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Several robust and efficient load flow techniques based on combined approach for ill-conditioned power systems

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ABSTRACT

This paper addresses the load flow (LF) problem of medium, large and very large-scale ill-conditioned power systems using several proposed techniques based on a combined approach. The proposed approach uses a novel combination among homotopic functions, numerical methods and the Newton's technique for efficiently solving ill-conditioned systems. Using the proposed approach, any numerical method can be adapted to develop efficient and robust LF techniques. In this work, two novel LF techniques based on this combined approach using Forward-Euler formula and Ralston method are proposed. The proposed techniques are tested on several medium and large-scale ill-conditioned systems, comparing their performance with other well-known LF methods. Results show that the proposed LF techniques are robust and efficient enough to address the issues related with medium and large-scale ill-conditioned systems and they are not significantly affected by the initial guess considered.

1. Introduction

Load flow (LF) is normally considered as the most-useful tool in planning and operation of power systems. LF raises a set of nonlinear equations which link the nodal voltages with the injected nodal powers. As results of LF analysis, the nodal voltage angles and magnitudes, power branch flows and total losses are calculated.

Due to the LF problem involves a set of nonlinear equations, numerical iterative techniques must be employed for solving it. Frequently, the well-known Newton-Raphson method (NR) [1], is considered as the most standard LF solver. Standard NR method offers a good balance between robustness and efficiency, however, it needs to update and factorize the Jacobian matrix each iteration. Fast-Decoupled-Load-Flow [2] and its BX version [3] were proposed to overcome the convergence issues of NR since they are based in constant matrices. These techniques are especially suitable for networks with low R/X ratios. Several linear versions of LF problem have been proposed [4–6]. The formulation of LF problem based on current injections have been exploited over years [7–9]. Several high-order Newton-like methods have been proposed in [10,11]. In these methods, the number of iterations of NR has been reduced by increasing its convergence rate (the standard NR has a convergence rate of 2). Similar to other energy systems (like natural gas networks), the LF has been formulated in terms of branch variables instead of nodal variables [12]. Recently, the formulation of LF problem has been raised in dq reference in [13].

These mentioned LF techniques normally converge without difficulty in well-conditioned cases. Oppositely, when the condition of system is ill (typically due to high R/X ratios or loading levels), the NR and other standard LF techniques might be affected if the initial guess point employed is far away the solution or outside of Region of Attraction [14].

Many authors have proposed robust load flow techniques to address the problem of ill-conditioned cases. The first robust techniques are based on the truncation of the corrector vector. In [15], it has proposed a methodology based on the computation of an optimal factor which modifies the corrector vector in order to avoid the divergence. This factor is calculated as a result of an optimization problem. This philosophy has been exploited in several works over decades [16–20]. These techniques are normally quite robust, but their convergence may be very slow.

The Continuous Newton's method [21] has been adapted to the LF problem by developing two techniques based on Forward-Euler method and 4th order Runge-Kutta formula in [22]. Continuous Newton's method establishes that the LF problem is analogue to a set of autonomous ordinary differential equations. On the basis of this analogy, any numerical method can be successfully adapted for solving the LF problem. These techniques have not been deeply exploited in the literature, but they seem very effective to large ill-conditioned systems. Recently,

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4th order Runge-Kutta LF method has been combined with Broyden's method in order to improve its computational performance in [23] and three novel LF methodologies based on the Adams-Bashforth's methods have been developed in [24].

Levenberg method is a nonlinear programming that has been successfully applied to different optimization problems like optimal power flow in [25]. This method has been recently exploited for solving the LF problem [26], in this reference, links between this methodology and Lyapunov theory are also established. Recently, several high order Levenberg-Marquadt methods have been adapted for solving the LF problem [27–30].

Most of the existing robust LF methods typically offer a good performance when the size of system is not too large (approximately less than 1000 buses), however, when the size is large, these techniques may face some convergence difficulties due to the following reasons:

- They may not be efficient enough to manage large matrices and vectors computations.
- Their convergence properties are affected if the guess point employed for initializing the algorithm is not close enough to the solution. This fact is normally reflected in large number of iterations or divergence.

The homotopy theorem was first presented by Hwrewicz in 1935 [31]. It was applied to find the solution of polynomial equations and later it was modified for solving systems with polynomial equations [32,33]. It has been proved that homotopy is a very effective technique to address the local convergence problem and solve set of nonlinear equations [34,35].

Homotopy theorem has been extensively employed in LF analysis, especially to find the multiple solutions of LF problem [36–38]. In [36], the adaptive Runge-Kutta method has been used to trace the solution path. In [37], the LF formulation has been raised in a polynomial form, then, homotopic has been extended to the complex space and continuation methods have been employed to find all the system solutions. The relationship between the topological characteristics of network and number of LF solutions has been tackled in [38]. On the other hand, the well-known Continuation Power-Flow tool [39], can be seen as a homotopy method [22]. It may be employed to trace the P-V curve of the system and, hence, finding all solutions of LF problem for different generation and load levels.

Finally, the holomorphic embedding method (HEM) has been recently applied for solving the LF problem in several works [40–42]. In [40], the HEM has been firstly applied to solve the LF problem in DC and nonlinear DC circuits. In [41], HEM represents a distinct class of nonlinear equation solvers which are recursive rather than iterative, therefore, an infinite number of formulations exist, each one with different numerical properties. In addition, a possible PV model compatible with HEM has been introduced. The computational performance of HEM has been improved in [42], by introducing two novel methodologies, which are flexible enough to take any point as initial start instead of the flat initial guess, thus, improved techniques become faster than other HEM methods.

Homotopy methods are likely to fail in the bifurcation points [43]. In order to avoid these points, the multiparameter homotopy has been employed to find the equilibrium points of differential-algebraic equations which model a nonlinear circuit [44]. In [45], the LF problem has been firstly solved by raising the linear problem (DC-LF), then, a homotopy function has been raised in order to, progressively, transform the linear problem into the original one. In [46], the homotopy has been used to calculate all the Nash equilibria of multiplayer games in the electricity markets. Homotopy has been also used to find all the equilibrium points [47,48], or to estimate the stable region of power system dynamic model [49].

This paper proposes a novel family of efficient and robust techniques based on a new combined approach for solving LF problem of medium, large and very large-scale ill-conditioned power systems. The proposed approach combines three basic ideas i.e. the efficiency of the Newton-like methods, the global convergence offered by homotopy methods and the possibility to employ high order numerical methods to improve the convergence. With the aim to explore the benefits of the proposed approach, two LF techniques using Forward-Euler formula [50], and Ralston method (which is a second order Runge-Kutta method) [51] are developed. Moreover, it is explained that any numerical method (e.g. Runge-Kutta formulas) can be adapted to the proposed approach for developing a family of robust and efficient LF techniques. The proposed LF techniques are validated using several ill-conditioned systems and compared with other well-known robust load flow techniques.

Remainder of this paper is organized as follows: In Section 2, the LF problem formulation is briefly described. The basic principles of homotopy are explained in Section 3. In Section 4, the proposed approach and LF techniques are presented. Test systems employed and simulations carried out are presented in Section 5. Finally, the most important conclusions are presented in Section 6.

2. LF problem formulation

The solution of LF problem provides the nodal voltages angles and magnitudes in each bus, the power branch flows and total power system losses. Firstly, the active and reactive power mismatches for each bus are calculated as [52]:

$$\Delta P_i = P_i^{sch} - \sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j) \quad (1)$$

$$\Delta Q_i = Q_i^{sch} - \sum\nolimits_{j=1}^{n} |V_i||V_j||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j) \quad (2)$$

where, ΔP_i and ΔQ_i are the active and reactive power mismatches at bus i, respectively, P^{sch} and Q^{sch} are the injected active and reactive power at bus i, respectively, $V_i\angle\delta_i$ are the complex voltage at bus i, $Y_{ij}\angle\theta_{ij}$ are the ij^{th} element of admittance matrix and n is the total number of buses.

In order to simplify the notation, compact version of (1) and (2) will be used onwards:

$$\boldsymbol{g}(\boldsymbol{x}) = 0 \quad (3)$$

where, $\boldsymbol{g}(\boldsymbol{x})$: $\mathbb{R}^m \mapsto \mathbb{R}^m$ are nonlinear, smooth equations defined in (1) and (2). The variables of LF problem are the voltage angles of PV buses, the voltage angles and magnitudes of PQ buses. The LF variable vector can be defined as follows:

$$\boldsymbol{x} = [\boldsymbol{\delta_{PV}}, \boldsymbol{\delta_{PQ}}, \boldsymbol{V_{PQ}}]^T \in \mathbb{R}^m \quad (4)$$

where, $\boldsymbol{\delta_{PV}}$ and $\boldsymbol{\delta_{PQ}}$ are the vectors of PV and PQ buses voltage angles, respectively and $\boldsymbol{V_{PQ}}$ is the vector of PQ buses voltage magnitudes. The algorithm stops if the following condition is satisfied:

$$||\boldsymbol{g}(\boldsymbol{x}^{(k)})||_\infty \le \epsilon \quad (5)$$

where, $\epsilon \in \mathbb{R}$ is a preset convergence parameter. Frequently the algorithm also stops if the number of iterations is greater than a given limit ($k > k_{MAX}$). In this case, the algorithm has likely failed to converge.

Since (3) are nonlinear and cannot be explicitly inverted, a numerical iterative method must be used to solve the LF problem. NR is considered the standard method for solving the LF problem. A generic k^{th} NR iteration for LF problem is defined as:

$$\Delta\boldsymbol{x}^{(k)} = -\boldsymbol{J_x}(\boldsymbol{x}^{(k)})^{-1}\boldsymbol{g}(\boldsymbol{x}^{(k)})$$

$$\boldsymbol{x}^{(k+1)} = \boldsymbol{x}^{(k)} + \Delta\boldsymbol{x}^{(k)} \quad (6)$$

where, $\boldsymbol{J_x} \in \mathbb{R}^{m\times m}$ is the Jacobian matrix of system and $(\bullet)^{-1}$ stands for the inverse operator, which is formed by partial derivative of (3) with respect to vector (4) and the superscript k enumerates the current iteration.

3. Basic principles of homotopy

Homotopy is a continuation-like method that used to find the solution of an equation or a set of equations due to its global convergence. With the aim of explaining the homotopy's procedure, set of nonlinear Eq. (3) are used. Let us assume that (3) is difficult to solve due to a lack of "reasonable" initial guess [43]. Assume we also have a set of equations $\bar{\boldsymbol{g}}(\boldsymbol{x}) = 0$ which is easy to solve. The homotopy allows us to solve (3) using $\bar{\boldsymbol{g}}(\boldsymbol{x}) = 0$ and constructing a chain of "intermediate" solutions known as solution path. In order to construct the solution path, it is necessary to provide a homotopy function $\boldsymbol{h}(\boldsymbol{x}, \lambda)$. It is worth to mention that homotopy function involves the exogenous parameter $\lambda \in [\lambda^0, \lambda^f]$; $(\lambda^0, \lambda^f) \in \mathbb{R}$. Therefore, the homotopy function constructs the solution path while λ evolves from λ^0 to λ^f as follows:

$$
\begin{aligned}
&\boldsymbol{h}(\boldsymbol{x}, \lambda^0) = \bar{\boldsymbol{g}}(\boldsymbol{x})\\
&\vdots\\
&\boldsymbol{h}(\boldsymbol{x}, \lambda)\\
&\vdots\\
&\boldsymbol{h}(\boldsymbol{x}, \lambda^f) = \boldsymbol{g}(\boldsymbol{x})
\end{aligned} \tag{7}
$$

As can be seen, the homotopic's process evolves from the easier version $\bar{\boldsymbol{g}}(\boldsymbol{x})$ to the original set of Eq. (3) while the parameter λ progresses from λ^0 to λ^f. Hence, the original set of Eq. (3) is solved at the end of the process. The main feature of homotopy is its global convergence, since it starts with an easy point. Homotopic techniques are more robust with respect to the initial guess point than other methods like Newton-like methods. For the sake of clarify, the generic procedure of the homotopy theorem is depicted in Fig. 1.

The main element of the homotopy is considered the homotopic function, hence several functions have been considered in the literature. In this paper, we use the following one [43,53]:

$$\boldsymbol{h}(\boldsymbol{x}, \lambda) = \lambda \boldsymbol{F}(\boldsymbol{x}) + (1 - \lambda)\boldsymbol{G}(\boldsymbol{x}), \lambda \in [0, 1] \tag{8}$$

where, $\boldsymbol{F}(\boldsymbol{x})$: $\mathbb{R}^m \mapsto \mathbb{R}^m$ is the original system and $\boldsymbol{G}(\boldsymbol{x})$: $\mathbb{R}^m \mapsto \mathbb{R}^m$ is the easier system, which can be defined in different ways. In the following section, $\boldsymbol{F}(\boldsymbol{x})$ and $\boldsymbol{G}(\boldsymbol{x})$ will be defined.

4. Proposed LF techniques

The main aim of this paper is to develop a new LF philosophy, thus, two novel Newton-Homotopic based LF techniques are proposed. Firstly, Forward-Euler method is used for updating the LF state vector through the solution path as follows:

$$\boldsymbol{x}^{k,\lambda+\Delta\lambda} = \boldsymbol{x}^{k,\lambda} + \Delta t * \boldsymbol{h}(\boldsymbol{x}^{k,\lambda}, \lambda), \quad For \quad \lambda = 0 \quad to \quad \lambda = 1 \tag{9}$$

where, $\Delta t \in \mathbb{R}$ is the step size of numerical method.

This methodology has been called the Forward-Euler-Homotopy method (FEH). Also, the second order Runge-Kutta formula called Ralston's method is used for developing the Ralston-Homotopy method (RH) as follows:

$$
\begin{aligned}
&\boldsymbol{k}_1 = \boldsymbol{h}(\boldsymbol{x}^{k,\lambda}, \lambda)\\
&\boldsymbol{x}_2^{k,\lambda} = \boldsymbol{x}^{k,\lambda} + 2/3\Delta t \boldsymbol{k}_1\\
&\boldsymbol{k}_2 = (\lambda + 2/3\Delta\lambda)\boldsymbol{k}_1 + (1 - (\lambda + 2/3\Delta\lambda))(\boldsymbol{x}_2^{k,\lambda} - \boldsymbol{x}^{k-1,\lambda^f})\\
&\boldsymbol{x}^{k,\lambda+\Delta\lambda} = \boldsymbol{x}^{k,\lambda} + \Delta t \frac{\boldsymbol{k}_1 + 3\boldsymbol{k}_2}{4}, \quad For \quad \lambda = 0 \quad to \quad \lambda = 1
\end{aligned} \tag{10}
$$

In the same manner, the proposed combined approach can be extended to other numerical methods like 4th order Runge-Kutta, Dormand-Prince or Runge-Kutta-Fehlberg. Hence, different LF techniques can be developed just by changing the numerical method employed to update the LF state vector. Proposing and testing more numerical methods is out of scope of this work. Nevertheless, the main features of the proposed combined approach can be proved by the proposed FEH and RH, which use a 1st and 2nd order Runge-Kutta formulas respectively.

As can be seen, the proposed combined approach employs the homotopic function $\boldsymbol{h}(\boldsymbol{x}, \lambda)$, to correct the value of unknown vector instead of that calculated in (6). Hence, unknown vector follows the path traces by the homotopic function.

To define $\boldsymbol{G}(\boldsymbol{x})$ which is appeared in (8), the fixed-point function [53], can be written as follows:

$$\boldsymbol{G}(\boldsymbol{x}^{k,\lambda}) = \boldsymbol{x}^{k,\lambda} - \boldsymbol{x}^{k-1,\lambda=1} \tag{11}$$

Fixed-point function is very simple since it just subtracts the final value of the unknown vector from previous iteration $\boldsymbol{x}^{k-1,\lambda=1}$, to the current value of the unknown vector on the solution path $\boldsymbol{x}^{k,\lambda}$. Therefore, it does not need to compute complicate operations like the factorization of the Jacobian matrix. On the other hand, the function $\boldsymbol{F}(\boldsymbol{x})$ will be the NR corrector vector $\Delta\boldsymbol{x}$ as follows:

$$\boldsymbol{F}(\boldsymbol{x}^{k,\lambda=0}) = -\boldsymbol{J}_x(\boldsymbol{x}^{k,\lambda=0})^{-1}\boldsymbol{g}(\boldsymbol{x}^{k,\lambda=0}) \tag{12}$$

The convergence criteria (5) is evaluated at the end of solution path. If (4) is not met, the algorithm continues with another solution path, taking the initial point of the end of the previous solution path:

$$\boldsymbol{x}^{k+1,\lambda=0} = \boldsymbol{x}^{k,\lambda=1} \tag{13}$$

Therefore, the proposed approach forces the state vector to evolve through the solution path from the easy function (11) to NR corrector vector (12). Furthermore, the proposed techniques can be considered as homotopy Newton-like methods, because they use the NR corrector vector (12) for supporting the evolution of state vector. The proposed family of LF techniques can be considered as artificial parameter homotopy [54]. Thus, only the final point of the homotopy path $(\boldsymbol{x}^{k,\lambda^f})$ is relevant.

For the sake of clarify, the evolution followed by the different LF variables in a 3-bus tutorial example (1 PQ bus and 1 PV bus), which has been taken from the system described in [55] (pp. 337–338) eliminating the bus#3, using the proposed FEH is shown in Fig. 2. As can be seen, unknowns follow the solution path each iteration (which are indicated by vertical lines). At the end of one iteration, if (5) is not met or the preset maximum number of iterations is not surpassed, another solution path starts taking as initial point the end of the previous path.

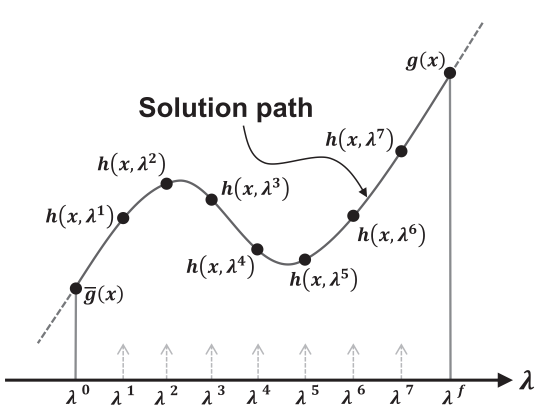

Fig. 1. Homotopy's principle.

4.1. Choosing the value of Δt

The proposed approach has one degree of freedom, the step size i.e. Δt. Its value is crucial for an optimal performance of the approach. However, choosing *a priori* a value for Δt may be very difficult. Moreover, this value should not be fixed, hence, at first iterations, the robustness of LF technique is the most important fact, therefore, Δt may

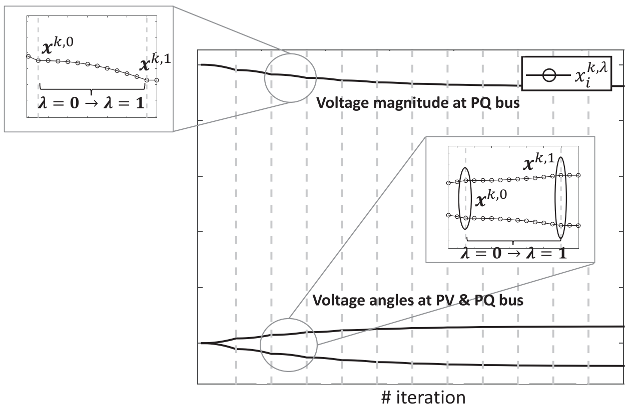

Fig. 2. Solution paths and unknown vector's progress in 3-bus tutorial example [55], using the proposed approach.

be fixed at a small value at the beginning of algorithm. Oppositely, when the algorithm finds a good direction to modify the state vector, the Δt may be increased in order to accelerate the convergence and reduce the total number of iterations. On the basis of this aspects, an adaptive mechanism for Δt is proposed. It uses two guess values of the LF state vector, which are calculated as follows:

$$\hat{\boldsymbol{x}}_1 = \boldsymbol{x}^{k+1,\lambda^0} + \Delta t/2\boldsymbol{F}(\boldsymbol{x}^{k+1,\lambda^0}) \tag{14}$$

$$\hat{\boldsymbol{x}}_2 = \boldsymbol{x}^{k+1,\lambda^0} + \Delta t/4\boldsymbol{F}(\boldsymbol{x}^{k+1,\lambda^0}) \tag{15}$$

Using these two values of state vector, the value of Δt can be computed according to the following heuristic rule:

$$\zeta = ||\frac{\hat{\boldsymbol{x}}_1 - \hat{\boldsymbol{x}}_2}{\Delta t}||_\infty \tag{16}$$

$$\textit{if} \quad \zeta > SF \quad \textit{then} \quad \Delta t \leftarrow max\{\sigma_1 \Delta t, \Delta t_{min}\}$$

$$\textit{if} \quad \zeta \leq SF \quad \textit{then} \quad \Delta t \leftarrow min\{\sigma_2 \Delta t, \Delta t_{max}\} \tag{17}$$

where, $SF \in \mathbb{R}$ is a security factor, $\sigma_1 \in \mathbb{R}$ and $\sigma_2 \in \mathbb{R}$ are damping coefficients and $\Delta t_{min} \in \mathbb{R}$ and $\Delta t_{max} \in \mathbb{R}$ are the minimum and the maximum incremental step sizes, respectively. It is worth to mention, that Δt and $\Delta\lambda$ are not the same parameters, while the first one is used in the numerical arrangement (in this case Forward-Euler or Ralston's method) and, therefore, it is related with a numerical method, the second one is directly linked with the homotopic function, and determine how the state vector evolves through the solution path. Furthermore, Δt_{min} and Δt_{max} do not bound the value of $\Delta\lambda$, which is taken fixed. The values of different aforementioned parameters are tuned based on our experience and the following simple criteria:

- The value of security factor should not be higher than 0.5
- The damping coefficients should not be much different of one. Moreover, it is recommendable that $\sigma_2 > \sigma_1$
- The minimum and maximum incremental step sizes mainly depend on the method used (FEH or RH). Typically, the proposed RH allows higher values of Δt than the FEH. Anyway, at the beginning of algorithm, the value of Δt should be equal to Δt_{min}

Based on the above ideas, Table 1 presents the parameters values used in the simulations.

With the aim to summarize, Fig. 3 shows the flowchart of the proposed approach.

4.2. Convergence analysis of proposed LF techniques

As mentioned before, the proposed techniques are based on the Newton's method, in other words, they are modifications of Newton's method. These techniques are proposed to take advantage of homotopic's principle and numerical arrangements in order to improve the robustness of the classical Newton's method. In other words, the proposed approach combines three basic ideas i.e. the efficiency of the Newton-like methods (it has quadratic convergence in the vicinity of solution, however, this feature is lost when the initial guess is outside of its basin of attraction), the global convergence offered by homotopy methods and the possibility to employ high order numerical methods to improve the convergence (Continuous Newton's method applied to LF problem in [22]).

However, one cannot claim the global convergence of proposed techniques since they are not in fact homotopic methods. Anyway, in this subsection the superior convergence properties of proposed techniques are analyzed through their regions of attraction. Fig. 4 shows the regions of attraction of the NR and the proposed FEH and RH LF techniques in the aforementioned 3-bus tutorial case. As can be seen, all methods are not much sensitive with respect the initial voltage magnitude, however, the proposed techniques show robust performance with respect the initial voltage angles. This feature of proposed techniques is further proved in the following section, solving various ill-conditioned systems using various initial guesses.

On the other hand, it is well-known that Runge-Kutta methods (Euler and Ralston's methods), are stable and they converge to the solution if the function is Lipschitz (in [22], it is demonstrated that $\boldsymbol{f}$ is asymptotically stable) and the sum of coefficients equals 1 (which is true in the Euler and Ralton's methods). Therefore, it can be said that the solution of LF problem is reachable by the proposed LF techniques if the initial guess lies inside of the ROA. Results show that the proposed FEH and RH LF techniques show wider ROA, therefore, one can affirm *a priori*, that the proposed techniques are more suitable for ill-conditioned cases and, therefore, they can be considered as robust LF solvers.

Table 1
Parameters values considered in simulations.

	FEH	RH
SF	0.3	0.3
σ_1	0.95	0.95
σ_2	1.05	1.05
Δt_{min}	0.05	0.1
Δt_{max}	0.1	1
$\Delta\lambda$	0.1	0.1

5. Tests and results

During this section, the two proposed LF techniques (FEH and RH) are validated using the following medium, large and very large-scale test systems:

- IEEE 118-bus system: This system has 186 branches and 54 generators. It is operated at 138, 161 and 345 kV. The total consumptions of this system are 424.2 MW and 143.8 MVar. More details about this system are given in [56,57].
- IEEE 300-bus system: This system has 411 branches and 69 generators. It is operated from 0.6 kV to 345 kV. Its total consumptions are 23.526 MW and 7.788 MVar. More details about this system are given in [56,57].
- 3012-bus system: This system is a snapshot of the Polish transmission system in the winter 2007–08 evening peak. It has 3.527 branches and 502 generators. It is operated from 16 kV to 400 kV. It has a total consumption of 27.170 MW and 10.201 MVar. All details about this system are given in [58].
- 9241-bus system: this system is a portion of European transmission system from the PEGASE project [59,60]. This system has 16.049 branches and 1.445 generators. It is operated from 110 kV to 750 kV. It has a consumption of 312.350 MW and 73.582 MVar.

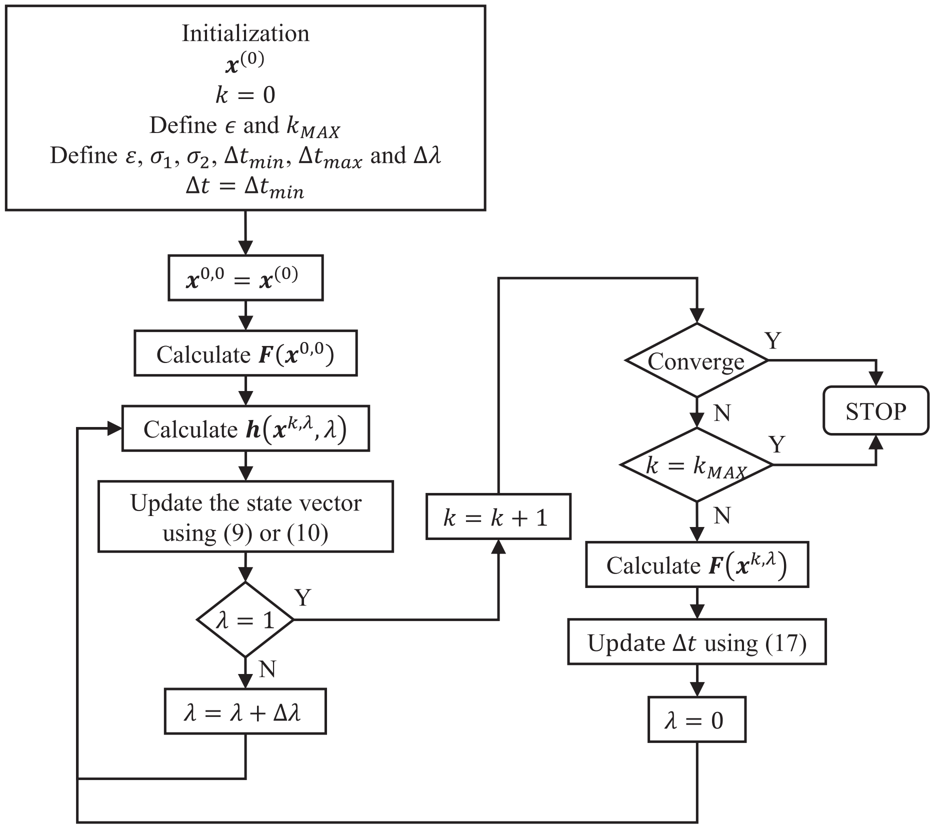

Fig. 3. Proposed approach flowchart.

- 13659-bus system: this system is a portion of European transmission system from the PEGASE project [59,60]. This system has 20.467 branches and 4.092 generators. It is operated from 400 V to 750 kV. It has a consumption of 381.431 MW and 98.523 MVar.

The aforementioned IEEE 118-bus, IEEE 300-bus and 9241-bus systems, are naturally well-conditioned. Since the main aim of this paper are the ill-conditioned systems, the condition of these systems is intentionally deteriorated by increasing their R/X ratios and loading levels. Moreover, in order to prove the superior convergence properties of proposed LF techniques with respect the standard NR method, a deteriorated version of the flat initial guess is considered for solving these systems. Thus, the flat initial guess point is modified by adding a real factor to the initial voltage magnitudes at PQ buses as follows:

$$\boldsymbol{V}_{PQ}^{0} = 1.0 + e \tag{18}$$

On the other hand, the Polish 3012-bus and the European 13659-bus systems can be considered as ill-conditioned and most of LF methods are not able to successfully solve them using a flat initial guess. Table 2 summarizes the conditions considered in the studies systems.

Table 2
The conditions considered in the studies systems.

	118-bus	300-bus	3012-bus	9241-bus	13659-bus
Loading level	120%	100%	100%	100%	100%
$\boldsymbol{R_{new}}$	$3xR_{old}$	$2xR_{old}$	$1xR_{old}$	$2xR_{old}$	$1xR_{old}$
$\boldsymbol{e}$	0.5	0.2	0	0.1	0

The proposed FEH and RH LF techniques are compared with the following standard and robust LF methods:

- Standard NR [1].
- Iwamoto's method [15].
- Three methods based on the Continuous Newton's principle: Forward-Euler [22], 4th order Runge-Kutta [22] and Heun's method [24].

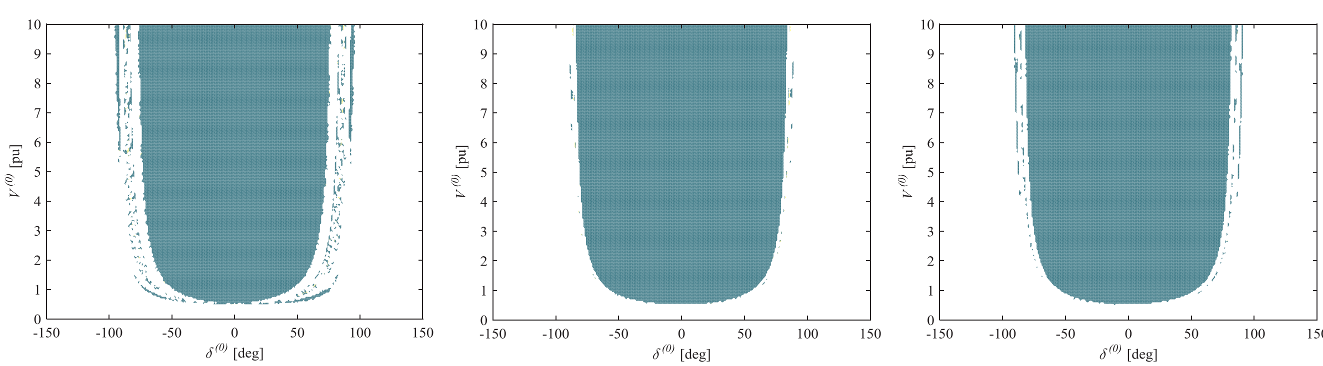

Fig. 4. Regions of attraction of standard NR (left) and proposed FEH (middle) and RH (right) in the 3-bus tutorial case.

Table 3
CPU time and total number of iterations employed for solving LF in the studied systems ($\epsilon = 10^{-6}$, $k_{MAX} = 50$).

Method		118-bus		300-bus		3012-bus		9241-bus		13659-bus	
		Time [s]	#iter	Time [s]	#iter	Time [s]	#iter	Time [s]	# iter	Time [s]	#iter
Standard NR [1]		Diverge	–	Diverge	–	Diverge	–	Diverge	–	Diverge	–
Iwamoto [15]		Fail*	–	Fail*	–	Fail*	–	Fail*	–	Fail**	–
Forward Euler [22]		Diverge	–	Diverge	–	Diverge	–	Diverge	–	Fail**	–
4th order Runge-Kutta [22]		0.1022	23	0.1822	23	Diverge	–	6.6675	26	8.5323	25
Heun's method [24]		Diverge	–	0.2199	49	1.7883	49	Fail*	–	5.1973	30
Levenberg's methods	[26]	Diverge	–	Fail*	–	Fail*	–	Fail*	–	Fail*	–
	[27]	0.0640	21	Fail***	–	Fail***	–	Fail***	–	Fail***	–
	[28]	Fail*	–	Fail*	–	Fail*	–	Fail*	–	Fail*	–
Proposed FEH		0.0503	30	0.0747	26	0.5049	26	2.1386	32	Fail**	–
Proposed RH		0.0316	18	0.0494	17	0.3167	16	1.2961	19	1.4721	16

* It did not converge in 50 iterations.
** It has obtained the low voltage solution.
*** It did not get results in a reasonable time.

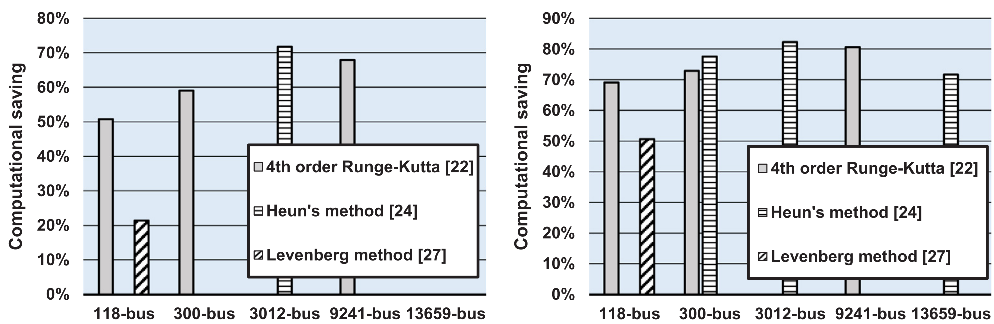

Fig. 5. Computational time saving [%] of proposed FEH (left) and RH (right) with respect other methods.

Table 4
Total number of iterations employed by the proposed LF techniques for solving 3012-bus system at different loading levels ($\epsilon = 10^{-6}$, $k_{MAX} = 50$).

ρ	Heun	FEH	RH
1.05	49	27	16
1.10	49	27	16
1.15	49	27	16
1.20	50	27	16
1.27	Fail*	28	17
1.28	Diverge	Diverge	Diverge

* It did not converge in 50 iterations.

Table 5
Total number of iterations employed by the proposed LF techniques for solving 3012-bus system at different R/X ratios ($R_{new} = \delta R_{old}$) ($\epsilon = 10^{-6}$, $k_{MAX} = 50$).

δ	Heun	FEH	RH
1.05	49	27	15
1.10	49	27	15
1.15	49	27	15
1.20	49	27	16

Table 6
Total number of iterations employed by the proposed LF techniques for solving 3012-bus system at different loading levels ($\epsilon = 10^{-6}$, $k_{MAX} = 50$).

e	Heun	FEH	RH
0.05	45	23	14
0.10	47	24	14
0.15	49	24	14

- Levenberg-Marquadt method [26].
- High-order Levenberg-Marquadt method [27].
- The corrected Levenberg method with non-monotone line search [28].

All simulations have been carried out using the software package MATPOWER 6.0 [61]. Furthermore, MATPOWER is open source, thus, the proposed algorithms can be easily translated to the MATPOWER's code.

Table 3 presents the performance of different LF techniques for solving the studied systems in term of computation time and number of iterations. The computational time is calculated as the mean value of 50 simulations. All simulations have been run on a machine with a 3.4 GHz Intel Core i5-7500 CPU (8.00 GB RAM).

From Table 3, it can be observed that most of well-known LF methods faced convergence difficulties for solving the studied ill-conditioned systems. On the other hand, the proposed FEH and RH load flow techniques are successfully converged in all cases (except the proposed FEH in the 13659-bus system which has obtained the low voltage solution). The 4th order Runge-Kutta has failed in the 3012-bus case while the Heun's method has failed in the 118-bus and 9241-bus systems in ill-conditions cases. On the other hand, the High order Levenberg-Marquadt method has only converged in the 118-bus system. In all cases, the proposed LF techniques have the lowest values of CPU time compared with the remainder methods. The proposed FEH load flow technique is very efficient but it needs many iterations to converge, this drawback is overcome using higher order schemes, thus, the proposed RH needs less iterations. It is worth to mention that the proposed RH needs less iterations than the remainder methods. Fig. 5 shows the computational time saving of proposed LF techniques with respect 4th order Runge-Kutta [22], Heun's [24] and High-Order Levenberg-Marquadt method [27]. The proposed FEH load flow technique is able to save up to 72% with respect to Heun's method while the proposed RH is able to reduce the CPU time up to 82%.

Now, let us analyse the effect of loading factor and R/X ratio on the performance of proposed LF technique. The loading level can be modified as follows:

$$P_i = \rho P_i, \forall\ i \in [\boldsymbol{PV}, \boldsymbol{PQ}]^T \tag{19}$$

$$Q_i = \rho Q_i, \forall\ i \in [\boldsymbol{PQ}]^T \tag{20}$$

where, $\rho \in \mathbb{R}$. Tables 4 and 5 present the total number of iterations employed by the proposed FEH and RH load flow techniques and the Heuns' method for solving the LF problem of 3012-bus system at different loading levels and R/X ratio, respectively. From Table 4, it can be observed that both loading level and R/X do not significantly affect the performance of proposed methods. Oppositely, Heun's method fail when the loading level is near to the maximum loadability point due to an excessive number of iterations.

Finally, the effect of initial guess on the performance of proposed LF techniques is studied. To do that, Eq. (18) is used to add a real factor e to the initial voltage magnitudes. This is applied on the studied 3012-bus case and total number of iterations are reported in Table 6. As can be seen, deteriorating the initial guess do not affect the performance of proposed techniques, curiously, they employ less iterations with respect the flat initial guess. It is explained in the fact that their Region of Attraction may be no symmetrical with respect the flat point, therefore, their performance can be improved by getting away from the flat initial guess.

6. Conclusions

In this paper, a novel family of robust and efficient LF techniques for solving ill-conditioned power systems have been proposed. The proposed techniques based on a developed combination approach which includes the following:

- The efficiency of Newton-like methods.
- The global convergence of homotopy functions.
- The possibility to use different numerical methods.

Owing to its features, the proposed approach is very promising for tackling ill-conditioned large-scale systems. Despite that several robust LF methods have been proposed in the literature over decades, they have bad performance in case of large-scale ill-conditioned systems. The proposed approach is able to overcome these difficulties. Using the proposed approach, several LF methods based on different numerical integration techniques such as superior order Runge-Kutta formulas, Adams-Bashforth methods, Bulirsch-Stoer method etc, can be developed.

To prove the proposed approach, the following two LF techniques have been proposed:

- Forward-Euler-Homotopy method (FEH)
- Ralston-Homotopy method (RH)

The proposed techniques have been tested on different ill-conditioned systems and compared with other robust LF methods presented in the literature.

The superiority of proposed techniques has been demonstrated since they are only able to converge in all studied systems. The obtained results shown good performance of proposed techniques compared with the other LF methods which faced different convergence difficulties. The proposed FEH and RH load flow techniques reduced the computational up to 69% and 80% with respect 4th order Runge-Kutta [22], 72% and 82% with respect the Heun's method [24] and 21% and 50% with respect the Levenberg's method [27], respectively. Moreover, it seems that the proposed techniques are quite scalable to systems with 10.000 buses and more.

Influence of loading level, R/X ratio and initial guess have been also explored. The proposed technique shown good performance in all studied scenarios.

Future works should be focused on raising different techniques based on the approach proposed in this paper. On the other hand, sophisticated schemes such as Bulirsch-Stoer and Romberg's methods should be adapted to the proposed approach. In addition, the proposed approach can be adapted for MV networks based on LV load measurements and estimations [62–67].

Appendix A. Supplementary material

Supplementary data to this article can be found online at https://doi.org/10.1016/j.ijepes.2019.03.035.

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