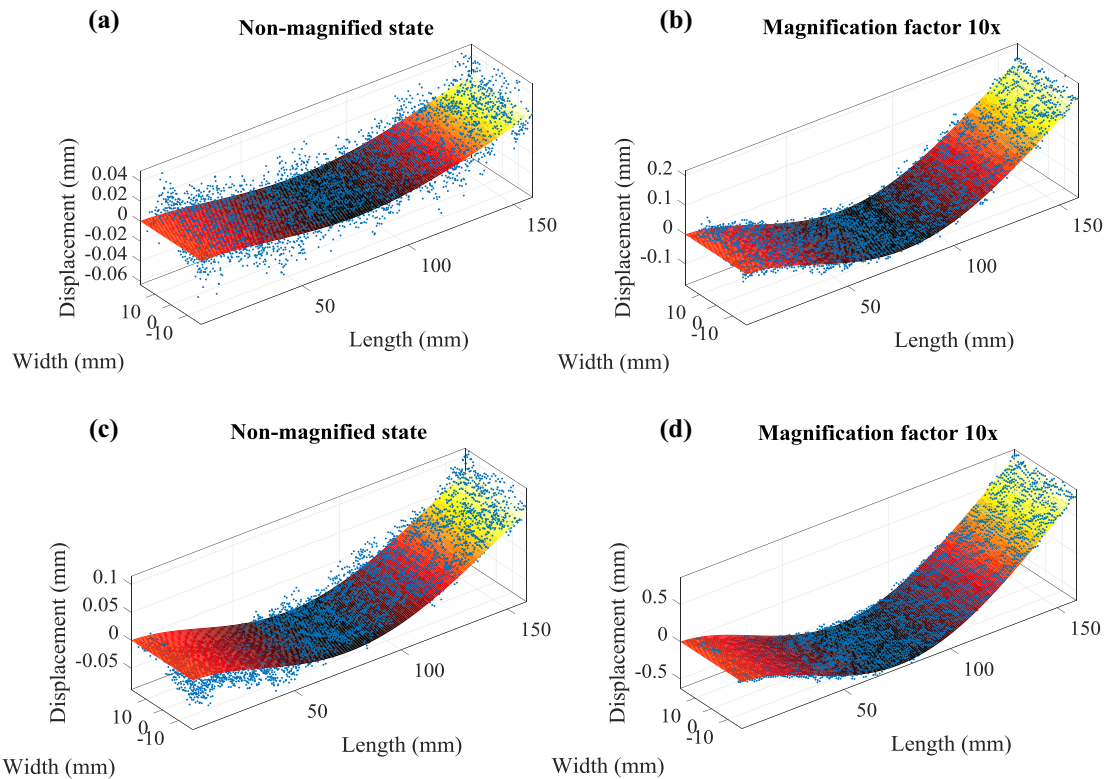
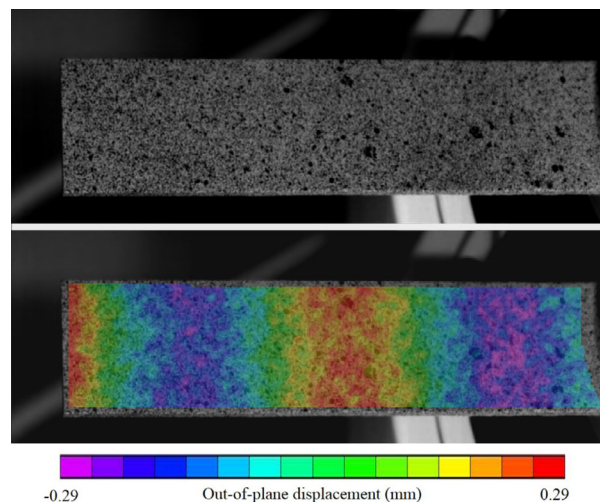


Fig. 11. Comparison of experimental (blue dots) and FEM mode shapes (contour surface) for the second mode. (a) Non-magnified state with low excitation. (b) Maximum magnification with low excitation. (c) Non-magnified state with high excitation. (d) Maximum magnification with high excitation. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

are similar in quality. Therefore, it is possible to obtain as good results with low excitation as the obtained with high excitation despite that the level of magnified displacements are almost the half.

In Video 1, a two windows video is presented corresponding to the motion of the fourth mode under high excitation level $100\times$ magnified. The first window illustrates the original images showing the motion of the beam. By observing this window, it is possible to discern the deformation of the beam but it is difficult to figure out the actual motion since displacements are purely out-of-plane. However, in the window below, 3D-DIC measurements are provided and now the interpretation of the motion turns out intuitive. In particular, this is a case in which both naked eye visualisation and DIC measurement provides information, which is one of the main advantages of the combination.



Video 1. Slow-motion video of the fourth mode with high excitation level and $100\times$ magnification.

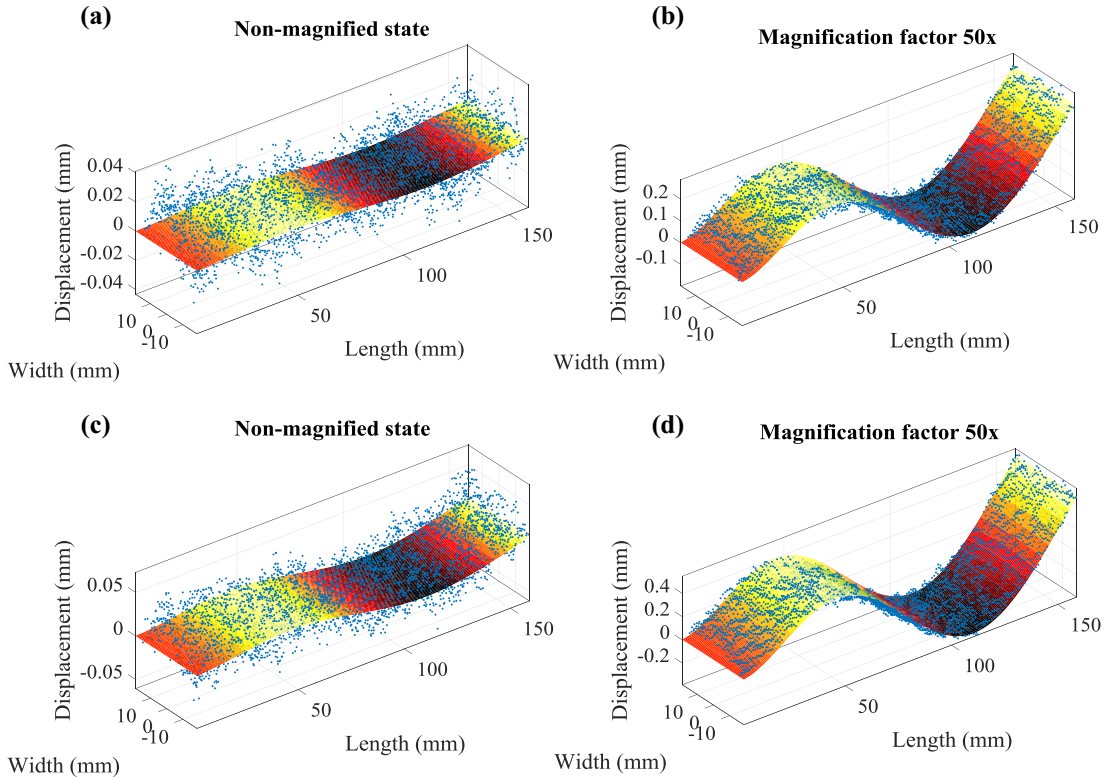


Fig. 12. Comparison of experimental (blue dots) and FEM mode shapes (contour surface) for the third mode. (a) Non-magnified state with low excitation. (b) Maximum magnification with low excitation. (c) Non-magnified state with high excitation. (d) Maximum magnification with high excitation. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

4. Methodology application on a three-dimensional large component

In this section the methodology is applied to a real component which present complex shape and modal behaviour. Although the methodology has been validated with the previous analysis, additional techniques could be employed, such as 3D scanning laser vibrometry, to validate the results. It is a large composite panel of 1.45 m high, 2.0 m wide and 0.6 m deep, which presents stiffening regions on the structure. The panel is curved and therefore in a 3D-shape (Fig. 16). As illustrated, frontal and lateral plane are almost perpendicular. Unlike the beam, the panel is expected to vibrate in more than one main component and thus, it shows the full capabilities of a stereoscopic system.

During the tests, the panel was hanged from a frame simulating free-free boundary conditions. A less powerful shaker was now employed, model Data Physics GW-V20/PA30E, forcing the panel from the rear face, using a stinger fixed at the bottom of the panel on the symmetry axis. Natural frequency identification was performed up to 300 Hz using an accelerometer on the corner, where the highest response is expected for many modes due to its geometry. The small exciter with low specifications (53 N of maximum sine force) was employed to check a limited exciter in a large element and hence evaluate motion magnification capabilities. Despite that, many modes were detected in the spectrum. Four peaks were selected to evaluate ODSs under low excitation level: 137 Hz, 190 Hz, 215 Hz and 285 Hz.

4.1. Image recording and 3D-DIC processing

As a consequence of the panel shape, it is difficult to make the whole panel visible by the cameras. An intermediate angle of view between the two principal planes of the panel was defined in order to monitor most of the surfaces. Focus was placed on half the panel under the assumptions of symmetry using lenses Nikkor AF-S 50 mm f/1.4D.

The same parameters were employed for every mode: 3600 fps, 1/3600 s of exposure time. For DIC processing 25 pixels facets were defined, overlapped 5 pixels. In Fig. 17, an image of the panel from one camera and the three-dimensional digitalisation of the surface are shown. The light supply was arranged to minimise the effect of brightness and it did not affect to the global results of this study.

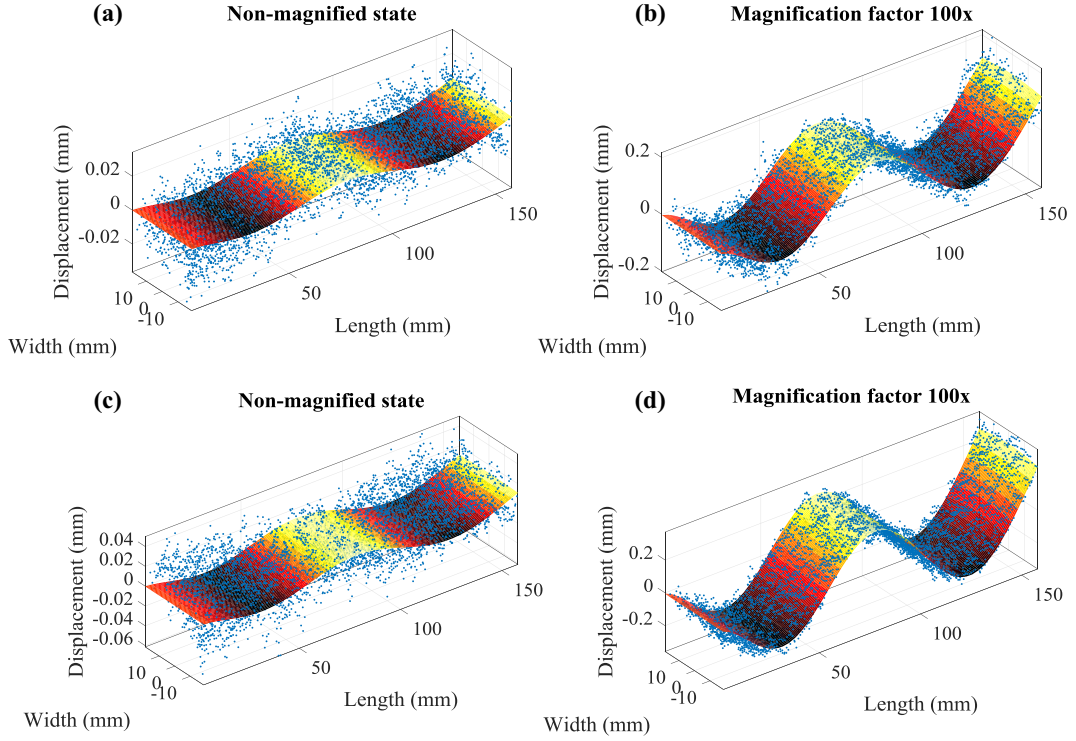


Fig. 13. Comparison of experimental (blue dots) and FEM mode shapes (contour surface) for the fourth mode. (a) Non-magnified state with low excitation. (b) Maximum magnification with low excitation. (c) Non-magnified state with high excitation. (d) Maximum magnification with high excitation. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

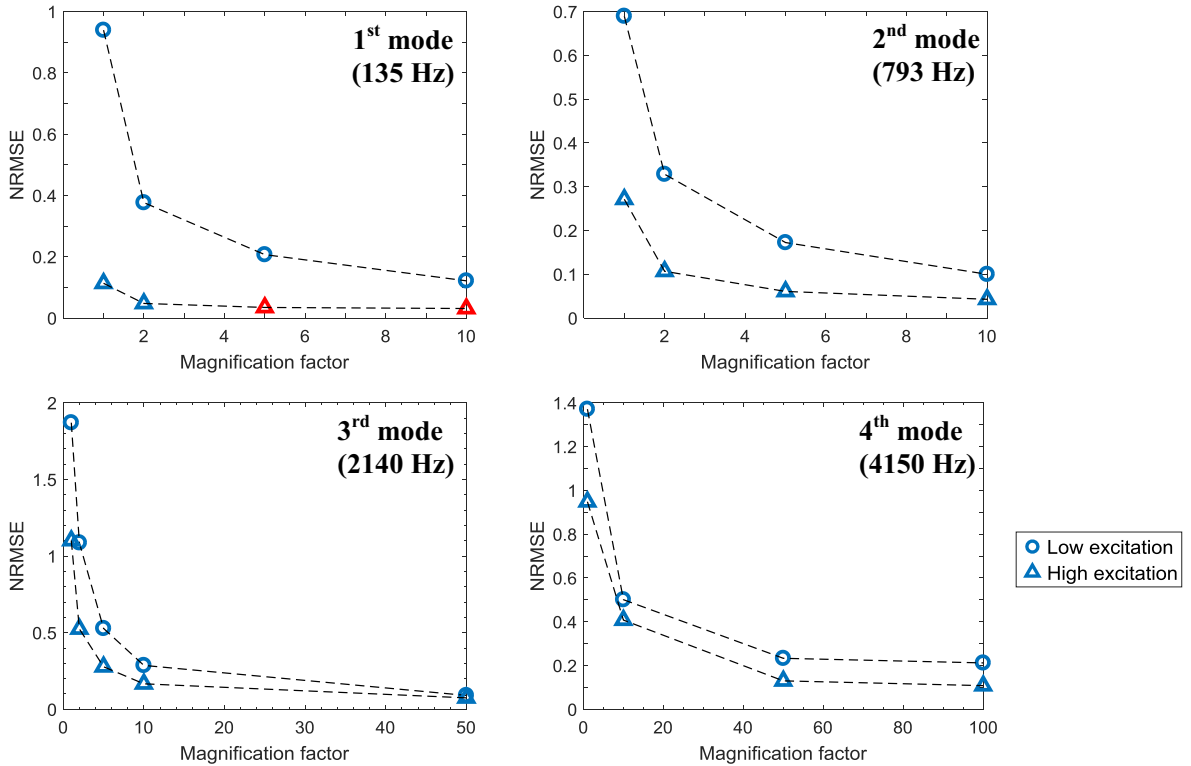


Fig. 14. Normalised Root Mean Squares Error versus magnification factor. Red markers indicate presence of non-correlated areas. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

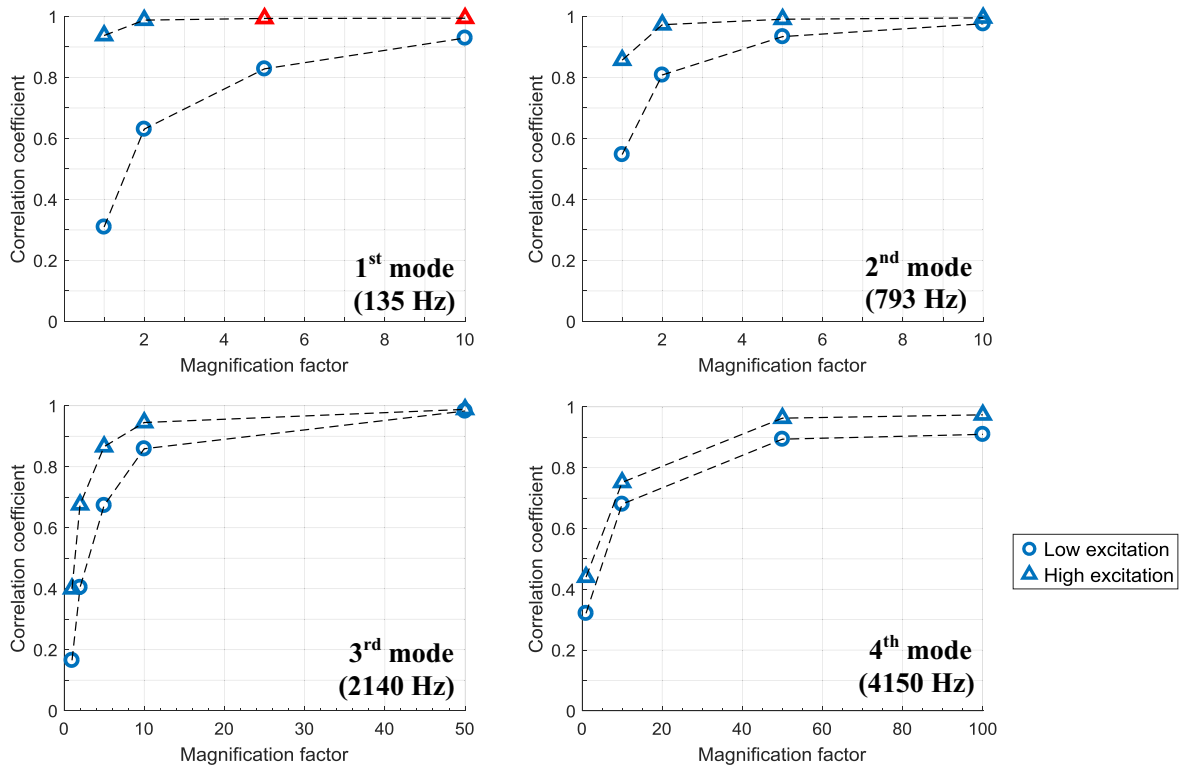


Fig. 15. Correlation coefficient between experimental and numerical mode shapes versus magnification factor. Red markers indicate presence of non-correlated areas. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

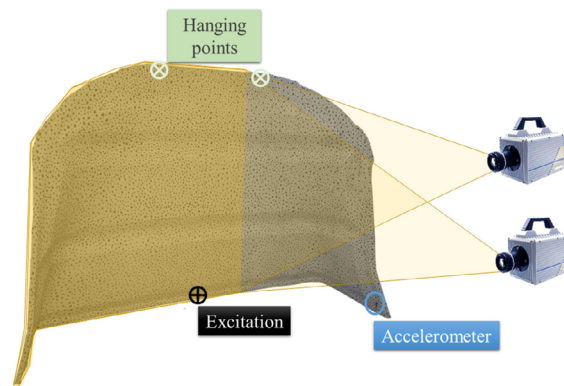


Fig. 16. Experimental setup for the panel with the high speed cameras focused on half the surface for ODSs characterisation and the accelerometers for natural frequencies identification.

4.2. Motion magnification

The employed shaker is not powerful enough to generate displacement in the panel that provides a clear visualisation of the ODSs using DIC. The study of the panel involved just one magnification factor to reveal ODSs from non-discernible ODSs tests. In Figs. 18–21 it is presented the results for the four modes. Each figure shows the ODSs corresponding to the three spatial components for the non-magnified and magnified states. The range of the colour bar was truncated to reveal minor displacements in the reinforcing ribs of the structure. In the figures, nothing is initially distinguishable in the non-magnified state and almost no relation with respect to the magnified results is observed. These results allow to see high order modes with complex shapes. Due to the shape of the panel, displacements are mainly registered normal to the surface, so U and W displacements are predominant, corresponding to the lateral and frontal planes. Likewise, some behaviours can be identified in one spatial component direction but not in others. For instance, U and W components of 190 Hz ODS in Fig. 19.

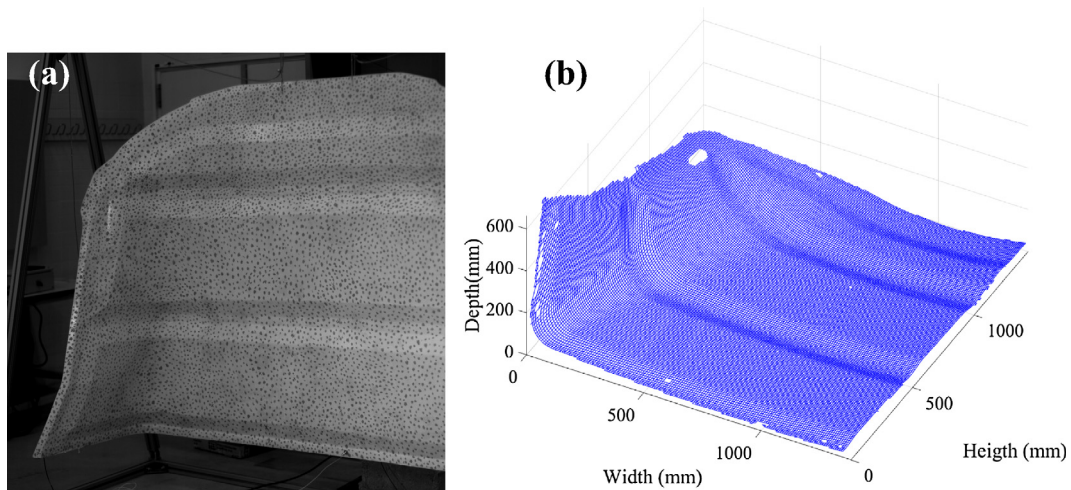


Fig. 17. (a) Image of the panel with 1024×1024 pixel resolution. (b) 3D digitalisation of the panel using DIC.

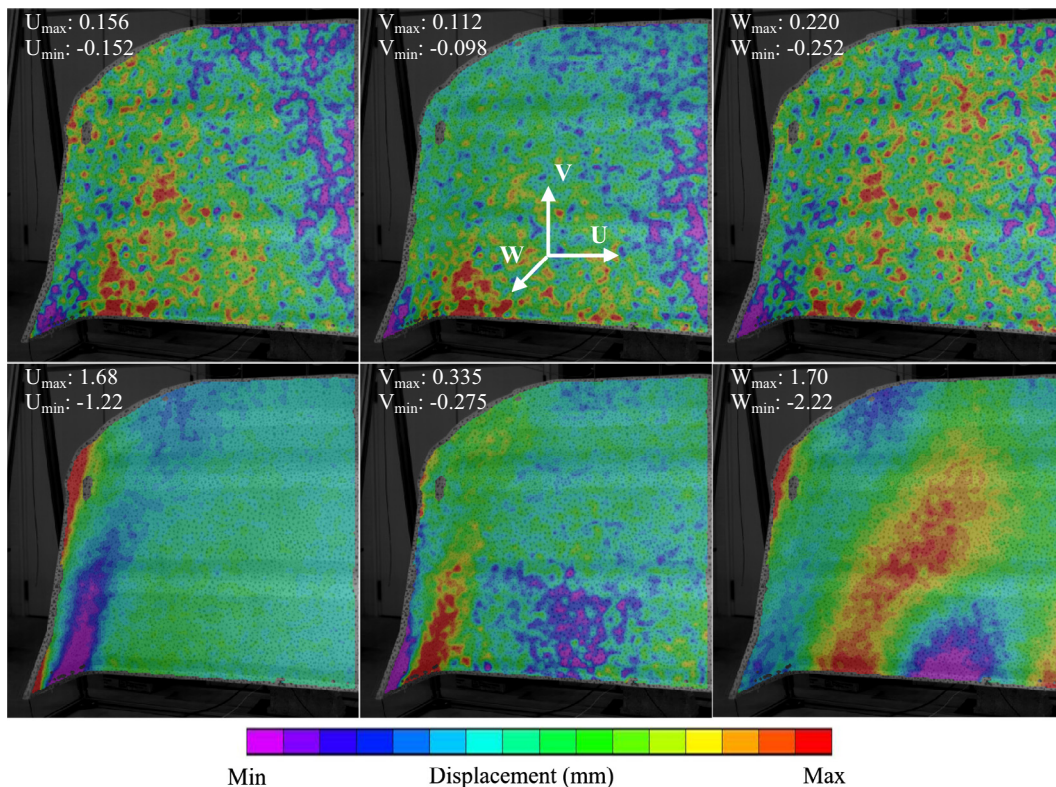


Fig. 18. Operational Deflection Shape of the composite panel at 137 Hz from non-magnified images (upper row) and $20\times$ magnified images (lower row).

As examples, videos are added for the modes at 190 Hz (Video 2) and 285 Hz (Video 3), including a raw video and U and W components. The motion in the naked video is visible but, once again, difficultly interpretable. The deformation is mainly perceived along the edge of the panel, especially in the lower corner. Furthermore, since the visualisation by naked eye is more sensitive to in-plane displacement than out-of-plane displacements, no interpretation can be extracted from the middle panel. In fact, previous works on modal characterisation using motion magnification employed edge detection algorithm instead of DIC to highlight the 2D shape on beam-like structures or other 1D elements [13–15], which is a more interpretable format than naked visualisation. Nevertheless, it is no useful in plate-like structures since no information would be extracted

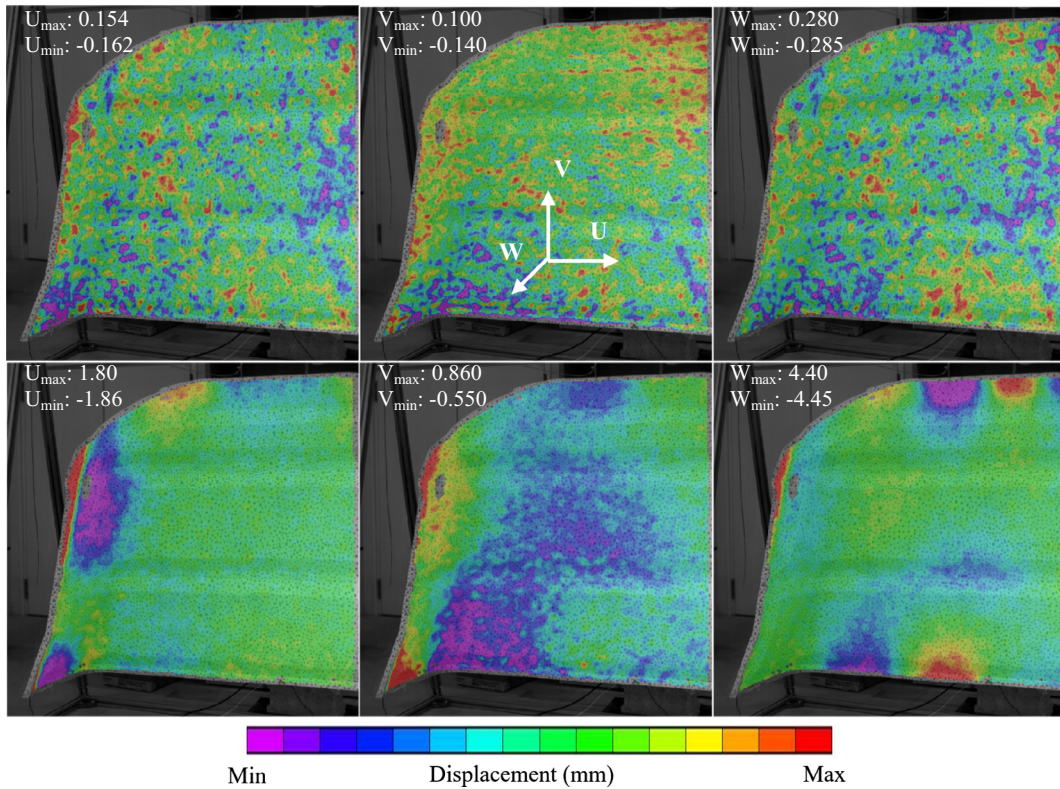


Fig. 19. Operational Deflection Shape of the composite panel at 190 Hz from non-magnified images (upper row) and 50× magnified images (lower row).

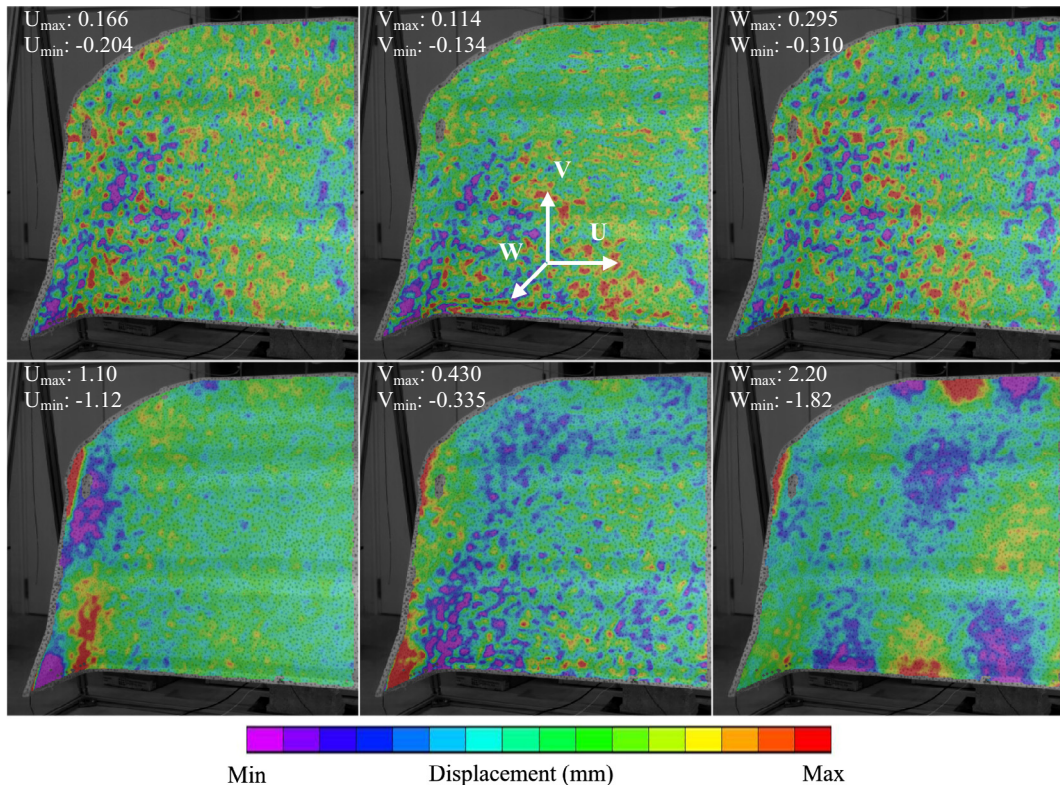


Fig. 20. Operational Deflection Shape of the composite panel at 215 Hz from non-magnified images (upper row) and 50× magnified images (lower row).

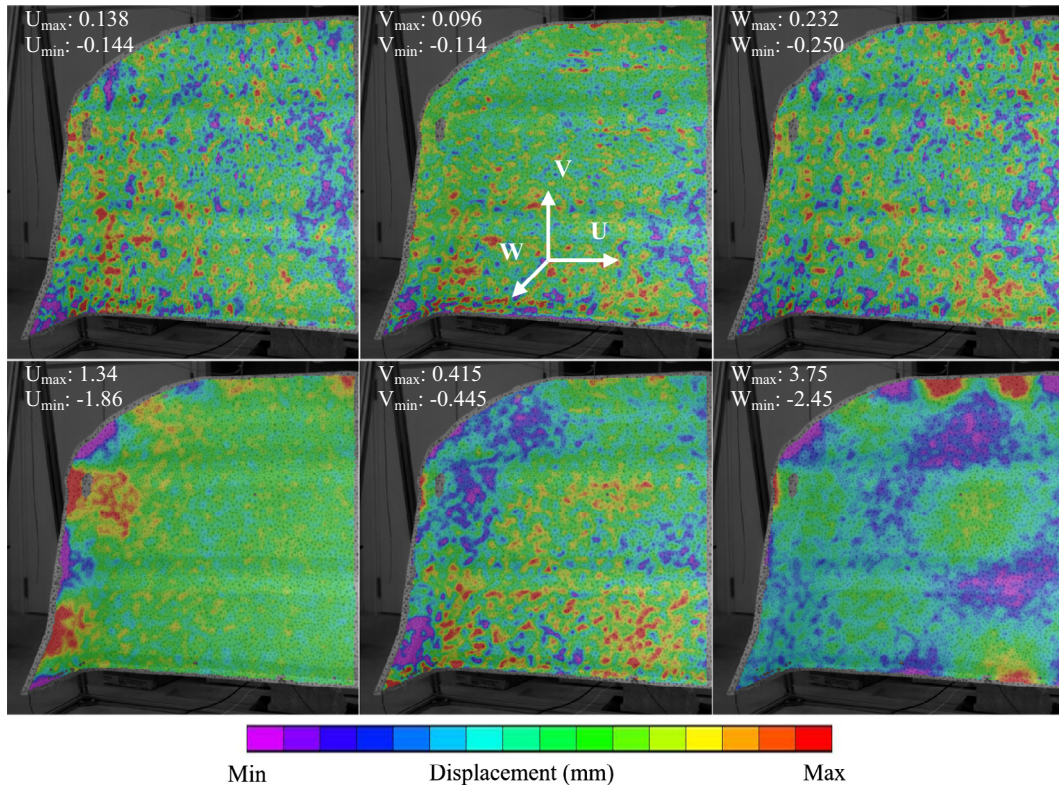


Fig. 21. Operational Deflection Shape of the composite panel at 285 Hz from non-magnified images (upper row) and 100× magnified images (lower row).

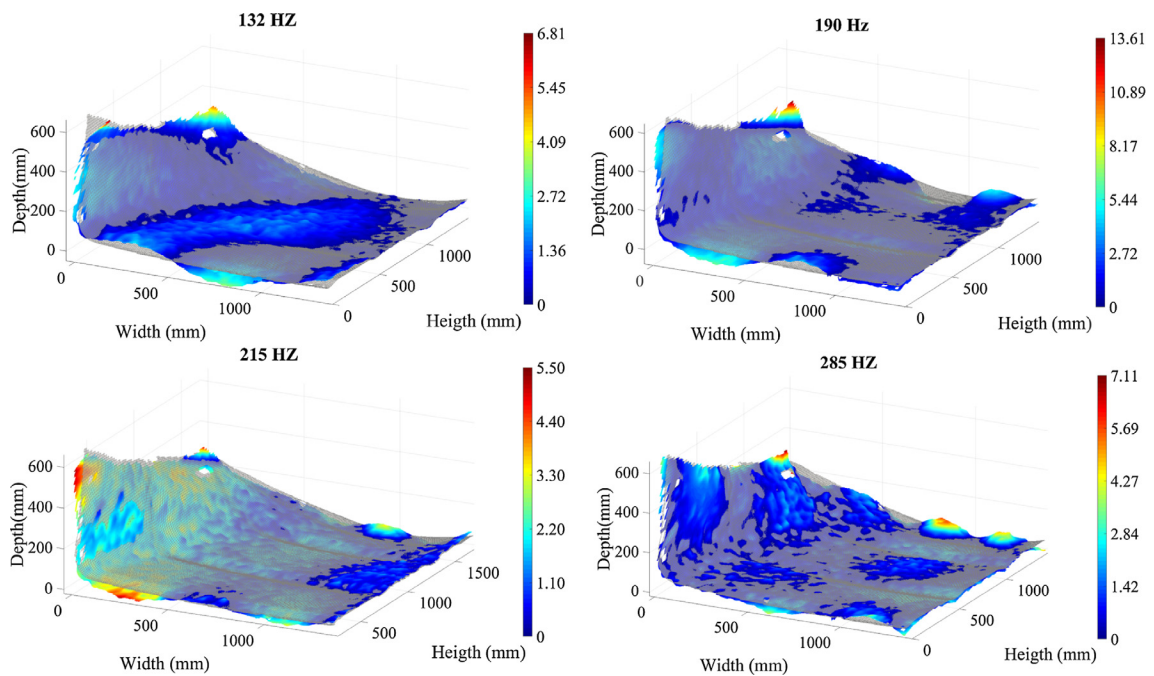
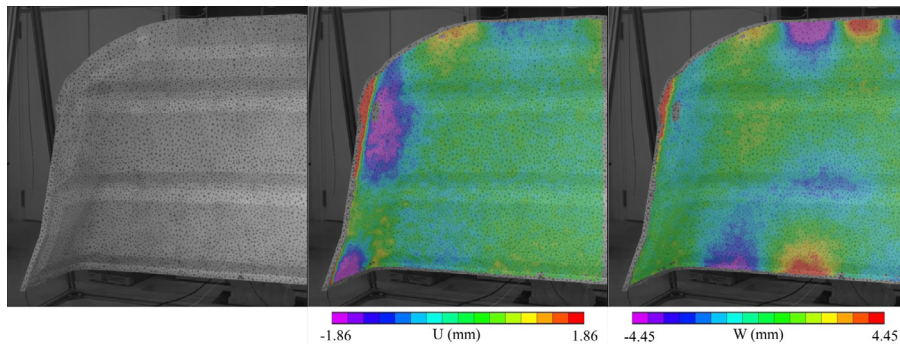
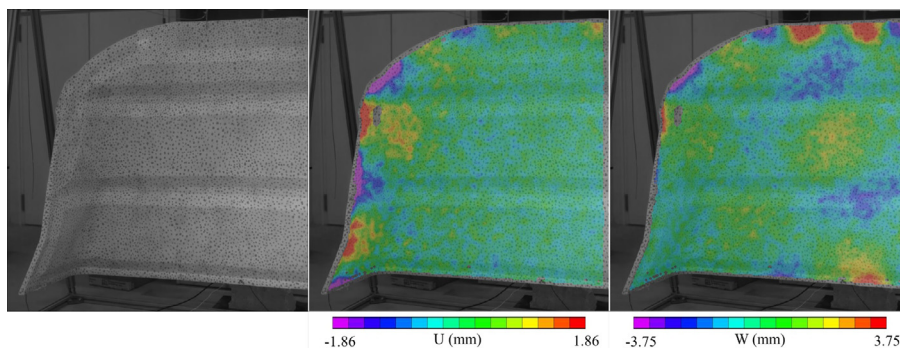


Fig. 22. Three dimensional representation of the absolute displacements of the four magnified ODSs along with the shaded non-deformed panel shape.

from the middle area of the panel. Three-dimensional displacement measurement employing 3D-DIC is hence demonstrated to be advantageous.



Video 2. Slow-motion video of the ODS at 190 Hz with 50× magnification.



Video 3. Slow-motion video of the ODS at 285 Hz with 100× magnification.

Despite these plots and videos of separate components go further than simple visualisation, now it is possible to depict 3D representations. Fig. 22 shows a scaled version of the absolute full-field displacement along with the non-deformed state as a grey shaded surface. The three spatial components are hence presented in conjunction, providing a full understanding of the deformation. To enhance the evaluation, these four illustrations are presented as [Supplementary material in Figure Matlab format](#), available online.

5. Conclusions

This study is the first attempt to combine phase-based motion magnification in a stereoscopic set of images to performed measurements using 3D-DIC. It has been based on the previous successful combination with 2D-DIC. Considering the challenge of dealing with couples of magnified images, the validation study have revealed important issues. Firstly, it has been proved that motion magnification helps to reveal ODSs using 3D-DIC from low amplitude tests. Actual shapes are not distorted by the magnification process. Consequently, this combination is especially intended for characterising ODSs in low amplitude events which could involve tests using non-powerful exciters or due to high frequencies. ODSs were presented at unprecedented high frequencies considering a 3D-DIC system.

On the other hand, 3D-DIC provides quantitative information to videos visualisation. The interpretation of motion by naked eye is confusing in many cases considering the complexity of the shapes. Thus, 3D-DIC improves this visualisation with intuitive full-field contour plots. But mainly, this work implies a significant advance regarding 2D previous works, probing the validity of 3D measurements. Moreover, edge detection, employed in previous phase-based motion magnification studies, presents evident limitations in plate-like structures. Thus, the use of motion magnification and its combination with DIC is extended to most practical real applications, enabling full-field comparison with FEM models.

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Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at <https://doi.org/10.1016/j.ymssp.2018.02.006>.

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