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Mixed static-dynamic protocol-based Tobit recursive filtering for stochastic nonlinear systems against random false data injection attacks

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***Abstract*—In this paper, the Tobit recursive filtering (TRF) issue is discussed for a class of time-varying stochastic nonlinear systems (SNSs) with censored measurements and random false data injection attacks (FDIAs) under the mixed static-dynamic protocol. The censored measurements considered are depicted by the Tobit Type I model and the phenomenon of the random FDIAs involved is governed by a set of Bernoulli random variables. Additionally, in order to reduce the communication burden and improve the data utilization efficiency, the mixed static-dynamic protocol is elaborately adopted to schedule the signal transmission, which is managed by the time-triggered and event-triggered rules to further increase the flexibility of the data scheduling. The main goal of this paper is to present a new TRF approach such that, in the presence of censored measurements, mixed static-dynamic protocol and random FDIAs, a minimized upper bound of the filtering error covariance (FEC) can be obtained. Moreover, a sufficient criterion from the theoretical analysis perspective is established to guarantee the desired uniform boundedness of the filtering error in the mean-square sense (MSS). Finally, some experiments with comparisons applicable for three-wheeled Ackerman turning model are conducted to show the applicability and advantages of newly proposed TRF scheme.**

***Index Terms*—Censored measurements, random false data injection attacks, FlexRay protocol, Tobit recursive filtering, algorithm design and performance analysis.**

I. Introduction

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THE design of filtering scheme has been acknowledged as a fundamental and appealing research subject in practical fields like signal processing and control engineering during the last decades. As it is well known, the Kalman filtering (KF) has been considered to be an efficient iterative algorithm of great significance with wide utilization in areas such as target tracking, computer vision and global positioning systems [1]–[6]. In general, the KF is recognized as an optimal algorithm in the least mean-square sense (MSS), which is applicable for handling the case that the linear systems with Gaussian white noises are considered and the system parameters are precisely known, thus posing some limitations. On the other hand, the censoring phenomenon is persistently encountered in various areas, such as bio-monitoring [7] and distributed detection [8], [9]. In fact, the censored measurements can be regarded as a special kind of nonlinearity caused by the saturation of sensor measurements due to the dynamic changes or interferences [10], [11]. It is noted that the measurement noise of the system near the censoring region is perceived as non-Gaussian [12], which violates the Gaussian hypothesis and then the traditional KF method is no longer suitable for systems subject to censored measurements. Consequently, there is an imperative necessity to further improve the KF algorithm from the performance-enhanced viewpoint with the aim of better handling the censored measurements.

Recently, the Tobit Kalman filtering (TKF) method has been seen as a powerful tool to tackle the filtering issue with censored measurements, in which the local approximation of uncensored probability has been exploited to provide efficient estimations of the state [13]–[15]. For instance, in [15], a Tobit Kalman estimator has been constructed to tackle the one-sided censored measurements, where the optimal feature in the unbiased minimum covariance sense has been illustrated with corresponding estimation scheme. In [16], the TKF approach has been generalized to the case where both left-censoring and right-censoring are present. Notably, it has been shown that the TKF can provide more efficient estimations than the unscented KF and the extended KF subject to the censored data condition [17]. To the best of the authors' knowledge, most of the reported results regarding the filtering issue with censored measurements are concentrated on linear systems. However, the nonlinearity is ubiquitous in real systems and, if it is not handled effectively, the filtering algorithm accuracy will be degraded [18], [19]. Consequently, the filtering issue for stochastic nonlinear systems (SNSs) has garnered increasing research interest. For example, the state-saturated recursive filtering strategy has been developed in [20] for stochastic nonlinear complex networks suffering from deception attacks. In [21], the distributed fusion filtering approach with resilience characteristic has been proposed for SNSs subject to missing measurements under dynamic event-triggered communication transmissions. Despite the case that the TKF scheme has been developed to a certain extent in handling the censored measurements, the filtering problem with censored measurements for SNSs has not yet been thoroughly addressed.

In comparison to the traditional control systems, the networked control systems (NCSs) primarily depend on networks as the medium for transmission. Nonetheless, owing to the vulnerabilities of NCSs, the attacks can be introduced into systems [22]–[24]. Generally, those attacks have the underlying intention of corrupting data via a shared network, which can lead to the oscillations or even destabilization of physical devices. Up to date, the addressed cyber-attacks can be typically divided into three categories: deception attacks [25], denial-of-service attacks [26] and replay attacks [27]. In particular, the false data injection attacks (FDIAs) as a special case of deception attacks are recognized as one of the well-studied malicious types of attacks, since the attacker can inject the false data into the system by tampering with the information during the data transmission, which in turn has an immeasurable impact on the system. In a follow-up study [28], the unscented KF problem has been tackled for saturated SNSs with random FDIAs, where the sensor saturation has been considered and properly handled during the filter design. Moreover, the distributed filtering approach has been presented in [29] under the FDIAs, in which an event-triggered strategy has been adopted to reduce the transmission of unreliable data over the network caused by the FDIAs.

In addition to the cyber-attacks, the communication protocols have become a major concern due to their essential role in regulating data transmission [30]–[33]. For the past few years, a mixed static-dynamic protocol has been developed based on both time-triggered and event-triggered mechanisms, i.e., the FlexRay protocol (FRP) as in [34]. Specifically, the static and dynamic protocols are combined by FRP to improve data utilization efficiency and reliability. It should be noted that only a few preliminary theoretical studies have been conducted about the FRP problem in the framework of NCSs. For example, the observer design problem for continuous systems has been addressed in [35], where the analysis has relied on the use of new hybrid Lyapunov function. To better follow the popular trend of digital communication, the FRP has been recently applied to discrete systems in [36], where the finite-horizon H_∞ filtering issue has been tackled over high-rate networks. Subsequently, the distributed recursive state estimation approach has been presented in [37] for multi-rate nonlinear systems. Nevertheless, the FRP-based filtering issue has rarely been discussed for SNSs with censored measurements, not to mention the case where random FDIAs are involved simultaneously. Hence, the primary objective of this paper is to develop a novel Tobit recursive filtering (TRF) algorithm with performance analysis criterion for addressed research issue with the hope of shortening such a gap.

Inspired by the above discussions, we aim to present a new TRF strategy for SNSs with FRP and random FDIAs. The encountered fundamental difficulties are recognized as: i) How to better handle the impacts caused by censored measurements and random FDIAs within the protocol regulation framework? ii) How to present an efficient filtering method by taking the censored case, the attack probability and the information of FRP scheduling into account? iii) How to provide a criterion to evaluate the boundedness performance of the presented TRF algorithm? To address these challenges, we have made the contributions that are listed below: 1) a novel TRF algorithm, which is capable of mitigating the effects of censored measurements, random FDIAs and FRP, is proposed that can be carried out recursively with advantages of online computations; 2) an appropriate approach is developed to design the explicit form of filter gain by fully considering the uncensored probability, the attack probability and the FRP scheduling, which can ensure the minimization of upper bound of the filtering error covariance (FEC); and 3) the performance discussion of the TRF approach is made by providing a sufficient criterion, which assures the uniform boundedness of the filtering error in the MSS.

Notations. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $\mathfrak{R} - \mathfrak{D}$ is nonnegative definite (positive definite) if $\mathfrak{R} \geq \mathfrak{D}$ ($\mathfrak{R} > \mathfrak{D}$), where $\mathfrak{R}$ and $\mathfrak{D}$ are symmetric matrices. For a matrix P, P^{-1}, P^T and $\mathrm{tr}(P)$ denote the inverse, the transpose and the trace of P, respectively. $\mathrm{sym}(P)$ represents $P + P^T$. I refers to an identity matrix with appropriate dimension. $\mathbb{E}\{*\}$ means the mathematical expectation of $*$. $\mathrm{col}\{a_1, a_2, \ldots, a_m\}$ describes a column vector with elements $a_1, a_2, \ldots, a_m$. "$\circ$" is the Hadamard product which is defined as $[\mathcal{V} \circ \mathcal{K}]_{i,j} = \mathcal{V}_{i,j}\mathcal{K}_{i,j}$ with $\mathcal{V}_{i,j}$ being the (i,j)th entry of $\mathcal{V}$ and $\mathcal{K}_{i,j}$ being the (i,j)th entry of $\mathcal{K}$. $\|\cdot\|$ depicts the Euclidean norm.

II. Problem Formulation

A. Nonlinear System Model

Consider the following class of discrete time-varying SNSs:

$$x_{h+1} = f(x_h) + D_h\omega_h, \tag{1}$$

$$y_h^* = g(x_h) + v_h, \tag{2}$$

where $x_h \in \mathbb{R}^n$ is the state vector, the initial value x_0 has mean $\bar{x}_0$ and covariance P_0, $y_h^* \in \mathbb{R}^{n_y}$ is the uncensored measurement output (can also be called latent variable). Both $f(x_h)$ and $g(x_h)$ are continuously differentiable nonlinear functions. $\omega_h \in \mathbb{R}^{n_\omega}$ and $v_h \in \mathbb{R}^{n_y}$ are zero-mean additive white Gaussian noises with covariances Q_h and R_h, respectively. D_h is a known matrix with proper dimension.

The nonlinearities $f(x_h) : \mathbb{R}^n \to \mathbb{R}^n$ and $g(x_h) : \mathbb{R}^n \to \mathbb{R}^{n_y}$ satisfy the following condition:

$$f(0) = 0, \ \|g(x_h)\| \leq \flat_{1,h}\|x_h\| + \flat_{2,h}, \tag{3}$$

where $\flat_{1,h}$ and $\flat_{2,h}$ are time-varying nonnegative scalars.

B. Censored Measurement Model

Let

$$y_h^* = \mathrm{col}\{\underbrace{y_{1,h}^*, \ldots, y_{s,h}^*}_{y_h^{*(I)}}, \underbrace{y_{s+1,h}^*, \ldots, y_{n_y,h}^*}_{y_h^{*(II)}}\},$$

$$\grave{y}_h = \mathrm{col}\{\underbrace{\grave{y}_{1,h}, \ldots, \grave{y}_{s,h}}_{\grave{y}_h^{(I)}}, \underbrace{\grave{y}_{s+1,h}, \ldots, \grave{y}_{n_y,h}}_{\grave{y}_h^{(II)}}\},$$

where $y_{i,h}^*$ $(i = 1, 2, \ldots, n_y)$ and $\grave{y}_{i,h}$ $(i = 1, 2, \ldots, n_y)$ are the i-th element of y_h^* and the censored measurement vector $\grave{y}_h$, respectively.

In practical engineering, the original measurement y_h^* may undergo the censoring phenomenon. Resorting to the Tobit Type I measurement model, the censoring phenomenon is depicted as follows:

$$\grave{y}_{i,h} = \begin{cases} y_{i,h}^*, & y_{i,h}^* > \mathcal{T}_i, \\ \mathcal{T}_i, & y_{i,h}^* \le \mathcal{T}_i, \end{cases} \tag{4}$$

where $\mathcal{T}_i$ $(i = 1, 2, \ldots, n_y)$ are constant thresholds.

Next, some Bernoulli random variables $\theta_{i,h}$ $(i = 1, 2, \ldots, n_y)$ are exploited to determine whether $y_{i,h}^*$ is censored or not, which can be expressed as follows:

$$\theta_{i,h} = \begin{cases} 1, & y_{i,h}^* > \mathcal{T}_i, \\ 0, & y_{i,h}^* \le \mathcal{T}_i, \end{cases} \tag{5}$$

satisfying the following statistical characteristics:

$$\mathrm{Prob}\{\theta_{i,h} = 1\} = \bar{\theta}_{i,h},\ \mathrm{Prob}\{\theta_{i,h} = 0\} = 1 - \bar{\theta}_{i,h},$$

where $\bar{\theta}_{i,h} \in (0, 1)$ $(i = 1, 2, \ldots, n_y)$ stand for uncensored probabilities.

As pointed out in [15], the concrete expression of the uncensored probability $\bar{\theta}_{i,h}$ can be obtained by the approximate means as follows:

$$\bar{\theta}_{i,h} = \Psi\left(\frac{g_i(x_h) - \mathcal{T}_i}{\sqrt{R_h^{(i,i)}}}\right) \approx \Psi\left(\frac{g_i(\hat{x}_{h|h-1}) - \mathcal{T}_i}{\sqrt{R_h^{(i,i)}}}\right), \tag{6}$$

where $g_i(x_h)$ is the i-th element of $g(x_h)$ and $\hat{x}_{h|h-1}$ is the one-step prediction at time $h-1$. $R_h^{(i,i)}$ is the (i,i)-th element of R_h, $\Psi(\cdot)$ is the cumulative distribution function of the standard normal distribution.

From (5), it is immediately deduced that (4) can be rewritten as follows:

$$\grave{y}_{i,h} = \theta_{i,h} y_{i,h}^* + (1 - \theta_{i,h})\mathcal{T}_i. \tag{7}$$

In order to obtain a unified form of censored measurement equation, denote $\mathcal{T} = \mathrm{col}\{\mathcal{T}_1, \mathcal{T}_2, \ldots, \mathcal{T}_{n_y}\}$ and $\Theta_h = \mathrm{diag}\{\theta_{1,h}, \theta_{2,h}, \ldots, \theta_{n_y,h}\}$. Hereafter, the actual measurement after censoring can be reorganized as $\grave{y}_h = \Theta_h y_h^* + (I - \Theta_h)\mathcal{T}$.

C. FlexRay Protocol

In many engineering applications, there is no doubt that the data congestion may inevitably occur if an excessive amount of data is transmitted simultaneously. Therefore, the FRP is introduced to schedule the data transmission and only one node has the access right via the communication network at each transmission moment. The FRP is a kind of mixed static-dynamic protocol, where the round-robin protocol (RRP) is adopted for the static segment and the weighted try-once-discard protocol (WTODP) is exploited for the dynamic segment. Without prejudice to generality, let the first s nodes be regulated by the RRP and the remaining nodes be regulated by the WTODP.

Firstly, let

$$\vec{y}_h = \mathrm{col}\{\underbrace{\vec{y}_{1,h}, \ldots, \vec{y}_{s,h}}_{\vec{y}_h^{(I)}}, \underbrace{\vec{y}_{s+1,h}, \ldots, \vec{y}_{n_y,h}}_{\vec{y}_h^{(II)}}\},$$

where $\vec{y}_h$ is the measurement obtained after the FRP scheduling. Subsequently, the scheduling behavior of the FRP will be discussed from the following two aspects:

1) Static Segment: In this segment, the RRP is exploited to schedule the data transmission, which is a time-based static protocol. The variable $\beta_h \in \{1, 2, \ldots, s\}$ is introduced to indicate the sensor node that the transmit measurement data at time h, which can be expressed by $\beta_h = \mathrm{mod}(h-1, s) + 1$, where $\mathrm{mod}(h-1, s)$ defines the remainder on division of $h-1$ by the positive integer s. The actual measurement $\vec{y}_{l,h}$ $(l = 1, 2, \ldots, s)$ under the zero-order holder strategy can be depicted as:

$$\vec{y}_{l,h} = \begin{cases} \grave{y}_{l,h}, & \beta_h = l, \\ \vec{y}_{l,h-1}, & \text{otherwise}. \end{cases}$$

Next, denote $\Phi_{\beta_h} = \mathrm{diag}\big\{\delta(\beta_h - 1), \delta(\beta_h - 2), \ldots, \delta(\beta_h - s)\big\} \in \mathbb{R}^{s \times s}$, where $\delta(\cdot)$ is the Kronecker delta function, i.e., $\delta(b) = 1$ when $b = 0$; otherwise $\delta(b) = 0$. Then, one has

$$\vec{y}_h^{(I)} = \sum_{\varrho=0}^{s-1} \Phi_{\beta_{h-\varrho}} \grave{y}_{h-\varrho}^{(I)}, \tag{8}$$

where $\beta_{h-\varrho} = \varrho$ and $\grave{y}_{h-\varrho} = \grave{y}_0$ for $h - \varrho \le 0$.

2) Dynamic Segment: The WTODP is applied in the dynamic protocol segment, which is an event-based dynamic protocol. At time h, the updated measurement will be assigned to the sensor with the highest priority. Next, let $\alpha_h \in \{s+1, s+2, \ldots, n_y\}$ be the selected sensor node. Then, α_h can be characterized as follows:

$$\alpha_h = \min\left\{\arg\max_{s+1 \le j \le n_y} (\grave{y}_{j,h} - \vec{y}_{j,h-1})^T W_j (\grave{y}_{j,h} - \vec{y}_{j,h-1})\right\},$$

where $\vec{y}_{j,h-1}$ $(j = s+1, s+2, \ldots, n_y)$ define the last successful transmitted measurement before time instant h of j-th node and $W_j > 0$ $(j = s+1, s+2, \ldots, n_y)$ are known weight coefficients of j-th node. The weight reflects the priority of the node; i.e., the greater the weight, the higher the priority is. Based on the above rule and combined with the zero-input strategy, $\vec{y}_{j,h}$ is denoted as follows:

$$\vec{y}_{j,h} = \begin{cases} \grave{y}_{j,h}, & \alpha_h = j, \\ 0, & \text{otherwise}. \end{cases}$$

As a result, the true measurement output $\vec{y}_h^{(II)}$ after transmission via the WTODP can be obtained in a simple form

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as:

$$\vec{y}_h^{(II)} = \Phi_{\alpha_h} \grave{y}_h^{(II)}, \tag{9}$$

where $\Phi_{\alpha_h} = \text{diag}\{\delta[\alpha_h-(s+1)], \delta[\alpha_h-(s+2)], \ldots, \delta(\alpha_h-n_y)\} \in \mathbb{R}^{(n_y-s)\times(n_y-s)}$. It follows from (8) and (9) that $\vec{y}_h$ can be determined by

$$\vec{y}_h = \mathcal{I}_1 \vec{y}_h^{(I)} + \mathcal{I}_2 \vec{y}_h^{(II)}, \tag{10}$$

where $\mathcal{I}_1 = \begin{bmatrix} I_{s\times s} & 0_{s\times(n_y-s)} \end{bmatrix}^T$ and $\mathcal{I}_2 = \begin{bmatrix} 0_{(n_y-s)\times s} & I_{(n_y-s)\times(n_y-s)} \end{bmatrix}^T$.

Remark 1 *The FRP consists of four core components: a static segment, a dynamic segment, a network idle time and a symbol window. Since the latter two are of relatively shorter duration, their lengths are typically set to 0 in practical applications. Therefore, the implementation focuses on the static and dynamic segments, where different network access techniques can be employed to cater specific communication demands. Specifically, the first thing executed during a FlexRay communication cycle is the RRP in a time-triggered way, which ensures that each node is endowed with the right to transmit the measurement data sequentially at a pre-determined time instant. Next, it is followed by the WTODP in an event-triggered way, in which the node with the larger absolute error between the current measurement value and the last updated value will be granted a higher priority to access the network. Particularly, with the aim of avoiding multiple nodes exhibiting the same absolute errors, we preferentially select the node with the minimum label value for data transmission. Of course, the node with the maximum label value can also theoretically be exploited as another way. The well-designed FRP mentioned above can effectively manage the network resources with aim to optimize the overall performance of NCSs.*

D. Random False Data Injection Attacks

A measurement model with random FDIAs is described by

$$y_{i,h} = \vec{y}_{i,h} + \gamma_{i,h}\eta_{i,h}, \tag{11}$$

where $y_{i,h}$ $(i = 1, 2, \ldots, n_y)$ are the signal received by the filter subject to the random FDIAs. $\eta_{i,h} \in \mathbb{R}$ $(i = 1, 2, \ldots, n_y)$ refer to the bounded false data injected by attackers. The probabilities of the Bernoulli random variables $\gamma_{i,h}$ $(i = 1, 2, \ldots, n_y)$ are $\text{Prob}\{\gamma_{i,h} = 1\} = \bar{\gamma}_{i,h}$ and $\text{Prob}\{\gamma_{i,h} = 0\} = 1-\bar{\gamma}_{i,h}$, where $\bar{\gamma}_{i,h} \in (0, 1)$ $(i = 1, 2, \ldots, n_y)$ are known constants called as attack probabilities. Moreover, suppose that x_0, $\theta_{i,h}$, $\gamma_{i,h}$, ω_h and v_h are uncorrelated with each other for all i and h.

For the convenience of subsequent calculations, define $y_h = \text{col}\{y_{1,h}, y_{2,h}, \ldots, y_{n_y,h}\}$, $\eta_h = \text{col}\{\eta_{1,h}, \eta_{2,h}, \ldots, \eta_{n_y,h}\}$ and $\Gamma_h = \text{diag}\{\gamma_{1,h}, \gamma_{2,h}, \ldots, \gamma_{n_y,h}\}$. Consequently, (11) can be transformed into the following equation:

$$y_h = \vec{y}_h + \Gamma_h\eta_h, \tag{12}$$

where η_h satisfies the condition $\eta_h^T\eta_h \le \bar{\eta}$ with $\bar{\eta}$ being a positive scalar. As in [38], the false data vector introduced in this paper is unknown but norm-bounded. This is a more general assumption that the impact of the attack is within a certain range rather than depending on specific knowledge of the attacker's behavior.

Remark 2 *In practice, the Bernoulli random variables are usually adopted to characterize the attack model owing to the random occurring characteristics of cyber-attacks. In addition, the process of the attack may be accompanied by some abnormal circumstances, and the statistical information about the attacks can be acquired by the defense via monitoring the system online for a brief period. Thus, it is reasonable to suppose that the upper bound and occurrence probability of the attacks are known. Moreover, it is noted that different values of $\gamma_{i,h}$ lead to different measurement results. Specifically, $\gamma_{i,h} = 1$ means that the attacker can inject false data into the i-th node, otherwise $\gamma_{i,h} = 0$. Particularly, $\Gamma_h = I$ implies that all nodes are attacked, and $\Gamma_h = 0$ indicates that no attack is injected into any node.*

E. Model Transformation

To simplify the derivation, $\vec{y}_h^{(I)}$ and $\vec{y}_h^{(II)}$ can be converted to the following equation:

$$\vec{y}_h^{(I)} = \sum_{\varrho=0}^{s-1} \Phi_{\beta_{h-\varrho}} \mathcal{I}_1^T \grave{y}_{h-\varrho}, \tag{13}$$

$$\vec{y}_h^{(II)} = \Phi_{\alpha_h} \mathcal{I}_2^T \grave{y}_h. \tag{14}$$

Next, substituting (13) and (14) into (10) and combining (12), we have

$$y_h = \sum_{\varrho=0}^{s-1} \mathfrak{L}_{h-\varrho} \grave{y}_{h-\varrho} + \mathfrak{P}_h \grave{y}_h + \Gamma_h\eta_h, \tag{15}$$

where $\mathfrak{L}_{h-\varrho} = \mathcal{I}_1 \Phi_{\beta_{h-\varrho}} \mathcal{I}_1^T$ and $\mathfrak{P}_h = \mathcal{I}_2 \Phi_{\alpha_h} \mathcal{I}_2^T$.

The following lemmas are essential to derive our results more directly.

Lemma 1 *[15] The mean of $\grave{y}_{i,h}$ based on Tobit regression model can be determined as follows:*

$$\begin{aligned} &\mathbb{E}\left(\grave{y}_{i,h} \mid x_h, R_h^{(i,i)}\right) \\ &= (1-\Psi(\vartheta_{i,h}))\left(g_i(x_h) + \sqrt{R_h^{(i,i)}}\varpi(\vartheta_{i,h})\right) + \Psi(\vartheta_{i,h})\mathcal{T}_i, \end{aligned}$$

where $\vartheta_{i,h} = \frac{\mathcal{T}_i - g_i(x_h)}{\sqrt{R_h^{(i,i)}}}$ and $\varpi(\vartheta_{i,h}) = \frac{\phi(\vartheta_{i,h})}{1-\Psi(\vartheta_{i,h})}$, $\phi(\cdot)$ is the probability density function of the standard normal distribution.

Lemma 2 *[39] For the matrices $\mathscr{G}$, $\mathscr{L}$, $\mathscr{K}$ and an unknown matrix $\mathscr{R}$ with $\mathscr{R}\mathscr{R}^T \le I$, if the condition $\mathfrak{p}^{-1}I - \mathscr{K}\Upsilon\mathscr{K}^T > 0$ is satisfied for any constant $\mathfrak{p} > 0$ and any matrix $\Upsilon > 0$, one has*

$$\begin{aligned} &(\mathscr{G} + \mathscr{L}\mathscr{R}\mathscr{K})\Upsilon(\mathscr{G} + \mathscr{L}\mathscr{R}\mathscr{K})^T \\ &\le \mathscr{G}(\Upsilon^{-1} - \mathfrak{p}\mathscr{K}^T\mathscr{K})^{-1}\mathscr{G}^T + \mathfrak{p}^{-1}\mathscr{L}\mathscr{L}^T. \end{aligned}$$

Lemma 3 *[21] For any two vectors $\lambda_1 \in \mathbb{R}^n$ and $\lambda_2 \in \mathbb{R}^n$, one has $\lambda_1\lambda_2^T + \lambda_2\lambda_1^T \leq \varepsilon\lambda_1\lambda_1^T + \varepsilon^{-1}\lambda_2\lambda_2^T$, where $\varepsilon > 0$ is a constant.*

Lemma 4 *[21] Let $\mathcal{Z} = [z_{i,j}]_{n\times n}$ be a real-valued matrix and $\mathcal{U} = \text{diag}\{u_1, u_2, \ldots, u_n\}$ be a stochastic matrix. Hence, one derives*

$$\mathbb{E}\{\mathcal{U}\mathcal{Z}\mathcal{U}^T\}=\begin{bmatrix} \mathbb{E}\{u_1^2\} & \mathbb{E}\{u_1u_2\} & \cdots & \mathbb{E}\{u_1u_n\} \\ \mathbb{E}\{u_2u_1\} & \mathbb{E}\{u_2^2\} & \cdots & \mathbb{E}\{u_2u_n\} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{E}\{u_nu_1\} & \mathbb{E}\{u_nu_2\} & \cdots & \mathbb{E}\{u_n^2\} \end{bmatrix} \circ \mathcal{Z}.$$

With the help of Lemma 1 and letting $\hat{\vartheta}_{i,h} = \frac{\mathcal{T}_i - g_i(\hat{x}_{h|h-1})}{\sqrt{R_h^{(i,i)}}}$, the prediction of the censored measurement $\grave{y}_{i,h}$ is derived by $\hat{\grave{y}}_{i,h+1|h} = \bar{\theta}_{i,h+1}\left(g_i(\hat{x}_{h+1|h}) + \sqrt{R_{h+1}^{(i,i)}}\varpi(\hat{\vartheta}_{i,h+1})\right) + (1-\bar{\theta}_{i,h+1})\mathcal{T}_i$. Denoting $\bar{\Theta}_h = \text{diag}\left\{\bar{\theta}_{1,h}, \bar{\theta}_{2,h}, \ldots, \bar{\theta}_{n_y,h}\right\}$, $\Delta_h = \text{diag}\left\{\varpi(\hat{\vartheta}_{1,h}), \varpi(\hat{\vartheta}_{2,h}), \ldots, \varpi(\hat{\vartheta}_{n_y,h})\right\}$ and $\mathcal{R}_h = \text{col}\left\{\sqrt{R_h^{(1,1)}}, \sqrt{R_h^{(2,2)}}, \ldots, \sqrt{R_h^{(n_y,n_y)}}\right\}$, we have

$$\hat{\grave{y}}_{h+1|h}=\bar{\Theta}_{h+1}\left(g(\hat{x}_{h+1|h})+\Delta_{h+1}\mathcal{R}_{h+1}\right)+(I-\bar{\Theta}_{h+1})\mathcal{T}. \quad (16)$$

Considering (1), (15) and (16), the Tobit recursive filter is constructed as:

$$\begin{aligned} \hat{x}_{h+1|h} &= f(\hat{x}_{h|h}), \quad (17)\\ \hat{x}_{h+1|h+1} &= \hat{x}_{h+1|h} + K_{h+1}\Bigg\{y_{h+1} - \sum_{\varrho=0}^{s-1}\mathfrak{L}_{h-\varrho+1}\big[\bar{\Theta}_{h-\varrho+1} \\ &\quad \times\big(g(\hat{x}_{h-\varrho+1|h-\varrho}) + \Delta_{h-\varrho+1}\mathcal{R}_{h-\varrho+1}\big) \\ &\quad +(I-\bar{\Theta}_{h-\varrho+1})\mathcal{T}\big] - \mathfrak{P}_{h+1}\big[\bar{\Theta}_{h+1}\big(g(\hat{x}_{h+1|h}) \\ &\quad +\Delta_{h+1}\mathcal{R}_{h+1}\big) + (I-\bar{\Theta}_{h+1})\mathcal{T}\big]\Bigg\}, \quad (18) \end{aligned}$$

where $\hat{x}_{h+1|h+1}$ is the estimation of the state x_{h+1}, and K_{h+1} is the filter gain to be designed.

Let $\tilde{x}_{h+1|h} = x_{h+1} - \hat{x}_{h+1|h}$ as well as $\tilde{x}_{h+1|h+1} = x_{h+1} - \hat{x}_{h+1|h+1}$ be the prediction error and filtering error. The prediction error covariance and FEC are described by $P_{h+1|h} = \mathbb{E}\left\{\tilde{x}_{h+1|h}\tilde{x}_{h+1|h}^T\right\}$ and $P_{h+1|h+1} = \mathbb{E}\left\{\tilde{x}_{h+1|h+1}\tilde{x}_{h+1|h+1}^T\right\}$, respectively. Now, we will state the main objectives of this paper. Firstly, a new TRF scheme is provided for SNSs that takes into account censored measurements, random FDIAs and FRP. Secondly, the upper bound of the FEC is obtained by exploiting the mathematical induction approach and the filter gain is calculated via minimizing such an upper bound. Finally, the boundedness analysis problem of the filtering error is discussed.

Remark 3 *It is evident to observe from (17)-(18) that the designed Tobit recursive filter has the structural similarities with the conventional Kalman filter in both the prediction step and the updating step. Nevertheless, notice the increased complexity of the addressed issue including the censored measurement and FRP, some additional terms related to the above phenomena are introduced in the filter design. Specifically, the first is the appearance of $\mathfrak{L}_{h-\varrho+1}$ and $\mathfrak{P}_{h+1}$ in updating step (18), which has revealed the nodes selected for data transmission in the static and dynamic segments of the FRP, respectively. Subsequently, there are new terms in the innovation due to the effect of the censored measurement, i.e., $\Delta_{h-\varrho+1}\mathcal{R}_{h-\varrho+1}$ and $\Delta_{h+1}\mathcal{R}_{h+1}$. These terms have been obtained by calculating the expectation of the Tobit regression model, which fully characterize the non-Gaussian noise in the vicinity of the censoring region. Moreover, the probabilities $\bar{\Theta}_{h-\varrho+1}$ and $\bar{\Theta}_{h+1}$ are also adopted to further reflect the effects of censored measurements.*

III. Main Results

In this section, both the prediction error covariance matrix and the FEC matrix are computed in accordance with the corresponding definitions. Next, the upper bound $\mathcal{P}_{h+1|h+1}$ for the FEC is determined by manipulating the uncertainty and cross terms, and the specific expression form of the optimized filter gain K_{h+1} is derived.

In the first place, it is deduced from (1) and (17) that $\tilde{x}_{h+1|h} = f(x_h) - f(\hat{x}_{h|h}) + D_h\omega_h$. Furthermore, in line with the Taylor series expansion for the nonlinear function $f(x_h)$ around $\hat{x}_{h|h}$, one gets

$$\tilde{x}_{h+1|h} = (\mathscr{G}_h + \mathscr{L}_{1,h}\mathscr{R}_{1,h}\mathscr{K}_{1,h})\tilde{x}_{h|h} + D_h\omega_h, \quad (19)$$

where $\mathscr{G}_h = \frac{\partial f(x_h)}{\partial x_h}|_{x_h=\hat{x}_{h|h}}$ is the Jacobian matrix, $\mathscr{L}_{1,h}$ represents a problem-dependent scaling matrix, $\mathscr{K}_{1,h}$ is employed to provide an additional degree of freedom to tune the filter, and the linearization errors are expressed by an unknown time-varying matrix $\mathscr{R}_{1,h}$ with $\mathscr{R}_{1,h}\mathscr{R}_{1,h}^T \leq I$.

Similar in (19), the filtering error can be derived as follows:

$$\begin{aligned} \tilde{x}_{h+1|h+1}=&\big[I - K_{h+1}(\mathfrak{L}_{h+1} + \mathfrak{P}_{h+1})\bar{\Theta}_{h+1}\Omega_{h+1}\big]\tilde{x}_{h+1|h} \\ &-\sum_{\varrho=1}^{s-1}K_{h+1}\mathfrak{L}_{h-\varrho+1}\bar{\Theta}_{h-\varrho+1}\Omega_{h-\varrho+1}\tilde{x}_{h-\varrho+1|h-\varrho} \\ &-\sum_{\varrho=0}^{s-1}K_{h+1}\mathfrak{L}_{h-\varrho+1}\Big[\tilde{\Theta}_{h-\varrho+1}g(x_{h-\varrho+1}) \\ &+\tilde{\Theta}_{h-\varrho+1}v_{h-\varrho+1} - \tilde{\Theta}_{h-\varrho+1}\mathcal{T} + \bar{\Theta}_{h-\varrho+1}\vec{v}_{h-\varrho+1}\Big] \\ &-K_{h+1}\mathfrak{P}_{h+1}\Big[\tilde{\Theta}_{h+1}g(x_{h+1}) + \tilde{\Theta}_{h+1}v_{h+1} \\ &-\tilde{\Theta}_{h+1}\mathcal{T} + \bar{\Theta}_{h+1}\vec{v}_{h+1}\Big] - K_{h+1}\Gamma_{h+1}\eta_{h+1}, \quad (20) \end{aligned}$$

where $\tilde{\Theta}_{h+1} = \Theta_{h+1} - \bar{\Theta}_{h+1}$, $\vec{v}_{h+1} = v_{h+1} - \Delta_{h+1}\mathcal{R}_{h+1}$ and $\Omega_{h+1} = \mathscr{F}_{h+1} + \mathscr{L}_{2,h+1}\mathscr{R}_{2,h+1}\mathscr{K}_{2,h+1}$. $\mathscr{L}_{2,h+1}$ is a known scaling matrix related to the specific problem, $\mathscr{K}_{2,h+1}$ is a given matrix to provide an extra degree of freedom to tune the filter, $\mathscr{F}_{h+1} = \frac{\partial g(x_{h+1})}{\partial x_{h+1}}|_{x_{h+1}=\hat{x}_{h+1|h}}$ and $\mathscr{R}_{2,h+1}$ denotes an unknown matrix satisfying $\mathscr{R}_{2,h+1}\mathscr{R}_{2,h+1}^T \leq I$.

Lemma 5 *The error covariance matrices of the one-step prediction and the filtering can be computed respectively as follows:*

$$P_{h+1|h} = (\mathscr{G}_h + \mathscr{L}_{1,h}\mathscr{R}_{1,h}\mathscr{K}_{1,h})P_{h|h}(\mathscr{G}_h + \mathscr{L}_{1,h}\mathscr{R}_{1,h}\mathscr{K}_{1,h})^T$$

$$+D_hQ_hD_h^T \tag{21}$$

and

$$\begin{aligned}
&P_{h+1|h+1}\\
=\ &\mathbb{E}\Big\{\left[I-K_{h+1}(\mathfrak{L}_{h+1}+\mathfrak{P}_{h+1})\bar{\Theta}_{h+1}\Omega_{h+1}\right]\tilde{x}_{h+1|h}\\
&\times\tilde{x}_{h+1|h}^T\left[I-K_{h+1}(\mathfrak{L}_{h+1}+\mathfrak{P}_{h+1})\bar{\Theta}_{h+1}\Omega_{h+1}\right]^T\Big\}\\
&+\mathbb{E}\Bigg\{\sum_{\varrho=0}^{s-1}K_{h+1}\mathfrak{L}_{h-\varrho+1}\tilde{\Theta}_{h-\varrho+1}g(x_{h-\varrho+1})\\
&\times\left(\sum_{\varrho=0}^{s-1}K_{h+1}\mathfrak{L}_{h-\varrho+1}\tilde{\Theta}_{h-\varrho+1}g(x_{h-\varrho+1})\right)^T\Bigg\}\\
&+\mathbb{E}\Bigg\{\sum_{\varrho=0}^{s-1}K_{h+1}\mathfrak{L}_{h-\varrho+1}\tilde{\Theta}_{h-\varrho+1}v_{h-\varrho+1}\\
&\times\left(\sum_{\varrho=0}^{s-1}K_{h+1}\mathfrak{L}_{h-\varrho+1}\tilde{\Theta}_{h-\varrho+1}v_{h-\varrho+1}\right)^T\Bigg\}\\
&+\mathbb{E}\left\{K_{h+1}\mathfrak{P}_{h+1}\tilde{\Theta}_{h+1}v_{h+1}v_{h+1}^T\tilde{\Theta}_{h+1}^T\mathfrak{P}_{h+1}^TK_{h+1}^T\right\}\\
&+\mathbb{E}\left\{K_{h+1}\mathfrak{P}_{h+1}\bar{\Theta}_{h+1}\vec{v}_{h+1}\vec{v}_{h+1}^T\bar{\Theta}_{h+1}^T\mathfrak{P}_{h+1}^TK_{h+1}^T\right\}\\
&+\mathbb{E}\left\{K_{h+1}\Gamma_{h+1}\eta_{h+1}\eta_{h+1}^T\Gamma_{h+1}^TK_{h+1}^T\right\}\\
&+\mathbb{E}\Big\{\mathrm{sym}\big(\mathfrak{C}_1\mathfrak{C}_2^T+\mathfrak{C}_1\mathfrak{C}_6^T+\mathfrak{C}_1\mathfrak{C}_{10}^T+\mathfrak{C}_1\mathfrak{C}_{11}^T+\mathfrak{C}_2\mathfrak{C}_6^T\\
&+\mathfrak{C}_2\mathfrak{C}_{10}^T+\mathfrak{C}_2\mathfrak{C}_{11}^T+\mathfrak{C}_3\mathfrak{C}_5^T+\mathfrak{C}_3\mathfrak{C}_7^T+\mathfrak{C}_3\mathfrak{C}_9^T+\mathfrak{C}_4\mathfrak{C}_8^T\\
&+\mathfrak{C}_5\mathfrak{C}_7^T+\mathfrak{C}_5\mathfrak{C}_9^T+\mathfrak{C}_6\mathfrak{C}_{10}^T+\mathfrak{C}_6\mathfrak{C}_{11}^T+\mathfrak{C}_7\mathfrak{C}_9^T+\mathfrak{C}_{10}\mathfrak{C}_{11}^T\big)\\
&+\mathfrak{C}_2\mathfrak{C}_2^T+\mathfrak{C}_5\mathfrak{C}_5^T+\mathfrak{C}_6\mathfrak{C}_6^T+\mathfrak{C}_7\mathfrak{C}_7^T+\mathfrak{C}_9\mathfrak{C}_9^T\Big\},
\end{aligned}\tag{22}$$

where

$$\begin{aligned}
\mathfrak{C}_1 &= \left[I-K_{h+1}(\mathfrak{L}_{h+1}+\mathfrak{P}_{h+1})\bar{\Theta}_{h+1}\Omega_{h+1}\right]\tilde{x}_{h+1|h},\\
\mathfrak{C}_2 &= -\sum_{\varrho=1}^{s-1}K_{h+1}\mathfrak{L}_{h-\varrho+1}\bar{\Theta}_{h-\varrho+1}\Omega_{h-\varrho+1}\tilde{x}_{h-\varrho+1|h-\varrho},\\
\mathfrak{C}_3 &= -\sum_{\varrho=0}^{s-1}K_{h+1}\mathfrak{L}_{h-\varrho+1}\tilde{\Theta}_{h-\varrho+1}g(x_{h-\varrho+1}),\\
\mathfrak{C}_4 &= -\sum_{\varrho=0}^{s-1}K_{h+1}\mathfrak{L}_{h-\varrho+1}\tilde{\Theta}_{h-\varrho+1}v_{h-\varrho+1},\\
\mathfrak{C}_5 &= \sum_{\varrho=0}^{s-1}K_{h+1}\mathfrak{L}_{h-\varrho+1}\tilde{\Theta}_{h-\varrho+1}\mathcal{T},\\
\mathfrak{C}_6 &= -\sum_{\varrho=0}^{s-1}K_{h+1}\mathfrak{L}_{h-\varrho+1}\bar{\Theta}_{h-\varrho+1}\vec{v}_{h-\varrho+1},\\
\mathfrak{C}_7 &= -K_{h+1}\mathfrak{P}_{h+1}\tilde{\Theta}_{h+1}g(x_{h+1}),\\
\mathfrak{C}_8 &= -K_{h+1}\mathfrak{P}_{h+1}\tilde{\Theta}_{h+1}v_{h+1},\\
\mathfrak{C}_9 &= K_{h+1}\mathfrak{P}_{h+1}\tilde{\Theta}_{h+1}\mathcal{T},\\
\mathfrak{C}_{10} &= -K_{h+1}\mathfrak{P}_{h+1}\bar{\Theta}_{h+1}\vec{v}_{h+1},\\
\mathfrak{C}_{11} &= -K_{h+1}\Gamma_{h+1}\eta_{h+1}.
\end{aligned}$$

Proof: Based on (19) and (20), the proof can be easily completed. ■

On the basis of the results in Lemma 5, in the following theorem, a specific expression is given for the upper bound of the FEC and the filter gain is determined accordingly.

Theorem 1 *Consider the SNSs characterized in (1) and (2) with the Tobit recursive filter in (17) and (18). Let* $\rho_{1,h}>0$, $\rho_{2,h-\varrho+1}>0$, $\flat_{1,h-\varrho+1}\geq 0$, $\flat_{2,h-\varrho+1}\geq 0$ $(\varrho=0,1,\ldots,s-1)$ *and* $\varepsilon_{i,h+1}>0$ $(i=1,2,\ldots,18)$ *be known constants and set the initial value* $\mathcal{P}_{0|0}\geq P_0$. *If the following matrix equations:*

$$\begin{aligned}
&\mathcal{P}_{h+1|h}\\
&=\mathscr{G}_h(\mathcal{P}_{h|h}^{-1}-\rho_{1,h}\mathscr{K}_{1,h}^T\mathscr{K}_{1,h})^{-1}\mathscr{G}_h^T+\rho_{1,h}^{-1}\mathscr{L}_{1,h}\mathscr{L}_{1,h}^T\\
&\quad+D_hQ_hD_h^T,
\end{aligned}\tag{23}$$

$$\begin{aligned}
&\mathcal{P}_{h+1|h+1}\\
&=(1+q_{1,h+1})\mathfrak{B}_{h+1}\mathfrak{D}_{h+1}\mathfrak{B}_{h+1}^T+(1+q_{1,h+1})\\
&\quad\times\rho_{2,h+1}^{-1}K_{h+1}(\mathfrak{L}_{h+1}+\mathfrak{P}_{h+1})\bar{\Theta}_{h+1}\mathscr{L}_{2,h+1}\mathscr{L}_{2,h+1}^T\\
&\quad\times\bar{\Theta}_{h+1}^T(\mathfrak{L}_{h+1}+\mathfrak{P}_{h+1})^TK_{h+1}^T+(1+q_{2,h+1})K_{h+1}\\
&\quad\times\mathfrak{A}_{h-\varrho+1}K_{h+1}^T+2q_{3,h+1}K_{h+1}\mathfrak{L}_{h+1}\mathfrak{F}_{h+1}\mathfrak{L}_{h+1}^TK_{h+1}^T\\
&\quad+2(1+\varepsilon_{8,h+1})\sum_{\varrho=0}^{s-1}K_{h+1}\mathfrak{L}_{h-\varrho+1}\mathfrak{F}_{h-\varrho+1}\mathfrak{L}_{h-\varrho+1}^TK_{h+1}^T\\
&\quad+\varepsilon_{16,h+1}K_{h+1}\mathfrak{L}_{h+1}\left(\breve{\Theta}_{h+1}\circ R_{h+1}\right)\mathfrak{L}_{h+1}^TK_{h+1}^T\\
&\quad+(1+\varepsilon_{16,h+1}^{-1})K_{h+1}\mathfrak{P}_{h+1}\left(\breve{\Theta}_{h+1}\circ R_{h+1}\right)\mathfrak{P}_{h+1}^TK_{h+1}^T\\
&\quad+K_{h+1}\sum_{\varrho=0}^{s-1}\mathfrak{L}_{h-\varrho+1}\left(\breve{\Theta}_{h-\varrho+1}\circ R_{h-\varrho+1}\right)\mathfrak{L}_{h-\varrho+1}^TK_{h+1}^T\\
&\quad+(1+\varepsilon_{8,h+1}^{-1})\sum_{\varrho=0}^{s-1}K_{h+1}\mathfrak{L}_{h-\varrho+1}\left[\breve{\Theta}_{h-\varrho+1}\circ\left(\mathcal{T}\mathcal{T}^T\right)\right]\\
&\quad\times\mathfrak{L}_{h-\varrho+1}^TK_{h+1}^T+q_{4,h+1}K_{h+1}\mathfrak{L}_{h+1}\left[\breve{\Theta}_{h+1}\circ\left(\mathcal{T}\mathcal{T}^T\right)\right]\\
&\quad\times\mathfrak{L}_{h+1}^TK_{h+1}^T+(1+q_{5,h+1})K_{h+1}\mathfrak{P}_{h+1}\\
&\quad\times\left[\breve{\Theta}_{h+1}\circ\left(\mathcal{T}\mathcal{T}^T\right)\right]\mathfrak{P}_{h+1}^TK_{h+1}^T+(1+q_{6,h+1})s\\
&\quad\times\sum_{\varrho=0}^{s-1}K_{h+1}\mathfrak{L}_{h-\varrho+1}\bar{\Theta}_{h-\varrho+1}\Big(R_{h-\varrho+1}+\Delta_{h-\varrho+1}\\
&\quad\times\mathcal{R}_{h-\varrho+1}\mathcal{R}_{h-\varrho+1}^T\Delta_{h-\varrho+1}^T\Big)\bar{\Theta}_{h-\varrho+1}^T\mathfrak{L}_{h-\varrho+1}^TK_{h+1}^T\\
&\quad+2(1+q_{7,h+1})K_{h+1}\mathfrak{P}_{h+1}\mathfrak{F}_{h+1}\mathfrak{P}_{h+1}^TK_{h+1}^T\\
&\quad+(1+q_{8,h+1})K_{h+1}\mathfrak{P}_{h+1}\bar{\Theta}_{h+1}\\
&\quad\times\Big(R_{h+1}+\Delta_{h+1}\mathcal{R}_{h+1}\mathcal{R}_{h+1}^T\Delta_{h+1}^T\Big)\bar{\Theta}_{h+1}^T\mathfrak{P}_{h+1}^TK_{h+1}^T\\
&\quad+(1+q_{9,h+1})K_{h+1}\left(\breve{\Gamma}_{h+1}\circ\bar{\eta}I\right)K_{h+1}^T
\end{aligned}\tag{24}$$

have solutions $\mathcal{P}_{h+1|h}>0$ *and* $\mathcal{P}_{h+1|h+1}>0$ *under the constraint conditions:*

$$\begin{aligned}
&\rho_{1,h}^{-1}I-\mathscr{K}_{1,h}\mathcal{P}_{h|h}\mathscr{K}_{1,h}^T>0,\\
&\rho_{2,h+1}^{-1}I-\mathscr{K}_{2,h+1}\mathcal{P}_{h+1|h}\mathscr{K}_{2,h+1}^T>0,\\
&\rho_{2,h-\varrho+1}^{-1}I-\mathscr{K}_{2,h-\varrho+1}\mathcal{P}_{h-\varrho+1|h-\varrho}\mathscr{K}_{2,h-\varrho+1}^T>0,
\end{aligned}$$

where

$$q_{1,h+1}=\varepsilon_{1,h+1}+\varepsilon_{2,h+1}+\varepsilon_{3,h+1}+\varepsilon_{4,h+1},$$

$$
\begin{aligned}
q_{2,h+1} &= \varepsilon_{1,h+1}^{-1} + \varepsilon_{5,h+1} + \varepsilon_{6,h+1} + \varepsilon_{7,h+1}, \\
q_{3,h+1} &= \varepsilon_{14,h+1} + \varepsilon_{15,h+1},\ q_{4,h+1} = \varepsilon_{17,h+1} + \varepsilon_{18,h+1}, \\
q_{5,h+1} &= \varepsilon_{11,h+1}^{-1} + \varepsilon_{15,h+1}^{-1} + \varepsilon_{18,h+1}^{-1}, \\
q_{6,h+1} &= \varepsilon_{2,h+1}^{-1} + \varepsilon_{5,h+1}^{-1} + \varepsilon_{9,h+1} + \varepsilon_{10,h+1}, \\
q_{7,h+1} &= \varepsilon_{11,h+1} + \varepsilon_{14,h+1}^{-1} + \varepsilon_{17,h+1}^{-1}, \\
q_{8,h+1} &= \varepsilon_{3,h+1}^{-1} + \varepsilon_{6,h+1}^{-1} + \varepsilon_{9,h+1}^{-1} + \varepsilon_{12,h+1}, \\
q_{9,h+1} &= \varepsilon_{4,h+1}^{-1} + \varepsilon_{7,h+1}^{-1} + \varepsilon_{10,h+1}^{-1} + \varepsilon_{12,h+1}^{-1}, \\
\Im_{h+1|h} &= (1+\varepsilon_{13,h+1})\mathcal{P}_{h+1|h} + (1+\varepsilon_{13,h+1}^{-1})\hat{x}_{h+1|h}\hat{x}_{h+1|h}^T, \\
\mathfrak{A}_{h+1} &= (s-1)\sum_{\varrho=1}^{s-1}\mathfrak{L}_{h+1}\bar{\Theta}_{h+1}\Big[\mathscr{F}_{h+1}\big(\mathcal{P}_{h+1|h}^{-1} \\
&\quad -\rho_{2,h+1}\mathscr{K}_{2,h+1}^T\mathscr{K}_{2,h+1}\big)^{-1}\mathscr{F}_{h+1}^T \\
&\quad +\rho_{2,h+1}^{-1}\mathscr{L}_{2,h+1}\mathscr{L}_{2,h+1}^T\Big]\bar{\Theta}_{h+1}^T\mathfrak{L}_{h+1}^T, \\
\mathfrak{B}_{h+1} &= I - K_{h+1}(\mathfrak{L}_{h+1}+\mathfrak{P}_{h+1})\bar{\Theta}_{h+1}\mathscr{F}_{h+1}, \\
\mathfrak{D}_{h+1} &= (\mathcal{P}_{h+1|h}^{-1} - \rho_{2,h+1}\mathscr{K}_{2,h+1}^T\mathscr{K}_{2,h+1})^{-1}, \\
\mathfrak{F}_{h+1} &= \breve{\Theta}_{h+1}\circ\big[\flat_{1,h+1}^2\mathrm{tr}(\Im_{h+1|h}) + \flat_{2,h+1}^2\big]I, \\
\breve{\Theta}_h &= \mathrm{diag}\big\{\bar{\theta}_{1,h}-\bar{\theta}_{1,h}^2, \bar{\theta}_{2,h}-\bar{\theta}_{2,h}^2, \ldots, \bar{\theta}_{n_y,h}-\bar{\theta}_{n_y,h}^2\big\}, \\
\breve{\Gamma}_h &= \begin{bmatrix} \bar{\gamma}_{1,h} & \bar{\gamma}_{1,h}\bar{\gamma}_{2,h} & \cdots & \bar{\gamma}_{1,h}\bar{\gamma}_{n_y,h} \\ \bar{\gamma}_{2,h}\bar{\gamma}_{1,h} & \bar{\gamma}_{2,h} & \cdots & \bar{\gamma}_{2,h}\bar{\gamma}_{n_y,h} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{\gamma}_{n_y,h}\bar{\gamma}_{1,h} & \bar{\gamma}_{n_y,h}\bar{\gamma}_{2,h} & \cdots & \bar{\gamma}_{n_y,h} \end{bmatrix}, \qquad (25)
\end{aligned}
$$

then it can be deduced that $\mathcal{P}_{h+1|h+1}$ is an upper bound of $P_{h+1|h+1}$, i.e., $P_{h+1|h+1} \le \mathcal{P}_{h+1|h+1}$ is true. Besides, if the filter gain K_{h+1} is designed as:

$$
\begin{aligned}
K_{h+1} \ &= \ (1+q_{1,h+1})\mathfrak{D}_{h+1}\mathscr{F}_{h+1}^T\bar{\Theta}_{h+1}^T(\mathfrak{L}_{h+1}+\mathfrak{P}_{h+1})^T \\
&\quad \times\nabla_{h+1}^{-1}, \qquad (26)
\end{aligned}
$$

where

$$
\begin{aligned}
&\nabla_{h+1} \\
=\ & (1+q_{1,h+1})\left[(\mathfrak{L}_{h+1}+\mathfrak{P}_{h+1})\bar{\Theta}_{h+1}\mathscr{F}_{h+1}\right]\mathfrak{D}_{h+1} \\
&\times\left[(\mathfrak{L}_{h+1}+\mathfrak{P}_{h+1})\bar{\Theta}_{h+1}\mathscr{F}_{h+1}\right]^T + (1+q_{1,h+1})\rho_{2,h+1}^{-1} \\
&\times(\mathfrak{L}_{h+1}+\mathfrak{P}_{h+1})\bar{\Theta}_{h+1}\mathscr{L}_{2,h+1}\mathscr{L}_{2,h+1}^T\bar{\Theta}_{h+1}^T(\mathfrak{L}_{h+1} \\
&+\mathfrak{P}_{h+1})^T + (1+q_{2,h+1})\mathfrak{A}_{h-\varrho+1} + 2q_{3,h+1}\mathfrak{L}_{h+1}\mathfrak{F}_{h+1} \\
&\times\mathfrak{L}_{h+1}^T + 2(1+\varepsilon_{8,h+1})\sum_{\varrho=0}^{s-1}\mathfrak{L}_{h-\varrho+1}\mathfrak{F}_{h-\varrho+1}\mathfrak{L}_{h-\varrho+1}^T \\
&+\varepsilon_{16,h+1}\mathfrak{L}_{h+1}\left(\breve{\Theta}_{h+1}\circ R_{h+1}\right)\mathfrak{L}_{h+1}^T + (1+\varepsilon_{16,h+1}^{-1}) \\
&\times\mathfrak{P}_{h+1}\left(\breve{\Theta}_{h+1}\circ R_{h+1}\right)\mathfrak{P}_{h+1}^T + \sum_{\varrho=0}^{s-1}\mathfrak{L}_{h-\varrho+1} \\
&\times\left(\breve{\Theta}_{h-\varrho+1}\circ R_{h-\varrho+1}\right)\mathfrak{L}_{h-\varrho+1}^T + (1+\varepsilon_{8,h+1}^{-1}) \\
&\times\sum_{\varrho=0}^{s-1}\mathfrak{L}_{h-\varrho+1}\left[\breve{\Theta}_{h-\varrho+1}\circ\left(\mathcal{T}\mathcal{T}^T\right)\right]\mathfrak{L}_{h-\varrho+1}^T \\
&+q_{4,h+1}\mathfrak{L}_{h+1}\left[\breve{\Theta}_{h+1}\circ\left(\mathcal{T}\mathcal{T}^T\right)\right]\mathfrak{L}_{h+1}^T + (1+q_{5,h+1}) \\
&\times\mathfrak{P}_{h+1}\left[\breve{\Theta}_{h+1}\circ\left(\mathcal{T}\mathcal{T}^T\right)\right]\mathfrak{P}_{h+1}^T + (1+q_{6,h+1})s \\
&\times\sum_{\varrho=0}^{s-1}\mathfrak{L}_{h-\varrho+1}\bar{\Theta}_{h-\varrho+1}\Big(R_{h-\varrho+1} + \Delta_{h-\varrho+1}\mathcal{R}_{h-\varrho+1} \\
&\times\mathcal{R}_{h-\varrho+1}^T\Delta_{h-\varrho+1}^T\Big)\bar{\Theta}_{h-\varrho+1}^T\mathfrak{L}_{h-\varrho+1}^T + 2(1+q_{7,h+1}) \\
&\times\mathfrak{P}_{h+1}\mathfrak{F}_{h+1}\mathfrak{P}_{h+1}^T + (1+q_{8,h+1})\mathfrak{P}_{h+1}\bar{\Theta}_{h+1} \\
&\times\Big(R_{h+1} + \Delta_{h+1}\mathcal{R}_{h+1}\mathcal{R}_{h+1}^T\Delta_{h+1}^T\Big)\bar{\Theta}_{h+1}^T\mathfrak{P}_{h+1}^T \\
&+(1+q_{9,h+1})\left(\breve{\Gamma}_{h+1}\circ\bar{\eta}I\right), \qquad (27)
\end{aligned}
$$

then the obtained upper bound $\mathcal{P}_{h+1|h+1}$ can be minimized.

Proof: The results of this theorem will be verified by employing the mathematical induction method and the stochastic analysis technique. Recalling the initial condition $P_0 \le \mathcal{P}_{0|0}$ and supposing $P_{h|h} \le \mathcal{P}_{h|h}$, it remains to show that the inequality $P_{h+1|h+1} \le \mathcal{P}_{h+1|h+1}$ holds.

To begin with, according to Lemma 2 and $P_{h|h} \le \mathcal{P}_{h|h}$, one has

$$
\begin{aligned}
&(\mathscr{G}_h + \mathscr{L}_{1,h}\mathscr{R}_{1,h}\mathscr{K}_{1,h})P_{h|h}(\mathscr{G}_h + \mathscr{L}_{1,h}\mathscr{R}_{1,h}\mathscr{K}_{1,h})^T \\
\le\ & \mathscr{G}_h(\mathcal{P}_{h|h}^{-1} - \rho_{1,h}\mathscr{K}_{1,h}^T\mathscr{K}_{1,h})^{-1}\mathscr{G}_h^T + \rho_{1,h}^{-1}\mathscr{L}_{1,h}\mathscr{L}_{1,h}^T,
\end{aligned}
$$

where $\rho_{1,h}$ is a positive real number. It is straightforward to obtain that

$$
\begin{aligned}
P_{h+1|h} \ &\le \ \mathscr{G}_h(\mathcal{P}_{h|h}^{-1} - \rho_{1,h}\mathscr{K}_{1,h}^T\mathscr{K}_{1,h})^{-1}\mathscr{G}_h^T + \rho_{1,h}^{-1}\mathscr{L}_{1,h}\mathscr{L}_{1,h}^T \\
&\quad +D_hQ_hD_h^T \triangleq \mathcal{P}_{h+1|h}. \qquad (28)
\end{aligned}
$$

Similarly, the inequality $P_{h-\varrho+1|h-\varrho} \le \mathcal{P}_{h-\varrho+1|h-\varrho}$ holds for any $\varrho \in \{1, 2, \ldots, s-1\}$.

In addition, some cross terms in (22) can be handled by employing Lemma 3 as follows:

$$
\begin{aligned}
\mathrm{sym}(\mathfrak{C}_1\mathfrak{C}_2^T) \ &\le \ \varepsilon_{1,h+1}\mathfrak{C}_1\mathfrak{C}_1^T + \varepsilon_{1,h+1}^{-1}\mathfrak{C}_2\mathfrak{C}_2^T, \\
\mathrm{sym}(\mathfrak{C}_1\mathfrak{C}_6^T) \ &\le \ \varepsilon_{2,h+1}\mathfrak{C}_1\mathfrak{C}_1^T + \varepsilon_{2,h+1}^{-1}\mathfrak{C}_6\mathfrak{C}_6^T, \\
\mathrm{sym}(\mathfrak{C}_1\mathfrak{C}_{10}^T) \ &\le \ \varepsilon_{3,h+1}\mathfrak{C}_1\mathfrak{C}_1^T + \varepsilon_{3,h+1}^{-1}\mathfrak{C}_{10}\mathfrak{C}_{10}^T, \\
\mathrm{sym}(\mathfrak{C}_1\mathfrak{C}_{11}^T) \ &\le \ \varepsilon_{4,h+1}\mathfrak{C}_1\mathfrak{C}_1^T + \varepsilon_{4,h+1}^{-1}\mathfrak{C}_{11}\mathfrak{C}_{11}^T, \\
\mathrm{sym}(\mathfrak{C}_2\mathfrak{C}_6^T) \ &\le \ \varepsilon_{5,h+1}\mathfrak{C}_2\mathfrak{C}_2^T + \varepsilon_{5,h+1}^{-1}\mathfrak{C}_6\mathfrak{C}_6^T, \\
\mathrm{sym}(\mathfrak{C}_2\mathfrak{C}_{10}^T) \ &\le \ \varepsilon_{6,h+1}\mathfrak{C}_2\mathfrak{C}_2^T + \varepsilon_{6,h+1}^{-1}\mathfrak{C}_{10}\mathfrak{C}_{10}^T, \\
\mathrm{sym}(\mathfrak{C}_2\mathfrak{C}_{11}^T) \ &\le \ \varepsilon_{7,h+1}\mathfrak{C}_2\mathfrak{C}_2^T + \varepsilon_{7,h+1}^{-1}\mathfrak{C}_{11}\mathfrak{C}_{11}^T, \\
\mathrm{sym}(\mathfrak{C}_3\mathfrak{C}_5^T) \ &\le \ \varepsilon_{8,h+1}\mathfrak{C}_3\mathfrak{C}_3^T + \varepsilon_{8,h+1}^{-1}\mathfrak{C}_5\mathfrak{C}_5^T, \\
\mathrm{sym}(\mathfrak{C}_6\mathfrak{C}_{10}^T) \ &\le \ \varepsilon_{9,h+1}\mathfrak{C}_6\mathfrak{C}_6^T + \varepsilon_{9,h+1}^{-1}\mathfrak{C}_{10}\mathfrak{C}_{10}^T, \\
\mathrm{sym}(\mathfrak{C}_6\mathfrak{C}_{11}^T) \ &\le \ \varepsilon_{10,h+1}\mathfrak{C}_6\mathfrak{C}_6^T + \varepsilon_{10,h+1}^{-1}\mathfrak{C}_{11}\mathfrak{C}_{11}^T, \\
\mathrm{sym}(\mathfrak{C}_7\mathfrak{C}_9^T) \ &\le \ \varepsilon_{11,h+1}\mathfrak{C}_7\mathfrak{C}_7^T + \varepsilon_{11,h+1}^{-1}\mathfrak{C}_9\mathfrak{C}_9^T, \\
\mathrm{sym}(\mathfrak{C}_{10}\mathfrak{C}_{11}^T) \ &\le \ \varepsilon_{12,h+1}\mathfrak{C}_{10}\mathfrak{C}_{10}^T + \varepsilon_{12,h+1}^{-1}\mathfrak{C}_{11}\mathfrak{C}_{11}^T,
\end{aligned}
$$

where $\varepsilon_{k,h+1}$ $(k = 1, 2, \ldots, 12)$ are known positive scalars.

Noting the definition of $\mathfrak{C}_1$ in (22) and together with Lemma 2, one has

$$
\begin{aligned}
\mathbb{E}\{\mathfrak{C}_1\mathfrak{C}_1^T\} \ &\le \ \mathfrak{B}_{h+1}\mathfrak{D}_{h+1}\mathfrak{B}_{h+1}^T + \rho_{2,h+1}^{-1}K_{h+1} \\
&\quad \times(\mathfrak{L}_{h+1}+\mathfrak{P}_{h+1})\bar{\Theta}_{h+1}\mathscr{L}_{2,h+1}\mathscr{L}_{2,h+1}^T\bar{\Theta}_{h+1}^T \\
&\quad \times(\mathfrak{L}_{h+1}+\mathfrak{P}_{h+1})^TK_{h+1}^T
\end{aligned}
$$

with $\rho_{2,h+1} > 0$ being a scalar.

It follows from Lemma 3 that

$$\begin{aligned}\mathbb{E}\{\mathfrak{C}_2\mathfrak{C}_2^T\} \leq\ & (s-1)\sum_{\varrho=1}^{s-1} K_{h+1}\mathfrak{L}_{h-\varrho+1}\bar{\Theta}_{h-\varrho+1}\Omega_{h-\varrho+1} \\ & \times P_{h-\varrho+1|h-\varrho}\Omega_{h-\varrho+1}^T\bar{\Theta}_{h-\varrho+1}^T\mathfrak{L}_{h-\varrho+1}^T K_{h+1}^T \\ \leq\ & K_{h+1}\mathfrak{A}_{h-\varrho+1}K_{h+1}^T, \\ \mathbb{E}\{\mathfrak{C}_6\mathfrak{C}_6^T\} \leq\ & s\sum_{\varrho=0}^{s-1} K_{h+1}\mathfrak{L}_{h-\varrho+1}\bar{\Theta}_{h-\varrho+1}\Big(R_{h-\varrho+1} \\ & +\Delta_{h-\varrho+1}\mathcal{R}_{h-\varrho+1}\mathcal{R}_{h-\varrho+1}^T\Delta_{h-\varrho+1}^T\Big) \\ & \times\bar{\Theta}_{h-\varrho+1}^T\mathfrak{L}_{h-\varrho+1}^T K_{h+1}^T,\end{aligned}$$

where $\mathfrak{A}_{h-\varrho+1}$ is defined by (25) and $\rho_{2,h-\varrho+1} > 0$ is a real number.

In light of (3), we have

$$\begin{aligned}& \mathbb{E}\left\{g(x_{h-\varrho+1})g^T(x_{h-\varrho+1})\right\} \\ \leq\ & \mathbb{E}\left\{(\flat_{1,h-\varrho+1}\|x_{h-\varrho+1}\| + \flat_{2,h-\varrho+1})^2\right\} I \\ \leq\ & 2\left[\flat_{1,h-\varrho+1}^2 \mathrm{tr}\left(\mathbb{E}\{x_{h-\varrho+1}x_{h-\varrho+1}^T\}\right) + \flat_{2,h-\varrho+1}^2\right] I, \quad (29)\end{aligned}$$

where $\flat_{1,h-\varrho+1}$ and $\flat_{2,h-\varrho+1}$ are known nonnegative scalars. Besides, according to $\tilde{x}_{h-\varrho+1|h-\varrho} = x_{h-\varrho+1} - \hat{x}_{h-\varrho+1|h-\varrho}$, it can be easily obtained that

$$\begin{aligned}& \mathbb{E}\{x_{h-\varrho+1}x_{h-\varrho+1}^T\} \\ \leq\ & (1+\varepsilon_{13,h-\varrho+1})P_{h-\varrho+1|h-\varrho} + (1+\varepsilon_{13,h-\varrho+1}^{-1}) \\ & \times\hat{x}_{h-\varrho+1|h-\varrho}\hat{x}_{h-\varrho+1|h-\varrho}^T \triangleq \mho_{h-\varrho+1|h-\varrho} \quad (30)\end{aligned}$$

with $\varepsilon_{13,h-\varrho+1}$ being a positive scalar. Meanwhile, on the basis of Lemma 4 and (29)-(30), one can calculate

$$\begin{aligned}\mathbb{E}\{\mathfrak{C}_3\mathfrak{C}_3^T\} \leq\ & 2\sum_{\varrho=0}^{s-1} K_{h+1}\mathfrak{L}_{h-\varrho+1}\Big\{\breve{\Theta}_{h-\varrho+1}\circ\big[\flat_{1,h-\varrho+1}^2 \\ & \times\mathrm{tr}(\mho_{h-\varrho+1|h-\varrho}) + \flat_{2,h-\varrho+1}^2\big]I\Big\}\mathfrak{L}_{h-\varrho+1}^T K_{h+1}^T \\ \triangleq\ & 2\sum_{\varrho=0}^{s-1} K_{h+1}\mathfrak{L}_{h-\varrho+1}\mathfrak{K}_{h-\varrho+1}\mathfrak{L}_{h-\varrho+1}^T K_{h+1}^T, \\ \mathbb{E}\{\mathfrak{C}_7\mathfrak{C}_7^T\} \leq\ & 2K_{h+1}\mathfrak{P}_{h+1}\Big\{\breve{\Theta}_{h+1}\circ\big[\flat_{1,h+1}^2\mathrm{tr}(\mho_{h+1|h}) \\ & +\flat_{2,h+1}^2\big]I\Big\}\mathfrak{P}_{h+1}^T K_{h+1}^T \\ \triangleq\ & 2K_{h+1}\mathfrak{P}_{h+1}\mathfrak{K}_{h+1}\mathfrak{P}_{h+1}^T K_{h+1}^T,\end{aligned}$$

where $\breve{\Theta}_{h+1}$ is defined in (25).

Again using Lemma 3 and Lemma 4 can lead to the following results:

$$\begin{aligned}& \mathbb{E}\left\{\mathrm{sym}(\mathfrak{C}_3\mathfrak{C}_7^T)\right\} \\ \leq\ & 2\varepsilon_{14,h+1}K_{h+1}\mathfrak{L}_{h+1}\mathfrak{K}_{h+1}\mathfrak{L}_{h+1}^T K_{h+1}^T + 2\varepsilon_{14,h+1}^{-1}K_{h+1} \\ & \times\mathfrak{P}_{h+1}\mathfrak{K}_{h+1}\mathfrak{P}_{h+1}^T K_{h+1}^T, \\ & \mathbb{E}\left\{\mathrm{sym}(\mathfrak{C}_3\mathfrak{C}_9^T)\right\} \\ \leq\ & 2\varepsilon_{15,h+1}K_{h+1}\mathfrak{L}_{h+1}\mathfrak{K}_{h+1}\mathfrak{L}_{h+1}^T K_{h+1}^T + \varepsilon_{15,h+1}^{-1} \\ & \times K_{h+1}\mathfrak{P}_{h+1}\left[\breve{\Theta}_{h+1}\circ\left(\mathcal{T}\mathcal{T}^T\right)\right]\mathfrak{P}_{h+1}^T K_{h+1}^T, \\ & \mathbb{E}\left\{\mathrm{sym}(\mathfrak{C}_4\mathfrak{C}_8^T)\right\} \\ \leq\ & \varepsilon_{16,h+1}K_{h+1}\mathfrak{L}_{h+1}\left(\breve{\Theta}_{h+1}\circ R_{h+1}\right)\mathfrak{L}_{h+1}^T K_{h+1}^T \\ & +\varepsilon_{16,h+1}^{-1}K_{h+1}\mathfrak{P}_{h+1}\left(\breve{\Theta}_{h+1}\circ R_{h+1}\right)\mathfrak{P}_{h+1}^T K_{h+1}^T, \\ & \mathbb{E}\left\{\mathrm{sym}(\mathfrak{C}_5\mathfrak{C}_7^T)\right\} \\ \leq\ & \varepsilon_{17,h+1}K_{h+1}\mathfrak{L}_{h+1}\left[\breve{\Theta}_{h+1}\circ\left(\mathcal{T}\mathcal{T}^T\right)\right]\mathfrak{L}_{h+1}^T K_{h+1}^T \\ & +2\varepsilon_{17,h+1}^{-1}K_{h+1}\mathfrak{P}_{h+1}\mathfrak{K}_{h+1}\mathfrak{P}_{h+1}^T K_{h+1}^T, \\ & \mathbb{E}\left\{\mathrm{sym}(\mathfrak{C}_5\mathfrak{C}_9^T)\right\} \\ \leq\ & \varepsilon_{18,h+1}K_{h+1}\mathfrak{L}_{h+1}\left[\breve{\Theta}_{h+1}\circ\left(\mathcal{T}\mathcal{T}^T\right)\right]\mathfrak{L}_{h+1}^T K_{h+1}^T \\ & +\varepsilon_{18,h+1}^{-1}K_{h+1}\mathfrak{P}_{h+1}\left[\breve{\Theta}_{h+1}\circ\left(\mathcal{T}\mathcal{T}^T\right)\right]\mathfrak{P}_{h+1}^T K_{h+1}^T,\end{aligned}$$

where $\varepsilon_{d,h+1}$ $(d = 14, 15, \ldots, 18)$ are known positive constants. Furthermore, by resorting to $\eta_h^T\eta_h \leq \bar{\eta}$, we can derive $\mathbb{E}\{\mathfrak{C}_{11}\mathfrak{C}_{11}^T\} \leq K_{h+1}\left(\breve{\Gamma}_{h+1}\circ\bar{\eta}I\right)K_{h+1}^T$, where $\breve{\Gamma}_{h+1}$ is defined in (25).

Based on the above calculations, it can be deduced that

$$\begin{aligned}& P_{h+1|h+1} \\ \leq\ & (1+q_{1,h+1})\mathfrak{B}_{h+1}\mathfrak{D}_{h+1}\mathfrak{B}_{h+1}^T + (1+q_{1,h+1}) \\ & \times\rho_{2,h+1}^{-1}K_{h+1}(\mathfrak{L}_{h+1}+\mathfrak{P}_{h+1})\bar{\Theta}_{h+1}\mathscr{L}_{2,h+1}\mathscr{L}_{2,h+1}^T \\ & \times\bar{\Theta}_{h+1}^T(\mathfrak{L}_{h+1}+\mathfrak{P}_{h+1})^T K_{h+1}^T + (1+q_{2,h+1})K_{h+1} \\ & \times\mathfrak{A}_{h-\varrho+1}K_{h+1}^T + 2q_{3,h+1}K_{h+1}\mathfrak{L}_{h+1}\mathfrak{F}_{h+1}\mathfrak{L}_{h+1}^T K_{h+1}^T \\ & +2(1+\varepsilon_{8,h+1})\sum_{\varrho=0}^{s-1}K_{h+1}\mathfrak{L}_{h-\varrho+1}\mathfrak{F}_{h-\varrho+1}\mathfrak{L}_{h-\varrho+1}^T K_{h+1}^T \\ & +\varepsilon_{16,h+1}K_{h+1}\mathfrak{L}_{h+1}\left(\breve{\Theta}_{h+1}\circ R_{h+1}\right)\mathfrak{L}_{h+1}^T K_{h+1}^T \\ & +(1+\varepsilon_{16,h+1}^{-1})K_{h+1}\mathfrak{P}_{h+1}\left(\breve{\Theta}_{h+1}\circ R_{h+1}\right)\mathfrak{P}_{h+1}^T K_{h+1}^T \\ & +K_{h+1}\sum_{\varrho=0}^{s-1}\mathfrak{L}_{h-\varrho+1}\left(\breve{\Theta}_{h-\varrho+1}\circ R_{h-\varrho+1}\right)\mathfrak{L}_{h-\varrho+1}^T K_{h+1}^T \\ & +(1+\varepsilon_{8,h+1}^{-1})\sum_{\varrho=0}^{s-1}K_{h+1}\mathfrak{L}_{h-\varrho+1}\left[\breve{\Theta}_{h-\varrho+1}\circ\left(\mathcal{T}\mathcal{T}^T\right)\right] \\ & \times\mathfrak{L}_{h-\varrho+1}^T K_{h+1}^T + q_{4,h+1}K_{h+1}\mathfrak{L}_{h+1}\left[\breve{\Theta}_{h+1}\circ\left(\mathcal{T}\mathcal{T}^T\right)\right] \\ & \times\mathfrak{L}_{h+1}^T K_{h+1}^T + (1+q_{5,h+1})K_{h+1}\mathfrak{P}_{h+1} \\ & \times\left[\breve{\Theta}_{h+1}\circ\left(\mathcal{T}\mathcal{T}^T\right)\right]\mathfrak{P}_{h+1}^T K_{h+1}^T + (1+q_{6,h+1})s \\ & \times\sum_{\varrho=0}^{s-1}K_{h+1}\mathfrak{L}_{h-\varrho+1}\bar{\Theta}_{h-\varrho+1}\Big(R_{h-\varrho+1} + \Delta_{h-\varrho+1} \\ & \times\mathcal{R}_{h-\varrho+1}\mathcal{R}_{h-\varrho+1}^T\Delta_{h-\varrho+1}^T\Big)\bar{\Theta}_{h-\varrho+1}^T\mathfrak{L}_{h-\varrho+1}^T K_{h+1}^T \\ & +2(1+q_{7,h+1})K_{h+1}\mathfrak{P}_{h+1}\mathfrak{F}_{h+1}\mathfrak{P}_{h+1}^T K_{h+1}^T \\ & +(1+q_{8,h+1})K_{h+1}\mathfrak{P}_{h+1}\bar{\Theta}_{h+1} \\ & \times\Big(R_{h+1} + \Delta_{h+1}\mathcal{R}_{h+1}\mathcal{R}_{h+1}^T\Delta_{h+1}^T\Big)\bar{\Theta}_{h+1}^T\mathfrak{P}_{h+1}^T K_{h+1}^T \\ & +(1+q_{9,h+1})K_{h+1}\left(\breve{\Gamma}_{h+1}\circ\bar{\eta}I\right)K_{h+1}^T, \quad (31)\end{aligned}$$

where $q_{j,h+1}$ $(j = 1, 2, \ldots, 9)$, $\Im_{h+1|h}$ and $\mathfrak{F}_{h+1}$ are defined by (25). Next, it is straightforward to obtain that $P_{h+1|h+1} \leq \mathcal{P}_{h+1|h+1}$ by combining (23), (24), (28) with (31).

The specific expression of K_{h+1} is derived by minimizing

$\mathrm{tr}(\mathcal{P}_{h+1|h+1})$ and we have

$$\begin{aligned}
&\frac{\partial \mathrm{tr}(\mathcal{P}_{h+1|h+1})}{\partial K_{h+1}}\\
=&-2(1+q_{1,h+1})\mathfrak{B}_{h+1}\mathfrak{D}_{h+1}\big[(\mathfrak{L}_{h+1}+\mathfrak{P}_{h+1})\\
&\times\bar{\Theta}_{h+1}\mathscr{F}_{h+1}\big]^T+2(1+q_{1,h+1})\rho_{2,h+1}^{-1}K_{h+1}(\mathfrak{L}_{h+1}\\
&+\mathfrak{P}_{h+1})\bar{\Theta}_{h+1}\mathscr{L}_{2,h+1}\mathscr{L}_{2,h+1}^T\bar{\Theta}_{h+1}^T(\mathfrak{L}_{h+1}+\mathfrak{P}_{h+1})^T\\
&+2(1+q_{2,h+1})K_{h+1}\mathfrak{A}_{h-\varrho+1}+4q_{3,h+1}K_{h+1}\mathfrak{L}_{h+1}\\
&\times\mathfrak{F}_{h+1}\mathfrak{L}_{h+1}^T+4(1+\varepsilon_{8,h+1})\sum_{\varrho=0}^{s-1}K_{h+1}\mathfrak{L}_{h-\varrho+1}\mathfrak{F}_{h-\varrho+1}\\
&\times\mathfrak{L}_{h-\varrho+1}^T+2\varepsilon_{16,h+1}K_{h+1}\mathfrak{L}_{h+1}\left(\breve{\Theta}_{h+1}\circ R_{h+1}\right)\mathfrak{L}_{h+1}^T\\
&+2q_{4,h+1}K_{h+1}\mathfrak{L}_{h+1}\left[\breve{\Theta}_{h+1}\circ\left(\mathcal{T}\mathcal{T}^T\right)\right]\mathfrak{L}_{h+1}^T\\
&+2(1+\varepsilon_{16,h+1}^{-1})K_{h+1}\mathfrak{P}_{h+1}\left(\breve{\Theta}_{h+1}\circ R_{h+1}\right)\mathfrak{P}_{h+1}^T\\
&+2K_{h+1}\sum_{\varrho=0}^{s-1}\mathfrak{L}_{h-\varrho+1}\left(\breve{\Theta}_{h-\varrho+1}\circ R_{h-\varrho+1}\right)\mathfrak{L}_{h-\varrho+1}^T\\
&+2(1+\varepsilon_{8,h+1}^{-1})\sum_{\varrho=0}^{s-1}K_{h+1}\mathfrak{L}_{h-\varrho+1}\left[\breve{\Theta}_{h-\varrho+1}\circ\left(\mathcal{T}\mathcal{T}^T\right)\right]\\
&\times\mathfrak{L}_{h-\varrho+1}^T+2(1+q_{5,h+1})K_{h+1}\mathfrak{P}_{h+1}\left[\breve{\Theta}_{h+1}\circ\left(\mathcal{T}\mathcal{T}^T\right)\right]\\
&\times\mathfrak{P}_{h+1}^T+2(1+q_{6,h+1})s\sum_{\varrho=0}^{s-1}K_{h+1}\mathfrak{L}_{h-\varrho+1}\bar{\Theta}_{h-\varrho+1}\\
&\times\Big(R_{h-\varrho+1}+\Delta_{h-\varrho+1}\mathcal{R}_{h-\varrho+1}\mathcal{R}_{h-\varrho+1}^T\Delta_{h-\varrho+1}^T\Big)\\
&\times\bar{\Theta}_{h-\varrho+1}^T\mathfrak{L}_{h-\varrho+1}^T+4(1+q_{7,h+1})K_{h+1}\mathfrak{P}_{h+1}\\
&\times\mathfrak{F}_{h+1}\mathfrak{P}_{h+1}^T+2(1+q_{8,h+1})K_{h+1}\mathfrak{P}_{h+1}\bar{\Theta}_{h+1}\\
&\times\Big(R_{h+1}+\Delta_{h+1}\mathcal{R}_{h+1}\mathcal{R}_{h+1}^T\Delta_{h+1}^T\Big)\bar{\Theta}_{h+1}^T\mathfrak{P}_{h+1}^T\\
&+2(1+q_{9,h+1})K_{h+1}\left(\breve{\Gamma}_{h+1}\circ\bar{\eta}I\right).
\end{aligned}$$

Finally, the gain matrix K_{h+1} can be determined via setting $\frac{\partial \mathrm{tr}(\mathcal{P}_{h+1|h+1})}{\partial K_{h+1}}=0$, which is the same as (26). The proof is complete. ■

Remark 4 *On the one hand, compared with the TRF problem under alternative communication protocols, a hybrid protocol has been adopted in this paper, i.e., the FRP. For the FRP, the measurement after the static protocol has been described in the form of (8) and the zero-input method has been employed to compensate for the signal in the dynamic protocol segment. This description obviously avoids the matrix augmentation and other unnecessary computations. On the other hand, after obtaining the gain matrix K_{h+1}, some new terms have emerged compared with [15]. These new terms have been induced by the nonlinearity, random FDIAs and FRP. For instance, the effects of the FRP and random FDIAs have been reflected by all summation terms in K_{h+1} and the term $\left(\breve{\Gamma}_{h+1}\circ\bar{\eta}I\right)$, respectively, and it can be observed that these new terms can be calculated recursively.*

The FRP-based TRF algorithm given by Theorem 1 can be summarized as in Algorithm 1.

Algorithm 1: The FRP-Based TRF Algorithm	
Step 1:	Set $h=0$ and initialize parameters.
Step 2:	Calculate $\hat{x}_{h+1|h}$ by (17).
Step 3:	Compute $\mathcal{P}_{h+1|h}$ by employing the matrix equation (23).
Step 4:	Substituting (23) into (26) yields the filter gain K_{h+1}.
Step 5:	Obtain the updated state $\hat{x}_{h+1|h+1}$ according to (18).
Step 6:	Calculate $\mathcal{P}_{h+1|h+1}$ in light of (24).
Step 7:	Let $h=h+1$. Go back to *Step* 2.

Remark 5 *So far, a new FRP-based TRF algorithm has been proposed and the implementation steps have been provided in Algorithm 1. To be more specific, the time instant $h=0$ is set and the system parameters including $\hat{x}_{0|0}=\bar{x}_0$, P_0 and $\mathcal{P}_{0|0}$ are first initialized such that the initial condition constraint $P_0\leq\mathcal{P}_{0|0}$ is satisfied. Next, the one-step prediction $\hat{x}_{h+1|h}$ can be calculated based on the prediction step (17), which further provides the upper bound of the prediction error covariance $\mathcal{P}_{h+1|h}$ in line with (23). Next, according to the obtained upper bound and (26), the gain matrix K_{h+1} can be given. Subsequently, the state estimation $\hat{x}_{h+1|h+1}$ can be obtained according to the designed updating step (18). At last, the upper bound of the FEC $\mathcal{P}_{h+1|h+1}$ can be found by solving the matrix equation (24). At this point, the state estimation of the time instant $h+1$ is available, and then the second to seventh steps are iteratively repeated to compute the estimation for the next time instant until the total number of iterations is reached.*

IV. Boundedness Analysis

In this section, a sufficient condition is established to ensure that the filtering error is uniformly bounded in the MSS. To achieve our purpose, the following assumption is firstly given.

Assumption 1 *There exist some positive real constants $\bar{g}$, $\bar{l}_1$, $\breve{\sigma}$, $\bar{\pi}_1$, $\bar{f}$, $\bar{l}_2$, $\bar{\pi}_2$, $\bar{d}$, $\bar{q}$, $\bar{\sigma}$, $\bar{r}$, $\bar{q}_i$, $\underline{q_i}$, $\underline{\rho_i}$, $\bar{\rho}_i$, $\underline{\iota}$, $\bar{\iota}$, $\bar{\delta}$, $\bar{t}$, $\bar{\chi}$, $\bar{\varepsilon}_8$, $\underline{\varepsilon_8}$, $\hat{r}$, $\bar{\varepsilon}_{13}$, $\underline{\varepsilon_{13}}$, $\bar{\varepsilon}_{16}$, $\underline{\varepsilon_{16}}$, $\bar{\mu}$ and $\bar{\xi}$ such that*

$$\begin{aligned}
&\mathscr{G}_{h-\varrho}\mathscr{G}_{h-\varrho}^T\leq\bar{g}I,\ \mathscr{L}_{1,h-\varrho}\mathscr{L}_{1,h-\varrho}^T\leq\bar{l}_1I,\ \breve{\Theta}_{h-\varrho+1}\leq\breve{\sigma}I,\\
&\mathscr{K}_{1,h-\varrho}^T\mathscr{K}_{1,h-\varrho}\leq\bar{\pi}_1I,\ \mathscr{F}_{h-\varrho+1}\mathscr{F}_{h-\varrho+1}^T\leq\bar{f}I,\\
&\mathscr{L}_{2,h-\varrho+1}\mathscr{L}_{2,h-\varrho+1}^T\leq\bar{l}_2I,\ \mathscr{K}_{2,h-\varrho+1}^T\mathscr{K}_{2,h-\varrho+1}\leq\bar{\pi}_2I,\\
&D_{h-\varrho}D_{h-\varrho}^T\leq\bar{d}I,\ Q_{h-\varrho}\leq\bar{q}I,\ \bar{\Theta}_{h-\varrho+1}\leq\bar{\sigma}I,\\
&R_{h-\varrho+1}\leq\bar{r}I,\ \underline{q_i}\leq q_{i,h+1}\leq\bar{q}_i,\ \underline{\rho_i}\leq\rho_{i,h-\varrho}\leq\bar{\rho}_i,\\
&\underline{\iota}I\leq\breve{\Gamma}_{h+1}\leq\bar{\iota}I,\ \Delta_{h-\varrho+1}\Delta_{h-\varrho+1}^T\leq\bar{\delta}I,\ \mathcal{T}\mathcal{T}^T\leq\bar{t}I,\\
&\hat{x}_{h-\varrho|h-\varrho}\hat{x}_{h-\varrho|h-\varrho}^T\leq\bar{\chi}I,\ \underline{\varepsilon_8}\leq\varepsilon_{8,h+1}\leq\bar{\varepsilon}_8,\\
&\mathcal{R}_{h-\varrho+1}\mathcal{R}_{h-\varrho+1}^T\leq\hat{r}I,\ \underline{\varepsilon_{13}}\leq\varepsilon_{13,h-\varrho+1}\leq\bar{\varepsilon}_{13},\\
&\underline{\varepsilon_{16}}\leq\varepsilon_{16,h+1}\leq\bar{\varepsilon}_{16},\ \flat_{1,h-\varrho+1}^2\leq\bar{\mu},\ \flat_{2,h-\varrho+1}^2\leq\bar{\xi}
\end{aligned}$$

hold for any $\varrho\in\{0,1,\ldots,s-1\}$.

Next, we denote

$$\begin{aligned}
\breve{p}&=(\tilde{p}^{-1}-\bar{\rho}_1\bar{\pi}_1)^{-1}\bar{g}+\underline{\rho_1}^{-1}\bar{l}_1+\bar{d}\bar{q},\\
\tilde{\chi}&=(\bar{\chi}^{-1}-\bar{\rho}_1\bar{\pi}_1)^{-1}\bar{g}+\underline{\rho_1}^{-1}\bar{l}_1,\\
\kappa&=n\left[(1+\bar{\varepsilon}_{13})\breve{p}+(1+\underline{\varepsilon_{13}}^{-1})\tilde{\chi}\right],
\end{aligned}$$

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$$
\begin{aligned}
\kappa_1 &= (s-1)^2\bar{\sigma}^2\left[\bar{f}(\check{p}^{-1}-\bar{\rho}_2\bar{\pi}_2)^{-1}+\underline{\rho_2}^{-1}\bar{l}_2\right],\\
\kappa_2 &= \left[(1+\underline{q_9})\underline{\iota}\bar{\eta}\right]^{-1},\ \chi=1+(\bar{\sigma}^2\bar{f}+1)(1+\mathcal{K}),\\
\mathcal{K} &= (1+\bar{q}_1^2)\bar{\sigma}^2\kappa_2^2(\check{p}^{-1}-\bar{\rho}_2\bar{\pi}_2)^{-2}\bar{f},\\
\imath &= (1+\bar{q}_1)(\check{p}^{-1}-\bar{\rho}_2\bar{\pi}_2)^{-1}\chi+(1+\bar{q}_1)\underline{\rho_2}^{-1}\bar{\sigma}^2\bar{l}_2\mathcal{K}\\
&\quad+(1+\bar{q}_2)\kappa_1\mathcal{K}+[2(1+\bar{\varepsilon}_8)s+2\bar{q}_3+2(1+\bar{q}_7)]\\
&\quad\times\breve{\sigma}(\bar{\mu}\kappa+\bar{\xi})\mathcal{K}+[(s+\bar{\varepsilon}_{16})+(1+\underline{\varepsilon_{16}}^{-1})]\breve{\sigma}\bar{r}\mathcal{K}\\
&\quad+[s(1+\underline{\varepsilon_8}^{-1})+\bar{q}_4+(1+\bar{q}_5)]\breve{\sigma}\bar{t}\mathcal{K}+[s^2(1+\bar{q}_6)\bar{\sigma}^2\\
&\quad+(1+\bar{q}_8)\bar{\sigma}^2](\bar{r}+\bar{\delta}\hat{r})\mathcal{K}+(1+\bar{q}_9)\bar{\iota}\bar{\eta}\mathcal{K}. \qquad (32)
\end{aligned}
$$

Theorem 2 *On the basis of Assumption 1 and (32), if the constraints $\imath\le\tilde{p}$, $\bar{\rho}_1\bar{\pi}_1<\tilde{p}^{-1}$, $\bar{\rho}_2\bar{\pi}_2<\check{p}^{-1}$ and $\bar{\pi}_1\bar{\chi}<\bar{\rho}_1^{-1}$ are satisfied under the initial value of the upper bound $\mathcal{P}_{0|0}\le\tilde{p}I$, then $\mathcal{P}_{h|h}\le\tilde{p}I$ holds for any $h\ge1$.*

Proof: Assume that $\mathcal{P}_{h|h}\le\tilde{p}I$ holds, then we will show that it also holds at the inductive step $h+1$. From Assumption 1, we obtain

$$
(\mathcal{P}_{h|h}^{-1}-\rho_{1,h}\mathscr{K}_{1,h}^T\mathscr{K}_{1,h})^{-1}\le(\tilde{p}^{-1}-\bar{\rho}_1\bar{\pi}_1)^{-1}I. \qquad (33)
$$

Moreover, it can be concluded from (23) that $\mathcal{P}_{h+1|h}\le(\tilde{p}^{-1}-\bar{\rho}_1\bar{\pi}_1)^{-1}\bar{g}I+\underline{\rho_1}^{-1}\bar{l}_1I+\bar{d}\bar{q}I=\check{p}I$. Notice that the initial value $\mathcal{P}_{0|0}\le\tilde{p}I$ is true, then $\mathcal{P}_{h-\varrho+1|h-\varrho}\le\check{p}I$ can be computed.

In line with the condition $f(0)=0$ and (17), we can derive $\hat{x}_{h-\varrho+1|h-\varrho}=(\mathscr{G}_{h-\varrho}+\mathscr{L}_{1,h-\varrho}\mathscr{R}_{1,h-\varrho}\mathscr{K}_{1,h-\varrho})\hat{x}_{h-\varrho|h-\varrho}$, and one has

$$
\begin{aligned}
&\hat{x}_{h-\varrho+1|h-\varrho}\hat{x}_{h-\varrho+1|h-\varrho}^T\\
&=(\mathscr{G}_{h-\varrho}+\mathscr{L}_{1,h-\varrho}\mathscr{R}_{1,h-\varrho}\mathscr{K}_{1,h-\varrho})\hat{x}_{h-\varrho|h-\varrho}\hat{x}_{h-\varrho|h-\varrho}^T\\
&\quad\times(\mathscr{G}_{h-\varrho}+\mathscr{L}_{1,h-\varrho}\mathscr{R}_{1,h-\varrho}\mathscr{K}_{1,h-\varrho})^T.
\end{aligned}
$$

It can be derived from $\bar{\pi}_1\bar{\chi}<\bar{\rho}_1^{-1}$ that $\rho_{1,h-\varrho}^{-1}I-\mathscr{K}_{1,h-\varrho}\hat{x}_{h-\varrho|h-\varrho}\hat{x}_{h-\varrho|h-\varrho}^T\mathscr{K}_{1,h-\varrho}^T>0$, which further verifies by Lemma 2 that $\hat{x}_{h-\varrho+1|h-\varrho}\hat{x}_{h-\varrho+1|h-\varrho}^T\le\left(\bar{\chi}^{-1}-\bar{\rho}_1\bar{\pi}_1\right)^{-1}\bar{g}I+\underline{\rho_1}^{-1}\bar{l}_1I=\tilde{\chi}I$. After the above derivation, we obtain $\mathrm{tr}\left(\Im_{h-\varrho+1|h-\varrho}\right)\le\mathrm{tr}\left\{\left[(1+\bar{\varepsilon}_{13})\check{p}+(1+\underline{\varepsilon_{13}}^{-1})\tilde{\chi}\right]I\right\}=\kappa$.

Along the same line with (33), one has $\mathfrak{D}_{h-\varrho+1}\le(\check{p}^{-1}-\bar{\rho}_2\bar{\pi}_2)^{-1}I$. Due to $\mathfrak{L}_{h-\varrho+1}\mathfrak{L}_{h-\varrho+1}^T\le I$, it can be found that

$$
\mathfrak{A}_{h-\varrho+1}\le(s-1)^2\bar{\sigma}^2\left[\bar{f}(\check{p}^{-1}-\bar{\rho}_2\bar{\pi}_2)^{-1}+\underline{\rho_2}^{-1}\bar{l}_2\right]I=\kappa_1I.
$$

Furthermore, (27) indicates that

$$
\begin{aligned}
\nabla_{h+1}^{-1}&\le\left[(1+q_{9,h+1})\left(\breve{\Gamma}_{h+1}\circ\bar{\eta}I\right)\right]^{-1}\\
&\le\left[(1+\underline{q_9})\underline{\iota}\bar{\eta}\right]^{-1}I=\kappa_2I.
\end{aligned}
$$

Noting the expression of the filter gain in (26) and $\mathfrak{P}_{h+1}\mathfrak{P}_{h+1}^T\le I$, it is not difficult to testify that

$$
K_{h+1}K_{h+1}^T\le(1+\bar{q}_1^2)\bar{\sigma}^2\kappa_2^2(\check{p}^{-1}-\bar{\rho}_2\bar{\pi}_2)^{-2}\bar{f}I=\mathcal{K}I.
$$

For the sake of brevity, letting $\varepsilon=1$ in Lemma 3 immediately yields

$$
\begin{aligned}
&\mathfrak{B}_{h+1}\mathfrak{B}_{h+1}^T\\
&\le I+\mathscr{F}_{h+1}^T\bar{\Theta}_{h+1}^T(\mathfrak{L}_{h+1}+\mathfrak{P}_{h+1})^T(\mathfrak{L}_{h+1}+\mathfrak{P}_{h+1})\bar{\Theta}_{h+1}\\
&\quad\times\mathscr{F}_{h+1}+K_{h+1}K_{h+1}^T+K_{h+1}(\mathfrak{L}_{h+1}+\mathfrak{P}_{h+1})\bar{\Theta}_{h+1}\\
&\quad\times\mathscr{F}_{h+1}\mathscr{F}_{h+1}^T\bar{\Theta}_{h+1}^T(\mathfrak{L}_{h+1}+\mathfrak{P}_{h+1})^TK_{h+1}^T\\
&\le\left[1+(\bar{\sigma}^2\bar{f}+1)(1+\mathcal{K})\right]I=\chi I.
\end{aligned}
$$

After the above algebraic manipulations, it is not difficult to prove that

$$
\begin{aligned}
&\mathcal{P}_{h+1|h+1}\\
&\le(1+\bar{q}_1)(\check{p}^{-1}-\bar{\rho}_2\bar{\pi}_2)^{-1}\chi I+(1+\bar{q}_1)\underline{\rho_2}^{-1}\bar{\sigma}^2\bar{l}_2\mathcal{K}I\\
&\quad+(1+\bar{q}_2)\kappa_1\mathcal{K}I+[2(1+\bar{\varepsilon}_8)s+2\bar{q}_3+2(1+\bar{q}_7)]\\
&\quad\times\breve{\sigma}(\bar{\mu}\kappa+\bar{\xi})\mathcal{K}I+[(s+\bar{\varepsilon}_{16})+(1+\underline{\varepsilon_{16}}^{-1})]\breve{\sigma}\bar{r}\mathcal{K}I\\
&\quad+[s(1+\underline{\varepsilon_8}^{-1})+\bar{q}_4+(1+\bar{q}_5)]\breve{\sigma}\bar{t}\mathcal{K}I+[s^2(1+\bar{q}_6)\bar{\sigma}^2\\
&\quad+(1+\bar{q}_8)\bar{\sigma}^2](\bar{r}+\bar{\delta}\hat{r})\mathcal{K}I+(1+\bar{q}_9)\bar{\iota}\bar{\eta}\mathcal{K}I=\imath I.
\end{aligned}
$$

It follows from the constraint $\imath\le\tilde{p}$ that $\mathcal{P}_{h+1|h+1}\le\tilde{p}I$ holds. The proof of this theorem is complete. ■

Theorem 3 *Under the conditions of Theorem 2, the following inequality holds:*

$$
\mathbb{E}\{\|\tilde{x}_{h|h}\|^2\}\le\tilde{p},
$$

and, consequently, $\tilde{x}_{h|h}$ is uniformly bounded in the MSS.

Proof: From Theorem 2, we have $P_{h|h}\le\mathcal{P}_{h|h}\le\tilde{p}I$, that is, $\mathbb{E}\{\tilde{x}_{h|h}\tilde{x}_{h|h}^T\}\le\tilde{p}I$. Next, $\mathbb{E}\{\tilde{x}_{h|h}^T\tilde{x}_{h|h}\}\le\tilde{p}$ is derived by exploiting the Schur complement lemma. Finally, it can be shown that $\mathbb{E}\{\tilde{x}_{h|h}^T\tilde{x}_{h|h}\}=\mathbb{E}\{\|\tilde{x}_{h|h}\|^2\}\le\tilde{p}$. This completes the proof. ■

Remark 6 *In Theorem 2, a sufficient condition is given to guarantee the uniform boundedness of the TRF algorithm, which is related to the corresponding information about the system matrix, covariance matrices of noises and some parameters about the Taylor series. It can be observed that Assumption 1 is a key prerequisite for performing the boundedness analysis of the filtering error, which is reasonably established based on physical constraints and operating conditions in real engineering scenarios. Moreover, additional efforts can be made to provide more looser sufficient criterion in future. Overall, this analytical process in Theorem 2 can reflect the real-world constraints and further provide theoretical support for the practical application of the proposed algorithm with guaranteed performance.*

Remark 7 *In this paper, an FRP-based recursive filtering method has been proposed for a class of time-varying SNSs with random FDIAs and censored measurements within the framework of TKF. In particular, in Theorem 1, the upper bound of the FEC has been given by employing the stochastic analysis technique and the filter gain has been obtained by minimizing such an upper bound. After that, a sufficient criterion has been given in Theorem 2 to guarantee the uniform boundedness of the filtering error in the MSS and the effectiveness of the developed algorithm has further been proved from the theoretical viewpoint. Eventually, it can be found that the significant factors involved in filter design include the time-varying system parameters, uncensored probability, the upper bound of the attack signal and the available information of the FRP, which have been adequately reflected in main results.*

V. An Illustrative Example

In this section, a practical example is provided to illustrate the advantages of the presented TRF scheme.

As in [40], a three-wheeled Ackerman turning model is considered for describing the kinematic characteristics of vehicles with three wheels and front-wheel turning functions to further illustrate the reliability of the developed FRP-based TRF scheme. The state model as well as the measurement model mentioned are denoted as follows:

$$x_{h+1} = \begin{bmatrix} \mathrm{x}_h + \frac{2E}{\tan(\zeta_h)}\sin(\mathfrak{q}_h)\cos(\mathfrak{m}_h + \mathfrak{q}_h) \\ \mathrm{y}_h + \frac{2E}{\tan(\zeta_h)}\sin(\mathfrak{q}_h)\sin(\mathfrak{m}_h + \mathfrak{q}_h) \\ \mathfrak{m}_h + \frac{\varsigma_h \Delta t \tan(\zeta_h)}{E} \\ \varsigma_h \\ \zeta_h \end{bmatrix} + D_h \omega_h,$$

$$y_h^* = \begin{bmatrix} \cos(\mathfrak{m}_h)(\mathrm{x_b} - \mathrm{x}_h) + \sin(\mathfrak{m}_h)(\mathrm{y_b} - \mathrm{y}_h) \\ -\sin(\mathfrak{m}_h)(\mathrm{x_b} - \mathrm{x}_h) + \cos(\mathfrak{m}_h)(\mathrm{y_b} - \mathrm{y}_h) \\ \mathfrak{m}_h \\ \varsigma_h \\ \zeta_h \end{bmatrix} + v_h,$$

where x_h, y_h, $\mathfrak{m}_h$, ς_h and ζ_h represent the vehicle coordinates, vehicle path angle deviation, linear velocity and turning angle, respectively. $\mathfrak{q}_h = \frac{\varsigma_h \Delta t \tan(\zeta_h)}{2E}$, E is the distance from the front wheel to the rear axle of the tricycle. Δt denotes the sampling interval. $\mathrm{x_b}$ and $\mathrm{y_b}$ are the coordinate positions of the sensor measurement beacon. Next, for all $\varrho \in \{0, 1, 2\}$, we will give the specific parameters about this model as follows:

$$\begin{aligned} \mathscr{L}_{1,h-\varrho} &= \mathrm{diag}\{0.95, 0.5, 2, 0.01, 4\}, \\ \mathscr{K}_{1,h-\varrho} &= \mathrm{diag}\{0.51, 1, 0.01, 0.01, 0.01\}, \\ \mathscr{L}_{2,h-\varrho+1} &= \mathrm{diag}\{0.09, 0.01, 0.91, 0.56, 0.04\}, \\ \mathscr{K}_{2,h-\varrho+1} &= \mathrm{diag}\{1.22, 1, 0.72, 1.1, 1.8\}. \end{aligned}$$

The initial state x_0 is selected as a set of random variables with mean $\bar{x}_0 = \begin{bmatrix} 0 & 0 & 5 & 0.5 & 1 \end{bmatrix}^T$ and variance $P_0 = \mathrm{diag}\{0.002, 0.002, 0.002, 0.002, 0.005\}$. Moreover, the initial value of the filtering is selected as $\hat{x}_{0|0} = \bar{x}_0$. The matrix $D_{h-\varrho}$ is chosen as $D_{h-\varrho} = \mathrm{diag}\{0.006, 0.006, 0.006, 0.008, 0.005\}$. The covariance matrices of the process noise and the measurement noise are given as $Q_{h-\varrho} = \mathrm{diag}\{0, 0, 0, 0.01, 0.015\}$ and $R_{h-\varrho+1} = 2.99 I_5$, respectively. η_h denotes the bounded false data obeying the uniform distribution over $[-\sqrt{1.9}, \sqrt{1.9}]$. Other parameters are set as $\mathcal{P}_{0|0} = 1.1 I_5$, $\mathcal{T} = \begin{bmatrix} -1.97 & -2.204 & 6.36 & -3.45 & 2 \end{bmatrix}^T$, $s = 3$, $E = 5$, $\mathrm{x_b} = 12$, $\mathrm{y_b} = 12$, $\Delta t = 1$, $\bar{\gamma}_{1,h+1} = 0.01$, $\bar{\gamma}_{2,h+1} = 0.09$, $\bar{\gamma}_{3,h+1} = 0.04$, $\bar{\gamma}_{4,h+1} = 0.04$, $\bar{\gamma}_{5,h+1} = 0.09$, $\bar{\eta} = 9.5$, $\varepsilon_{1,h+1} = 0.01$, $\varepsilon_{2,h+1} = 0.01$, $\varepsilon_{3,h+1} = 0.01$, $\varepsilon_{4,h+1} = 0.1$, $\varepsilon_{5,h+1} = 0.4$, $\varepsilon_{6,h+1} = 0.2$, $\varepsilon_{7,h+1} = 0.6$, $\varepsilon_{8,h+1} = 0.01$, $\varepsilon_{9,h+1} = 0.01$, $\varepsilon_{10,h+1} = 0.01$, $\varepsilon_{11,h+1} = 0.79$, $\varepsilon_{12,h+1} = 0.09$, $\varepsilon_{13,h-\varrho+1} = 2.99$, $\varepsilon_{14,h+1} = 2.01$, $\varepsilon_{15,h+1} = 0.98$, $\varepsilon_{16,h+1} = 0.09$, $\varepsilon_{17,h+1} = 0.05$, $\varepsilon_{18,h+1} = 0.98$, $\flat_{1,h-\varrho+1} = 3.1$ and $\flat_{2,h-\varrho+1} = 2.7$.

The mean-square error (MSE) is introduced to evaluate the superiority of the presented TRF algorithm, which is defined as $\mathrm{MSE} = \frac{1}{M}\mathbb{E}\left\{\sum_{j=1}^{M}\sum_{\ell=1}^{5}\left(x_h^{\ell,j} - \hat{x}_{h|h}^{\ell,j}\right)^2\right\}$, where $M = 300$ stands for the number of experiments, $x_h^{\ell,j}$ and $\hat{x}_{h|h}^{\ell,j}$ represent the real state and the estimate of x_h^{ℓ} in the j-th simulation run, respectively. Hence, the primary results obtained are shown in Figs. 1-8. The trajectories of the true state and its estimates for the three-wheeled Ackerman turning model are exhibited in Figs. 1-5, it can be seen that even in the presence of random FDIAs, censored measurements and FRP, the designed TRF algorithm in this paper can still estimate the true state well. The relationship between the MSE and the upper bound of the FEC is plotted in Fig. 6, it can be easily observed that the MSE keeps below the upper bound, which is fully consistent with the theoretical derivation of this paper. To further illustrate the effect of random FDIAs on the accuracy of the TRF algorithm, Table I and Fig. 7 provide the values of MSE under three different attack probabilities, i.e., $\bar{\gamma}_{i,h}$ are selected as 0.1, 0.5 and 0.65, respectively. It is obvious that as the attack probability increases, the MSE also increases, which means that a higher attack probability causes the degraded accuracy of the TRF algorithm. The sensor nodes selected by the time-triggered and event-triggered rules under the FRP are shown in Fig. 8. In conclusion, the above simulations show the validity of the proposed TRF approach.

TABLE I
MSE with Different $\bar{\gamma}_{i,h}$.

Time(h)	$\cdots$	59	60	61	$\cdots$	139	140	141	$\cdots$
$\bar{\gamma}_{i,h} = 0.1$	$\cdots$	0.121	0.107	0.096	$\cdots$	0.048	0.052	0.054	$\cdots$
$\bar{\gamma}_{i,h} = 0.5$	$\cdots$	0.921	1.204	1.56	$\cdots$	0.144	0.124	0.124	$\cdots$
$\bar{\gamma}_{i,h} = 0.65$	$\cdots$	1.06	1.64	2.383	$\cdots$	27.959	27.101	25.944	$\cdots$

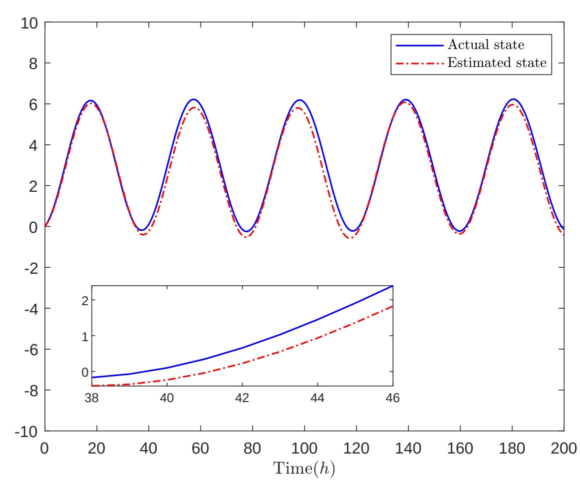

Fig. 1. State x_h and its estimation.

VI. Conclusions

In this paper, the FRP-based TRF issue has been investigated for time-varying SNSs subject to censored measurements and random FDIAs. The Tobit Type I model has been exploited to characterize the censored measurements and some random variables obeying the Bernoulli distribution have been introduced to depict the phenomenon of random FDIAs. In

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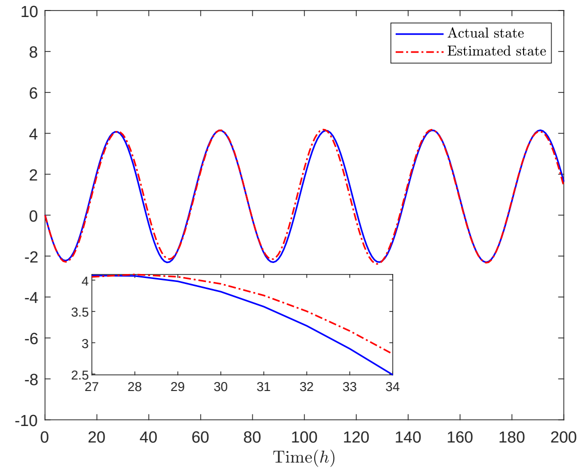

Fig. 2. State y_h and its estimation.

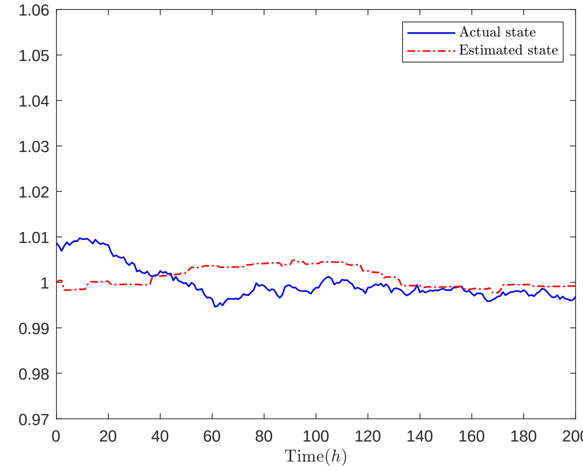

Fig. 5. State ζ_h and its estimation.

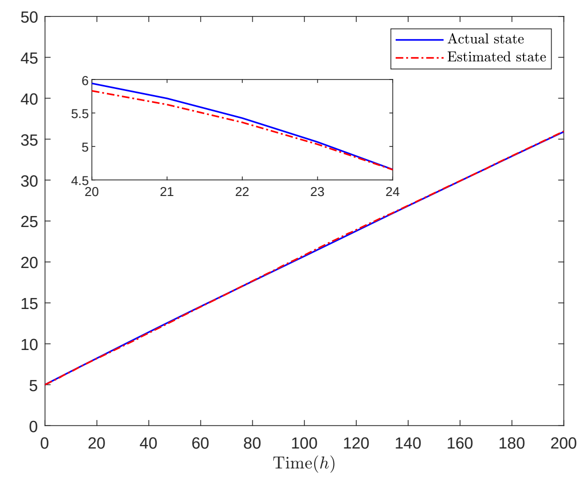

Fig. 3. State $\mathfrak{m}_h$ and its estimation.

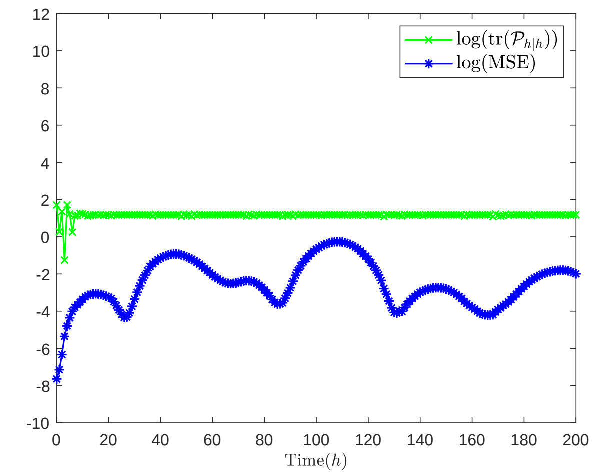

Fig. 6. The logarithm of MSE and $\mathrm{tr}(\mathcal{P}_{h|h})$.

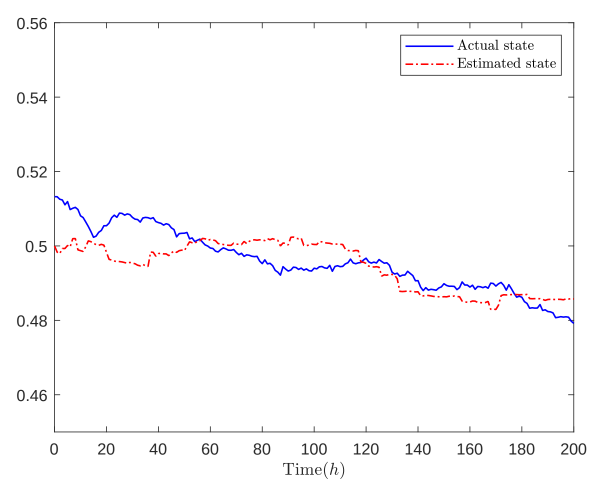

Fig. 4. State ς_h and its estimation.

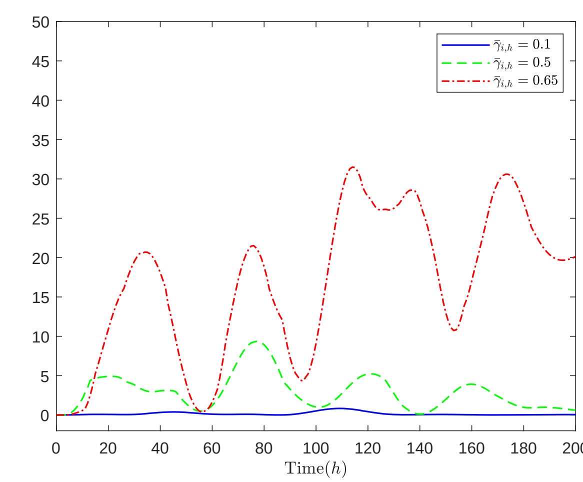

Fig. 7. MSE with different $\bar{\gamma}_{i,h}$.

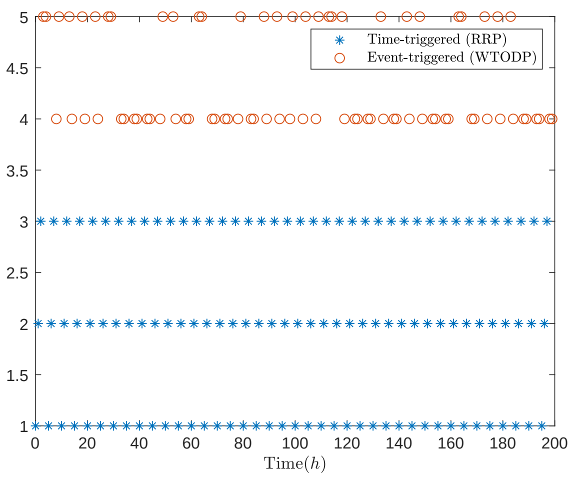

Fig. 8. The selected sensor node by the FRP.

addition, the FRP scheduling has been adopted to mitigate the data conflicts during the transmission, thereby reducing the communication costs and enhancing the data utilization efficiency. The upper bound of the FEC has been obtained and the specific form of the optimized filter gain has been calculated by minimizing the trace of this upper bound. Furthermore, a sufficient criterion has been given to guarantee the uniform boundedness of the filtering error in the MSS. Finally, the usefulness of the presented TRF algorithm has been demonstrated by conducting some comparative simulations. In future, we can extend the currently designed filtering technique to accommodate the multi-sensor data fusion scenarios under the covariance intersection fusion criterion as in [41].

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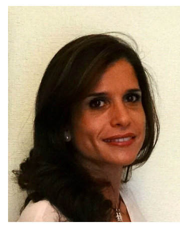

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