UNIVERSITY OF JAÉN

School of Engineering and Computing
Computer Science Department

Group Decision Making Dealing With Hesitant Fuzzy Linguistic Information

Thesis Memory Presented by

HUIHUI SONG

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to obtain the PhD Degree in Computer science

Supervisors

Dr. Luis Martínez López
Dr. Álvaro Labella Romero

JAÉN, MARCH, 2025

The thesis entitled *Group decision making dealing with hesitant fuzzy linguistic information*, presented by Dª Huihui Song to obtain the Ph.D. degree in Computer Science, has been carried out in the Computer Science Department of the University of Jaén with the supervisors Dr. Luis Martínez López and the Dr. Álvaro Labella Romero. To be evaluated, this research memory is presented as a set of published articles, according to Article 23, point 3, Regulation of Doctoral Studies of the University of Jaén, approved in February 2012.

Jaén, March, 2025

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Acknowledgements

I arrived in Spain on November 9th, 2022, and as I reflect on that day, it feels like it was just yesterday. As I approach my graduation with a PhD from the University of Jaén, I am filled with deep gratitude for all those who have supported me throughout my journey here.

First and foremost, I want to express my deepest thanks to my supervisor, Professor Luis Martínez, whose invitation letter gave me the chance to pursue my PhD at the University of Jaén. This opportunity has not only helped me develop my research skills but also introduced me to a wonderful team. I have been truly inspired by his academic excellence and integrity. He is not just an exceptional scholar, but also a fantastic supervisor. His dedication and hard work have motivated me a lot, and his patient guidance throughout my PhD has been invaluable in shaping my understanding of research. He is also a close friend, he helped us with accommodation issues and even invited us to his home for lunch. I have to say, his paella is the best I have ever had. I am incredibly grateful for his support and care, it is an honor to be his student.

I am also incredibly grateful to my co-director, Professor Álvaro Labella. He is very rigorous and focused in his research, always pushing for innovation, and I always look up to him as a role model. I really appreciate all the time and effort he put into guiding me through my thesis. He cared about my progress and carefully helped me revise my thesis, even if it meant revising it multiple times. In daily life, he is also a very kind and approachable person who always greets everyone with a warm smile. Every conversation with him is inspiring, and his positive energy always makes me feel encouraged.

I would like to extend my heartfelt gratitude to my collaborators and dear friends, Bapi and Diego. I deeply appreciate their selfless support and invaluable guidance throughout my research journey. Their patience in sharing their knowledge and research experiences, as well as their assistance in paper writing and research methodology selection, has been indispensable. Under their influence, I have truly come to understand the importance of responsibility and precision in academic research. I am also sincerely grateful for the time we spent together in Jaén. Whether it was walking together, cooking, exploring new restaurants, or traveling, these shared experiences have greatly enriched my life. I will always cherish the warmth and care they showed me, and I will never forget the sense of belonging I felt during that time.

I also want to thank my friends Rosa, Bruno, Jessi, Wen, Xiang, Yefan, Zijian, and Yutian. Spending holidays and important moments with them made my time in Jaén both enjoyable and fulfilling, I will always treasure the memories of our shared experiences.

I am forever grateful to my parents, whose love and support have been unwavering. They have always been there for me, believing in me and encouraging me to follow my path, no matter how challenging. Their selflessness has given me the strength to keep moving forward. I also want to thank my younger brother and sister-in-law, who have taken on the responsibility of caring for our family, allowing me to focus on my studies. Of course, I also want to thank my two lovely nieces, who have brought a lot of joy and happiness to our family. Their laughter and innocence always inspire me to maintain a positive attitude towards life.

Thank all the people who supported and helped me in my life and studies career. May everyone be healthy and happy.

Contents

Chapter 1

Introduction

This chapter outlines the motivation and objectives of this research, followed by an overview of the structure of this research memory.

1.1 Motivation

Decision theory [1] is the discipline that studies how to select the optimal solution from a range of possible choices. It involves evaluating, selecting, and analyzing the logic and methods behind decision-making behavior. In practical applications, the decision-making process extends beyond the selection of a single objective; it frequently involves handling complex scenarios where multiple evaluative attributes must be considered simultaneously. This decision-making approach is referred to as multi-attribute decision-making (MADM) [2, 3]. With the advancement of theory, numerous methods and models have been proposed to assist in decision-making within a multi-attribute context. These methods and models help systematically process and analyze decision problems [4, 5].

In real-world decision-making, decision-makers (DMs) are inherently constrained by their individual knowledge, skills, and experience. These constraints can introduce cognitive biases and result in incomplete or subjective evaluations. Furthermore, the complexity of decision-making scenarios, coupled with the vagueness and incompleteness of available information, often exceeds the cognitive and analytical capacities of a single individual. Such limitations significantly increase the risk of errors, thereby diminishing the robustness and reliability of decisions made solely by an individual. To address these challenges, group decision-making (GDM) [6, 7] has been introduced as an effective tool to solve this complex decision problems. In such a context of GDM, multiple DMs collaboratively discuss and analyze the same decision-making problem, integrating diverse perspectives and preferences to form a collective judgment. Based on this collective evaluation, alternatives are ranked and selected. GDM mitigates individual biases and harnesses the collective intelligence of the group, substantially improving the quality and objectivity of the decisions.

In addressing GDM problems, DMs are often required to evaluate multiple alternatives by expressing their preferences. However, due to the inherent complexity of real-world scenarios, the uncertainty in decision-making contexts, and the subjective nature of individual judgments, providing precise numerical evaluations for each alternative can be highly challenging. Therefore, DMs often rely on natural language descriptions because linguistic expressions allow them to articulate uncertainty, nuance, and subjective preferences in a flexible way that rigid numerical scales cannot fully capture. Language is inherently flexible and aligns with how humans process and communicate complex information, making it a preferred medium for qualitative assessments. This reliance on linguistic expressions gives rise to linguistic group decision-making (LGDM) problems [8, 9, 10]. By using linguistic expressions to convey evaluative information, LGDM better aligns with human cognitive processes, enabling more intuitive and accessible decision-making.

Extant LGDM studies often assume that DMs express their preferences by using a single linguistic term, which may fail to capture the full complexity of their evaluations. In real-world decision-making, particularly in uncertain contexts, DMs may hesitate when making judgments due to limitations in their experience, knowledge, or abilities. To address this limitation, Rodríguez et al. [11] introduced the concept of hesitant fuzzy linguistic term sets (HFLTSs), which enable DMs to express their evaluations using a range of consecutive linguistic terms, thereby providing a more nuanced and comprehensive representation of their preferences. By offering greater flexibility and precision, HFLTSs make the decision-making process more reflective of practical needs. Consequently, LGDM methods based on hesitant fuzzy linguistic information, referred to as HLGDM, have attracted widespread attention and become a vital representation tool for effectively managing uncertainty and ambiguity in complex decision-making scenarios [12, 13]. As shown in Figure 1.1, the general framework of the HLGDM consists of, but is not limited to, the following three phases:

Phase 1: One or more linguistic term sets with suitable granularity, syntax, and semantics are selected [14], enabling DMs to effectively express hesitant fuzzy linguistic information.

Phase 2: Some appropriate functions or methods must be selected to operate on hesitant fuzzy linguistic information.

Phase 3: Different methods or models must be developed to select the optimal alternative; this stage generally involves, but is not limited to, two phases:

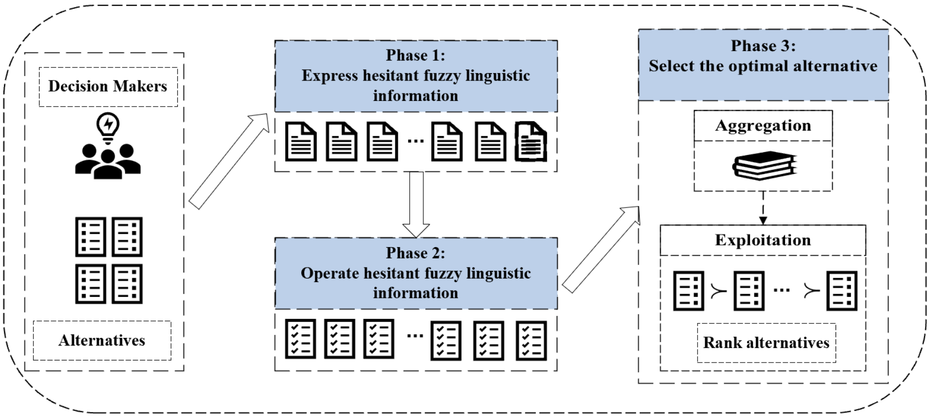


Figure 1.1: The general scheme of the HLGDM

(a) **Aggregation process:** The hesitant linguistic opinions are aggregated using a selected operator or method to produce a collective opinion.

(b) **Exploitation process:** Decision-making techniques are applied to evaluate and rank alternatives based on collective opinion, and ultimately select the optimal alternative based on the ranking results.

As societal and technological development accelerates, therefore management and decision-making processes have become increasingly complex and variable. Meanwhile, advancements in technology have significantly enhanced data processing capabilities and communication efficiency, making it feasible and effective for large-scale groups to participate in complex decision-making scenarios. Traditionally, GDM methods were classified as large-scale group decision making (LSGDM) when the number of DMs reached or exceeded 20 [15]. However, this definition has become obsolete with the advent of technologies such as social networks and e-democracy, where the number of DMs can reach significantly higher scales [16, 17]. This shift highlights the need for new methodologies and frameworks to address the challenges posed by modern LSGDM environments. The LSGDM framework has been widely implemented across various fields, including business management [18], healthcare [19], engineering [20], green supplier selection [21] and so on . By incorporating a broader range of perspectives and expertise, LSGDM methods effectively mitigate individual biases, while improving decision accuracy, adaptability, and overall robustness. Therefore, LSGDM has become an indispensable approach for addressing the increasingly complex challenges of modern industries and the management practices face.

As illustrated in Figure 1.2, the linguistic-based LSGDM framework comprises, but is not limited to, the following four phases:

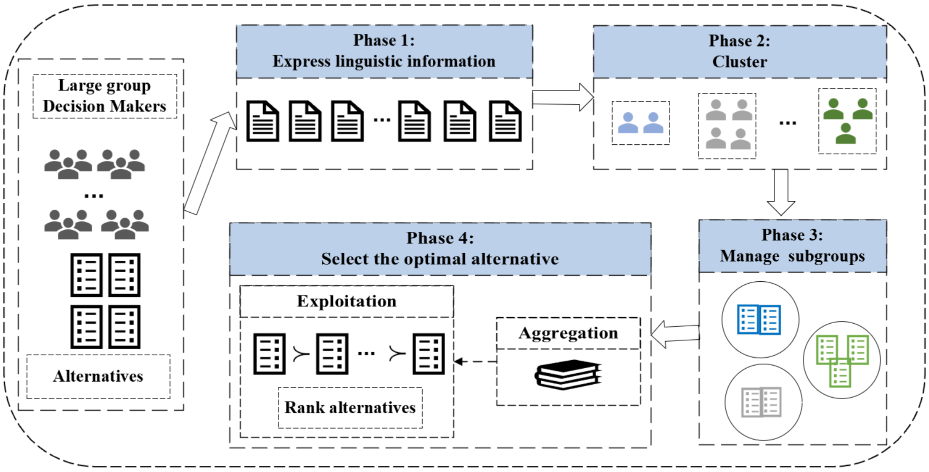


Figure 1.2: The general linguistic-based LSGDM framework

Phase 1: One or more linguistic term sets, designed with suitable granularity, syntax, and semantics, are selected to allow DMs to precisely express their linguistic preferences.

Phase 2: Suitable clustering algorithms or methods are developed and used to partition the large group into smaller and more manageable subgroups.

Phase 3: Effective subgroup management strategies are established to coordinate and refine the preferences within each subgroup, ensuring balanced representation and facilitating the consolidation of subgroup opinions.

Phase 4: Advanced methodologies or models are developed to determine the optimal alternative. This phase typically comprises two key processes:

(a) **Aggregation process:** The subgroup opinions are fused using an aggregation operator or method to produce a collective opinion.

(b) **Exploitation process:** Decision-making techniques are applied to analyze and rank the alternatives based on the collective opinions, ultimately identifying the optimal alternative according to the ranking results.

In GDM, DMs often encounter disagreements due to differences in perspectives, expertise, and preferences. This is a natural outcome when individuals with diverse backgrounds are involved in the evaluation of the alternatives. However, in

LSGDM, in which the number of DMs is considerably larger, these disagreements become more pronounced and complex to manage. The increased diversity of opinions in LSGDM not only amplifies the level of conflict but also complicates the process of reaching an agreement [16]. The consensus reaching process (CRP) plays a critical role in overcoming these challenges. It provides a structured approach for DMs to engage in communication and negotiation, allowing them to adjust their preferences in pursuit of a collective agreement [22]. Through iterative discussions and adjustments, CRPs help to reduce conflicts, bridge gaps among polarized and conflicting opinions, and align preferences towards an agreed and shared decision. Achieving consensus ensures that the final decision is not only acceptable to all participants but also, reflects the collective judgment of the group. Figure 1.3 illustrates the LSGDM framework with CRP, which includes four phases. Phases 1, 2, and 4 are consistent with those phases introduced in Figure 1.2 and are not reiterated here. Notably, Phase 3 in Figure 1.3 focuses on developing consensus models or mechanisms to address disagreements among DMs or subgroups, thereby facilitating the achievement of group consensus.

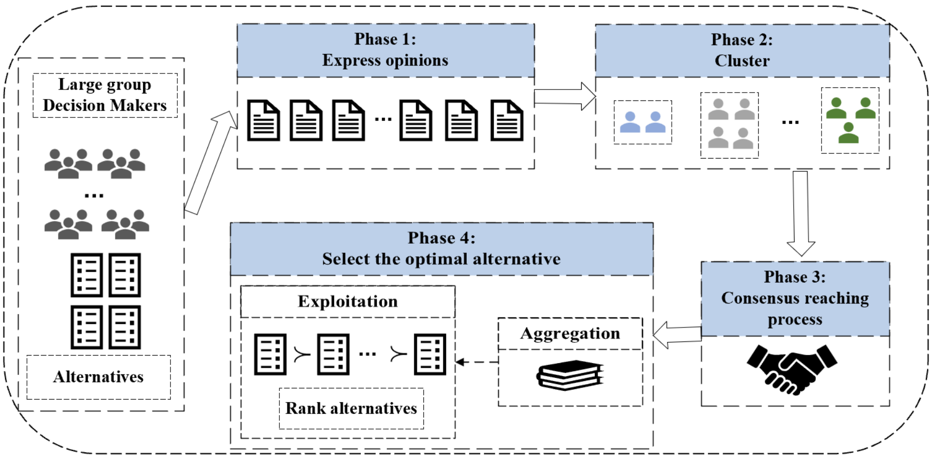


Figure 1.3: The general LSGDM framework with CRP

Even though, numerous models and approaches have been proposed by various researchers to address GDM and LSGDM challenges, and their contributions have significantly advanced the field of decision-making. However, the current research remains incomplete for tackling complex, real-world decision-making problems. Emerging difficulties and challenges highlight the need for deeper exploration and refinement of existing methods. Therefore, there are several challenges ahead in the GDM and LSGDM process that are the main motivations for this research:

(1) Current LGDM problems [23, 24, 25, 26] have been addressed using non-parametric models such as Data Envelopment Analysis (DEA) [27], yet their effectiveness

in handling HLGDM remains overlooked. Existing DEA-based LGDM methods [28, 29] are often constrained by single-granularity linguistic term sets, limiting both the accuracy and flexibility of evaluations. Moreover, these methods typically assume fully rational DMs, neglecting psychological factors that influence real-world decision-making. These limitations highlight the need for extended DEA models that can better account for the complexities of HLGDM, ensuring more realistic and adaptable decision-making processes.

(2) Existing DEA-based LGDM studies often converted linguistic information into precise numerical values [30, 31], which compromises linguistic nuances and the inherent fuzziness of LGDM environments. Additionally, DEA models rely on diverse assessments from multiple DMs to evaluate alternatives, requiring an effective aggregation mechanism to produce final efficiency of alternatives. However, existing DEA-based evaluation models in GDM [32, 33] primarily aggregate raw data without ensuring consensus at the level of evaluation efficiency, undermining the representativeness and reliability of final decisions. These limitations underscore the need for improving GDM framework that retains linguistic richness and integrates consensus to enhance decision reliability and acceptance.

(3) The existing CRP models for LSGDM face some notable limitations. A key challenge is the lack of clear and actionable guidance for DMs during the consensus process [34, 35], reducing both interpretability and the effectiveness of CRPs. Additionally, studies on LSGDM with hesitant fuzzy linguistic information often overlook the impact of non-cooperative behaviors [19, 36, 37], where some DMs resist aligning with the group consensus, such behaviors obstruct decision-making and reduce CRP efficiency. Therefore, these limitations highlight the need for improved interpretability and strategies to manage non-cooperative behavior, ensuring a more reliable and effective CRP in LSGDM.

(4) Most existing clustering approaches in LSGDM [38, 39, 40, 41] assume DMs belong to a single subgroup, overlooking the reality that DMs often participate in multiple subgroups simultaneously. This oversight weakens the CRP, as the influence of overlapping structures on consensus formation remains unaddressed. Furthermore, while feedback mechanisms are proposed to align DMs with group consensus, they often fail to account for individual acceptance of the recommendations [34, 42, 43], leading to resistance and hindering CRPs. These gaps underscore the need for a more nuanced approach that integrates overlapping structures of social network and enhances feedback acceptance, ultimately improving the practicality and reliability of CRP in LSGDM.

In summary, in the face of increasingly complex, ambiguous, and uncertain decision-making environments in modern economic and management activities, in-depth research into GDM methods based on hesitant fuzzy linguistic information is an important topic worth further exploration in the field of modern decision science.

1.2 Objectives

Considering the motivations discussed in the previous section, the aim of this Ph.D. thesis is to develop novel GDM methods and consensus models based on hesitant fuzzy linguistic information. To achieve this aim, the following objectives are set:

(1) To develop a novel GDM method based on extended DEA models for handling multi-granular hesitant fuzzy linguistic information. This method combines the strengths of DEA cross-efficiency models and regret theory, aiming to enhance the precision of linguistic evaluations while incorporating the regret-avoidance tendencies of DMs in the GDM process.

(2) To develop a novel GDM method using fuzzy DEA cross-efficiency models to handle hesitant fuzzy linguistic information. The approach preserves the fuzziness of the original data by converting it into fuzzy envelopes, minimizing information loss in DEA calculations. An optimization model will be disigned to determine the optimal weights of DMs, ensuring accurate aggregation and rationality in the GDM process.

(3) To propose a new CRP that addresses non-cooperative behavior in LSGDM with hesitant fuzzy linguistic information. This objective aims to build the consensus optimization models using Extended Comparative Linguistic Expression with Symbolic Translation (ELICIT), enhancing the transparency and explainability of the CRP. Additionally, a robust non-cooperative behavior management mechanism will be designed to effectively promote the decision-making process.

(4) To propose a new CRP that accounts for the overlapping structure of social trust networks in LSGDM. The approach will identify overlapping DMs and develop consensus optimization models that integrate bounded confidence and ELICIT information, enhancing the interpretability of hesitant fuzzy linguistic information and increasing the acceptance of adjustment suggestions.

1.3 Structure

This doctoral research aims to achieve the objectives outlined in the previous section. To do so, it presents a collection of articles authored by the doctoral candidate. This way of presenting the doctoral thesis complies with Article 23, Point 3, of the current Doctoral Studies regulations at the University of Jaén, as outlined in RD 99/2011. This memory includes four contributions, all published in internationally recognized journals listed in the Journal Citation Reports (JCR) database.

A brief summary of the structure of this research memory is presented below:

- **Chapter 2:** This chapter outlines the theoretical foundations of the research, including the fuzzy linguistic approach, computing with words (CWW), and linguistic computational models such as the 2-tuple linguistic model, HFLTSs, and ELICIT. It also discusses the framework of LSGDM and CRP. Additionally, classical decision theories and methods, including DEA, regret theory, bounded confidence theory, and social network analysis, are introduced.

- **Chapter 3:** This chapter provides a concise and comprehensive overview of the published articles that form the core of this research memory. For each contribution, a brief discussion of the key results obtained are presented.

- **Chapter 4:** This chapter includes the four previously mentioned papers, along with detailed information about the journals in which they were published.

- **Chapter 5:** This chapter outlines the final conclusions of the research and discusses potential future directions for further research development presented in this study.

Chapter 2

Basic Concepts and Methods

This chapter presents the conceptual framework and tools essential for understanding the research presented in this document. While the papers within this collection provide a detailed introduction and review of the research background, this chapter offers a concise overview of the key theoretical concepts and methodologies related to our work. These include the fuzzy linguistic approach, covering linguistic term sets, the 2-tuple linguistic model, hesitant fuzzy linguistic term sets, and extended comparative linguistic expressions with symbolic translation. Additionally, it explores linguistic decision-making frameworks, including both linguistic group decision-making and large-scale linguistic group decision-making. Key methods and theories relevant to this research are also discussed, including DEA methods, regret theory, bounded confidence theory, and social network analysis.

2.1 Fuzzy linguistic approach and computing with words

This section provides an overview of fundamental concepts related to decision-making under uncertainty, including fuzzy logic and the fuzzy linguistic approach [44, 45, 46]. These methods utilize linguistic variables to effectively represent and handle uncertainty, demonstrating significant success in various applications.

2.1.1 Fuzzy logic and fuzzy linguistic approach

Fuzzy logic theory [47], introduced by Zadeh, was developed to address uncertainty and imprecision in various systems. In this approach, Zadeh expanded the concept of crisp sets to include fuzzy sets, enabling the representation of elements with degrees of membership rather than strict boundaries. A crisp set [48] is a collection of elements that either fully belong or do not belong to the set, with no intermediate states. Membership is strictly defined, and the binary membership function provides clear boundaries for the set.

Definition 1 *[48] Any crisp set A in a universe of discourse X can be defined as:*

$$A = \{(x, \mu_A(x))|x \in X\}$$

where x is an element in the universe X, and $\mu_A(x)$ is the membership function, which takes values of either 0 or 1 for a crisp set. Specifically, for any element $x \in X$:

- *If $x \in A$, then $\mu_A(x) = 1$;*
- *If $x \notin A$, then $\mu_A(x) = 0$.*

A fuzzy set is an extension of a crisp set where each element has a degree of membership that reflects the extent to which it belongs to the set. Rather than having a binary membership function, a fuzzy set uses a membership function that assigns to each element a value in the interval [0, 1].

Definition 2 *[47] A fuzzy set $\tilde{A}$ in a universe of discourse X is defined as:*

$$\tilde{A} = \{(x, \tilde{\mu}_A(x))|x \in X\}$$

where:

- *x is an element in the universe X,*
- *$\tilde{\mu}_A(x)$ is the membership function of the fuzzy set, mapping each element x to a value in the interval [0, 1], indicating the degree of membership of x in $\tilde{A}$.*

In fuzzy set theory, various types of membership functions are employed to quantify the degree of membership of elements within a fuzzy set. Notable examples include L-R type membership functions [49] and quasi-trapezoidal fuzzy numbers [50]. Among these, the trapezoidal fuzzy number (TrFN) [47] is particularly popular due to its simplicity and flexibility in modeling uncertainty. The definition of a TrFN is as follows:

Definition 3 *[47, 51] For a fuzzy set , if its membership function is as Eq.(2.1), then it is called a trapezoidal fuzzy number. If parameters b and c are equal , then it is called a triangular fuzzy number (TFN):*

$$\mu_{\tilde{A}}(x) = \begin{cases} (x-a)/(b-a), & \text{if } a \leq x \leq b \\ 1, & \text{if } b \leq x \leq c \\ (d-x)/(d-c), & \text{if } c \leq x \leq d \\ 0, & \text{otherwise} \end{cases} \tag{2.1}$$

The fuzzy linguistic approach [44, 45, 46] is a method used in fuzzy logic to handle uncertainty and imprecision by representing information with "linguistic variables" rather than precise numbers. This approach is particularly useful in decision-making and modeling human cognition. A linguistic variable is an abstraction used to represent qualitative or vague information in a structured form. By assigning words or phrases to different levels within the universe of discourse, linguistic variables allow fuzzy logic systems to model real-world situations in which uncertainty or imprecision is present.

Definition 4 *[44, 45, 46] A linguistic variable can be described as a quintuple* $(H, T(H), U, G, M)$*, where:*

- *H: Refers to the name of the variable.*
- *T(H): Represents the set of linguistic values associated with the variable.*
- *U: Denotes the universe of discourse, defining the range of possible values for the variable.*
- *G: Specifies the syntactic rule used to generate the linguistic terms in T(H).*
- *M: Represents the semantic rule that links each linguistic term in T(H) to a fuzzy subset of U, thereby assigning meaning to the terms.*

Each linguistic variable is characterized by its name, a set of possible values (known as linguistic terms), a defined domain, and syntactic and semantic rules that govern the generation of each term. This framework enables a more flexible representation of complex, uncertain information.

Definition 5 *[52] Let* $S = \{s_0, s_1, \ldots, s_g\}$ *represent a linguistic term set, then* $g+1$ *is called the granularity of* S*, where* g *is the number of linguistic terms in* S*,* s_i *represents the* i^{th} *linguistic term in* S*. For any two terms* s_i *and* s_j *, the following conditions are satisfied:*

(1) Ordered property: If $i > j$*, then* $s_i > s_j$*;*

(2) Negation operator: If $i = g - j$*, then* $Neg(s_i) = s_j$*;*

(3) Maximization operator: If $s_i > s_j$*, then* $max(s_i, s_j) = s_i$*;*

(4) Minimization operator: If $s_i > s_j$*, then* $min(s_i, s_j) = s_j$*.*

The characteristics of a linguistic variable are a linguistic label and a semantic value, as shown in Figure 2.1. A linguistic label is a word belonging to a set of linguistic terms, while the semantics is represented by a fuzzy set in the domain.

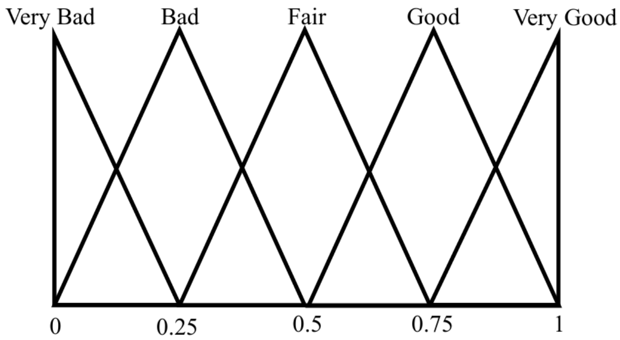


Figure 2.1: A 5-granularity linguistic term set and its semantics

2.1.2 Computing with words (CWW)

CWW is a paradigm based on the fuzzy logic, aimed at handling imprecision and uncertainty in information expressed through natural language. Instead of relying on precise numerical inputs and outputs, CWW operates with linguistic terms or words, enabling the modeling of human reasoning and decision-making processes more closely. The general CWW scheme [53, 54] is illustrated in Figure 2.2, consists of three primary steps, which can be outlined as follows:

- **Translation**: Transforming natural linguistic terms or phrases into their corresponding fuzzy sets or other formal representations.
- **Manipulation**: Performing computations and operations on the encoded fuzzy sets using fuzzy logic and associated mathematical techniques.
- **Retranslation**: Translating the results back into linguistic terms that are meaningful and interpretable for humans.

Figure 2.2: The CWW scheme

CWW is a powerful framework for bridging the gap between human reasoning and computational systems. It provides a flexible and intuitive approach for tackling decision-making problems under uncertain environments. The main advantages of CWW can be summarized as follows:

- **Human-like reasoning**: Mimics the way humans process qualitative information and make decisions.

- **Improved interpretability**: Results are presented in natural language terms, making them easier to understand for non-technical users.
- **Flexibility**: Handles uncertainty and vagueness effectively, making it suitable for complex, real-world problems.

2.2 Linguistic computational models

In complex real-world decision-making scenarios, the inherent ambiguity and uncertainty often render precise numerical data inadequate for capturing the full scope of decision alternatives. Consequently, DMs frequently rely on qualitative information to express their judgments and preferences, as linguistic information more accurately represents their cognitive processes, especially in situations where subjective judgments are involved [55, 56]. To address these needs, a range of linguistic representation models and its operational frameworks have been developed and refined over time [57, 58], allowing for nuanced handling of linguistic information. These models facilitate more robust decision-making processes by enabling DMs to articulate preferences that aligns more closely with human reasoning and perception.

2.2.1 2-tuple linguistic model

Herrera and Martínez [59] developed the 2-tuple linguistic model to improve the interpretability and precision of linguistic representations. This model employs the concept of symbolic translation, using 2-tuples to convey nuanced linguistic information. Adhering to the framework of CWW [60], it operates within a principle where both inputs and outputs are linguistic terms, minimizing potential inaccuracies. Unlike traditional models that rely on a discrete linguistic domain [61, 62], this approach treats the linguistic domain as continuous, enhancing interpretative depth. The model's interpretability benefits from combining linguistic terms with symbolic translation, allowing it to perform computational operations directly on linguistic labels and thereby preserving the original information with minimal loss.

Definition 6 *[59] Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set, and let $\bar{S} = S \times [-0.5, 0.5)$ denote the set of 2-tuple linguistic values associated with each linguistic term. Here, s_i represents any term in S, and $\beta \in [0, g]$ denotes the outcome of the symbolic computation on the linguistic terms, which is then mapped to an equivalent 2-tuple semantic value (s_i, α) via a function Δ_S:*

$$\Delta_S(\beta) = (s_i, \chi), where \begin{cases} i = round(\beta) \\ \alpha = \beta - i \end{cases}$$

where the function round$(\cdot)$ returns the nearest integer value, and $\alpha \in [-0.5, 0.5)$ represents the symbolic translation, as shown in Figure 2.3.

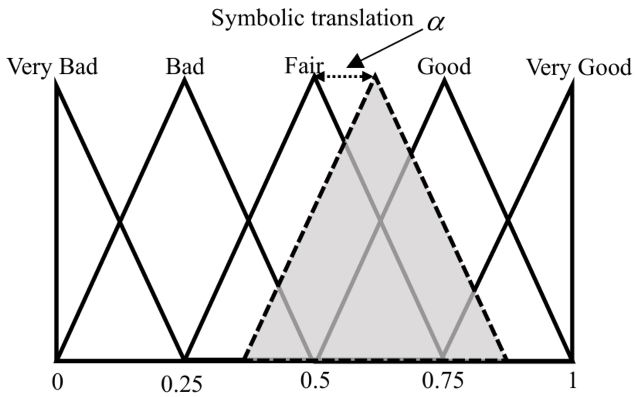


Figure 2.3: The symbolic translation

Definition 7 *[59] Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set, and let (s_i, χ) represent the 2-tuple linguistic value associated with S. The inverse function Δ^{-1}, which converts (s_i, α) into an equivalent real number $\beta \in [0, g]$, is defined as:*

$$\Delta^{-1} : \bar{S} \to [0, g]$$
$$\Delta_S^{-1}(s_i, \alpha) = \alpha + i = \beta$$

Remark 1 *Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set. Any linguistic term s_i in S can be represented as a 2-tuple in the form $(s_i, 0)$, where $s_i \in S$.*

2.2.2 Hesitant fuzzy linguistic term sets (HFLTSs)

In complex decision-making scenarios, DMs may experience hesitation in selecting single linguistic terms to convey their opinions. To address this, Rodríguez et al. [11] introduced the concept of HFLTSs, which are designed to quantify and model this hesitation. HFLTSs are particularly effective for capturing the DM's uncertainty when choosing among several linguistic terms, allowing the inclusion of multiple potential values to reflect this ambiguity in the decision-making process. This model is especially useful in GDM contexts, where multiple DMs may hold differing perspectives or an individual may have reservations regarding a particular assessment. By providing a richer and more flexible structure, HFLTSs enable a more nuanced expression of uncertainty and ambiguity [63, 64].

Definition 8 *[11] Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set, a HFLTS over S is defined as an ordered subset containing a finite sequence of consecutive linguistic terms, represented as $H_S = \{s_i, s_{i+1}, \ldots, s_j\}$*

Thus, for all $0 \leq i \leq j \leq g$, any possible HFLTS can be represented as:

$$H_S = \begin{cases} \{s_i\} & if \quad i = j; \\ \{s_0, s_1, \ldots, s_j\} & if \quad i = 0; \\ \{s_i, s_{i+1}, \ldots, s_g\} & if \quad j = g; \\ \{s_i, s_{i+1}, \ldots, s_j\} & if \quad 1 \leq i \leq j \leq g-1. \end{cases} \tag{2.2}$$

DMs may employ HFLTSs to convey multiple linguistic values directly. However, such expressions often diverge from the natural language typically used in human communication. To bridge this gap, Comparative Linguistic Expressions (CLEs) [12] based on HFLTSs have been developed. CLEs are constructed by using a context-free grammar, it provides a structured framework that not only models the hesitation of DMs but also enables the generation of linguistic expressions more aligned with common human language, thereby enhancing clarity in decision-making contexts.

Definition 9 *[12] Let G_H be a context-free grammar which serves to generate CLEs based on HFLTSs, and $S = \{s_0, s_1 \ldots s_g\}$ a linguistic term set. The elements of $G_H = (V_N, V_T, I, P)$ are defined as follows:*

$$\begin{aligned} V_N = \{&(primary\ term),\ (composite\ term),\ (unary\ relation), (binary\ relation), \\ &(conjunction)\} \\ V_T = \{&at\ least,\ at\ most,\ between,\ and,\ s_0, \ldots, s_g\} \\ I \in\ & V_N \end{aligned}$$

$$\begin{aligned} P = \{I\ &::=\ (primary\ term)|(composite\ term) \\ &(composite\ term)\ ::=\ (unary\ relation)(primary\ term)| \\ &\qquad (binary\ relation)(primary\ term)(conjunction)(primary\ term) \\ &(primary\ term)\ ::=\ s_0|s_1|\ldots|s_g \\ &(unary\ relation)\ ::=\ at\ least|at\ most \\ &(binary\ relation)\ ::=\ between \\ &(conjunction)\ ::=\ and\} \end{aligned}$$

Definition 10 *[12] Let ll_S be a CLE, generated by a context-free grammar G_H over a given linguistic term set S. A transformation function E_{G_H} is defined as a mapping*

$$E_{G_H} : ll_S \mapsto H_S$$

CLEs can be converted into HFLTSs through the following transformations:

$$E_{G_H}(ll_S) = \begin{cases} \{s_i\} & \text{if } ll_S = s_i; \\ \{s_0, s_1, \ldots, s_j\} & \text{if } ll_S = \text{at most } s_j; \\ \{s_i, s_{i+1}, \ldots, s_g\} & \text{if } ll_S = \text{at least } s_i; \\ \{s_i, s_{i+1}, \ldots, s_j\} & \text{if } ll_S = \text{between } s_i \text{ and } s_j. \end{cases} \tag{2.3}$$

Various operators and models [12, 65] have been developed to process HFLTSs or CLEs, enabling the extraction of precise values through specific functions. However, these models often produce values that are neither fuzzy numbers nor linguistic expressions, potentially resulting in information loss. To address this issue, Liu and Rodríguez [66] proposed a novel procedure of obtaining the fuzzy envelope for HFLTS/CLE, represented as a TrFN.

Definition 11 *[66] Let H_S represent the HFLTS, $T(a,b,c,d)$ represent the trapezoidal fuzzy membership function, then the fuzzy envelope is defined as the trapezoidal fuzzy membership function as follows:*

$$env_F(H_S) = T(a,b,c,d)$$

The general procedure for constructing the fuzzy envelope of HFLTS is illustrated in Figure 2.4 and can be summarized as follows [66]:

Step 1: Gather the elements required for aggregation;

Step 2: Calculate the four parameters (a,b,c,d) for $T(a,b,c,d)$;

Step 3: Use the OWA operator [67] to determine the parameters b and c .

Step 4: Obtain the the fuzzy envelope $T(a,b,c,d)$ for H_S.

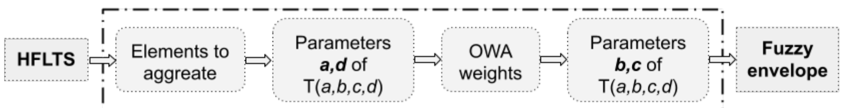


Figure 2.4: The general procedure for constructing the fuzzy envelope of HFLTS

2.2.3 Extended comparative linguistic expressions with symbolic translation (ELICIT)

Although, the linguistic representation models and their extensions [68, 69] effectively capture the uncertainty and hesitancy in linguistic information, they still present limitations, particularly regarding result interpretability and information loss in the CWW process, which constrains their applicability in linguistic decision-making contexts. To address these issues, Labella et al. [70] developed the ELICIT framework, which combines the strengths of the 2-tuple linguistic model and HFLTSs. ELICIT extends CLEs generated by context-free grammars into the continuous domain by using the concept of symbolic translation defined for linguistic 2-tuples, enabling a more comprehensive representation of linguistic information. This framework leverages the interpretability of CLEs and replaces linguistic terms with linguistic 2-tuple representations where needed, allowing accurate CWW processes. This approach thus enhances both accuracy and interpretability, providing clearer and more reliable results.

Definition 12 *[70] Let G_H be a context-free grammar which serves to generate CLEs based on HFLTSs, and $S = \{s_0, \ldots, s_g\}$ be a linguistic term set, and the elements of $G_H = (V_N, V_T, I, P)$ given by:*

$$\begin{aligned} V_N = \{&(\textit{continuous primary term}),\ (\textit{composite term}),\ (\textit{unary relation}),\\ &(\textit{binary relation}), (\textit{conjunction})\}\\ V_T = \{&\textit{at least},\ \textit{at most},\ \textit{between},\ \textit{and},\ (s_0,\alpha)^\gamma, (s_1,\alpha)^\gamma, \ldots, (s_g,\alpha)^\gamma\}\\ I \in{} & V_N \end{aligned}$$

$$\begin{aligned} P =\{&I\ ::=\ (\textit{continuous primary term})|(\textit{composite term})\\ &(\textit{composite term})\ ::=\ (\textit{unary relation})(\textit{continuous primary term})|\\ &(\textit{binary relation})(\textit{continuous primary term})(\textit{conjunction})(\textit{continuous primary term})\\ &(\textit{continuous primary term})\ ::=\ (s_0,\alpha)^\gamma|(s_1,\alpha)^\gamma|\ldots|(s_g,\alpha)^\gamma\\ &(\textit{unary relation})\ ::=\ \textit{at least}|\textit{at most}\\ &(\textit{binary relation})\ ::=\ \textit{between}\\ &(\textit{conjunction})\ ::=\ \textit{and}\} \end{aligned}$$

where $\gamma \in \left[-\frac{1}{2g}, \frac{1}{2g}\right)$ serves as an extra parameter to preserve information and enhance the accuracy of ELICIT outcomes..

The original ELICIT scheme was introduced using a context-free grammar, which made its computational process relatively complex. Subsequently, its mathematical formulation was enhanced with a simple yet rigorous notation [71], facilitating the construction and manipulation of ELICIT expressions.

Definition 13 *[71] The ELICIT information is represented by the expression $[\bar{s}_i, \bar{s_j}]_{\gamma_1,\gamma_2}$, where $\bar{s}_i, \bar{s}_j \in \bar{S}, i < j$ are two 2-tuple values. The parameters γ_1, γ_2 ensure no loss* *occurs during calculations. Furthermore, can be equivalently expressed as an ELICIT value, as demonstrated in Figure 2.5.*

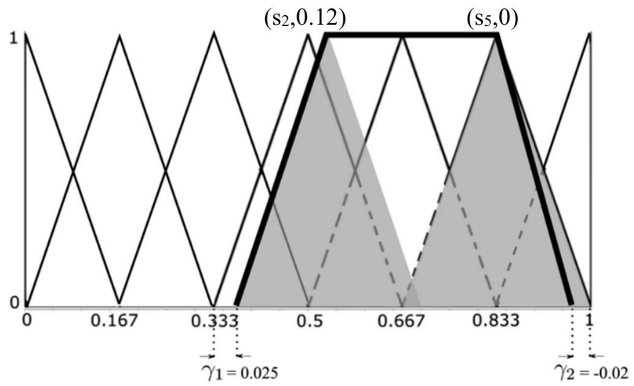


Figure 2.5: An example of ELICIT expression: Between $(s_3, 0.12)^{0.025}$ and $(s_5, 0)^{-0.02}$

The ELICIT scheme, illustrated in Figure 2.6, can be described through the following processes:

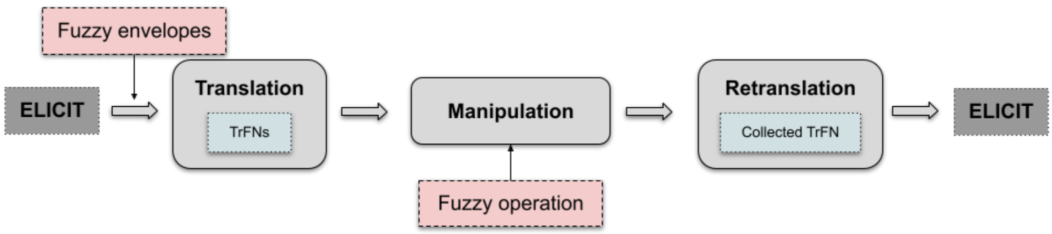


Figure 2.6: The ELICIT scheme

(1) **Translation process:** This step involves converting ELICIT expressions into their equivalent TrFNs using the fuzzy envelope function ζ^{-1} definition, as outlined below:

Definition 14 *[71] The fuzzy envelope of an ELICIT expression, also known as TrFNs, is defined as a mapping*

$$\zeta^{-1} : \overline{\overline{S}} \longrightarrow \overline{T}$$
$$[(s_l, \alpha_l), (s_r, \alpha_r)]_{\gamma_l, \gamma_r} \mapsto T(a, b, c, d)$$

and is computed as

$$
\begin{aligned}
a &= \gamma_l + \max\{\frac{\Delta_S^{-1}(s_l,\alpha_l) - \frac{1}{g}}{g}, 0\}, \quad b = \frac{\Delta_S^{-1}(s_l,\alpha_l)}{g}, \\
d &= \gamma_r + \min\{\frac{\Delta_S^{-1}(s_r,\alpha_r) + \frac{1}{g}}{g}, 1\}, \quad c = \frac{\Delta_S^{-1}(s_r,\alpha_r)}{g}.
\end{aligned}
\tag{2.4}
$$

(2) **Manipulation process:** This involves performing fuzzy aggregation by applying an aggregation operator to fuse the TrFNs, resulting in a unified collective TrFN.

(3) **Retranslation process:** This step uses the function ζ, the inverse of ζ^{-1}, to convert the collective TrFN back into its corresponding ELICIT value.

Definition 15 *[71] The function ζ is defined as a mapping, i.e.,*

$$
\begin{aligned}
\zeta : \overline{T} &\longrightarrow \overline{\overline{S}} \\
T(a,b,c,d) &\mapsto [(s_l,\alpha_l),(s_r,\alpha_r)]_{\gamma_l,\gamma_r}
\end{aligned}
$$

and processed by

$$
\begin{aligned}
(s_l,\alpha_l) &= \Delta_S(gb) \quad \textit{with} \quad \gamma_l = a - \max\{b - \frac{1}{g^2}, 0\} \\
(s_r,\alpha_r) &= \Delta_S(gc) \quad \textit{with} \quad \gamma_r = d - \min\{c + \frac{1}{g^2}, 1\}
\end{aligned}
\tag{2.5}
$$

In applying the ELICIT computational model within decision-making contexts, it becomes essential to compare various ELICIT expressions to derive conclusive decision outcomes. Given that each ELICIT expression corresponds to an equivalent TrFN, comparisons can be effectively facilitated by examining these associated TrFNs, often through an established TrFN ranking methodology. Labella et al. [70] recommended evaluating ELICIT expressions based on the concept of *magnitude* [72] which is a ranking index for TrFNs.

Definition 16 *[70] Let $[\bar{s_i},\bar{s_j}]_{\gamma_1,\gamma_2}$ represent an ELICIT expression with an equivalent fuzzy representation in terms of TrFNs $T(a,b,c,d)$. The magnitude of this ELICIT expression can then be calculated as follows:*

$$
Mag([\bar{s_i},\bar{s_j}]_{\gamma_1,\gamma_2}) = Mag(T(a,b,c,d)) = \frac{a+5b+5c+d}{12} \tag{2.6}
$$

Additionally, Labella et al.[70] proposed using the geometric distance between the corresponding TrFNs to measure the distance between two ELICIT values.

Definition 17 *[70] Let t* $T_1 \equiv (a_1, b_1, c_1, d_1)$ *and* $T_2 \equiv (a_2, b_2, c_2, d_2)$ *represent two TrFNs associated with ELICIT expressions. The distance between these TrFNs can be defined as follows:*

$$d(T_1, T_2) = \frac{1}{4}(|a_1 - a_2| + |b_1 - b_2| + |c_1 - c_2| + |d_1 - d_2|) \tag{2.7}$$

Remark 2 *[71] Any linguistic term* $s_i \in S$ *can be transformed into an ELICIT expression* $(s_i, 0)_0 \equiv [(s_i, 0), (s_i, 0)]_{00}$. *Similarly, a HFLTS* $\{s_i, s_{i+1}, \ldots, s_j\}$ *can also be represented as an ELICIT expression* $[(s_i, 0), (s_j, 0)]_{00}$.

2.3 Large-scale group decision making

GDM traditionally involves multiple participants, generally two or more individuals, working collaboratively to select the optimal choice from a set of alternatives [73, 74]. However, as decision-making environments have become more sophisticated with technological advancements, the need for coordinated input from a larger number of participants has grown, especially when dealing with complex, multifaceted problems. Traditionally, LSGDM has emerged as a specialized framework, applied when the number of DMs reaches 20 [15]. With advancements in technology, however, the scale of DMs has expanded significantly, enabling its application to much larger groups [16, 17]. The basis of LSGDM lies in its structure:

- A set of alternatives, $A = \{A_1, A_2, \ldots, A_m\}$ where $(m \geq 2)$,representing the possible options available for consideration in the decision problem.
- A large group of DMs, $E = \{E_1, E_2, \ldots, E_K\}$ with $K \geq 20$ traditionally, each provides their preferences regarding the alternatives under each attribute.
- A set of attributes, $C = \{c_1, c_2, \ldots, c_n\}$, used to evaluate and compare the performance of the alternatives.

LSGDM is particularly advantageous for addressing the intricate and diverse challenges that characterize modern decision contexts [75, 76], as it brings together a broad array of perspectives, knowledge bases, and specialized expertise from individuals with different backgrounds and professional fields. This diversity of input not only enhances the comprehensiveness of the decision-making process but also adds a layer of scientific rigor and objectivity, ultimately contributing to more balanced and equitable decision outcomes. By integrating such extensive inputs, LSGDM helps mitigate individual biases and supports decisions that are both robust and widely acceptable, aligning well with the needs of complex decision environments.

In the current framework of LSGDM [17], the following key aspects are frequently studied, though the scope is not limited to these, as illustrated in Figure 2.7.

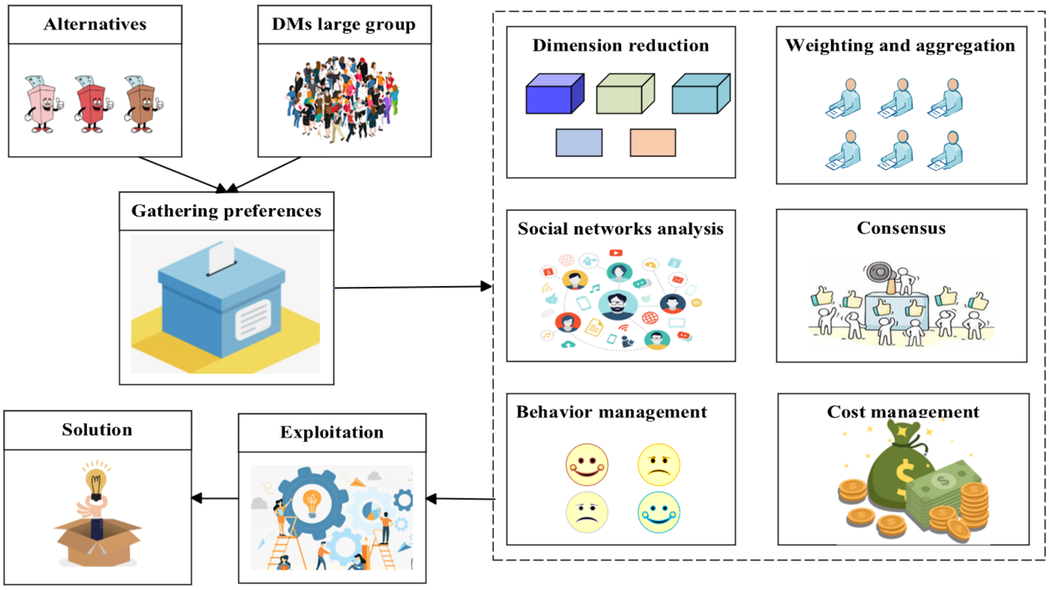


Figure 2.7: The key aspects of LSGDM

- *Dimension reduction*: Simplify the dimensions of large-scale decision-making groups to enhance process efficiency [77].
- *Consensus*: Facilitate a shared agreement or accepted collective decision among group members [22].
- *Social networks analysis*: Incorporate social connections among DMs, examining how these relationships influence decision-making outcomes [78].
- *Behavior management*: Develop mechanisms to identify and address non-cooperative behavior among DMs, minimizing disruptions in the LSGDM process [22].
- *Cost management*: Allocate human, financial, and time resources effectively to support the development of models for complex decision-making scenarios [79].
- *Weighting and aggregation*: Evaluate the influence of each participant in the decision-making process and aggregate their preferences into a unified outcome [80].

2.4 Consensus reaching process

Both GDM and LSGDM problems, DMs often start the resolution process with opinions that can be significantly divergent. A CRP is generally required to guide the group toward a shared agreement on the selected alternatives [16, 81]. Ideally, consensus reflects full and unanimous agreement among all DMs on each viable alternative. However, achieving complete agreement means unanimity that is challenging in practice, leading to the concept of ”soft” consensus [82, 83], which is common in real-world decision-making. Reaching consensus involves iterative rounds of discussion, where DMs adjust their initial opinions to align more closely with those of the broader group.

Consensus can be attained through processes either involving feedback or without it [16]. In CRPs without feedback, adjustments to the initial assessments are made autonomously, without input from DMs. Conversely, in feedback-based CRPs, DMs engage in iterative discussions, modifying their initial opinions to align progressively with the collective group perspective. A moderator often oversees this feedback process, recommending adjustments to assessments that significantly deviate from the emerging consensus, based on established identification and directional guidelines [84, 85]. The general framework of a CRP [16] is depicted in Figure 2.8.

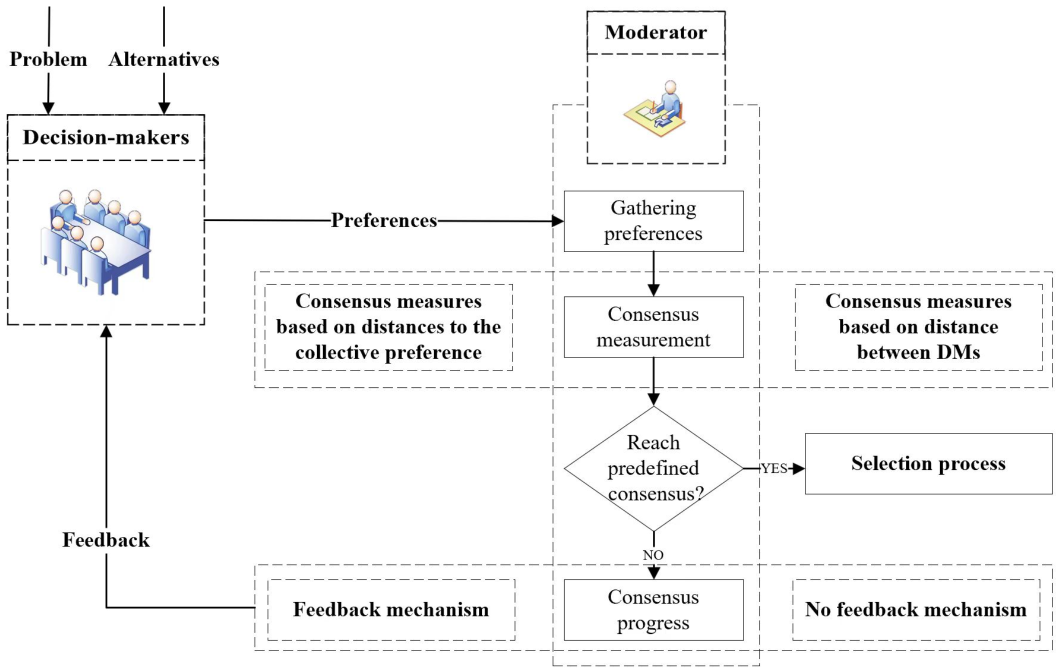


Figure 2.8: The general framework of CRP

While CRPs without feedback mechanisms are faster and simpler, they often fail to achieve a high level of consensus or effectively resolve conflicts, but can guide and show ways to achieve an agreement to the DMs involved in the GDM. In contrast, CRPs with feedback mechanisms are dynamic and interactive, resulting in higher-quality decisions by fostering collaboration and mutual understanding among

DMs. Existing studies typically employ two main approaches [81]: identification and direction rules and optimization-based consensus models.

- **Identification and Direction Rules**: Identifies specific DMs, alternatives, or preferences that require adjustment. After identification, it provides recommendations or directions for modifications [22].
- **Optimization-Based Consensus Model**: Adopts a mathematical perspective, seeking to minimum adjustments required or the associated costs while achieving the desired consensus level [81].

Simulation experiments [81] have shown that optimization-based consensus models outperform identification and direction rules in terms of efficiency, demonstrating their ability to achieve consensus with minimal effort and adjustments. The specialized literature frequently employs two prominent optimization-based consensus models to handle linguistic information, as outlined below [86, 87]:

(1) **Minimum Adjustment Consensus Model (MACM)**

The MCAM aims to minimize the disparity between DMs' initial opinions and their adjusted ones, or alternatively, to reduce the number of adjustments needed during the CRP to attain the desired level of agreement.

Consider a set of DMs, denoted as $E = \{E_1, E_2, \ldots, E_K\}$. Let $O = \{o_1, o_2, \ldots, o_K\}$ and $\bar{O} = \{\bar{o}_1, \bar{o}_2, \ldots, \bar{o}_K\}$ represent the initial and adjusted preferences of the DMs, respectively. The MCAM for achieving group consensus based on linguistic assessments can be formulated as follows [87]:

$$\begin{array}{l} \min \sum_{k=1}^{K} d(o_k, \bar{o}_k) \\ s.t. \begin{cases} \bar{o}^c = F_A(\bar{o}_1, \bar{o}_2, \ldots, \bar{o}_K), \\ CD(\bar{o}_k, \bar{o}^c) \leq \alpha, \forall k = 1, \ldots, K. \end{cases} \end{array} \tag{2.8}$$

where F_A represents different aggregation functions, CD is the consensus measures, and α is the predefined consensus threshold.

(2) **Minimum Cost Consensus Model (MCCM)**

Unlike the MACM model, the MCCM incorporates the cost of persuading each DM to adjust their opinion, factoring in a unit adjustment cost. This concept defines the adjustment cost as the product of the unit adjustment cost and the adjustment distance required [88].

$$\begin{array}{l} \min \sum_{k=1}^{K} c_k \cdot d(o_k, \bar{o^k}) \\ s.t. \left\{ CD(\bar{o}_k, \bar{o}^c) \leq \alpha, \forall k = 1, \ldots, K. \right. \end{array} \tag{2.9}$$

where c_k is the unit adjustment cost.

In the previous model, the solution yields the optimal adjusted opinions, which can then be used to derive a collective opinion. However, Model (2.9) lacks a specified approach for obtaining the collective opinion from these adjusted opinions. To address this limitation, an extended version of Model (2.9) was developed [89], incorporating an operator to aggregate the opinions of DMs as follows:

$$\begin{aligned} &\min \sum_{k=1}^{K} c_k \cdot d(o_k, \bar{o^k}) \\ &s.t. \begin{cases} \bar{o}^c = F_A(\bar{o}_1, \bar{o}_2, \ldots, \bar{o}_K), \\ CD(\bar{o}_k, \bar{o}^c) \leq \alpha, \forall k = 1, \ldots, K. \end{cases} \end{aligned} \tag{2.10}$$

where c_k denotes the unit adjustment cost, F_A represents various aggregation functions, CD is the consensus measures, and α is the predefined consensus threshold.

In the context of GDM and LSGDM, certain DMs or subgroups may exhibit non-cooperative behavior, where they are reluctant to adjust their perspectives to facilitate CRP. Such non-cooperative attitudes can disrupt the consensus process and, in some cases, cause a complete standstill in decision-making, thereby compromising the quality of the final outcomes. In extreme scenarios, when DMs rigidly adhere to their own opinions, the CRP may reach a deadlock, preventing the achievement of effective decision results [22, 90, 91]. The non-cooperative behaviors may include [92]:

- **Persistent Resistance**: Refusal to adjust opinions despite clear evidence or group trends.
- **Strategic Manipulation**: Providing misleading or insincere evaluations to influence outcomes.
- **Indifference**: Lack of active participation or engagement in the decision-making process.
- **Conflict Escalation**: Introducing unnecessary conflicts or amplifying disagreements.

The detection and management of non-cooperative behaviors in the CRP are essential for maintaining the effectiveness and efficiency of GDM, particularly in LSGDM contexts. Detection methods typically include behavioral analysis, which involves tracking historical opinion adjustments to identify anomalies or types of non-cooperative behavior [93, 94], and consistency metrics, which use measures such as opinion consistency or deviation to detect discrepancies among DMs [22, 95]. Management strategies for addressing non-cooperative behaviors often involve dynamic weight adjustment [96, 97], where the influence of opinions from DMs exhibiting non-cooperative behavior is reduced, and encouraging cooperation through mechanisms like rewards for active participation and adherence to group objectives [96, 98]. These approaches ensure the decision-making process remains fair, collaborative, and aligned with the group's aims.

2.5 Methods and theories

In this section, we will revise several methods and theories that are relevant to our research, including DEA method, regret theory, non-cooperative behavior, bounded confidence theory and social network analysis.

2.5.1 DEA methods in multi-criteria decision making

Unlike other decision-making methods that rely on predefined information [99, 100], DEA [27], as a non-parametric approach, operates without requiring preset decision models or assumptions about specific data relationships. This characteristic minimizes subjectivity and circumvents the limitations associated with model assumptions, making DEA a robust tool for addressing complex decision-making problems [24, 25]. In decision-making, alternatives are viewed as decision-making units (DMUs), with cost attributes represented as inputs and benefit attributes as outputs. Let us suppose that, there are m alternatives, denoted by $A = \{A_1, A_2, \ldots, A_m\}$, each evaluated under R cost attributes and S benefit attributes. Let $x_{ir}(r = 1, \ldots, R)$ and $y_{is}(s = 1, \ldots, S)$ denote the assessment for A_i. For the specific alternative under evaluation, A_d, its efficiency can be calculated using the CCR model [27]:

$$\begin{aligned} &\max E_{dd} = \sum_{s=1}^{S} u_{ds} y_{ds} \\ &s.t. \begin{cases} \sum_{r=1}^{R} v_{dr} x_{dr} = 1, \\ \sum_{s=1}^{S} u_{ds} y_{is} - \sum_{r=1}^{R} v_{dr} x_{ir} \leq 0, i = 1, \ldots, m, \\ u_{ds} \geq 0, v_{dr} \geq 0, s = 1, \ldots, S, r = 1, \ldots, R. \end{cases} \end{aligned} \tag{2.11}$$

where $u^*_{sd}(s = 1, \ldots, S)$ and $v^*_{rd}(p = 1, \ldots, R)$ represent the optimal weights for outputs and inputs associated with A_d. The efficiency E^*_{dd} is relative efficiency or self-efficiency. However, Model (2.11) encounters the issue of non-unique optimal weights, making it challenging to fully distinguish efficient alternatives. To overcome this limitation, several studies have introduced secondary objective functions, including benevolent, aggressive [101, 102], and neutral cross-efficiency [103] models, which aim to achieve a unique solution while preserving self-efficiency.

In the context of decision-making, the benevolent cross-efficiency model is shown as below. Its objective is to maximize the cross-efficiency values of other alternatives while maintaining the optimal self-efficiency of the alternative under evaluation [102].

$$
\begin{aligned}
&\max E_{dd} = \sum_{s=1}^{S} u_{ds} \sum_{i=1,i\neq d}^{m} y_{is} \\
&s.t. \begin{cases}
\sum_{r=1}^{R} v_{dr} \sum_{i=1,i\neq d}^{m} x_{ir} = 1, \\
\sum_{s=1}^{S} u_{ds} y_{ds} - E_{dd}^{*} \sum_{r=1}^{R} v_{dr} x_{dr} = 0, \\
\sum_{s=1}^{S} u_{ds} y_{is} - \sum_{r=1}^{R} v_{dr} x_{ir} \leq 0, i = 1, \ldots, m; i \neq d, \\
u_{ds} \geq 0, v_{dr} \geq 0, s = 1, \ldots, S, r = 1, \ldots, R.
\end{cases}
\end{aligned}
\tag{2.12}
$$

The aggressive cross-efficiency model is created by altering the objective function of Model (2.12) from maximization to minimization, highlighting a competitive dynamic among the alternatives. In contrast, the neutral cross-efficiency model [103] posits that an alternative should not emphasize a benevolent or aggressive stance toward others. Instead, it should optimize its weight allocation. The corresponding optimization model is outlined as follows:

$$
\begin{aligned}
&\max \delta \\
&s.t. \begin{cases}
\sum_{r=1}^{R} v_{dr} \sum_{i=1,i\neq d}^{m} x_{ir} = 1, \\
\sum_{s=1}^{S} u_{ds} y_{ds} = E_{dd}^{*}, \\
\sum_{s=1}^{S} u_{ds} y_{is} - \sum_{r=1}^{R} v_{dr} x_{ir} \leq 0, i = 1, \ldots, m; i \neq d, \\
u_{ds} y_{ds} - \delta \geq 0, s = q, \ldots, S, \\
\delta \geq 0, u_{ds} \geq 0, v_{dr} \geq 0, s = 1, \ldots, S, r = 1, \ldots, R.
\end{cases}
\end{aligned}
\tag{2.13}
$$

where δ, $u_{ds}(s = 1, \ldots, S)$ and $v_{dr}(r = 1, \ldots, R)$ are variables.

Conventional DEA models require precisely defined input and output values, limiting their applicability in situations involving uncertainty or imprecise data. To overcome this challenge, various fuzzy DEA models [104, 105, 106] have been developed to manage ambiguous or fuzzy inputs and outputs. To save space, here we only review the fuzzy DEA models with inputs and outputs as intervals [105].

The fuzzy DEA method assesses the maximum and minimum self-efficiency for A_d using interval data. In the context of decision-making, let the evaluations for the cost and benefit attributes of A_d be represented as intervals, denoted by $[x_{dr}^L, x_{dr}^U](r = 1, \ldots, R)$ and $[y_{ds}^L, y_{ds}^U](s = 1, \ldots, S)$ respectively. The maximin self-efficiency E_{dd}^U for A_d can then be determined using the following model:

$$\max E_{dd}^U = \sum_{s=1}^{S} u_{ds} y_{ds}^U$$
$$s.t. \begin{cases} \sum\limits_{r=1}^{R} v_{dr} x_{dr}^L = 1, \\ \sum\limits_{s=1}^{S} u_{ds} y_{ds}^U - \sum\limits_{r=1}^{R} v_{dr} x_{dr}^L \geq 0, \\ \sum\limits_{s=1}^{S} u_{ds} y_{is}^L - \sum\limits_{r=1}^{R} v_{dr} x_{ir}^U \geq 0, i = 1, \ldots, m, i \neq d, \\ u_{ds} \geq 0, v_{dr} \geq 0, s = 1, \ldots, S, r = 1, \ldots, R. \end{cases} \tag{2.14}$$

Likewise, the minimum self-efficiency E_{dd}^L of the evaluated A_d can be derived using the following model:

$$\max E_{dd}^L = \sum_{s=1}^{S} u_{ds} y_{ds}^L$$
$$s.t. \begin{cases} \sum\limits_{r=1}^{R} v_{dr} x_{dr}^u = 1, \\ \sum\limits_{s=1}^{S} u_{ds} y_{ds}^L - \sum\limits_{r=1}^{R} v_{dr} x_{dr}^U \geq 0, \\ \sum\limits_{s=1}^{S} u_{ds} y_{is}^U - \sum\limits_{r=1}^{R} v_{dr} x_{ir}^L \geq 0, i = 1, \ldots, m, i \neq d, \\ u_{ds} \geq 0, v_{dr} \geq 0, s = 1, \ldots, S, r = 1, \ldots, R. \end{cases} \tag{2.15}$$

2.5.2 Regret theory

Regret theory [107, 108], a foundational concept in behavioral decision theory, is designed to model the regret-averse tendencies of DMs. In complex decision-making environments, DMs may face challenges in selecting the optimal choice, often resulting in regret through comparative evaluation. Regret theory formalizes this response by means of three distinct stages:

- **Comparison stage:** DMs assess alternatives in pairs based on potential outcomes. A sense of satisfaction arises if the chosen alternative proves superior; otherwise, the DM experiences regret. This response is captured by the regret-rejoice function (Definition 19).

- **Evaluation stage:** DMs assign a perceived utility value (Definition 18) to each alternative by integrating traditional utility with the regret-rejoice outcome, thus reflecting the anticipated impact of each choice.

- **Selection stage:** Alternatives with higher perceived utility values are prioritized, signaling stronger alignment with the DM's preferences and objectives.

Definition 18 *[109] Let v_i and v_j be the evaluation values of alternatives A_i and A_j respectively. Then, the perceived utility value of alternative A_i is defined as:*

$$U(v_i, v_j) = V(v_i) + W(V(v_i) - V(v_j)) \tag{2.16}$$

where $U(\cdot)$ denotes a utility function, while $W(\cdot)$ represents the regret-rejoice function. Typically, the utility function is a monotonically increasing function, often represented by a power function $V(x) = x^\gamma$, where the parameter γ serves as the risk aversion coefficient. Figure 2.9 illustrates the shape of the utility function for various values of the risk aversion coefficient γ.

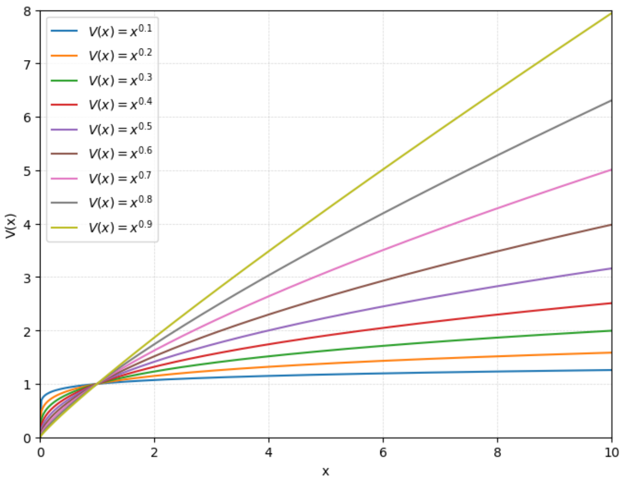


Figure 2.9: The utility function $V(x) = x^\gamma$

Definition 19 *[110] Let v_i and v_j be the evaluation values of alternatives A_i and A_j respectively. Then, the regret-rejoice function is defined as:*

$$\begin{aligned} &W(\Delta V) = 1 - exp(-\zeta * \Delta V), \zeta > 0 \\ &\Delta V = V(v_i) - V(v_j) \end{aligned} \tag{2.17}$$

Here, ζ denotes the regret aversion coefficient. When $W(\Delta V) > 0$, DMs feel satisfaction from selecting option A_i instead of A_j. Conversely, if $W(\Delta V) < 0$, they experience regret. Notably, when $\Delta V > 0$, DMs may exhibit a heightened sensitivity to negative outcomes (i.e., $-\Delta V$) compared to positive ones, indicating an aversion to regret. Figure 2.10 illustrates the variation in the regret-rejoice function curve across different values of the regret aversion coefficient.

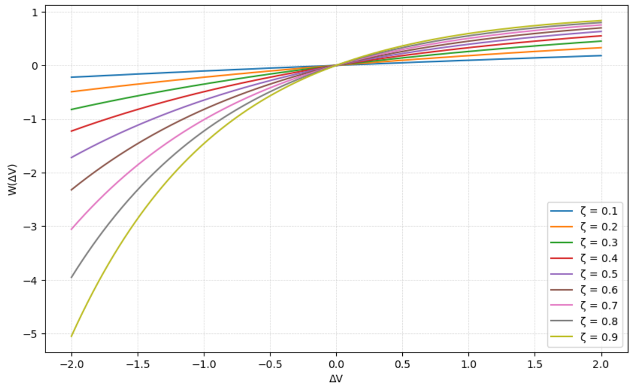


Figure 2.10: The regret-rejoice function $W(\Delta V) = 1 - exp(-\zeta * \Delta V)$

2.5.3 Bounded confidence theory

In CRP, it is frequently presumed that participants will adopt feedback and adjust their perspectives during the opinion adjustment phase. However, this assumption often neglects the degree to which individuals are actually willing to modify their opinions [111, 112]. If the recommended adjustments diverge substantially from a DM's current position, they may disregard the feedback, thereby hindering the convergence of the consensus process [43, 113, 114]. This phenomenon, commonly referred to as bounded confidence behavior in opinion dynamics research [115, 116], highlights the influence of cognitive boundaries on individual responses. Models that incorporate bounded confidence, such as the Hegselmann-Krause model [117] and the Deffuant-Weisbuch model [118], typically assume that participants will only consider opinions that fall within a specific "tolerance threshold," reflecting the acceptable range of deviation from their initial opinions.

Definition 20 *[43] Let $R^{k*} = (r_{ij}^{k*})$ denote the adjustment recommendations provided to DM E_K during the feedback phase of the CRP, and let $R^k = (r_{ij}^k)$ represent E_K's initial opinions. Additionally, let $\eta_k \in [0, 1]$ define the bounded confidence level of E_K. With this setup, E_K will only consider modifying its opinion according to the adjustment recommendation if the Manhattan distance between the recommended adjustment R^{k*} and the initial opinion R^k is less than or equal to η_k, as specified in Eq. (2.18)*

$$dis(R^{k*}, R^k) = \frac{1}{m \times n} \sum_{i=1}^{m} \sum_{j=1}^{n} |r_{ij}^{k*} - r_{ij}^k| \leq \eta_k \tag{2.18}$$

2.5.4 Social network analysis (SNA)

Social networks represent a stable structural framework arising from the interactions among various social entities, including individuals, groups, and organizations [119]. As intricate network topologies, social networks encompass numerous nodes and edges, where nodes signify individual actors or social collectives, and edges denote diverse social relationships, such as trust, collaboration, and shared identity. SNA is a fundamental research methodology and analytical tool designed to explore and interpret the relational patterns within these networks [120]. Through SNA, it becomes possible not only to identify critical nodes within the network but also to assess the intensity and nature of the connections between them, thereby providing a theoretical basis for investigating the mechanisms of collective consensus formation [121, 122].

Currently, there are three mainstream methods to describe the relationship between nodes in a social network [123], as shown in Figure 2.11. The specific descriptions are as follows:

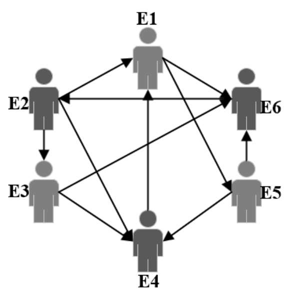
Graph	Matrix	Binary relation
	$T=\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$	E_1RE_5 E_1RE_6 E_2RE_1 E_2RE_3 E_2RE_4 E_3RE_4 E_3RE_6 E_4RE_1 E_5RE_4 E_5RE_6 E_6RE_2

Figure 2.11: Different representation schemes in social network analysis

(1) **Graph Theory**: Represent the structure of a social network using directed or undirected graphs, where nodes symbolize individuals and edges depict the relationships between them.

(2) **Adjacency Matrix Method**: Employ a two-dimensional adjacency matrix to capture the relationships among individuals, with "1" indicating the presence of a connection and "0" signifying its absence.

(3) **Algebraic Approach**: Use algebraic expressions to formally represent the relationships between nodes within the network structure.

Definition 21 *[34] A social trust network can be described as an undirected graph, denoted as $G(E, L)$, where $E = \{E_1, E_2, \ldots, E_K\}$ represents the set of DMs, and $L = \{L_{bd} | E_b, E_d \in E\}$ represents the set of edges. An edge L_{bd} between nodes $E_b \in E$ and $E_d \in E$ indicates a reciprocal trust relationship between these DMs within the social network.*

Definition 22 *[123, 124] Let $E = \{E_1, E_2, \ldots, E_K\}$ denote the set of DMs, and let $T = (t_{bd})_{K\times K}$ represent the fuzzy adjacency matrix associated with E. This matrix is defined by the membership function $\mu : E \times E \rightarrow [0, 1]$, where each entry $t_{bd} := \mu(E_b, E_d)$ represents the trust level from E_b to E_d. Each element of the matrix falls within the interval $[0, 1]$, with 0 indicating complete distrust and 1 indicating full trust. Then the trust score of E_b is defined as:*

$$TS_b = \frac{1}{K-1} \sum_{b=1, b\neq d}^{K} t_{bd} \tag{2.19}$$

Remark 3 *Trust relationships are often derived from subjective judgments or from incomplete trust matrices provided by DMs [123, 125]. However, in large-scale groups, the complex network structure makes it difficult to establish these relationships, which makes it challenging for participants to comprehensively evaluate trust throughout the entire network [126]. Consequently, trust can be more effectively inferred from objective indicators such as historical interactions, participation records, user evaluations, mutual reciprocity, and common interests. This approach provides a practical and scalable solution for establishing trust relationships in LSGDM, especially within social media environments.*

Chapter 3

Discussion of Results

This chapter presents a brief summary of the main proposals developed in this research, ensuring alignment with the objectives of the doctoral thesis. It also highlights the research contributions and discusses the results achieved for each proposal, as detailed below:

1. A novel GDM framework integrating DEA cross-efficiency models and regret theory to manage multi-granular hesitant fuzzy linguistic information.

2. A new method for maximum consensus aggregation of evaluation results in fuzzy DEA cross-efficiency models under hesitant fuzzy linguistic information.

3. New consensus optimization models in LSGDM based on ELICIT information, featuring two key proposals:

    - Identification and management of non-cooperative behaviors within the CRP.
    - Detection of overlapping communities and incorporation of bounded confidence in CRP.

3.1 GDM based on DEA cross-efficiency models and regret theory

The granularity of a linguistic term set refers to the number of linguistic terms it contains. Given that different decision makers may perceive uncertainty differently when addressing the same decision problem, employing multi-granular linguistic term sets becomes essential. These sets allow for a more precise representation of individual evaluations, thereby enhancing the accuracy of assessments and contributing to improved decision outcomes [127, 128].

DEA is a suitable tool for evaluating the performance of available alternatives in complex GDM scenarios, as it can handle multiple attribute indicators without the need for pre-assigned weights. Extensions like fuzzy DEA allow the incorporation of linguistic and hesitant information, which is useful when uncertainty or ambiguity arises in GDM [31, 129, 130].

However, current DEA-based GDM methods typically rely on single-granular linguistic terms, limiting the flexibility and clarity of linguistic information. Furthermore, most of these studies operate under the assumption that DMs are entirely rational, which fails to reflect the complexities of real-world decision-making, where psychological factors significantly influence behavior. In practice, DMs cannot always ensure the effectiveness of their choices or guarantee the optimality of their selected alternatives. Consequently, when they recognize that a better-performing alternative is available, they may experience regret, a common psychological response in decision-making.

To address these challenges, we propose a novel DEA-based GDM method that incorporates regret theory to capture the regret-avoidance behavior of DMs during the decision-making process. This method extends the DEA cross-efficiency models by integrating regret theory, enabling the handling of GDM problems involving multi-granular hesitant fuzzy linguistic information. The specific contributions of this approach are outlined as follows:

1. To collect evaluation data from DMs across various domains using multi-granular HFLTSs, thereby increasing the flexibility of linguistic representation. A method is proposed to unify these multi-granular HFLTSs into a unified granularity scale.

2. The application of the regret theory to the DEA models and define the corresponding total regret-rejoice utility values to capture the regret-aversion tendencies of DMs. Using these utility values, cross-efficiency models are developed, establishing the lower and upper bounds of efficiency for alternatives from both aggressive and benevolent perspectives.

3. To introduce an extended stochastic neutral DEA cross-efficiency model to aggregate expert assessments and derive the final ranking of alternatives.

The paper associated with this proposal is:

Song, H. H., García-Zamora, D. G., Labella, Á. L., Jia, X., Wang, Y. M., & Martínez, L. (2023). Handling multi-granular hesitant information: A group decision-making method based on cross-efficiency with regret theory. Expert Systems with Applications, 227, 120332.

3.2 Maximum consensus aggregation for fuzzy cross-efficiency

When incorporating DEA into GDM with linguistic information, many existing studies [30] rely on converting linguistic assessments into precise numerical values to facilitate the construction of DEA models. However, such approaches risk losing essential linguistic information, which is critical for preserving the inherent fuzziness and subjectivity characteristic of GDM processes. Such a loss may compromise the ability of the analysis to fully represent the complexity and ambiguity embedded in the decision-making context.

Many linguistic-based Data Envelopment Analysis models rely heavily on self-evaluation [29, 131], which often results in limited differentiation among alternatives. Cross-efficiency methods address this issue by integrating self-evaluation with peer-evaluation, offering a more comprehensive and balanced assessment framework. However, existing fuzzy DEA cross-efficiency models for group decision-making often handle fuzzy variables by focusing only on interval endpoints [132]. Consequently, the determination of optimal and worst-performing cross-efficiencies relies solely on the upper and lower bounds of the evaluation data, which limits the precision and overall effectiveness of these models.

Moreover, in DEA-based GDM models, some studies [26] adopt a strategy where evaluation data from multiple DMs are aggregated into a comprehensive assessment, and the efficiency of alternatives is calculated to identify the optimal choice. While effective in producing collective decision outcomes, this approach often overlooks the individual preferences of DMs regarding the alternatives. Furthermore, during the aggregation process, it is crucial to assign weights that account for the varying levels of influence among DMs, rather than assuming all participants have equal significance.

To address these challenges, we have proposed fuzzy DEA cross-efficiency models for solving GDM problems. This approach applies fuzzy set theory to convert hesitant fuzzy linguistic information into fuzzy envelopes, ensuring that the data for

DEA cross-efficiency models preserve the inherent fuzziness, thereby better capturing the uncertainty characteristics in GDM. Additionally, we develop a maximum group consensus optimization model based on cross-efficiency to determine the optimal weights for each DM, ensuring that the final decision is both recognized and accepted by all participants. This method does not only minimize the loss of fuzzy information during the GDM process but also enhances the rationality and robustness of the aggregated evaluation results. The specific contributions of this approach are summarized as follows:

1. It utilizes fuzzy envelopes to transform HFLTSs into TrFNs-based representations, preserving the richness of the original linguistic information. This approach leverages the computational efficiency of numerical values while maintaining the underlying uncertainty and fuzziness of the evaluation data within the context of fuzzy numbers.

2. It applies the α-level sets to generate intervals from TrFNs-based assessments across different attributes, forming the basis for fuzzy DEA cross-efficiency models. These models help identify both the optimal and worst cross-efficiency values for alternatives, providing essential intervals for evaluating their performance. By handling inputs and outputs as fuzzy variables, the model takes into account the full range of possible attribute values within these intervals at various α-levels, enabling a more comprehensive and refined evaluation of the alternatives.

3. It establishes DMs' weights by creating a maximum consensus model based on individual cross-efficiency intervals. These optimal weights are crucial for aggregating the cross-efficiency intervals from each DM's evaluation, producing collective intervals that are more acceptable to all individuals, improving the overall consensus and facilitating a more reliable ranking of the alternatives.

The paper related to this proposal is the following one:

Song, H. H., Wang, Y. M., & Martínez, L. (2024). Enhancing group decision-making: Maximum consensus aggregation for fuzzy cross-efficiency under hesitant fuzzy linguistic information. Computers & Industrial Engineering, 110622.

3.3 Consensus optimization models in LSGDM based on ELICIT information

In LSGDM scenarios involving a large number of DMs, achieving consensus is essential for mitigating potential conflicts within the group [22, 81]. Reaching a group consensus not only enhances the rationality of the decision but also strengthens its authority and persuasiveness. Additionally, ensuring the accuracy and interpretability of linguistic information is critical in the CRP. In response to these needs, we propose consensus optimization models for LSGDM based on ELICIT information, aimed at optimizing decision outcomes and improving decision-making efficiency.

3.3.1 Identification and management of non-cooperative behaviors within the CRP

In current LSGDM consensus models, different linguistic representations are introduced during the CRP, but improper handling of these linguistic terms often leads to the loss of important information [42]. Although, feedback mechanisms can be used to guide DMs to adjust their opinions to promote group consensus, many existing methods lack precision and interpretability, resulting in participants being unable to intuitively understand the opinions of the group after consensus is reached. Furthermore, the identification and management of non-cooperative behaviors in LSGDM remain problematic [92, 133]. Some studies focus on subgroup-level non-cooperative behaviors, neglecting that individuals may not adjust their opinions completely according to the recommendations. The identification rules for such behaviors are often subjective [134, 135], relying heavily on personal judgment. This subjectivity makes it challenging to distinguish between different types of non-cooperative behaviors.

After group consensus is reached, the focus shifts to selecting alternatives. While some studies [26, 136] have applied DEA methods to derive the final rankings of alternatives, most rely on the traditional CCR model [27]. However, the CCR model frequently results in non-unique rankings due to the possibility of multiple sets of optimal weights, which undermines the reliability and consistency of the final ranking outcomes. This calls for more robust methods to ensure greater accuracy and consistency in ranking decisions.

To address these challenges, we have developed the novel consensus optimization models for LSGDM by considering the non-cooperative behavior. In addition, we have introduced the new mechanisms for identifying and managing non-cooperative behaviors among DMs. The key contributions of this proposal are outlined below:

1. Convert the original evaluation into ELICIT information and represented by TrFNs to better follow framework of computing with words. ELICIT not only retains the ambiguity in linguistic expression, but also generates results that are easier to understand and explain, improving the transparency and explainability of the decision-making process.

2. Establish a feedback mechanism using two consensus optimization models to

provide optimal adjustments for subgroups and DMs. Introduce a mechanism to identify and manage non-cooperative behavior, setting an objective threshold based on a normal distribution function that accounts for DM differences. Implement a cooperation level function for weight penalties, ensuring fair and effective decision-making by considering evaluation results and trust relationships, leading to more reliable solutions.

3. Employ the DEA cross-efficiency method to evaluate each alternative's efficiency based on the final group opinion, overcoming the limitations of single-function evaluations and providing a more comprehensive assessment. By ranking alternatives objectively based on their performance, this method enables DMs to make more informed and robust choices.

The paper associated with this proposal is as follows:

Song, H. H., Dutta, B., García-Zamora, D., Wang, Y. M., & Martínez, L. (2024). Managing non-cooperative behaviors in consensus reaching process: A novel multistage linguistic LSGDM framework. Expert Systems with Applications, 240, 122555.

3.3.2 Detection of overlapping communities and incorporation of bounded confidence in CRP

In LSGDM scenarios, managing the opinions of numerous DMs efficiently requires the application of dimension-reduction techniques, such as clustering methods. These techniques help to reduce resource consumption while streamlining the process of forming cohesive decision-making solutions. One common dimension-reduction approach involves partitioning a large group into several subgroups based on social network structures. In this context, community detection methods have proven effective for dividing a group into meaningful and manageable subgroups. However, most clustering algorithms in LSGDM employ non-overlapping detection techniques, assigning each DM to only one subgroup, which can not reflect the reality of multiple group memberships. In practice, individuals often participate in several communities or subgroups simultaneously, making it essential for LSGDM approaches to differentiate between non-overlapping and overlapping DMs to better manage subgroup interactions.

Accurately determining the weights of individuals or subgroups is crucial for effectively aggregating opinions and generating representative decisions in LSGDM. The influence of individuals and subgroups is primarily shaped by their social connections. However, a critical aspect often overlooked in some studies is that when individuals are divided into specific subgroups, they may lose certain connections with other groups, which can significantly alter their influence and relevance within the broader network. As a result, DMs within subgroups may not fully reflect their true importance in the whole network. Therefore, constructing an appropriate weighting strategy that accounts for these differences is essential to accurately capture the varying levels of importance that DMs hold across different networks.

Additionally, while some studies have introduced feedback mechanisms to align individuals' opinions with collective consensus, these approaches often overlook the psychological difficulties associated with adjusting opinions. When DMs' opinions deviate significantly from the recommended ones, they may resist making the necessary modifications, which can hinder the group from reaching a unified decision. This challenge is particularly evident among overlapping DMs, whose opinions are influenced by their involvement in multiple communities. Therefore, it is necessary to develop more advanced feedback mechanisms that take into account both the psychological barriers to opinion adjustment and the structural complexities of overlapping DMs.

To address these challenges, we propose several innovative consensus optimization models that incorporate bounded confidence theory and ELICIT information, alongside specialized feedback mechanisms tailored for overlapping DMs. These improvements aim to improve the acceptance of recommendations, ultimately facilitating smoother and more cohesive decision-making processes. The specific contributions of this proposal are summarized as follows:

1. Employ the Lancichinetti-Fortunato method (LFM) to identify DMs who participate in one or more communities. This not only deepened our understanding of overlapping structure, but also lays the foundation for developing more targeted feedback mechanisms for overlapping DMs.

2. Evaluate the influence of DMs from both global and local levels through a two-layer perspective, more accurately capturing the importance of individuals in social networks at different levels and clearly revealing the difference between the global and local contributions of each participant.

3. Integrate ELICIT information and bounded confidence theory to construct consensus optimization models. These models not only improve DMs' acceptance of adjustment suggestions, but also enhances their understanding of the consensus process and its results, thereby creating a transparent and explainable decision-making environment.

4. Design the feedback mechanism for overlapping DMs by fully considering the influence of multiple subgroups and effectively responds to their challenges when facing multiple adjustment suggestions. This mechanism not only promotes the

formation of consensus, but also fills the gap in existing research on guiding overlapping DMs to adjust their opinions.

The paper associated with this proposal is as follows:

Wang, Y. M., Song, H. H., Dutta, B., García-Zamora, D., & Martínez, L. (2024). Consensus reaching in LSGDM: Overlapping community detection and bounded confidence-driven feedback mechanism. Information Sciences, 679, 121104.

Chapter 4

Publications

In accordance with Article 25, Section 2, of the current Doctoral Studies regulations at the University of Jaén, under the RD 99/2011 program, this chapter presents the core publications that form the foundation of this doctoral thesis.

These publications consist of four scientific articles published in international journals indexed in the Journal Citation Reports (JCR) database, produced by Clarivate Analytics.

4.1 Handling multi-granular hesitant information: A group decision-making method based on cross-efficiency with regret theory

- State: Published.
- Title: Handling multi-granular hesitant information: A group decision-making method based on cross-efficiency with regret theory.
- Authors: Hui-Hui Song, Diego García Zamora, Álvaro Labella Romero, Xiang Jia, Ying-Ming Wang and Luis Martínez.
- Journal: Expert Systems With Applications.
- Volume: 227. Page: 120332. Date: 6 May 2023.
- DOI: https://doi.org/10.1016/j.eswa.2023.120332.
- ISSN: 0957-4174.
- Impact factor (JCR 2023): 7.5.
  - Quartiles:
    * Quartile 1 COMPUTER SCIENCE, ARTIFICIAL INTELLIGENCE 24/197.
    * Quartile 1 ENGINEERING, ELECTRICAL & ELECTRONIC 25/352.
    * Quartile 1 OPERATIONS RESEARCH & MANAGEMENT SCIENCE 6/106.

4.2 Enhancing group decision-making: Maximum consensus aggregation for fuzzy cross-efficiency under hesitant fuzzy linguistic information

- State: Published.
- Title: Enhancing group decision-making: Maximum consensus aggregation for fuzzy cross-efficiency under hesitant fuzzy linguistic information.
- Authors: Hui-Hui Song, Ying-Ming Wang and Luis Martínez.
- Journal: Computers & Industrial Engineering.
- Volume: 197. Page: 110622. Date: 4 October 2024.
- DOI: https://doi.org/10.1016/j.cie.2024.110622.
- ISSN:
- Impact factor (JCR 2023): 6.7.
  - Quartiles:
    * Quartile 1 COMPUTER SCIENCE, INTERDISCIPLINARY APPLICATIONS 20/169.
    * Quartile 1 ENGINEERING, INDUSTRIAL 11/69.

4.3 Managing non-cooperative behaviors in consensus reaching process: A novel multi-stage linguistic LSGDM framework

- State: Published.
- Title: Managing non-cooperative behaviors in consensus reaching process: A novel multi-stage linguistic LSGDM framework.
- Authors: Hui-Hui Song, Bapi Dutta, Diego García Zamora, Ying-Ming Wang and Luis Martínez.
- Journal: Expert Systems With Applications.
- Volume: 240. Page: 122555. Date: 13 November 2023.
- DOI: https://doi.org/10.1016/j.eswa.2023.122555.
- ISSN: 0957-4174.
- Impact factor (JCR 2023): 7.5.
  - Quartiles:
    * Quartile 1 COMPUTER SCIENCE, ARTIFICIAL INTELLIGENCE 24/197.
    * Quartile 1 ENGINEERING, ELECTRICAL & ELECTRONIC 25/352.
    * Quartile 1 OPERATIONS RESEARCH & MANAGEMENT SCIENCE 6/106.

4.4 Consensus reaching in LSGDM: Overlapping community detection and bounded confidence-driven feedback mechanism

- State: Published.
- Title: Consensus reaching in LSGDM: Overlapping community detection and bounded confidence-driven feedback mechanism.
- Authors: Ying-Ming Wang, Hui-Hui Song, Bapi Dutta, Diego García Zamora and Luis Martínez.
- Journal: Information Sciences.
- Volume: 679. Page: 121104. Date: 28 June 2024.
- DOI: https://doi.org/10.1016/j.ins.2024.121104.
- ISSN: 0020-0255.
- Impact factor (JCR 2022): 8.1.
  - Quartiles:
    * Quartile 1 COMPUTER SCIENCE, INFORMATION SYSTEMS 17/251.

Chapter 5

Conclusions and Future Works

Finally, this chapter provides a comprehensive summary of the research presented in this thesis, reviews the key proposals and findings, and outlines potential directions for future studies based on the results obtained.

5.1 Conclusions

This research addresses GDM problems under hesitant linguistic information, with the decision-making process in complex environments serving as the central focus. The study targets key stages in the decision-making process, including information expression, conversion, fusion, clustering, and the CRPs. Additionally, the scale of DMs progressively expands from small-scale to large-scale settings. The main aim of this research is to systematically enhance the efficiency and rigor of decision-making, providing practical solutions and greater application value for addressing complex GDM challenges. The results and conclusions are summarized as follows:

(1) Considering the diverse cognition and background knowledge of DMs from different fields, multi-granular hesitant fuzzy linguistic term sets have been used to capture this variation and hesitation, accurately reflecting DMs' opinions and enhancing linguistic flexibility in complex GDM scenarios. Furthermore, the integration of regret theory with the DEA cross-efficiency model optimizes the GDM process. Unlike traditional methods like TOPSIS and VIKOR, the DEA models require no prior information, ensuring a more objective and fair evaluation. By factoring in DMs' psychological behaviors, such as regret and avoidance, this approach brings the evaluation of alternatives closer to real-world conditions, significantly improving its practical applicability. This proposal achieves the first objective of this thesis.

(2) To avoid the loss of critical information that often results from converting linguistic data into precise numerical values in DEA-based GDM methods, we transformed hesitant linguistic information into fuzzy envelopes and represented them as TrFNs, preserving the inherent fuzziness of the original linguistic data. Subsequently, fuzzy DEA cross-efficiency models were developed, considering all possible values of fuzzy inputs and outputs, thereby ensuring objectivity and comprehensiveness in evaluating both the optimal and worst cross-efficiencies of alternatives. Additionally, a maximum group consensus model was introduced, using cross-efficiency intervals to accurately determine DMs' weights, ensuring that the resulting comprehensive cross-efficiency interval is widely accepted by the group. These enhanced DEA models offer effective tools for handling GDM in fuzzy environments, successfully achieving the second objective of this thesis.

(3) The ELICIT scheme was employed to manage hesitant fuzzy linguistic information using TrFNs, ensuring that calculations adhere to the framework of CWW. This scheme enhances the interpretability of the decision-making process and ensures that the results remain closely aligned with the original input data. Additionally, two consensus optimization models based on ELICIT information have been developed, offering targeted adjustment suggestions for each subgroup and DM. A feedback mechanism has been implemented to identify and manage non-cooperative behaviors. By constructing a structured and comprehensive approach to address LSGDM challenges, this proposal optimizes the processing of linguistic information, effectively manages non-cooperative behaviors, and employs the DEA method for robust ranking, thus ensuring the scientific of the decision-making process. These improvements offer a reliable solution for complex LSGDM problems and successfully achieves the third objective of this thesis.

(4) Recognizing overlapping structures within large-scale social networks, the LFM algorithm was applied to identify DMs involved in multiple communities. The influence of each participant was evaluated globally and locally, highlighting differences in their roles and contributions at different levels. By combining ELICIT information with bounded confidence theory, a two-stage consensus optimization model was developed, which greatly improved individuals' acceptance of adjustments and clarified the consensus process, creating a more transparent and interpretable decision-making environment. The feedback mechanism considered the influence of multiple subgroups, effectively addressing challenges overlapping DMs may face with varied adjustment suggestions. These improvements bridge a gap in guiding overlapping DMs to refine their opinions and offer

a systematic method for handling complex LSGDM challenges. This approach achieves the final objective of this thesis.

Notably, all the objectives outlined at the beginning of this thesis have been achieved, providing state-of-the-art technology and laying the foundation for future research investigating new tools, models, and results, as described in the following sections.

5.2 Future Works

Based on the results of this research, several proposals emerge for continuing the work undertaken in this doctoral thesis. Several possible future research directions are as follows:

(1) To develop the advanced DEA-based GDM methods capable of handling multi-source heterogeneous information. As society and technology progress, the channels through which information is acquired have become more diverse, resulting in decision-making information that is vast and highly heterogeneous. Future research should focus on standardizing this multi-source, heterogeneous data and creating corresponding DEA methods tailored to address complex and dynamic GDM environments effectively.

(2) To develop dynamic clustering algorithms for LSGDM. Real-world LSGDM scenarios involve numerous DMs, often in uncertain environments where information may be incomplete, and trust relationships are not always fully established. Additionally, as DMs' opinions evolve, trust relationships may also shift dynamically. Therefore, Future research should focus on designing clustering algorithms that accommodate incomplete and evolving trust relationships, including trust propagation, to better address the complexities of real decision-making scenarios.

(3) To incorporate game theory into the CRPs within LSGDM contexts. In reality, different groups may pursue distinct objectives. For instance, a moderator may aim to minimize decision-making costs, while individuals or subgroups may seek to maximize their benefits, leading to potential conflicts of interest. Future research can apply game theory to facilitate consensus among groups, resolving conflicts and enhancing the fairness and rationality of the decision-making process.

Appendix A

Resumen escrito en Español

A.1 Motivación

La teoría de decisión [1] es la disciplina que estudia cómo seleccionar la solución óptima entre una serie de opciones posibles. Implica evaluar, seleccionar y analizar la lógica y los métodos que sustentan el proceso de la toma de decisiones. En aplicaciones prácticas, el proceso de toma de decisiones va más allá de la selección de un único objetivo; con frecuencia implica el manejo de escenarios complejos en los que deben considerarse simultáneamente múltiples atributos evaluables. Este enfoque se denomina toma de decisiones multiatributo (TDM) [2, 3]. Debido al avance de los estudios teóricos, se han propuesto numerosos métodos y modelos para la TDM. Estos métodos y modelos que ayudan a procesar y analizar sistemáticamente los problemas de decisión [4, 5].

En el mundo real, los responsables de la toma de decisiones están limitados por sus conocimientos, habilidades y experiencia individuales. Estas limitaciones pueden introducir sesgos cognitivos y dar lugar a evaluaciones incompletas o subjetivas. Además, la complejidad de los escenarios de toma de decisiones, junto con la vaguedad y el carácter incompleto de la información disponible, superan a menudo las capacidades cognitivas y analíticas de un solo individuo. Tales limitaciones aumentan significativamente el riesgo de errores, disminuyendo así la solidez y fiabilidad de las decisiones tomadas únicamente por un individuo. Para hacer frente a estos retos, la toma de decisiones en grupo (TDG) [6, 7] se ha introducido como una herramienta eficaz para resolver estos complejos problemas de decisión. En este contexto de TDG, varios decisores discuten y analizan conjuntamente el mismo problema de toma de decisiones, integran diferentes puntos de vista y preferencias y forman juicios colectivos. A partir de esta evaluación colectiva, se clasifican y seleccionan las alternativas. La TDG mitiga los sesgos individuales y aprovecha la inteligencia colectiva del grupo, mejorando sustancialmente la calidad y objetividad de las decisiones.

En los problemas de TDG, a menudo se pide a los responsables de la toma de decisiones que evalúen múltiples alternativas expresando sus preferencias mediante valoraciones numéricas precisas. Sin embargo, debido a la complejidad inherente de los escenarios del mundo real, la incertidumbre en los contextos de toma de decisiones y la naturaleza subjetiva de los juicios individuales, proporcionar evaluaciones numéricas precisas para cada alternativa puede suponer un gran reto. Por ello, los responsables de la toma de decisiones recurren a menudo a valoraciones expresadas en lenguaje natural, ya que les permiten modelar la incertidumbre, los matices y las preferencias subjetivas de un modo que las rígidas escalas numéricas no pueden captar plenamente. El lenguaje es intrínsecamente flexible y se ajusta a la forma en que los seres humanos procesan y comunican información compleja, lo que lo convierte en el medio preferido para las evaluaciones cualitativas. Este uso de las expresiones lingüísticas da lugar a los problemas de toma de decisión en grupo lingüísticos (TDGL) [8, 9, 10]. Al utilizar expresiones lingüísticas para transmitir información evaluable, la TDGL se ajusta mejor a los procesos cognitivos humanos, lo que permite una toma de decisión más intuitiva y accesible.

Los estudios existentes sobre TDGL suelen presuponer que los decisores expresan sus preferencias utilizando un único término lingüístico, lo que puede no reflejar la complejidad de sus evaluaciones. En la toma de decisiones del mundo real, especialmente en condiciones de incertidumbre, los decisores pueden dudar a la hora de emitir juicios debido a limitaciones en su experiencia, conocimientos o habilidades. Para abordar esta limitación, Rodríguez et al. [11] introdujeron el concepto de conjuntos de términos lingüísticos difusos dudosos (CTLDDs), que permiten a los decisores expresar sus evaluaciones utilizando una serie de términos lingüísticos consecutivos, en lugar de solamente uno, proporcionando así una representación más matizada y completa de sus preferencias. Al ofrecer mayor flexibilidad y precisión, los CTLDD hacen que el proceso de toma de decisiones refleje mejor las opiniones de los decisores. En consecuencia, los métodos TDGL basados en información lingüística difusa dudosa, denominados TDGLD, han atraído una amplia atención y se han convertido en una herramienta de representación clave para gestionar eficazmente la incertidumbre y la ambigüedad en escenarios complejos de toma de decisiones [12, 13]. Como se muestra en la Figura A.1, el marco general de la TDGLD consiste en, pero no se limita a, las tres fases siguientes:

Fase 1: Se seleccionan uno o más conjuntos de términos lingüísticos con granularidad, sintaxis y semántica adecuadas [14], lo que permite a los responsables de las decisiones expresar eficazmente la indecisión en la información lingüística difusa dudosa.

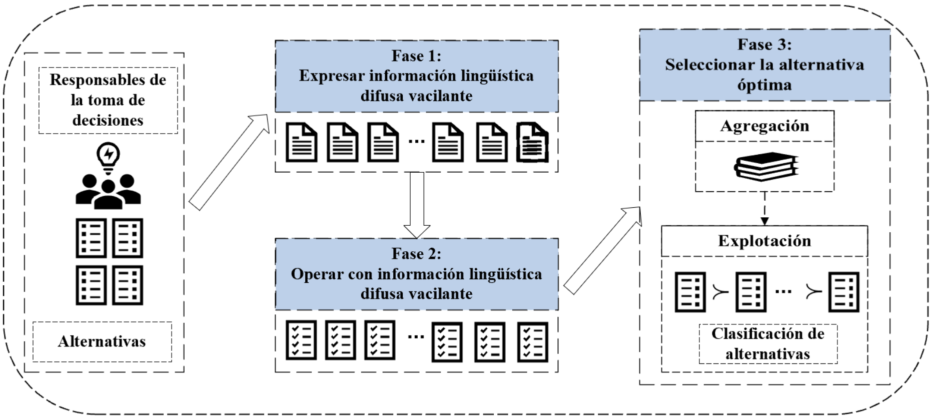


Figura A.1: Esquema general del TDGLD

Fase 2: Deben seleccionarse algunas funciones o métodos apropiados para operar con información lingüística difusa dudosa.

Fase 3: Para seleccionar la alternativa óptima hay que desarrollar distintos métodos o modelos; esta etapa suele constar de dos fases, aunque no se limita a ellas:

(a) **Proceso de agregación:** Las opiniones lingüísticas dudosas se agregan utilizando un operador o método seleccionado para obtener una opinión colectiva.

(b) **Proceso de explotación:** Se aplican técnicas de toma de decisiones para evaluar y clasificar las alternativas en función de la opinión colectiva y, en última instancia, seleccionar la alternativa óptima basándose en los resultados de la clasificación.

A medida que se acelera el desarrollo social y tecnológico, los procesos de gestión y toma de decisiones se han vuelto cada vez más complejos y variables. Al mismo tiempo, los avances tecnológicos han mejorado significativamente la capacidad de procesamiento de datos y la eficiencia de las comunicaciones, haciendo posible y eficaz la participación de grupos a gran escala en escenarios complejos de toma de decisiones. Tradicionalmente, los problemas de TDG se clasificaban como toma de decisiones en grupo a gran escala (TDGGE) cuando el número de decisores alcanzaba o superaba los 20 [15]. Sin embargo, esta definición ha quedado obsoleta con el desarrollo de tecnologías como las redes sociales y la democracia electrónica, en las que el número de decisores puede alcanzar escalas significativamente superiores [16, 17]. Este cambio pone de relieve la necesidad de nuevas metodologías y marcos para hacer frente a los retos que plantean los entornos en TDGGE modernos. Destacar que el marco de TDGGE se ha aplicado ampliamente en diversos campos, como la gestión empresarial [18], la atención sanitaria [19], la ingeniería [20], selección de proveedores verdes [21], etc. Al incorporar una gama más amplia de perspectivas y conocimientos, los métodos la TDGGE mitigan eficazmente los sesgos individuales, al tiempo que mejoran la precisión de las decisiones, su adaptabilidad y la solidez general. Por lo tanto, el TDGGE se ha convertido en un enfoque indispensable para abordar los retos cada vez más complejos a los que se enfrentan las industrias y las prácticas de gestión modernas.

Como se ilustra en la Figura A.2, el marco en TDGGE basado en información lingüística comprende, entre otras, las cuatro fases siguientes:

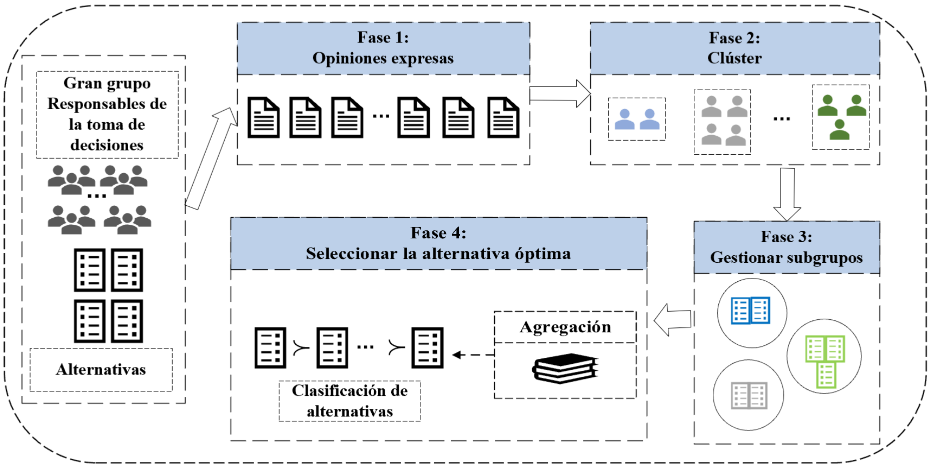


Figura A.2: El marco general de en TDGGE basado en la lingüística

Fase 1: Se seleccionan uno o varios conjuntos de términos lingüísticos, diseñados con la granularidad, la sintaxis y la semántica adecuadas, para que los decisores puedan expresar con precisión sus preferencias lingüísticas.

Fase 2: Se desarrollan algoritmos o métodos de agrupación adecuados para dividir el gran grupo de decisores en subgrupos más pequeños y manejables.

Fase 3: Se establecen estrategias eficaces de gestión de subgrupos para coordinar y afinar las preferencias dentro de cada subgrupo, garantizando una representación equilibrada y facilitando la consolidación de las opiniones de los subgrupos.

Fase 4: Se desarrollan metodologías o modelos avanzados para determinar la alternativa óptima. Esta fase suele constar de dos procesos clave:

(a) **Proceso de agregación:** Las opiniones de los subgrupos se agregan utilizando un operador o método seleccionado para producir una opinión colectiva.

(b) **Proceso de explotación:** Se aplican técnicas de toma de decisiones para analizar y clasificar las alternativas en función de la opinión colectiva, identificando en última instancia la alternativa óptima según los resultados de la clasificación.

En la TDG, los responsables de la toma de decisiones a menudo se encuentran con desacuerdos debidos a diferencias de perspectivas, experiencia y preferencias. Se trata de un resultado natural cuando se encarga a personas con formación diversa que evalúen alternativas. Sin embargo, en TDGGE, donde el número de participantes es considerablemente mayor, estos desacuerdos se hacen más pronunciados y complejos de gestionar. La mayor diversidad de opiniones en TDGGE no sólo amplifica el nivel de conflicto, sino que también complica el proceso de alcanzar un consenso [16]. Un proceso de alcance de consenso (PAC) desempeña un papel fundamental en la superación de estos retos. Proporciona un enfoque estructurado para que los responsables de la toma de decisiones entablen comunicación y negociación, permitiéndoles ajustar sus preferencias en busca de un acuerdo colectivo [22]. Mediante debates iterativos y ajustes, los PACs ayudan a reducir conflictos, reducir las distancias entre opiniones opuestas y alinear las preferencias hacia una decisión consensuada. Alcanzar el consenso garantiza que la decisión final no sólo sea aceptable para todos los participantes, sino que también refleje el juicio colectivo del grupo. La Figura A.3 ilustra el marco en TDGGE con PAC, que incluye cuatro fases. Las fases 1, 2 y 4 coinciden con las de la Figura A.2 y no se reiteran aquí. En particular, la fase 3 de la Figura A.3 se centra en el desarrollo de modelos o mecanismos de consenso para abordar los desacuerdos entre los responsables de la toma de decisiones o subgrupos, facilitando así el logro del consenso del grupo.

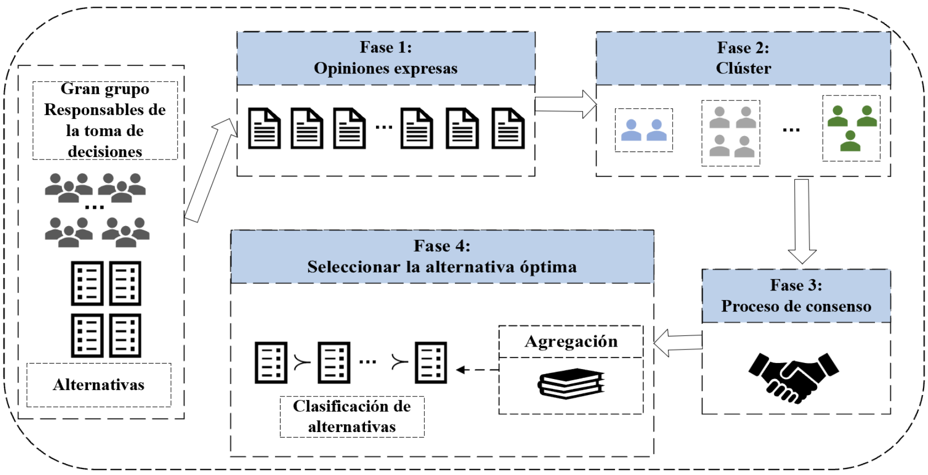


Figura A.3: El marco general en TDGGE con PAC

Varios investigadores han propuesto numerosos modelos y enfoques para abordar los retos de la TDG y la en TDGGE, que han hecho avanzar significativamente el campo de la toma de decisiones. Sin embargo, la investigación actual sigue siendo inadecuada para abordar problemas complejos de toma de decisiones en el mundo real. Las dificultades y los retos emergentes ponen de manifiesto la necesidad de profundizar en la exploración y el perfeccionamiento de los métodos existentes. Por lo tanto, hay varios desafíos en el proceso de TDG y TDGGE, que son los principales motivos de este estudio:

(1) Los problemas actuales de TDGL [23, 24, 25, 26] se han abordado utilizando modelos no paramétricos como el Análisis Envolvente de Datos (DEA) [27], sin embargo, su eficacia en el manejo de TDGLD sigue siendo ignorada. Los métodos existentes de TDGL basados en DEA [28, 29] suelen estar restringidos por conjuntos de términos lingüísticos de granularidad única, lo que limita tanto la precisión como la flexibilidad de las evaluaciones. Además, estos métodos a menudo suponen que los responsables de la toma de decisiones son totalmente racionales, sin tener en cuenta los factores psicológicos que influyen en la toma de decisiones en el mundo real. Estas limitaciones ponen de manifiesto la necesidad de modelos DEA ampliados que tengan más en cuenta las complejidades del TDGLD y garanticen procesos de toma de decisiones más realistas y adaptables.

(2) Los estudios existentes de TDGL basados en DEA a menudo convierten la información lingüística en valores numéricos precisos [30, 31], lo que compromete los matices lingüísticos y la imprecisión inherente a los entornos de TDGL. Además, los modelos DEA se basan en diversas evaluaciones de múltiples responsables de la toma de decisiones para evaluar alternativas, lo que requiere un mecanismo de agregación eficaz para obtener la eficiencia final de las alternativas. Sin embargo, los actuales modelos basados en DEA en TDG [32, 33] agregan datos brutos sin garantizar el consenso entre los decisores, lo que perjudica la representatividad y la fiabilidad de las decisiones finales. Estas limitaciones subrayan la necesidad de mejorar el marco de TDG empleando información lingüística e integrando el consenso para mejorar la fiabilidad y la aceptación de las decisiones.

(3) Los modelos de PAC existentes para TDGGE se enfrentan a algunas limitaciones notables. Un reto clave es la falta de orientación clara y procesable para los responsables de la toma de decisiones durante el proceso de consenso [34, 35], lo que reduce tanto la interpretabilidad como la eficacia de los PACs. Además, los estudios sobre TDGGE con información lingüística difusa dudosa suelen pasar por alto el impacto de los comportamientos no cooperativos [19, 36, 37], en los que algunos responsables de la toma de decisiones se resisten a alinearse con el

consenso del grupo. Tales comportamientos obstruyen la toma de decisiones y reducen la eficacia de los PAC. Por lo tanto, estas limitaciones ponen de relieve la necesidad de mejorar la interpretabilidad y las estrategias para gestionar el comportamiento no cooperativo, garantizando un PAC más fiable y eficaz en TDGGE.

(4) La mayoría de los enfoques de agrupación existentes en TDGGE que aplican técnicas de redes sociales para modelar las relaciones entre los decisores, [38, 39, 40, 41] asumen que dichos decisores pertenecen a un único subgrupo dentro de la red, pasando por alto la posibilidad que estos puedan formar parte de múltiples subgrupos simultáneamente. Este descuido debilita el PAC, ya que no se tiene en cuenta la influencia de las estructuras superpuestas en la formación del consenso. Además, aunque se proponen mecanismos de recomendación para alinear a los responsables de la toma de decisiones con el consenso del grupo, a menudo no tienen en cuenta la aceptación individual de las recomendaciones [34, 42, 43], lo que provoca resistencia y obstaculiza los PACs. Estas lagunas subrayan la necesidad de un enfoque más matizado que integre las estructuras superpuestas de la red social y mejore la aceptación de la recomendación, mejorando en última instancia la practicidad y la fiabilidad de la PAC en TDGGE.

En resumen, ante los entornos de toma de decisiones cada vez más complejos, ambiguos e inciertos, la investigación en profundidad de los métodos de TDG basados en información lingüística difusa dudosa es un tema importante que merece la pena seguir explorando en el campo de la ciencia de la decisión moderna.

A.2 Objetivos

Teniendo en cuenta las motivaciones expuestas en el apartado anterior, el objetivo de esta tesis doctoral es desarrollar nuevos métodos de TDG y modelos de consenso basados en información lingüística difusa dudosa. Para ello, se plantean los siguientes objetivos:

(1) Desarrollar un nuevo método de TDG basado en modelos DEA ampliados para manejar información lingüística difusa dudosa multigranular. Este método combina los puntos fuertes de los modelos DEA de eficiencia cruzada y la teoría del arrepentimiento, con el objetivo de mejorar la precisión de las evaluaciones lingüísticas al tiempo que incorpora técnicas para evitar el arrepentimiento de los responsables de la toma de decisiones en el proceso de TDG.

(2) Desarrollar un método TDG novedoso que utilice modelos DEA difusos de eficiencia cruzada para manejar información lingüística difusa dudosa. El enfoque

preserva el carácter difuso de los datos originales convirtiéndolos en envolventes difusas, minimizando la pérdida de información en los cálculos DEA. Se diseñará un modelo de optimización para determinar las pesos óptimos de los responsables de la toma de decisiones, garantizando una agregación precisa y la racionalidad en el proceso de TDG.

(3) Proponer un nuevo PAC que aborde el comportamiento no cooperativo en TDGGE con información lingüística difusa dudosa. Este objetivo pretende construir los modelos de optimización de consenso utilizando Expresiones lingüísticas comparativas extendidas con Traslación Simbólica (ELICIT), mejorando la transparencia y explicabilidad del PAC. Además, se diseñará un mecanismo robusto de gestión del comportamiento no cooperativo para ayudar al proceso de toma de decisiones.

(4) Proponer un nuevo PAC que tenga en cuenta la estructura solapada de las redes de confianza social en TDGGE. El enfoque identificará a los decisores solapados y desarrollará modelos de optimización del consenso que integren la confianza limitada y la información ELICIT, mejorando la interpretabilidad de la información lingüística difusa dudosa y aumentando la probabilidad de aceptación de las sugerencias de cambio en las opiniones de los decisores.

A.3 Estructura

Esta investigación doctoral pretende alcanzar los objetivos expuestos en el apartado anterior. Para ello, presenta una recopilación de artículos elaborados por el doctorando. Esta forma de presentar la tesis doctoral se ajusta a lo establecido en el artículo 23, punto 3, de la normativa vigente de Estudios de Doctorado de la Universidad de Jaén, recogida en el RD 99/2011. Esta memoria incluye cuatro contribuciones, todas ellas publicadas en revistas de reconocido prestigio internacional recogidas en la base de datos Journal Citation Reports (JCR).

A continuación se presenta un breve resumen de la estructura de esta memoria de investigación:

- **Capítulo 2:** En este capítulo se esbozan los fundamentos teóricos de la investigación, incluido el enfoque lingüístico difuso, la computación con palabras (CWW) y los modelos lingüísticos computacionales, como el modelo lingüístico de 2 tuplas, los CTLDD y ELICIT. También se analiza el marco de en TDGGE y PAC. Además, se introducen teorías y métodos de decisión clásicos, como DEA, la teoría del arrepentimiento, la teoría de la confianza limitada y el análisis de redes sociales.

- **Capítulo 3:** Este capítulo ofrece una visión concisa y completa de los artículos publicados que constituyen el núcleo de esta investigación. Para cada contribución, se presenta un breve análisis de los principales resultados obtenidos.
- **Capítulo 4:** Este capítulo incluye los cuatro trabajos mencionados anteriormente, junto con información detallada sobre las revistas en las que se publicaron.
- **Capítulo 5:** En este capítulo se esbozan las conclusiones finales de la investigación y se abordan posibles direcciones futuras que pudieran seguir desarrollando la investigación presentada en esta tesis doctoral.

A.4 Resumen

La TDG es una actividad crucial en la investigación de operaciones para los sistemas de información modernos, con aplicaciones en política, economía, cultura y otros campos. Los problemas de decisión del mundo real a menudo se caracterizan por la complejidad y la incertidumbre, lo que dificulta a los decisores alcanzar una solución. Esta complejidad e incertidumbre también crean mayores desafíos para los decisores a la hora de evaluar alternativas utilizando valores numéricos precisos. Para abordar este problema, los decisores suelen confiar en evaluaciones cualitativas expresadas en lenguaje natural, que están más alineadas con los procesos cognitivos humanos. Sin embargo, en entornos de decisión complejos y ambiguos, los decisores a menudo muestran duda al expresar opiniones utilizando términos lingüísticos debido a una experiencia limitada o un conocimiento incompleto. En este contexto, los CTLDD ofrecen una herramienta eficaz para capturar la duda y la incertidumbre al permitir a los decisores utilizar múltiples términos lingüísticos para evaluar alternativas, reflejando mejor tanto la imprecisión de la información de evaluación como la duda experimentada por los decisores. Aunque los métodos TDG basados en información lingüística difusa dudosa han logrado avances significativos en los últimos años, siguen existiendo varios desafíos sin resolver frente a entornos de decisión cada vez más complejos. Para abordar estos desafíos, este estudio tiene como objetivo avanzar en los fundamentos teóricos de la TDG en marcos lingüísticos difusos dudosos y propone metodologías innovadoras de TDG. Las principales contribuciones de la investigación incluyen:

(1) El uso de términos lingüísticos con un único nivel de granularidad a menudo no logra captar los matices de las opiniones de los decisores en TDG. Esta limitación surge de la diversidad en el conocimiento, los antecedentes culturales y las perspectivas de los decisores, lo que lleva a la imprecisión en las evaluaciones.

Además, los decisores no son completamente racionales durante la toma de decisiones y pueden experimentar arrepentimiento por sus elecciones. También se vuelve un desafío para los participantes identificar una solución óptima, cuando la clasificación de las alternativas no es única. Para abordar estos problemas, se propone un nuevo método de TDG que combina DEA y la teoría del arrepentimiento, utilizando información lingüística difusa dudosa de granularidad múltiple. Inicialmente, las alternativas se evalúan utilizando CTLDD de granularidad múltiple, que representan de manera efectiva la ambigüedad y la incertidumbre en problemas complejos. Para garantizar un procesamiento consistente de la información, las evaluaciones lingüísticas se convierten en una forma numérica unificada utilizando funciones de utilidad. A continuación, se desarrollan modelos de eficiencia cruzada basados en la teoría del arrepentimiento para evitar el arrepentimiento de los individuos durante el proceso de toma de decisiones. Luego se introduce un modelo DEA de eficiencia cruzada neutral estocástico para clasificar completamente las alternativas e identificar la óptima. Al integrar conjuntos de términos lingüísticos de granularidad múltiple, teoría del arrepentimiento y DEA, el método propuesto mejora la precisión de la evaluación, captura factores psicológicos y garantiza una evaluación objetiva en TDG. Este enfoque mejora significativamente la racionalidad y la precisión de los resultados de la toma de decisiones, lo que proporciona una solución sólida para manejar escenarios de evaluación complejos e inciertos.

(2) TDGL, los métodos basados en DEA existentes suelen convertir la información lingüística en valores numéricos precisos, lo que elimina la ambigüedad inherente de los datos lingüísticos y difumina el significado de las expresiones lingüísticas. Además, al agregar los resultados de la evaluación de los individuos, a menudo se pasa por alto la influencia del peso de cada decisor, lo que difumina la contribución real del decisor y la importancia de sus opiniones en el resultado final. Para abordar estas limitaciones, este estudio propone un marco de TDG basado en modelos de eficiencia cruzada DEA difusos que utilizan envolventes difusas para preservar la incertidumbre inherente de la información lingüística difusa dudosa. Primero, los CTLDD se transforman en números difusos trapezoidales (TrFNs), manteniendo la incertidumbre inherente en esta representación. Luego, se construyen modelos de eficiencia cruzada DEA difusos para obtener los intervalos de eficiencia de las alternativas. Para mejorar la agregación de los resultados de la evaluación, se desarrolla un modelo de consenso grupal máximo para determinar los pesos de los decisores, asegurando que los intervalos de eficiencia cruzada agregados sean ampliamente aceptados y reflejen la opinión de cada decisor. Finalmente, las alternativas se clasifican en función de los interva-

los de eficiencia cruzada agregados, lo que lleva a la selección de la opción más óptima. Esta metodología no solo mitiga la pérdida de información durante el proceso de transformación, sino que también mejora la precisión y la racionalidad de TDGL a través de la fusión optimizada de los intervalos de eficiencia cruzada.

(3) En TDGGE, los algoritmos de agrupamiento y PACs son áreas de investigación clave. Muchos PACs existentes, presuponen que los decisores aceptarán los cambios sugeridos y modificarán sus opiniones siempre. Sin embargo, en la práctica, los individuos pueden exhibir comportamientos no cooperativos. Además, los PACs existentes a menudo pasan por alto la precisión e interpretabilidad de la información lingüística, lo que puede perjudica la confiabilidad y claridad de los resultados de la toma de decisiones en aplicaciones prácticas. Para abordar estos desafíos, este estudio propone un marco en TDGGE que incorpora el comportamiento no cooperativo y construye modelos de optimización de consenso basados en la información ELICIT. Primero, el algoritmo de agrupamiento K-Means se adapta para tener en cuenta tanto la información de evaluación proporcionada por los decisores como las relaciones de confianza entre ellos, lo que permite la división de un grupo en subgrupos más pequeños para mejorar la eficiencia computacional. En segundo lugar, se desarrollan modelos de optimización de consenso basados en la información ELICIT que proporcionan sugerencias lingüísticas personalizadas de para ajustar las preferencias de los individuos, lo que garantiza un enfoque de CWW que mejora la claridad e interpretabilidad del PAC. En tercer lugar, se identifican y gestionan los comportamientos no cooperativos, introduciendo mecanismos de penalización adecuados para mitigarlos. Por último, se emplea un enfoque de eficiencia cruzada DEA para clasificar las alternativas en función de la opinión final del grupo, determinando así la mejor opción. Estas mejoras aumentan la transparencia y la interpretabilidad del PAC, proporcionando una solución más fiable para en TDGGE.

(4) En la investigación existente sobre en TDGGE, si bien algunos algoritmos de agrupamiento tienen en cuenta las conexiones existentes en las redes sociales, a menudo pasan por alto que pueden existir estructuras superpuestas dentro de estas redes. Además, muchos modelos de consenso suponen que los decisores aceptarán incondicionalmente los cambios recomendados, obviando el posible rechazo de estos ajustes por parte de los decisores, lo que puede afectar la validez y la satisfacción de la decisión final. Para abordar estas cuestiones, se propone un nuevo método en TDGGE basado en información ELICIT y comunidades superpuestas. Primero, se utiliza el algoritmo del método Lancichinetti-Fortunato

(LFM) para identificar la estructura de las comunidades superpuestas dentro de una red de confianza social, identificando a los decisores que pertenecen a múltiples subgrupos simultáneamente. A continuación, se utiliza el algoritmo PageRank para calcular la influencia de los individuos tanto a nivel global como local, lo que facilita la integración efectiva de las opiniones tanto colectivas como en los subgrupos. A continuación, se desarrollan los modelos de optimización de consenso de dos etapas basados en la confianza limitada que proporcionan sugerencias de ajuste personalizadas para los individuos utilizando información ELICIT, mejorando la aceptación de estas recomendaciones por parte de los decisores. Además, se introducen mecanismos de recomendación para distintos tipos de decisores, que especifican la dirección y la magnitud de los ajustes de opinión. Este enfoque no sólo mejora la interpretabilidad de la información lingüística difusa y dudosa durante el PAC, sino que también aumenta la aceptación de las recomendaciones por parte de los decisores al definir de forma precisa los ajustesde sus opiniones, mejorando así el funcionamiento general del PAC.

A.5 Conclusiones y trabajos futuros

Por último, este capítulo ofrece un resumen exhaustivo de la investigación presentada en esta tesis, repasa las principales propuestas y conclusiones, y esboza posibles direcciones para futuros trabajos basados en los resultados obtenidos.

A.5.1 Conclusiones

Esta investigación tiene como pilar central los problemas de TDG bajo información lingüística dudosa, en entornos complejos. El estudio se centra en etapas clave del proceso de toma de decisiones, como el modelado de la información, la su transformación, fusión, clasificación y los PAC. Además, el número de decisores ha pasado gradualmente de la pequeña escala a entornos a gran escala. El objetivo principal de esta investigación es mejorar sistemáticamente la eficiencia y el rigor de la toma de decisiones, proporcionando soluciones prácticas y un mayor valor de aplicación para abordar los complejos retos de la TDG. Los resultados y conclusiones se resumen a continuación:

(1) Teniendo en cuenta la diversidad de conocimientos y antecedentes de los responsables de la toma de decisiones en distintos campos, se han utilizado CTLDDs multigranulares para capturar la duda en las opiniones de los decisores, reflejando con precisión dichas opiniones y mejorando la flexibilidad lingüística en

escenarios complejos de TDG. Además, la integración de la teoría del arrepentimiento con el modelo de eficiencia cruzada DEA optimiza el proceso de TDG. A diferencia de los métodos tradicionales como TOPSIS y VIKOR, los modelos DEA no requieren información previa, lo que garantiza una evaluación más objetiva y justa. Al tener en cuenta los comportamientos psicológicos de los responsables de la toma de decisiones, como el arrepentimiento y su aversión, este enfoque acerca la evaluación de alternativas a las condiciones del mundo real, mejorando significativamente su aplicabilidad práctica. Con esta propuesta se alcanza el primer objetivo de esta tesis.

(2) Para evitar la pérdida de información crítica que suele producirse al convertir datos lingüísticos en valores numéricos precisos en los métodos de TDG basados en DEA, transformamos la información lingüística dudosa en envolventes difusas y las representamos como TrFNs, preservando la incertidumbre inherente en los datos lingüísticos originales. Posteriormente, se desarrollaron modelos difusos de eficiencia cruzada DEA, considerando todos los valores posibles de entradas y salidas difusas, garantizando así la objetividad y la exhaustividad en la evaluación tanto de las eficiencias cruzadas óptimas como de las peores alternativas. Además, se introdujo un modelo de consenso máximo de grupo, utilizando intervalos de eficiencia cruzada para determinar con precisión los pesos de los decisores, garantizando que el intervalo de eficiencia cruzada global resultante sea ampliamente aceptado por el grupo. Estos modelos DEA mejorados ofrecen herramientas eficaces para el manejo de TDG en entornos difusos, logrando con éxito el segundo objetivo de esta tesis.

(3) Se empleó el esquema ELICIT para gestionar información lingüística difusa dudosa utilizando TrFNs, asegurando que los cálculos siguieran el esquema de CWW. Este esquema mejora la interpretabilidad del proceso de toma de decisiones y asegura que los resultados permanezcan estrechamente alineados con los datos de entrada originales. Además, se han desarrollado dos modelos de optimización de consenso basados en información ELICIT, que ofrecen recomendaciones de ajuste en preferencias específicas para cada subgrupo y decisor. Se implementa un mecanismo de recomendación para identificar y gestionar comportamientos no cooperativos. Al construir un enfoque estructurado e integral para abordar los desafíos de en TDGGE, esta propuesta optimiza el procesamiento de la información lingüística, gestiona de manera efectiva los comportamientos no cooperativos y emplea el método DEA para una clasificación robusta, asegurando así la validez del proceso de toma de decisiones. Estas mejoras ofrecen una solución confiable para problemas complejos de en TDGGE alcanzando con

éxito el tercer objetivo de esta tesis.

(4) Para considerar las estructuras superpuestas dentro de las redes sociales a gran escala, se aplicó el algoritmo LFM que identifica a los decisores que pertenecen a múltiples comunidades. La influencia de cada participante se evaluó global y localmente, destacando las diferencias en sus roles y contribuciones en diferentes niveles. Al combinar la información de ELICIT con la teoría de confianza limitada, se desarrolló un modelo de optimización de consenso de dos etapas, que mejoró en gran medida la aceptación de los ajustes por parte de los individuos y, por lo tanto, aclaró el PAC, creando un entorno de toma de decisiones más transparente e interpretable. El mecanismo de recomendación consideró la influencia de múltiples subgrupos, abordando de manera efectiva el desafio de hacer recomendaciones a decisores que pertenecen a varias comunidades simultáneamente. Estas mejoras hacen más precisas las opiniones de los decisores superpuestos y ofrecen un método sistemático para manejar los desafíos complejos en TDGGE. De esta forma se logra el objetivo final de esta tesis.

Cabe destacar que se han logrado todos los objetivos planteados al inicio de esta tesis, aportando propuestas innovadoras y sentando las bases para futuras investigaciones. Algunas de ellas se describen en la siguiente sección.

A.5.2 Trabajos futuros

En esta sección, se plantean varias propuestas para continuar el trabajo realizado en esta tesis doctoral. Estas posibles líneas de investigación futuras son:

(1) Desarrollar métodos avanzados de TDG basados en DEA capaces de manejar información heterogénea de múltiples fuentes. A medida que la sociedad y la tecnología progresan, los canales a través de los cuales se adquiere información se han vuelto más diversos, lo que da como resultado una gran cantidad de información altamente heterogénea. Las investigaciones futuras deberían centrarse en la estandarización de estos datos heterogéneos de múltiples fuentes y en la creación de métodos DEA adaptados para abordar entornos TDG complejos y dinámicos de manera eficaz.

(2) Desarrollar algoritmos de agrupamiento dinámico para TDGGE. Los escenarios de en TDGGE del mundo real involucran a numerosos decisores, a menudo en entornos inciertos donde la información puede ser incompleta y las relaciones de confianza no siempre están completamente establecidas. Además, a medida que evolucionan las opiniones de los decisores, las relaciones de confianza también pueden cambiar dinámicamente. Por lo tanto, una posible investigación futura

debería centrarse en el diseño de algoritmos de agrupamiento que se adapten a las relaciones de confianza incompletas y a su evolución, incluida la propagación de la confianza, para abordar mejor las complejidades de los escenarios de toma de decisiones reales.

(3) Incorporar la teoría de juegos a los PAC en contextos de TDGGE. En realidad, diferentes grupos pueden perseguir objetivos distintos. Por ejemplo, un moderador puede apuntar a minimizar los costes de la toma de decisiones, mientras que individuos o subgrupos pueden buscar maximizar sus beneficios, lo que lleva a posibles conflictos de intereses. En investigaciones futuras se podría aplicar la teoría de juegos para facilitar el consenso entre grupos, resolver conflictos y mejorar la imparcialidad y racionalidad del proceso de toma de decisiones.

List of Figures

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