

## Dealing with Contradictory Objectives in Energy Communities: a Game-oriented Trilevel Approach

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**Abstract.** Energy communities empower end users to partake actively in the operation of the system while lowering energy procurement through optimal sharing resources. The main objective of energy communities is reducing the collective bill by maximizing the usage of local assets such as photovoltaic and storage systems. However, the different community members may raise particular objectives that may eventually lie in contradiction with the reduction of the electricity cost. For example, prosumers may be interested in incrementing their consumption above a benchmark point in order to increase their comfort and satisfaction. Such contradictory objectives should be considered in energy management of communities in order to ensure its social stability and successful. To this end, a novel game-based trilevel day-ahead approach for cooperative communities is developed, in which two secondary objectives can be accommodated together with the cost minimization original target. As a sake of example, the developed tool tailors in this paper to the case in which prosumers aim at maximizing their consumption while storage pretend to minimize the degradation of assets. The original trilevel structure is reduced to a solvable single-level problem that provide an equilibrium point in the Nash sense. A number of results is provided in 5 and 15-bus cases in order to validate the new approach. Results show that the new proposal can be easily implemented in a variety of scenarios, showing a case-independent performance. The hierarchical decision-logic procedure has been illustrated and validated analysing the total community cost under different users' preferences. Finally, it is shown that the developed methodology scales well with the storage capability and community size.

**Keywords.** Battery; Energy Communities; Prosumer; Sustainable Cities

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## Nomenclature

<i>Indices (sets)</i>	
$i$ ( $\mathbb{I}$ )	Prosumer
$b$ ( $\mathbb{B}$ )	Battery energy storage
$t$ ( $\mathbb{T}$ )	Time
$j, k$ ( $\mathbb{J}$ )	Bus
$\Omega_j$	Prosumers connected to the $j^{\text{th}}$ node in the network
$\Xi_j$	Batteries connected to the $j^{\text{th}}$ node in the network
$\Psi_j/\Phi_j$	Set of nodes downstream/upstream from the $j^{\text{th}}$ node in the network
<i>Superscripts</i>	
$M$	Community manager
$BES, c/d$	Battery energy storage in charging/discharging mode
$F$	Flexible demand
$i/e$	Importing/exporting
$P/Q$	Make mention to active/reactive power flows or injections
$\underline{(\cdot)}/\overline{(\cdot)}$	Minimum/maximum value of a variable or parameter
<i>Parameters</i>	
$\lambda$	Parameter to modulate the weight of objectives (€/kWh)
$D/PV$	Base-demand/photovoltaic generation (kW)
$\eta$	Efficiency (pu)
$C$	Retailer electricity cost (€/kWh)
$R/X$	Branch resistance/inductance (ohm)
$V^0$	Voltage at the root bus (V)
$\sigma$	Active to reactive power conversion factor (kvar/kW)
$\gamma$	Auxiliary parameter to linearize thermal power flow limit (-)
<i>Decision variables (primal)</i>	
$p/q$	Active/reactive power (kW/kvar)
$\varepsilon$	Energy (kWh)
$y$	Commitment status (binary)
$f$	Power flow (kW/kvar)
$V$	Nodal voltage (V)
<i>Decision variables (dual)</i>	
$\varphi/\phi$	Dual variables linked to equality constraints (€/kWh)
$\underline{(\zeta/\bar{\zeta})}/\underline{(\mu/\bar{\mu})}$	Dual variables linked to inequality constraints (€/kWh)

## 1 – Introduction

### 1.1 – Context and Motivation

In the last years, a remarkable increase in the photovoltaic (PV) power installed in residential areas has been observed [1, 2]. In this regard, there is a growing interest on enabling coordination mechanisms among prosumers in order to optimize the use of PV resources. Moreover, intermittent feature of PV systems makes necessary the installation of energy storage in order to reduce power curtailment. In this context, Energy Communities (ECs) emerge as a valuable framework to provide coordination channels among prosumers in order to further harness local resources. This way, ECs offer economic advantages by lowering energy procurement and increase the efficiency of the system reducing renewable curtailment [3].

One of the main barriers to the deployment of ECs is the lack of fair and straightforward coordination mechanisms among community members [4]. In this sense, control schemes between prosumers and the so-called community manager should be developed. Coordination agreements typically pose conflicts among agents. Insofar as reducing collective cost is a common objective fairly agreed among agents, the way in which this target is achieved may result conflicting as prosumers are keen on maximizing their own satisfaction, whereas battery owners may limit the access to storage assets in order to avoid degrading their equipment.

The way in which contrary interests are considered in ECs remains as a grey research topic. This paper focuses on this issue.

### 1.2 – Literature Review

A review of the literature shows that ECs have attracted huge attention in recent years. The number of works focused on energy management and control strategies have boosted up remarkably, encompassing different aspects like storage technologies, local markets, uncertainty modelling or solution techniques.

Energy management in ECs typically casts as an optimization problem seeking for reducing the collective electricity bill. Zhou et al [5] propose an energy management strategy for ECs based on reinforcement learning. The proposed strategy envisages for ECs encompassing smart and non-intelligent homes and establishes a pricing mechanism to encourage local transactions. In [6], coalitions among prosumers are promoted in order to increase their payoff by engaging in local peer-to-peer trading. Theoretical evidences are provided showing that bigger coalitions contribute further to lower procurement. Lilla et al [7] develop an energy management model for cooperative ECs based on the alternating direction of multipliers, reducing information exchanging among prosumers. Different cost allocation mechanisms are proposed in [8], by which the collective costs are particularized among prosumers according to different metrics. The proposed strategies were applied to a real test-case showing that costs can be fairly allocated according the individual demand and PV production. Similarly, Laurini et al [9] propose a design model for renewable energy communities, with the objective of increasing shared energy within the community. The developed optimization model was applied to a real public-private EC in Italy. A solution technique based on conic projection is developed in [10], by which privacy of community members is preserved. Wu and Conejo [11] propose a market-based EC strategy based on looking for equilibrium points. To this end, prosumers and conventional consumers engage in distribution market mechanisms under local marginal prices. In [12], a scalable planning tool for collective PV plants in cooperative ECs is developed. The authors develop a data-driven methodology and propose different reduction strategies such as clustering methods and the application of decomposition techniques.

Intermittent PV generation provoke uncertainty in energy management decisions of ECs, but also local demand is subjected to a high-degree of uncertainty due to unpredictable human decisions. In this sense, different references focus on uncertainty modelling of ECs. Vespermann et al [13] propose a market-oriented model for risk trading in ECs. This proposal uses well-known

metrics (e.g. the conditional value at risk) and the Arrow-Debreu securities to take trading decisions in day-ahead and intraday markets under uncertain PV generation. Orozco et al [14] develop a two-stage scheduling strategy for cooperative ECs, in which day-ahead decisions are derived at the first stage, while intraday decisions are taken at the second stage. Intraday scheduling is performed under uncertainty, modelling the realization of uncertain parameters through scenarios, thus resulting in a stochastic-based procedure. Whereas previous references model uncertain parameters via scenarios, other alternative models have been explored for ECs. Thus, interval optimization was applied to cooperative ECs in [15]. The developed optimization model casts as a multi-stage procedure in which uncertainties in demand and PV generation are modelled using intervals rather than discretized scenarios. Similarly, Information Gap Decision Theory (IGDT) was applied to ECs in [16]. This approach models uncertainties using intervals and parametrized risk levels, resulting in a multi-step procedure. In this case, IGDT was hybridized with conventional stochastic programming. More particularly, wholesale electricity prices were modelled using scenarios, while IGDT was employed to model the inherent uncertainty of electric vehicles, to which a tailored uncertainty modelling based on storage bounds was developed.

Commonly, PV systems are installed individually but storage assets are more economically viable when harnessing collectively [17]. In this sense, [18, 19] studied different strategies for providing individual access to collective battery energy systems (BESs). Particularly, RePro, distributive and proportional strategies were proposed and analysed. Through the RePro approach, each agent accesses to collective assets under different sharing portions. On contrary, the distributive approach establishes that each stakeholder has the same access quota. Finally, the proportional strategy depicts energy equilibrium restrictions by which each agent needs to return the energy imported from collective BESs. Other references consider individual storage assets, developing market-driven strategies to regulate the access to batteries. Particularly, Jo et al [20] propose scheduling mechanism through which storage capability is exchanged by means of bilateral contracts. In [21], different financial mechanisms are modelled and studied to enable access to individual assets. Thus, the authors considered financial and physical storage rights. Through the former, storage systems are collectivized and scheduled in a central fashion, while the latter provides physical access to an individual BES through peer-to-peer (P2P) market trading. Faria et al [22] propose a sizing methodology for ECs including batteries. The model casts as a multi-objective optimization approach, which takes into account self-sufficiency and economy using a heuristic methodology. Likewise, different collective storage schemes are analysed in [23], paying special attention to their role in local markets. Thus, competitive and cooperative storage market arrangements are modelled on the basis of the economical character of the storage provider agent.

Battery systems are, by far, the most common storage solution in residential installations. However, other references have explored other storage technologies. More specifically, [24-27] study hydrogen-based technologies in ECs, mainly installing gaseous vessels. Particularly, ref. [24] focuses on uncertainty modelling, proposing an IGDT-based day-ahead scheduling tool for cooperative ECs involving hydrogen chain formed by electrolysis, fuel-cell and storage. On the other hand, Cheng and Ruiz [25] describe a risk-based local market mechanism wherein where the aggregated shortage or excess of energy within the community can be traded in the presence of hydrogen technologies (electrolysis, storage and fuel-cell vehicles). In [26], different energy management models for near-zero ECs with hybrid energy storage encompassing hydrogen and BESs. Brusco et al, explore in [27] the role of hybrid energy storage in providing ancillary services from ECs. To this end, a day-ahead tool is proposed for ECs encompassing hydrogen and electrical-based storage systems.

### *1.3 – Main Limitations of the Existing Literature*

As commented in Section 1.1, this work focuses on how to integrate contradictory decisions in energy management for in ECs. We gave an illustrative example for the case of prosumers and battery-owners, whose targets may be in conflict with the collective objective of reducing the electricity bill. Indeed, let us assume that a particular prosumer is keen on maximizing her consumption in order to increase her particular satisfaction. This behaviour is clearly in contradiction with the collective objective of reducing the overall energy consumption. Similarly, battery-owners may be reluctant to provide full access to their BESs if it supposes a rapid degradation of their assets, which reduces the energy arbitrage capability of the community and may result in larger power curtailments.

Ignoring such contradiction in particular preferences may result in negative effects. For instance, a particular prosumer could leave the community if her preferences are not attended properly, while rapid degradation of batteries may discourage investments in storage assets. In the literature, this problem is typically ignored or partially addressed. For example, references principally devoted on storage systems [18-21, 27] ignore the degradation of assets or simply include a corresponding maintenance cost. On the other hand, users' preferences are typically hideout behind the collective objective. For example, flexible demands are considered in [15, 24], such as air conditioner units which eventually increase the comfort of users. However, these units are assumed to be operated in a central fashion and therefore the users' preferences are inevitably subordinated to the common objective of lowering energy procurement.

Also, we call the reader attention on the fact that community members undoubtedly perceive some objectives as more important than others. This feature depicts a hierarchical decision framework that should be considered. For the sake of simplicity, Table 1 shows a summary of the relevant literature.

Table 1 – Summary of the relevant literature

<b>Ref.</b>	<b>Model</b>	<b>Objectives</b>	<b>Objectives sorting</b>
[6]	Coalition	Satisfaction	No
[7, 14]	Algorithm	Economy	No
[8, 9, 20, 21, 27]	MILP	Economy	No
[10]	Algorithm	Economy	No
[11, 25]	Equilibrium	Economy	No
[13, 23]	Complementarity	Risk trading	No
[15, 24]	Algorithm	Self-sufficiency Economy	No
[22]	Heuristic	Self-sufficiency Economy	No
[26]	Heuristic	Self-sufficiency Economy Environment	No
Present	MILP	Economy Satisfaction Degradation	Yes

MILP: Mixed-Integer Linear Programming

There are only few works that deal with multiple objectives in ECs. In this sense, the references [15, 22, 24] simultaneously optimize economy and self-sufficiency, whereas [26] also deals with environmental indicators. In all cases, the multi-objective approach requires the identification of the so-called Pareto solutions by means of iterative procedures. For example, the references [15, 22, 26] use the so-called weighting functions while [22] considers a metaheuristic to find up those solutions in the Pareto front. In both cases, such multi-objective models fail to sort objectives or depends excessively of input parameters, being in consequence case-dependent and not easily implementable [28]. Moreover, both [22, 26] use metaheuristic approaches, which may fail to achieve the global optimum. In addition, metaheuristic models are typically case-dependent formulations and their adaptability to different layouts and cases studied is not trivial [29].

Therefore, there is a clear lack in the literature on how to deal with contradictory objectives in ECs. Note, however, that this problematic is frequent in ECs, where prosumers and other agents (e.g. storage providers) are keen on reducing the collective bill, but not at expense of deteriorating their particular targets notably. Hence, one can conclude that the existing literature regarding energy management in ECs fails to address this issue, as seen in Table 1. Actually, the reviewed literature typically focuses on reducing the collective bill. Therefore, other particular objectives are undoubtedly neglected, which may discourage users to partake in the community. Moreover, sorting objectives according to their importance is pivotal in ECs, as the economy should be firstly optimized in any case. In this sense,

#### *1.4 – Main Objectives*

Due to the existence of multiple agents with different objectives in ECs, energy management models need to seek trade-off solutions in order to improve the collective welfare. Therefore, energy management in ECs is, in fact, a multi-objective optimization problem involving different targets. Conventional multi-objective approaches might fail when dealing with contradictory objectives or are, typically, very sensitive to hyperparameters tuning, resulting in case-dependent approaches [28]. In this sense, developing a fair and easily implementable methodology, capable to deal with multiple objectives in contradiction, might result valuable for ECs.

#### *1.5 – Specific Contributions and Paper Organization*

The specific contributions of this paper are numerated below:

- Developing optimization models for prosumers, BESs and community manager envisioned individual objectives. Thus, the proposed prosumers' modelling aims at maximizing users' satisfaction by increasing the energy consumed by flexible loads, assuming that prosumers are firstly keen on satisfying a base-load dedicated to necessary human actions, while a portion of the total demand is rather devoted on increasing the individual comfort (e.g. thermal comfort). On the other hand, the battery modelling considers degradation as a function of the total energy exchanged with the community.
- Integrating the different objectives into a holistic framework through an original tri-level formulation, which aligns with the approach developed in [30]. However, its use in that reference is far away of the use in the considered methodology, which is the main novelty of this work. Indeed, while this reduction technique was employed in [30] for market clearing, it has been used in this paper for dealing with contradictory objectives within ECs. The new proposal presents three salient features. Firstly, it allows to take into consideration the preferences of the different users easily, thus improving existing approaches rather devoted on single-objectives. Secondly, the original tri-level model allows dealing with contradictory objectives easily, reducing the influence or hyperparameter tuning as in conventional multi-objective techniques. Thirdly, the developed tri-level problem allows sorting the different objectives according to their perceived importance. Thus, the objective at the lower level is not satisfied at expenses of the targets above.

In the rest of this paper, Section 2 describes the considered community, states the problem and gives preliminary notations and assumptions. Section 3 develops optimization models for prosumers, battery managers and community manager. Section 4 describes the developed tri-level model for integration of the different agents as well as explains its reduction into a tractable single-level framework. Section 5 provides different case studies with results. Finally, the paper is concluded with Section 6.

## **2 – Preliminaries**

### *2.1 – Assumptions*

We consider a cooperative EC in which local markets are disabled and therefore P2P power exchanges are performed among prosumers without expecting a direct counterpart. In this sense, we assume that community establish some compensatory mechanisms, *a posteriori*, with the object of allocating costs among prosumers. Additionally, it is assumed that prosumers and batteries are connected to the same network within the community, which is operated in a centralized manner by a community manager. Thus, we assume that prosumers are located near each other.

## 2.2 – Basic Notations

We consider a cooperative EC formed by  $i \in \mathbb{I}$  prosumers who install rooftop PV panels. In line with [18, 19], storage is provided collectively through  $b \in \mathbb{B}$  battery systems. Prosumers and BESs are connected to the same network formed by  $j \in \mathbb{J}$  buses. Scheduling decisions are day-ahead taken over a 24-h time horizon divided into  $t \in \mathbb{T}$  time slots. In order to avoid an abuse of notation, we use the same symbols for cardinality and sets, this way  $\mathbb{T} = |\mathbb{T}|$ .

## 2.3 – Community Structure

Fig. 1 sketches the structure of the considered EC and the agents involved, as well as the main interactions among them. Scheduling decisions are determined by the community manager, who seeks for minimizing the overall energy cost by trading energy with the distribution network. Due to the direct participation of ECs in wholesale markets is rather unlikely [13], we assume that transactions with the grid are agreed with a local retailer under time-of-use (TOU) tariffs.

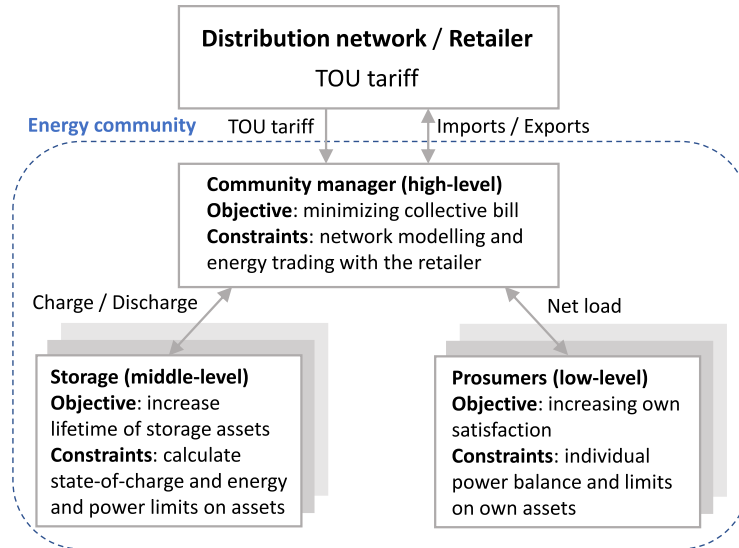


Fig. 1 – Main agents involved in the considered EC as well as their objectives and main interactions

The community manager is also responsible of operating the local network, in order to ensure its safe operation while maintaining the quality of service above an acceptable limit. Thereby, the community manager aims at satisfying the prosumers' demand at minimum cost, for which can leverage energy arbitrage provided by local storage assets. We assume that storage capability is provided collectively but partaking as independent agents who pursue their own objectives. In this case, we consider a virtual agent called storage-provider who only focus on maximizing the lifetime of batteries. In this regard, we consider that battery degradation is a function of the energy exchanged with the community [31].

Finally, the prosumers aim at maximizing their own satisfaction by increasing their consumption above a base load. In this sense, we consider that total prosumers' demand is divided into two types of loads. On the one hand, base demand must be satisfied entirely in order to ensure a minimum level of comfort. On the other hand, the so-called flexible demand rather increases the users' satisfaction above minimum requirements. Note that this idea aligns with the concept

of non-critical or thermostatically-controlled appliances typically considered in home energy management systems [32]. This way, we assume that the users' satisfaction is directly proportional to the amount of flexible demand satisfied.

It is worth noting that the objectives of prosumers and the storage-provider lies in contradiction with the objective raised by the community manager. The main aim of this paper is developing an energy management tool for ECs capable to deal with such contradictory targets. In this paper, we have considered users' satisfaction and battery lifetime expansion as secondary objectives, but the philosophy followed in this paper could be applied to any other casuistry like peak-shaving or strategic energy reserve [33].

#### 2.4 – Problem Statement

Considering that  $\mathbf{x}^M$  and  $\mathbf{y}^M$  are the continuous and binary variables of the manager, respectively; while  $\mathbf{x}_b$  and  $\mathbf{x}_i$  are the continuous variables of BESs and prosumers, respectively, the tri-level problem to be solved has the structure below:

$$\min_{\mathbf{x}^M, \mathbf{y}^M} g^M \quad (1a)$$

Subject to:

$$\mathbf{A}^M \mathbf{x}^M + \mathbf{B}^M \mathbf{y}^M + \mathbf{b}^M = \mathbf{0} \quad (1b)$$

$$\mathbf{F}^M \mathbf{x}^M + \mathbf{G}^M \mathbf{y}^M + \mathbf{d}^M \leq \mathbf{0} \quad (1c)$$

$$\mathbf{x}^M \in \mathcal{R}^{m_1}, \mathbf{y}^M \in \mathcal{Z}^{m_2} \quad (1d)$$

$$\min_{\mathbf{x}_b} g^{BES} \quad (1e)$$

Subject to:

$$\mathbf{A}_b \mathbf{x}_b + \mathbf{b}_b = \mathbf{0} : \quad \boldsymbol{\varphi}_b; \forall b \in \mathbb{B} \quad (1f)$$

$$\mathbf{F}_b \mathbf{x}_b + \mathbf{d}_b \leq \mathbf{0} : \quad \boldsymbol{\mu}_b; \forall b \in \mathbb{B} \quad (1g)$$

$$\mathbf{x}_b \in \mathcal{R}^{m_3}; \quad \forall b \in \mathbb{B} \quad (1h)$$

$$\min_{\mathbf{x}_i} g_i; \quad \forall i \in \mathbb{I} \quad (1i)$$

Subject to:

$$\mathbf{A}_i \mathbf{x}_i + \mathbf{b}_i = \mathbf{0} : \quad \boldsymbol{\phi}_i; \quad \forall i \in \mathbb{I} \quad (1j)$$

$$\mathbf{F}_i \mathbf{x}_i + \mathbf{d}_i \leq \mathbf{0} : \quad \boldsymbol{\zeta}_i; \quad \forall i \in \mathbb{I} \quad (1k)$$

$$\mathbf{x}_i \in \mathcal{R}^{m_4}; \quad \forall i \in \mathbb{I} \quad (1l)$$

where  $\mathbf{A}$ ,  $\mathbf{B}$ ,  $\mathbf{F}$  and  $\mathbf{G}$  are matrix of real coefficients, whereas  $\mathbf{b}$  and  $\mathbf{d}$  are vectors of independent real coefficients.

The high-level (1a)-(1d) conforms the Mixed-Integer Linear Programming (MILP) model of the manager, who aims at minimizing the EC procurement cost and encompasses the network modelling, thermal power flow limits, voltage constraints and nodal power balances. The middle-level (1e)-(1h) responds to the storage provider model aiming at expanding the lifetime of assets including state-of-charge (SOC) dynamics. Finally, the low-level (1i)-(1l) corresponds to prosumers, who are keen on maximizing their own satisfaction and are capable to manage own assets such as PV rooftop panels or flexible demand.

In (1), the objectives are shortened according their perceived importance, thus, we consider that community members are firstly keen on reducing the collective bill, while battery lifetime expansion and users' satisfaction are considered secondly and thirdly, respectively. It is worth noting that the middle and low levels are linear and therefore can be replaced by their equivalent first-order optimality conditions [34], for which the dual variables are shown at the right-hand side of their corresponding constraints.

### 3 – Mathematical Models

#### 3.1 – Modelling of Prosumers

The detailed modelling of prosumers writes as:

$$\max_{\mathbf{x}_i} \sum_{t \in \mathbb{T}} \lambda^F p_{i,t}^F \quad (2a)$$

Subject to:

$$p_{i,t} + PV_{i,t} - D_{i,t} - p_{i,t}^F = 0: \phi_{i,t}; \forall t \in \mathbb{T} \quad (2b)$$

$$\sum_{t \in \mathbb{T}} p_{i,t}^F \leq \bar{\varepsilon}_i^F: \bar{\zeta}_i^{\varepsilon^F} \quad (2c)$$

$$0 \leq p_{i,t}^F \leq \bar{p}_{i,t}^F: \underline{\zeta}_{i,t}^F, \bar{\zeta}_{i,t}^F; \forall t \in \mathbb{T} \quad (2d)$$

$$-\bar{p}_{i,t} \leq p_{i,t} \leq \bar{p}_{i,t}: \underline{\zeta}_{i,t}, \bar{\zeta}_{i,t}; \forall t \in \mathbb{T} \quad (2e)$$

where  $\mathbf{x}_i = [p_{i,t}, p_{i,t}^F]$ .

The prosumers aim at maximizing their total flexible demand in (2a), assuming that flexible loads are devoted on increasing the perceived satisfaction. Note that this assumption aligns with real-life cases, as controllable (i.e. flexible) loads encompass thermostatically controlled devices or similar. In this regard, we assume that more comfort can be achieved by directly increasing the consumption of flexible loads. On the other hand, we consider a base-load  $D$  that needs to be necessarily satisfied. This parameter coherently includes critical or uncontrollable loads such as fridges, lighting or television. (2b) ensures individual power balance, including the action of the rooftop PV panels, while (2c) ensures that the total flexible demand is below a given limit, thus ensuring physically realizable solutions and avoiding unbounded problems. Finally, (2d) and (2e) bind instantaneous flexible demand and power exchanged with the community, respectively.

Note that the objective function is parameterized via  $\lambda^F$ . This parameter is tuned by the manager and allows modifying the importance given to the users' satisfaction with respect to the other objectives in (1). Actually,  $\lambda^F$  can be considered as an artificial satisfaction cost (€/kWh). In addition, as the proposed model minimizes energy cost in the community at the high-level, maximization of self-consumption is ensured by assuming that renewable energy has not marginal cost. Thus, despite that the objective of the prosumers do not specifically maximize self-consumption, this is actually maximized at a higher level, when optimizing the cost in the community. Finally, it is worth noting that (2) is a linear programming (LP).

#### 3.2 – Storage Provider Modelling

Battery degradation can be formulated as a quadratic function of the total energy exchanged (total discharged plus total charged) [35]. However, assuming a linear dependency results plausible in many conditions, as demonstrated in [31, 36-38]. Keeping this on mind, the model for the storage provider reads as:

$$\min_{\mathbf{x}_b} \lambda^{BES} \sum_{t \in \mathbb{T}} \sum_{b \in \mathbb{B}} \left\{ p_{b,t}^{BES,c} \eta^{BES} + \frac{p_{b,t}^{BES,d}}{\eta^{BES}} \right\} \quad (3a)$$

Subject to:

$$\varepsilon_{b,t}^{BES} = \varepsilon_{b,t-1}^{BES} + p_{b,t}^{BES,c} \eta^{BES} - \frac{p_{b,t}^{BES,d}}{\eta^{BES}}: \varphi_{b,t}^{BES}; \forall b \in \mathbb{B} \wedge t \in \mathbb{T} \setminus \{1\} \quad (3b)$$

$$\varepsilon_{b,(t=1)}^{BES} = \varepsilon_{b,(t=\mathbb{T})}^{BES} = 0: \varphi_{b,(t=1)}^{BES}, \varphi_b^{BES,end}; \forall b \in \mathbb{B} \quad (3c)$$

$$0 \leq \varepsilon_{b,t}^{BES} \leq \bar{\varepsilon}_b^{BES}: \underline{\mu}_{b,t}^{BES}, \bar{\mu}_{b,t}^{BES}; \forall b \in \mathbb{B} \wedge t \in \mathbb{T} \quad (3d)$$

$$0 \leq p_{b,t}^{BES,c} \leq \bar{p}_b^{BES}: \underline{\mu}_{b,t}^{BES,c}, \bar{\mu}_{b,t}^{BES,c}; \forall b \in \mathbb{B} \wedge t \in \mathbb{T} \quad (3e)$$

$$0 \leq p_{b,t}^{BES,d} \leq \bar{p}_b^{BES}: \underline{\mu}_{b,t}^{BES,d}, \bar{\mu}_{b,t}^{BES,d}; \forall b \in \mathbb{B} \wedge t \in \mathbb{T} \quad (3f)$$

where  $\mathbf{x}_b = [p_{b,t}^{BES,c}, p_{b,t}^{BES,d}, \varepsilon_{b,t}^{BES}]$ .

The objective (3a) expresses the total energy exchanged by all the batteries connected to the EC. Thereby and assuming the conclusions in [31, 36-38], the objective (3a) can be considered to estimate the total battery degradation. Note that (3a) is parameterized by  $\lambda^{BES}$  in a similar way to (2a). (3b) models the SOC dynamics as a function of the SOC at the previous time instant and the total energy exchanged. Note that (3b) includes battery inefficiency, which has been considered the same for charging and discharging, as customary for Li-ion batteries [36]. On the other, (3c) fixes the initial and final SOC to avoid battery depletion and complete the model (3b). Finally, (3d)-(3f) bind variables according to rated values. Specifically, total energy stored is limited by the total capacity of batteries in (3e), while (3e) and (3f) limit charging and discharging power, respectively. It is worth noting that rated power of battery assets typically depends on the total storage capacity and the so-called energy-to-power ratio [36], which depends on the battery technology, mainly.

Note that neglecting binary commitment statuses in storage systems may result in unrealizable solutions with simultaneous charging-discharging statuses [39]. To solve this issue, we will include such binary variables in the manager modelling instead of (3), thus preserving its linearity.

### 3.3 – Community Manager

The community manager aims at minimizing the collective energy cost while exchanging power with the distribution network, thus resulting in the following model:

$$\min_{\mathbf{x}^M, \mathbf{y}^M} \sum_{t \in \mathbb{T}} \{C_t^i p_t^i - C_t^e p_t^e\} \quad (4a)$$

Subject to:

$$0 \leq p_t^i \leq y_t^P \bar{p}^i; \forall t \in \mathbb{T} \quad (4b)$$

$$0 \leq p_t^e \leq (1 - y_t^P) \bar{p}^e; \forall t \in \mathbb{T} \quad (4c)$$

$$0 \leq q_t^i \leq y_t^Q \bar{q}^i; \forall t \in \mathbb{T} \quad (4d)$$

$$0 \leq q_t^e \leq (1 - y_t^Q) \bar{q}^e; \forall t \in \mathbb{T} \quad (4e)$$

$$0 \leq p_t^{BES,c} \leq y_{b,t}^{BES} \bar{p}_b^{BES}; \forall b \in \mathbb{B} \wedge t \in \mathbb{T} \quad (4f)$$

$$0 \leq p_t^{BES,d} \leq (1 - y_{b,t}^{BES}) \bar{p}_b^{BES}; \forall b \in \mathbb{B} \wedge t \in \mathbb{T} \quad (4g)$$

$$p_t^i - p_t^e = f_{(j=0),t}^P; \forall t \in \mathbb{T} \quad (4h)$$

$$q_t^i - q_t^e = f_{(j=0),t}^Q; \forall t \in \mathbb{T} \quad (4i)$$

$$f_{j,t}^P = \sum_{i \in \Omega_j} p_{i,t} + \sum_{b \in \Xi_j} (p_t^{BES,c} - p_t^{BES,d}) + \sum_{k \in \Psi_j} f_{k,t}^P; \forall j \in \mathbb{J} \setminus \{0\} \wedge t \in \mathbb{T} \quad (4j)$$

$$f_{j,t}^Q = \sum_{i \in \Omega_j} q_{i,t} + \sum_{k \in \Psi_j} f_{k,t}^Q; \forall j \in \mathbb{J} \setminus \{0\} \wedge t \in \mathbb{T} \quad (4k)$$

$$\sqrt{(f_{j,t}^P)^2 + (f_{j,t}^Q)^2} \leq \bar{f}_j; \forall j \in \mathbb{J} \wedge t \in \mathbb{T} \quad (4l)$$

$$V_{j,t} = \sum_{k \in \Phi_j} V_{k,t} - \frac{R_j f_{j,t}^P + X_j f_{j,t}^Q}{V_{(j=0),t}}; \forall j \in \mathbb{J} \setminus \{0\} \wedge t \in \mathbb{T} \quad (4m)$$

$$V_{(j=0),t} = V^0; \forall t \in \mathbb{T} \quad (4n)$$

$$0.95V^0 \leq V_{j,t} \leq 1.05V^0; \forall j \in \mathbb{J} \wedge t \in \mathbb{T} \quad (4o)$$

$$q_{i,t} = p_{i,t} \sigma; \forall i \in \mathbb{I} \wedge t \in \mathbb{T} \quad (4p)$$

$$\mathbf{y}^M \in \{0,1\} \quad (4q)$$

where  $\mathbf{x}^M = [p_t^i, p_t^e, f_{j,t}^P, f_{j,t}^Q, V_{j,t}]$  and  $\mathbf{y}^M = [y_t^P, y_t^Q, y_{b,t}^{BES}]$ .

The objective (4a) stands for the electricity community cost as a function of the energy traded with the distribution network under known TOU tariffs. Constraints (4b)-(4e) limit the active and reactive power exchanged with the distribution network, including complementarity conditions

in order to avoid simultaneous imports and exports. Likewise, (4f) and (4g) are analogue to (3e) and (3f) but including binary commitment in order to avoid simultaneous charges and discharges.

The group of constraints (4h)-(4k) model the local network using the well-known LinDistFlow model [40]. In particular, (4h) and (4i) establish active and reactive power flows at the root node (distribution network) while (4j) and (4k) ensure power balances in the rest of the buses composing the network. (4l) imposes thermal power flow limits across branches while (4m) calculates nodal voltages as a function of power flows and lines resistance and inductance. The voltage at the root node is fixed by (4n) while voltages at the rest of nodes are enforced to be within acceptable limits in (4o). (4p) assumes that reactive power from prosumers is a fixed portion ( $0 \leq \sigma \leq 1$ ) of their net active power, in line with [4], while binary variables are properly declared in (4q).

Model (4) is a MILP except for the thermal power flow limits (4l), which are nonlinear. To linearize these constraints, we use the inner polygon approach described in [41]. Through this linearization trick, the set of constraints (4l) can be replaced by the following linear relationship:

$$\gamma_n^P f_{j,t}^P - \gamma_n^Q f_{j,t}^Q - \gamma_n^F \bar{f}_j \leq 0; \forall j \in \mathbb{J} \wedge t \in \mathbb{T} \wedge n \in \mathbb{N} \quad (5)$$

The parameters  $\gamma$  can be easily calculated following the guidelines in [42]. Note that (5) is not a relaxation but an inner approximation, establishing an equivalency with (4l) in such a way that enforcing (5) is equivalent to enforce (4l) and therefore both set of constraints are interchangeable.

## 4 – Solution of the Proposed Trilevel Model

### 4.1 – Foundations

Solving multi-level optimization problems is NP hard and unaffordable for off-the-shelf solvers. This is due, mainly, to the coexistence of various objective functions which forces to use equilibrium or game-based approaches seeking for coherent solutions. The most conventional strategy for solving bilevel optimization problems is replacing the low-level by its equivalent first-order optimality conditions. Indeed, first-order optimality conditions constitute a set of equations with objective function, whose resolution, eventually, leads to the same solution of the original problem and therefore both formulations are exchangeable. Uniquely, a condition of convexity is required in the original optimization problem to be equivalent to its optimality conditions.

However, the proposed methodology deals with a tri-level problem and therefore this idea is not directly implementable. The main issue is the presence of complementarity nonlinear conditions in the optimality conditions, whose makes them non-convex in nature [43]. Therefore, once the low-level has been reduced to a single-level model using its first-order optimality equations, it cannot be further reduced due to its non-convexity and thus the resulting framework remains as a bi-level model.

To circumvent this issue and in line with [30], we propose to replace the low-level problem by its equivalent primal-dual formulation [44]. This approach replaced complementarity terms by the so-called strong duality equality, which forces primal and dual objectives to take the same value. As said in [30, 44], this primal dual formulation is equivalent to the first-order optimality, but avoiding the use of complementarity constraints. This way, the joint middle-low level remains convex and can be further reduced, thus resulting in a tractable single-level Mathematical Problem with Equilibrium Constraints (MPEC). For simplicity, Fig. 2 sketches the proposed solution strategy.

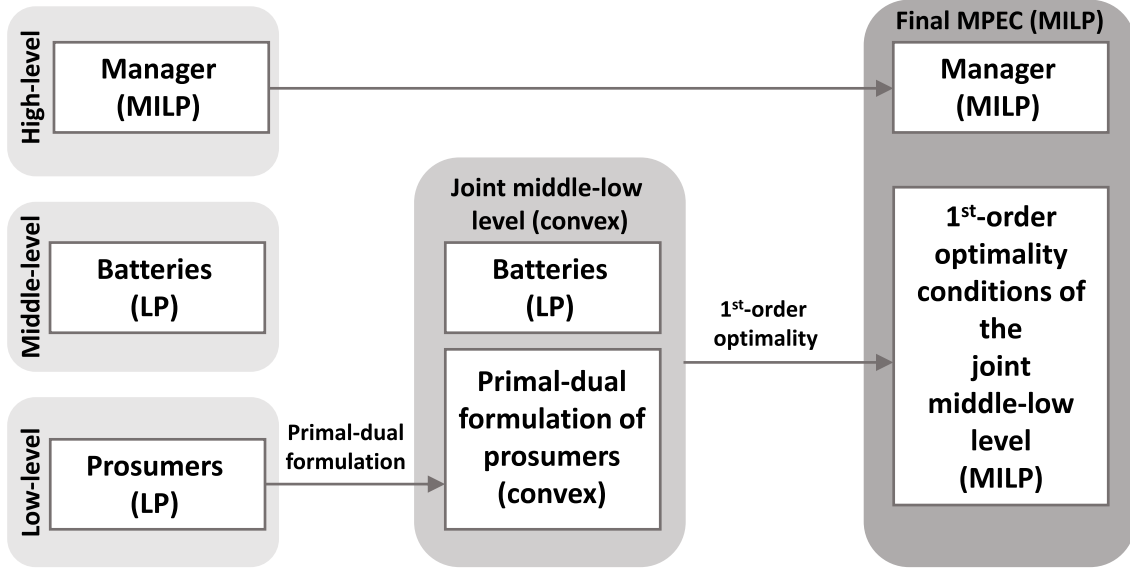


Fig. 2 – Sketch of the proposed solution of the trilevel model (1)

It is worth noting that (1), together with the solution strategy adopted, results in an equilibrium problem nesting two Stackelberg games [45]. Firstly, the storage provider acts as the leader in the joint middle-low level, while prosumers play as followers. On the other hand, the manager plays as the leader in the single-level MPEC. Thereby, objectives of lower levels are subordinated to those at higher-levels, thus responding to the hierarchical structure discussed in Section 2.

Below, we present the mathematical formulation corresponding to the joint middle-low level and the final MPEC. In order to ease the notation, formulation below is presented in compact form, following the notation of (1), whereas detailed formulation is provided in Appendix A.

#### 4.2 – Primal-dual Formulation of the Low-level

The primal-dual formulation of the low-level (prosumers) writes as:

$$\mathbf{A}_i \mathbf{x}_i + \mathbf{b}_i = \mathbf{0}; \varphi_i^{(2)} \quad (6a)$$

$$\lambda^F + \mathbf{F}_i^T \boldsymbol{\zeta}_i + \mathbf{A}_i^T \boldsymbol{\phi}_{i,t} = \mathbf{0}; \boldsymbol{\varphi}_i; \forall t \in \mathbb{T} \quad (6b)$$

$$\boldsymbol{\zeta}_i \geq \mathbf{0}; \boldsymbol{\mu}_i \quad (6c)$$

$$\boldsymbol{\phi}_{i,t}; \text{ free}; \forall t \in \mathbb{T} \quad (6d)$$

$$g_i = -(\mathbf{b}_i^T \boldsymbol{\phi}_{i,t} + \mathbf{d}_i^T \boldsymbol{\zeta}_i); \varphi_i^{SD} \quad (6e)$$

where  $\boldsymbol{\zeta}_i = [\underline{\zeta}_{i,t}, \bar{\zeta}_{i,t}, \underline{\zeta}_{i,t}^F, \bar{\zeta}_{i,t}^F, \bar{\zeta}_i^{\varepsilon F}]$  and  $\boldsymbol{\mu}_i = [\underline{\mu}_{i,t}, \bar{\mu}_{i,t}, \underline{\mu}_{i,t}^F, \bar{\mu}_{i,t}^F, \bar{\mu}_i^{\varepsilon F}]$ . The primal-dual formulation of (2) is formed by the primal feasibility constraints (6a), its stationary conditions (6b), the dual feasibility constraints (6c) and (6d), and the strong duality equality (6d). For a further explanation about the primal-dual formulation, the reader is referred to references [30, 43].

#### 4.3 – The Joint Middle-low Level

The joint middle-low level reads as:

$$\min_{\mathbf{x}_i, \mathbf{x}_b, \boldsymbol{\phi}_{i,t}, \boldsymbol{\zeta}_i} g^{BES} \quad (7a)$$

Subject to:

$$(3b)-(3f) \quad (7b)$$

$$(6); \forall i \in \mathbb{I} \quad (7c)$$

As seen, (7) shares objective with the storage provider model (3). This is due to the joint middle-low level problem is actually a Stackelberg game model wherein the storage provider acts

as the leader. Thus, the prosumers' model is actually replaced by its equivalent primal-dual formulation (6), while the rest of constraints are the same as in (3). Note that (7) expands the decision space by adding the primal and dual variables of the prosumers' model (2).

Next, the joint middle-low level (7) needs to be further reduced in order to reduce all the framework (1) into a single-level model. As seen, (7) is linear and therefore equivalent to its first-order optimality conditions given below:

$$\mathbf{A}_i \mathbf{x}_i + \mathbf{b}_i = \mathbf{0}; \forall i \in \mathbb{I} \quad (8a)$$

$$g_i = -(\mathbf{b}_i^\top \phi_{i,t} + \mathbf{d}_i^\top \zeta_i); \forall i \in \mathbb{I} \quad (8b)$$

$$\mathbf{A}_b \mathbf{x}_b + \mathbf{b}_b = \mathbf{0}; \forall b \in \mathbb{B} \quad (8c)$$

$$\varphi_i^{(2)} = 0; \forall i \in \mathbb{I} \quad (8d)$$

$$\varphi_i^{(2)} - \lambda^F \varphi_i^{SD} = 0; \forall i \in \mathbb{I} \quad (8e)$$

$$\lambda^{BES} + \mathbf{F}_b^\top \boldsymbol{\mu}_b + \mathbf{A}_b^\top \boldsymbol{\varphi}_b = 0; \forall b \in \mathbb{B} \quad (8f)$$

$$0 \leq \boldsymbol{\mu}_i \perp \zeta_i \geq 0; \forall i \in \mathbb{I} \quad (8g)$$

$$0 \leq \mathbf{d}_b - \mathbf{F}_b \mathbf{x}_b \perp \boldsymbol{\mu}_b \geq 0; \forall b \in \mathbb{B} \quad (8h)$$

$$\boldsymbol{\mu}_i, \zeta_i, \boldsymbol{\mu}_b \geq \mathbf{0} \quad (8i)$$

$$\boldsymbol{\varphi}: \text{free} \quad (8j)$$

$$\text{where} \quad \boldsymbol{\mu}_b = \left[ \underline{\mu}_{b,t}^{\varepsilon^{BES}}, \overline{\mu}_{b,t}^{\varepsilon^{BES}}, \underline{\mu}_{b,t}^{BES,c}, \overline{\mu}_{b,t}^{BES,c}, \underline{\mu}_{b,t}^{BES,d}, \overline{\mu}_{b,t}^{BES,d} \right], \quad \boldsymbol{\varphi} =$$

$$\left[ \varphi_{i,t}, \varphi_{i,t}^F, \varphi_{i,t}^{SD}, \varphi_{i,t}^{(2)}, \varphi_{b,t}^{BES}, \varphi_b^{BES,end} \right].$$

The first-order optimality conditions of the joint middle-low level encompasses the primal feasibility conditions (equality constraints) of the low-level (8a), the strong duality equality of the low-level (8b), the primal feasibility conditions of the middle-level (8c), the stationary conditions (8d)-(8f), the complementarity conditions (8g) and (8h) and the dual feasibility conditions (8i) and (8j). For a more detailed explanation about how to derive first-order optimality conditions of convex problems, the reader is referred to [46, 47].

Complementarity conditions are nonlinear but can be linearized using the well-known big-M approach [44]. Typically, tuning the value of the big-M's is challenging and case-dependent [48]. However, in our particular problem, they can be tuned arbitrarily large ( $\sim 10^6$ ) without altering the results.

#### 4.4 – Final Single-level Model

In the resulting MPEC model, the community manager acts as the leader, while the prosumers and storage provider play as followers. In this sense, the final single-level problem incorporates the high-level problem completely, including its objective function, while the followers are included via their optimality conditions, as explained in previous sections. Thus, the resulting single-level MPEC reads as:

$$\min_{\substack{x_i, x_b, x^M, y^M, \\ \phi_{i,t}, \zeta_i, \boldsymbol{\varphi}, \boldsymbol{\mu}}} \sum_{t \in \mathbb{T}} \{C_t^i p_t^i - C_t^e p_t^e\} \quad (9a)$$

Subject to:

$$(2b); \forall i \in \mathbb{I} \quad (9b)$$

$$(3b), (3c) \quad (9c)$$

$$(4b)-(4q) \quad (9d)$$

$$(8) \quad (9e)$$

The model (9) shares objective with the community manager, who acts as the leader of the Stackelberg game formed by the manager, storage provider and prosumers. (9b) and (9c) include equality constraints of the low and middle levels, respectively, in order to ensure their feasibility. (9d) collects the manager's model with all the constraints in (4), while (9e) gathers the first-order

optimality conditions of the joint middle-low level. Note that the decision space of (9) incorporates primal and dual variables of all the levels forming the trilevel framework (1).

Since (9) constitutes a Stackelberg game model, its solution constitutes a Nash equilibrium point among players. In this sense, the solution of (9) firstly minimizes its own objective (i.e. the community cost) but the other objectives are intrinsically considered. Thus, the equilibrium point attained is assumed to be a trade-off solution among players, but preserving the original tri-level hierarchical structure, thus allowing to sort objectives.

After linearizing the complementarity conditions, the single-level optimization model (9) is a MILP easily manageable by off-the-shelf solvers and conventional computers.

## 5 – Cases Studied

This section presents a variety of results in different cases studied. To this end, the developed optimization model (9) was implemented in Matlab R2021b and solved using Gurobi [49]. All the simulations were run on an Intel Core i7-10700K CPU 3.80GHz 3.79 GHz with 32 GB RAM.

We will validate the developed methodology on two different cases. Firstly, an illustrative case is studied. This case involves only three prosumers and a BES, connected to a small network with 5 buses. Secondly, the developed methodology is further validated on a more real-life community with 15 buses, with 11 prosumers and 3 BES. Thereby, the new proposal is validated on a variety of scenarios and systems, taking into account that, typically, communities involve a few prosumers [13] and larger communities are infrequent or even infeasible from a management or legislative perspective.

### 5.1 – 5-bus Case

We start with the small-scale 5-bus case depicted in Fig. 3. The per-unit PV potential, individual base demand and TOU tariff are the same as in [23] and plotted in Fig. 4 for the sake of self-sufficiency. The exporting price was considered 0.7 times the importing one [50], while the data regarding flexible demands are collected in Table 2. The system installs a 10-kWh BES at bus #2, with an efficiency of 95 % and a rated power of 5 kW, typical parameters of Li-ion batteries [51]. Table 1 also shows PV data of each prosumer, who can exchange 10 kW with the community, while  $\sigma = 0.15$ .

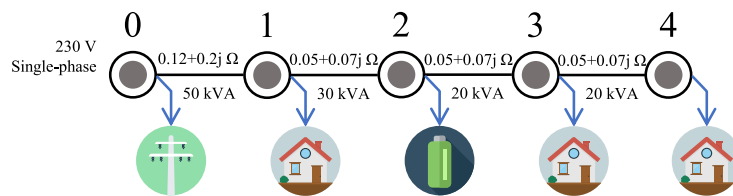


Fig. 3 – The 5-bus network considered in simulations

Table 2 – Prosumer data for the 5-bus case

# bus	Flexible (kWh)	Flexible (kW)	PV rated (kW)
1	7	2.5	2.5
3	5	1.5	4.5
4	3	0.5	1.5

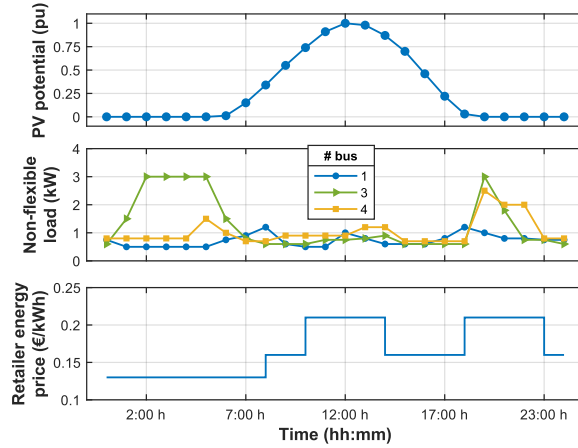


Fig. 4 – Per-unit PV potential (top), individual base demand (middle) and TOU tariff (bottom) considered in the 5-bus case

a) *Assessing the effect of the  $\lambda$ 's*: the parameters  $\lambda^F$  and  $\lambda^{BES}$  allow giving relative importance to the prosumers and storage provider objectives, respectively. In this regard, Fig. 5 plots the total flexibility demand satisfied and energy exchanged by batteries for various values of these parameters. As expected, the impact of the  $\lambda$ 's on particular targets is clear, increasing the value of each objective function as each corresponding  $\lambda$  increases. Specifically, when  $\lambda^F$  takes high values, the level of flexible demand satisfied grows thus enhancing the users' satisfaction. The effect is similar in the case of  $\lambda^{BES}$ , reducing the energy exchanged by batteries (and therefore reducing their degradation) as the value of this parameter increases. It is worth noting that the value of objectives is not highly sensitive to the value of the  $\lambda$ 's. As observed, both objectives keep stable for a wide range of values of the  $\lambda$ 's. This feature differentiates our methodology of others based on optimal or Pareto front solutions, as discussed later.

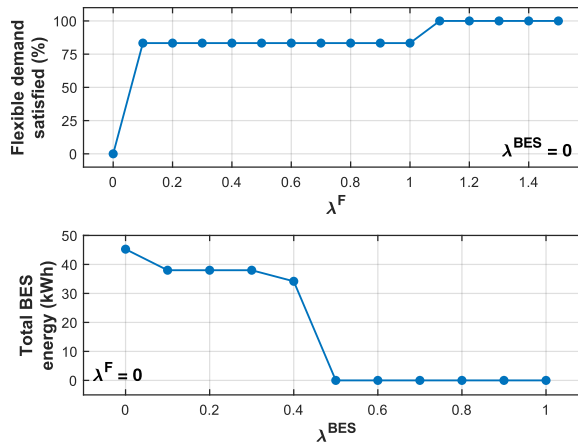


Fig. 5 – Total flexible demand satisfied (top) and energy exchanged by BESs (bottom) for various values of the  $\lambda$ 's in the 5-bus case

b) *Community cost*: the proposed energy management tool allows sorting objectives according to their perceived importance. In our particular approach, the high-level objective (and therefore the most important one) is the total community cost. To better analyse this point, Fig. 6 shows the total EC cost for different values of the  $\lambda$ 's. As observed, when secondary objectives gain importance (i.e. the value of the  $\lambda$ 's increase), the community cost grows. This result clearly demonstrates that the objectives raised by the low and middle levels are in contradiction with the objective of the manager, which has motivated the developed game-based structure. It is important

to see that the results in Fig. 5 keeps quite stable with the value of the  $\lambda$ 's. It demonstrates that the developed tool prioritizes the primary objective (i.e. the EC cost) against the others, until a particular value of the  $\lambda$ 's achieves a critical importance for a secondary objective. Then, the developed tool seeks for an equilibrium point in which the secondary objective improves according to its increasing level of importance, at expenses of increasing the cost. Thereby, the final results do not present a high degree of volatility with respect the value of the weights  $\lambda$ . Note that this is an important drawback of other multi-objective approaches, as explained later on. Moreover, one can see that both weights are tuned in the same scale regardless the relative value of each objective.

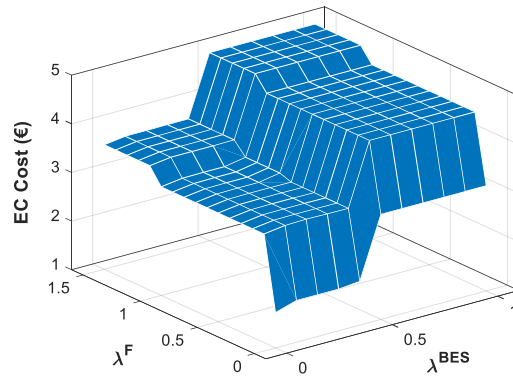


Fig. 6 – Total community cost for various values of the  $\lambda$ 's in the 5-bus case

c) *Effect of flexible demand on voltage profile*: as seen in Fig. 5, increasing the value of  $\lambda^F$  leads to give more importance to users' satisfaction. In our particular problem, the users' satisfaction is measured by the amount of flexible demand satisfied, assuming that users gain in comfort by consuming more. Evidently, increasing the value of  $\lambda^F$  entails a higher consumption in the community, with the consequence impact on the community network. To validate the network modelling employed in this paper, Fig. 7 plots nodal voltages for different values of  $\lambda^F$  representing different equilibrium points. As seen, as the value of  $\lambda^F$  increases, the voltage profile deteriorates notably, as expected. It is also worth noting that voltage decreases monotonously across the network due to its radiality.

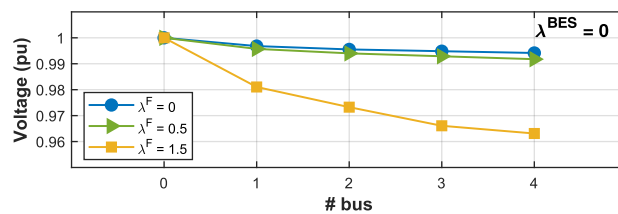


Fig. 7 – Nodal voltages for various values of  $\lambda^F$  in the 5-bus case

d) *Effect of battery capacity*: Fig. 8 shows the value of the storage provider's objective for different value of  $\lambda^{BES}$  and total storage capacity. To this end, the total capacity and rated power were multiplied by a real factor  $\rho$ . As observed, the different profiles are similar and follow similar trends to that shown at the bottom of Fig. 5. This result demonstrates that the developed tool effectively attains equilibrium points for various storage sizes, proving its scalability. This feature allows tuning the value of  $\lambda^{BES}$  coarsely even for different storage capacities, thus further demonstrating its simple implementation. Moreover, it is worth noting that the total energy exchanged falls to zero when  $\lambda^{BES} = 0.5$  in all the studied cases, thus demonstrating that the developed methodology is rather based on finding equilibrium points among players.

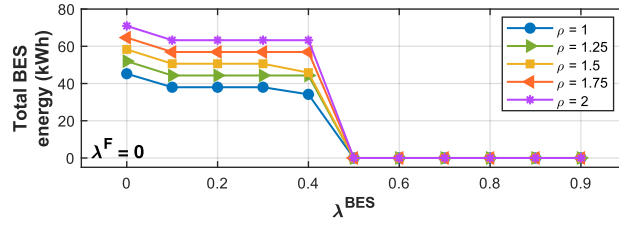


Fig. 8 – Total energy exchanged by batteries for various values of  $\lambda^{BES}$  and battery sizes in the 5-bus case

### 5.2 – 15-bus Case

Next, we further validate the developed tool in a larger 15-bus network as plotted in Fig. 9. This network is a modified version of those grid models used in [4, 41]. The parameters are the same as in [52] except for thermal limits, which were adapted to the particular loads in this study. PV peak powers and flexible load features are shown in Table 3. Three batteries are installed at buses #1, #5 and #14, whose data are collected in Table 4.

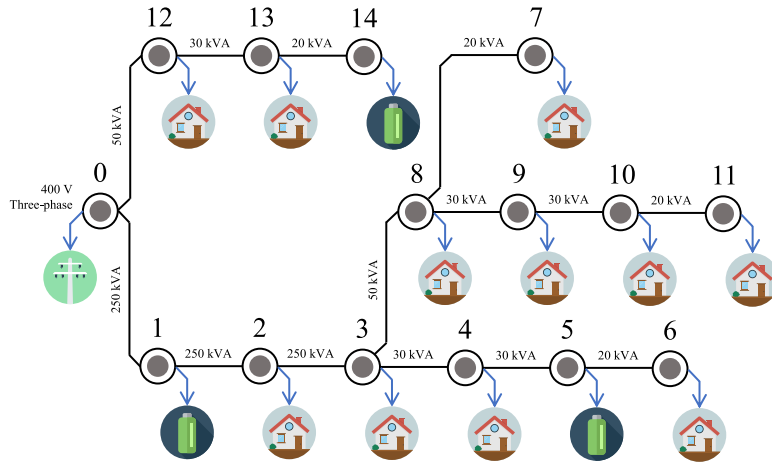


Fig. 9 – The 15-bus network considered in simulations

Table 3 – Prosumer data for the 15-bus case

# bus	Energy (kWh)	Power (kW)	PV rated (kW)
2	7	2.5	2.5
3	5	1.5	4.5
4	3	0.5	1.5
5	3	2.5	4
7	5	1.5	3
8	3	1.5	2.5
9	7	2.5	1.5
10	7	0.5	3
11	5	1	2.5
12	3	2	4
13	5	2.5	2

Table 4 – Battery data for the 15-bus case

# bus	Capacity (kWh)	Power (kW)
1	7	3.5
5	10	5
14	5	2.5

a) *Assessing the effect of the  $\lambda$ 's*: firstly, we repeat the analysis in Fig. 5 for the 15-bus case, showing the results obtained in Fig. 10. As seen, one can conclude the same for this case, further validating the developed approach. Nevertheless, some particular results are worth to be further discussed. In particular, the total energy exchanged by batteries grow when  $\lambda^{BES}$  increases from

0.3 to 0.4. This result may seem surprising but actually demonstrates the hierarchical structure of the developed game-based model. Indeed, the developed tool gives more importance to the high-level objective, i.e. the community cost in our case, and therefore aims at optimizing it firstly. In this sense, the manager takes advantage of energy arbitrage provided by batteries in order to reduce the total energy imported from the distribution network. However, increasing the value of  $\lambda^{BES}$  enforces to reduce the battery usage showing a contradictory objective with the manager. In such a case, an equilibrium point is attained between the manager and the storage provider, by which the total storage energy grows slightly just to reduce the community cost. This result further demonstrates the capability of the developed tool to sort objectives effectively.

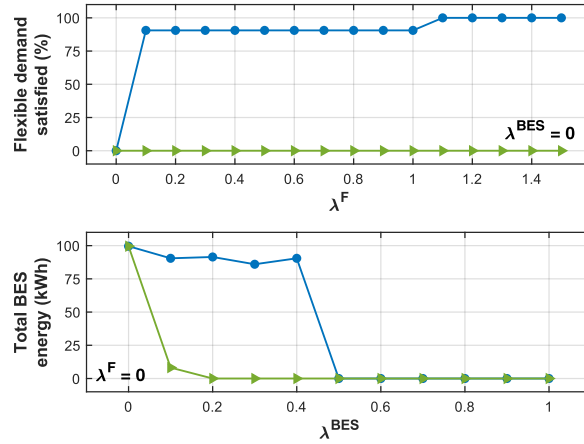


Fig. 10 – Total flexible demand satisfied (top) and energy exchanged by BESs (bottom) for various values of the  $\lambda$ 's in the 15-bus case using the developed methodology (blue line) and conventional multi-objective optimization (green line)

Additionally, Fig. 10 provides a comparison with conventional multi-objective optimization as explained in Appendix B. As observed, the total energy exchanged by batteries seem very sensitive to the value of  $\lambda^{BES}$  and the multi-objective model does not allow intermediate solution points, rapidly varying from 100 kWh to almost zero for  $\lambda^{BES} = 0.2$ . More remarkably, the total flexible demand remains equal to zero for any value of  $\lambda^F$ , thus demonstrating that the conventional multi-objective approach presents issues when dealing with contradictory objectives. In this sense, conventional methodologies might be not easily implementable as their performance strongly depends on the value of input hyperparameters, which, in turn, may be at different scales depending on the units or relative value of each objective. Furthermore, this feature makes difficult sorting objectives as the importance of each one depends significantly on the value of such hyperparameters. Hence, these results demonstrate that our approach is more easily implementable as it is not very sensitive to the value of the  $\lambda$ 's, which can be tuned coarsely.

b) *Community cost*: next, the community cost is studied as in Fig. 6, showing the results obtained in the 15-bus case in Fig. 11. As observed, the results obtained are rather similar to those reported in Fig. 6. These results further demonstrate that the developed methodology is based on equilibrium rather than optimum solutions. It allows tuning hyperparameters coarsely, easing its implementation. Moreover, results in Fig. 11 demonstrates that the developed tool prioritizes the high-level objective, i.e. the community cost, which only increases when the importance of a secondary objective (measured by its corresponding  $\lambda$ ) increases sufficiently to displace the solution until a new equilibrium point.

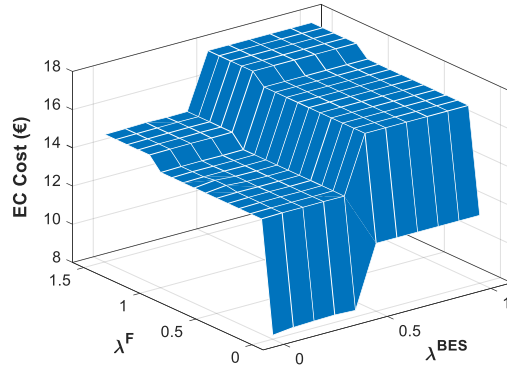


Fig. 11 – Total community cost for various values of the  $\lambda$ 's in the 15-bus case

Thus, the new proposal seems to be case-independent as the final values are based on equilibrium points. Indeed, one can check that results in Figs. 6 and 11 show a similar influence of the weights  $\lambda$ . This feature supposes an advantage with respect other approaches which, as demonstrated in the previous sub-section, strongly depends on the value of input hyperparameters.

c) *Effect of flexible demand on voltage profile*: results in Fig. 12 are analogue to those in Fig. 7 and devote on further validating the network modelling. In this sense, the considered 15-bus network can be considered a more realistic layout for conventional communities, including ramifications but preserving radial topology. Not surprisingly, nodal voltages deteriorate with the value of  $\lambda^F$ . Moreover, it is interesting to see that the effect of  $\lambda^F$  almost vanished across the branch 12-14, where only two prosumers are connected. Thus, increasing the total flexible demand has a marginal impact on this branch. On the other hand, ramifications are properly modelled. As observed, the voltage at node 8 is similar to that at its upstream node (node 3). Likewise, the voltage at node 12 approaches 1 p.u. due to its proximity to the root node. Therefore, the considered network model works well in community systems, where short branches and radial topology are typical features.

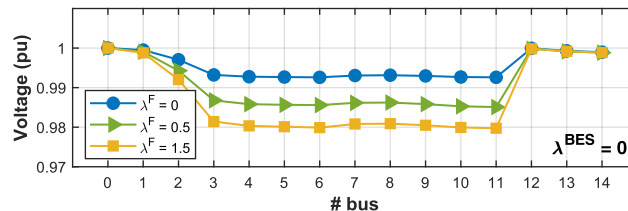


Fig. 12 – Nodal voltages for various values of  $\lambda^F$  in the 15-bus case

d) *Effect of battery capacity*: we also analyse how the developed tool scale with the size of batteries. In this case, the capacity and rated powers of the three installed BESs were multiplied by the same real factor  $\rho$  and the results are plotted in Fig. 13. As observed, the developed tool shows the same particular equilibrium-ruled behaviour discussed in Fig. 10, drawing stable results for different values of  $\lambda^{BES}$ , thus reducing its dependency on the value of hyperparameters. Moreover, one can see that in all cases the energy exchanged by batteries fall to zero with  $\lambda^{BES} = 0.5$ , showing the exact same behaviour that in the 5-bus case. This result allows us to hypothesize that the performance of the developed methodology is case-independent, easing its implementation in a wide variety of scenarios and networks.

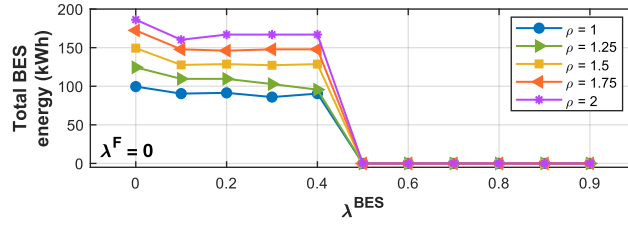


Fig. 13 – Total energy exchanged by batteries for various values of  $\lambda^{BES}$  and battery sizes in the 15-bus case

e) *Effect of storage in imports*: energy storage provides energy arbitrage by optimally charging-discharging storage assets, thus further contributing to reduce energy costs in the community. In order to validate the storage modelling employed in this paper, Fig. 14 shows community imports for two extreme values of  $\lambda^{BES}$ . As expected, when  $\lambda^{BES}$  decreases storage assets are more profusely exploited with a clear impact on power imports. Specifically, the community imports more power at night when  $\lambda^{BES} = 0$  to charge batteries, while they are discharged at evening to allows peak-shaving effect. In contrast, when the value of  $\lambda^{BES}$  increases, the storage provider limits the total energy exchanged by storage assets, increasing the consumption at evening but reducing it at night.

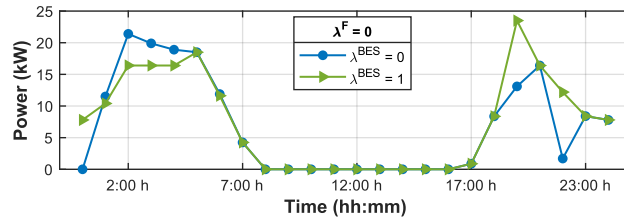


Fig. 14 – Power imported for the community for various values of  $\lambda^{BES}$  in the 15-bus case

f) *Flexible demand satisfaction*:

finally, Fig. 15 plots the accumulated flexible energy satisfied for two extreme values of  $\lambda^F$ . As observed, the main differences in both cases are observed at 7:00 h and 17:00 h, when the community satisfies more flexible demand with  $\lambda^F = 1.5$ . Particularly, this behaviour contributes to reduce the total community cost. Indeed, the retailer price is low at 8:00 h just when PV potential starts to increase, while at 16:00 h there is still a high PV potential and the retailer price decreases compared to midday. The community manager leverages these two favourable periods to satisfy a high amount of flexible demand. Contrarily, flexible demand is not further satisfied at midday, when PV potential is maximum. Instead, the community prefers exporting surplus renewable generation in order to obtain an extra monetary income and thus reducing its cost. This result further demonstrates the hierarchical decision-logic follows by the developed tool, prioritizing the reduction of the electricity bill.

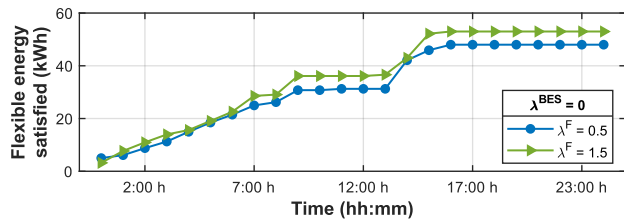


Fig. 15 – Total energy demand satisfied for various values of  $\lambda^F$  in the 15-bus case

### 5.3 – Limitations

The proposed model is applicable to a wide of cases and different objectives can be included following the same ideas drawn in this paper. However, when implementing the developed tool, two points should be taken into account:

- The mathematical models of the middle and low levels should be linear and continuous in order to be equivalent to their optimality conditions. Note that this avoids the use of binary variables. Nevertheless, the inclusion of binary commitment variables is possible at the high-level (see (4)). Therefore, the proposed mathematical model still enables the incorporation of different technologies that require the use of binary commitment statuses such as diesel engines or heat pumps, but keeping in mind that such models must be included at the high-level.
- The proposed methodology is limited to three contradictory objectives, while other approaches should be considered in case of including more targets.

## 6 – Conclusions and Future Works

This paper presented a novel methodology to deal with contradictory objectives in ECs. As a sake of example, the formulation developed tailors for the case in which prosumers aim at increasing their consumption, while storage assets aim at expanding their lifetime, lying in contradiction with the procurement minimization aimed by the community manager. The new proposal casted as a trilevel game-based framework, for which a solution strategy based on a single-level reduction approach has been proposed. The developed tool draws a hierarchical structure allowing to sort objectives according to their perceived importance. Moreover, objectives are parametrized in order to modulate their weight in the overall framework. The main advantage of the proposed approach lies in the fact that it is based on equilibrium rather than optimization. This way, the performance of the new proposal is not strongly affected by input hyperparameters, which may be difficult to tune a priori as in the case of other multi-objective approaches considered.

The new tool has been validated in two case studies (5-bus and 15-bus), showing some particular good features. Firstly, it has been shown that input parameters can be tuned coarsely, as the new proposal is based on equilibrium rather than optimization. In this sense, the results show that particular objectives keep stable for a wide range of hyperparameters, which eases its implementation. This particular behaviour has been further demonstrating comparing the results with a conventional multi-objective approach, which shows high variability depending on the input parameters, but also difficulties to satisfy some particular objectives. Secondly, the hierarchical decision-logic has been proved, showing that those objectives at the high-level are optimized firstly, while the others are optimized when their particular weight increases sufficiently to move towards another particular equilibrium point. It has been also proved by observing the behaviour of secondary objectives, which eventually deteriorated only to optimize further the objective at the high-level. Thirdly, the community mathematical model has been also validated analysing the impact of flexible demand and storage in different aspects such as voltages and power imports. Finally, the impact of battery sizing has been assessed, showing up the developed tool keeps its features for different storage capacity and network size.

Although the new proposal has been validated for three particular objectives (manager, BESs and prosumers), we hypothesize that it will adapt well to other different objectives, which will be analysed in future works. Moreover, the proposed idea could be applied to other similar problems where different objectives lie in contradiction. For instance, the developed tool may adapt well to home energy management problems seeking for maximizing thermal comfort while reducing electricity bill, as in [28].

### CRedit authorship contribution statement

**Marcos Tostado-Véliz:** Conceptualization, Methodology, Software, Investigation, Data curation, Writing – Original Draft, Writing - Review & Editing, Visualization, Funding acquisition. **Hany M. Hasanien:** Conceptualization, Methodology, Investigation, Validation, Formal analysis, Writing - Original Draft, Writing - Review & Editing, Supervision. **Carlos Cruz:** Resources, Software, Data curation, Writing - Original Draft, Visualization, Supervision.

**Francisco Jurado:** Formal analysis, Visualization, Supervision, Project administration, Funding acquisition.

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## Appendix A. Detailed Formulation

In the following, we detail the formulation corresponding to the developed optimization framework.

### A.1 – Primal-dual Formulation of the Low-level

The primal-dual formulation of the low-level (prosumers) writes as:

$$p_{i,t} + PV_{i,t} - D_{i,t} - p_{i,t}^F = 0; \varphi_{i,t}^{(2)}; \forall t \in \mathbb{T} \quad (\text{A1})$$

$$\frac{\partial \mathcal{L}}{\partial p_{i,t}} = \phi_{i,t} - \underline{\zeta}_{i,t} + \bar{\zeta}_{i,t} = 0; \varphi_{i,t}; \forall t \in \mathbb{T} \quad (\text{A2})$$

$$\frac{\partial \mathcal{L}}{\partial p_{i,t}^F} = -\lambda^F - \phi_{i,t} + \bar{\zeta}_i^{\varepsilon^F} - \underline{\zeta}_{i,t} + \bar{\zeta}_{i,t} = 0; \varphi_{i,t}^F; \forall t \in \mathbb{T} \quad (\text{A3})$$

$$-\sum_{t \in \mathbb{T}} \lambda^F p_{i,t}^F = \sum_{t \in \mathbb{T}} \left\{ \phi_{i,t} (PV_{i,t} - D_{i,t}) - \bar{\zeta}_{i,t}^F \bar{p}_{i,t}^F - \bar{p}_{i,t} (\underline{\zeta}_{i,t} + \bar{\zeta}_{i,t}) \right\} - \bar{\zeta}_i^{\varepsilon^F} \bar{\varepsilon}_i^F; \varphi_i^{SD} \quad (\text{A4})$$

$$\bar{\zeta}_i^{\varepsilon^F} \geq 0; \bar{\mu}_i^{\varepsilon^F} \quad (\text{A5})$$

$$\underline{\zeta}_{i,t}, \bar{\zeta}_{i,t}, \underline{\zeta}_{i,t}^F, \bar{\zeta}_{i,t}^F \geq 0; \underline{\mu}_{i,t}, \bar{\mu}_{i,t}, \underline{\mu}_{i,t}^F, \bar{\mu}_{i,t}^F; \forall t \in \mathbb{T} \quad (\text{A6})$$

$$\phi_{i,t}: \text{free}; \forall t \in \mathbb{T} \quad (\text{A7})$$

### A.2 – First-order Optimality Conditions of the Joint Middle-low Level

The joint middle-low level (7) is linear and therefore equivalent to its first-order optimality conditions given below:

$$p_{i,t} + PV_{i,t} - D_{i,t} - p_{i,t}^F = 0; \forall i \in \mathbb{I} \wedge t \in \mathbb{T} \quad (\text{A8})$$

$$-\sum_{t \in \mathbb{T}} \lambda^F p_{i,t}^F = \sum_{t \in \mathbb{T}} \left\{ \phi_{i,t} (PV_{i,t} - D_{i,t}) - \bar{\zeta}_{i,t}^F \bar{p}_{i,t}^F - \bar{p}_{i,t} (\underline{\zeta}_{i,t} + \bar{\zeta}_{i,t}) \right\} - \bar{\zeta}_i^{\varepsilon^F} \bar{\varepsilon}_i^F; \forall i \in \mathbb{I} \quad (\text{A9})$$

$$\varepsilon_{b,t}^{BES} = \varepsilon_{b,t-1}^{BES} + p_{b,t}^{BES,c} \eta^{BES} - \frac{p_{b,t}^{BES,d}}{\eta^{BES}}; \varphi_{b,t}^{BES}; \forall b \in \mathbb{B} \wedge t \in \mathbb{T} \setminus \{1\} \quad (\text{A10})$$

$$\varepsilon_{b,(t=1)}^{BES} = \varepsilon_{b,(t=\mathbb{T})}^{BES} = 0; \varphi_{b,(t=1)}^{BES}, \varphi_b^{BES,end}; \forall b \in \mathbb{B} \quad (\text{A11})$$

$$\frac{\partial \mathcal{L}}{\partial p_{i,t}} = \varphi_{i,t}^{(2)} = 0; \forall i \in \mathbb{I} \wedge t \in \mathbb{T} \quad (\text{A12})$$

$$\frac{\partial \mathcal{L}}{\partial p_{i,t}^F} = -\varphi_{i,t}^{(2)} - \lambda^F \varphi_i^{SD} = 0; \forall i \in \mathbb{I} \wedge t \in \mathbb{T} \quad (\text{A13})$$

$$\frac{\partial \mathcal{L}}{\partial p_{b,t}^{BES,c}} = \eta^{BES} (\lambda^{BES} - \varphi_{b,t}^{BES}) - \underline{\mu}_{b,t}^{BES,c} + \bar{\mu}_{b,t}^{BES,c} = 0; \forall b \in \mathbb{B} \wedge t \in \mathbb{T} \quad (\text{A14})$$

$$\frac{\partial \mathcal{L}}{\partial p_{b,t}^{BES,d}} = \frac{1}{\eta^{BES}} (\lambda^{BES} + \varphi_{b,t}^{BES}) - \underline{\mu}_{b,t}^{BES,d} + \bar{\mu}_{b,t}^{BES,d} = 0; \forall b \in \mathbb{B} \wedge t \in \mathbb{T} \quad (\text{A15})$$

$$\frac{\partial \mathcal{L}}{\partial \varepsilon_{b,t}^{BES}} = \varphi_{b,t}^{BES} - \varphi_{b,(t+1)}^{BES} - \underline{\mu}_{b,t}^{\varepsilon^{BES}} + \bar{\mu}_{b,t}^{\varepsilon^{BES}} = 0; \forall b \in \mathbb{B} \wedge t \in \mathbb{T} \setminus \{\mathbb{T}\} \quad (\text{A16})$$

$$\frac{\partial \mathcal{L}}{\partial \varepsilon_{b,(t=\mathbb{T})}^{BES}} = \varphi_{b,(t=\mathbb{T})}^{BES} - \varphi_b^{BES,end} - \underline{\mu}_{b,(t=\mathbb{T})}^{\varepsilon^{BES}} + \bar{\mu}_{b,(t=\mathbb{T})}^{\varepsilon^{BES}} = 0; \forall b \in \mathbb{B} \quad (\text{A17})$$

$$\frac{\partial \mathcal{L}}{\partial \phi_{i,t}} = \phi_{i,t} - \varphi_{i,t}^F - \varphi_i^{SD} (PV_{i,t} - D_{i,t}) = 0; \forall i \in \mathbb{I} \wedge t \in \mathbb{T} \quad (\text{A18})$$

$$\frac{\partial \mathcal{L}}{\partial \underline{\zeta}_{i,t}} = -\varphi_{i,t} - \underline{\mu}_{i,t} + \varphi_i^{SD} \bar{p}_{i,t} = 0; \forall i \in \mathbb{I} \wedge t \in \mathbb{T} \quad (\text{A19})$$

$$\frac{\partial \mathcal{L}}{\partial \bar{\zeta}_{i,t}} = \varphi_{i,t} - \bar{\mu}_{i,t} + \varphi_i^{SD} \bar{p}_{i,t} = 0; \forall i \in \mathbb{I} \wedge t \in \mathbb{T} \quad (\text{A20})$$

$$\frac{\partial \mathcal{L}}{\partial \bar{\zeta}_i^{\varepsilon^F}} = \varphi_{i,t}^F + \varphi_i^{SD} \bar{\varepsilon}_i^F - \bar{\mu}_i^{\varepsilon^F} = 0; \forall i \in \mathbb{I} \wedge t \in \mathbb{T} \quad (\text{A21})$$

$$\frac{\partial \mathcal{L}}{\partial \zeta_{i,t}^F} = -\varphi_{i,t}^F - \underline{\mu}_{i,t}^F = 0; \forall i \in \mathbb{I} \wedge t \in \mathbb{T} \quad (\text{A22})$$

$$\frac{\partial \mathcal{L}}{\partial \bar{\zeta}_{i,t}^F} = \varphi_{i,t}^F + \varphi_i^{SD} \bar{p}_{i,t}^F - \bar{\mu}_{i,t}^F = 0; \forall i \in \mathbb{I} \wedge t \in \mathbb{T} \quad (\text{A23})$$

$$0 \leq \varepsilon_{b,t}^{BES} \perp \underline{\mu}_{b,t}^{\varepsilon^{BES}} \geq 0; \forall b \in \mathbb{B} \wedge t \in \mathbb{T} \quad (\text{A24})$$

$$0 \leq \bar{\varepsilon}_b^{BES} - \varepsilon_{b,t}^{BES} \perp \bar{\mu}_{b,t}^{\varepsilon^{BES}} \geq 0; \forall b \in \mathbb{B} \wedge t \in \mathbb{T} \quad (\text{A25})$$

$$0 \leq p_{b,t}^{BES,c} \perp \underline{\mu}_{b,t}^{BES,c} \geq 0; \forall b \in \mathbb{B} \wedge t \in \mathbb{T} \quad (\text{A26})$$

$$0 \leq \bar{p}_b^{BES} - p_{b,t}^{BES,c} \perp \bar{\mu}_{b,t}^{BES,c} \geq 0; \forall b \in \mathbb{B} \wedge t \in \mathbb{T} \quad (\text{A27})$$

$$0 \leq p_{b,t}^{BES,d} \perp \underline{\mu}_{b,t}^{BES,d} \geq 0; \forall b \in \mathbb{B} \wedge t \in \mathbb{T} \quad (\text{A28})$$

$$0 \leq \bar{p}_b^{BES} - p_{b,t}^{BES,d} \perp \bar{\mu}_{b,t}^{BES,d} \geq 0; \forall b \in \mathbb{B} \wedge t \in \mathbb{T} \quad (\text{A29})$$

$$0 \leq \zeta_{i,t} \perp \underline{\mu}_{i,t} \geq 0; \forall i \in \mathbb{I} \wedge t \in \mathbb{T} \quad (\text{A30})$$

$$0 \leq \bar{\zeta}_{i,t} \perp \bar{\mu}_{i,t}^F \geq 0; \forall i \in \mathbb{I} \wedge t \in \mathbb{T} \quad (\text{A31})$$

$$0 \leq \bar{\zeta}_i^{\varepsilon^F} \perp \bar{\mu}_i^{\varepsilon^F} \geq 0; \forall i \in \mathbb{I} \quad (\text{A32})$$

$$0 \leq \zeta_{i,t}^F \perp \underline{\mu}_{i,t}^F \geq 0; \forall i \in \mathbb{I} \wedge t \in \mathbb{T} \quad (\text{A33})$$

$$0 \leq \bar{\zeta}_{i,t}^F \perp \bar{\mu}_{i,t}^F \geq 0; \forall i \in \mathbb{I} \wedge t \in \mathbb{T} \quad (\text{A34})$$

$$\phi_{i,t}: \text{free}; \forall i \in \mathbb{I} \wedge t \in \mathbb{T} \quad (\text{A35})$$

$$\boldsymbol{\mu}_i, \boldsymbol{\zeta}_i, \boldsymbol{\mu}_b \geq \mathbf{0} \quad (\text{A36})$$

$$\boldsymbol{\varphi}: \text{free} \quad (\text{A37})$$

## Appendix B. Multi-objective Optimization

The multi-objective model employed for comparison is given below:

$$\min_{x_i, x_b, x^M, y^M} \sum_{t \in \mathbb{T}} \{C_t^i p_t^i - C_t^e p_t^e\} + \sum_{t \in \mathbb{T}} \sum_{i \in \mathbb{I}} \lambda^F p_{i,t}^F + \lambda^{BES} \sum_{t \in \mathbb{T}} \sum_{b \in \mathbb{B}} \left\{ p_{b,t}^{BES,c} \eta^{BES} + \frac{p_{b,t}^{BES,d}}{\eta^{BES}} \right\} \quad (\text{B1})$$

Subject to:

$$(2b)-(2e); \forall i \in \mathbb{I} \quad (\text{B2})$$

$$(3b)-(3f) \quad (\text{B3})$$

$$(4b)-(4q) \quad (\text{B4})$$

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