

Development of different load flow methods for solving large-scale ill-conditioned systems

Marcos Tostado-Véliz¹ | Salah Kamel^{2,3} | Francisco Jurado¹ 

¹Department of Electrical Engineering, University of Jaén, 23700 EPS Linares, Jaén, Spain

²Electrical Engineering Department, Faculty of Engineering, Aswan University, 81542 Aswan, Egypt

³State Key Laboratory of Power Transmission Equipment and System Security and New Technology, Chongqing University, Chongqing, China

Correspondence

Francisco Jurado, Department of Electrical Engineering, University of Jaén, 23700 EPS Linares, Jaén, Spain.
Email: fjurado@ujaen.es

Summary

In this paper, several methods are proposed to find the load flow (LF) solution of large-scale ill-conditioned systems. These methods are based on the Adams-Bashforth formulation. With the use of the proposed methods, the convergence problems of the most popular LF methods are addressed especially when the flat initial guess point is considered. The proposed methods are validated using three ill-conditioned systems: the modified IEEE 300-bus system, the 3375-bus Polish system, and a 13 659-bus portion of the European transmission system. The obtained results prove that the proposed methods constitute a family of robust and efficient methods for the LF problem.

KEYWORDS

Adams-Bashforth, ill-conditioned, large-scale systems, robust load flow methods

1 | INTRODUCTION

The load flow (LF) problem can be considered as the most useful tool in planning and operation of power systems. The LF solution is the starting point for many studies such as continuation power flow, optimal power flow, and online applications, which required fast computational time.^{1,2} Consequently, LF has been the object of many research efforts over the decades.

From a mathematical point of view, LF raises the nonlinear relationship between the nodal voltages and the nodal power injections. This set of nonlinear equations is usually solved using an iterative technique. Classical techniques employed to solve the LF problem are Gauss-Seidel,³ Newton-Raphson (NR),⁴ and fast-decoupled LF (FD).⁵ Among

Abbreviations: LF, Load-flow; NR, Newton-Raphson; FD, Fast-decoupled-load-flow; SRM, Simple robust method; RK4, Fourth-order Runge-Kutta load-flow method; LM, Levenberg-Marquardt; ODEs, Autonomous ordinary differential equations; AB2, Second-order Adams-Bashforth method; JAB, Jacobian adjusted Adams-Bashforth method; HLM, High-order Levenberg-Marquardt method; NM-LM, Nonmonotone line search with corrected LM method; LMJ, Jacobian Adjusted Levenberg-Marquardt method

Symbols: $\Delta P_i \in \mathbb{R}$, Active power mismatch at i^{th} bus; $\Delta Q_i \in \mathbb{R}$, Reactive power mismatch at i^{th} bus; $V_i \angle \delta_i$, Complex voltage at i^{th} bus; $Y_{ij} \angle \theta_{ij}$, ij^{th} Element of the admittance matrix; $n \in \mathbb{R}$, Number of buses; $\delta_{PV} \in \mathbb{R}^{n_{PV}}$, Vector of voltage angles at PV buses; $\delta_{PQ} \in \mathbb{R}^{n_{PQ}}$, Vector of voltage angles at PQ buses; $\mathbf{V}_{PQ} \in \mathbb{R}^{n_{PQ}}$, Vector of voltage magnitudes at PQ buses; $\mathbf{x} \in \mathbb{R}^m$, Load-flow state vector; $\mathbf{g} : \mathbb{R}^m \mapsto \mathbb{R}^m$, Load-flow equations; $\epsilon \in \mathbb{R}$, Convergence tolerance; $\mathbf{J}_x \in \mathbb{R}^m \times m$, Load-flow Jacobian matrix; $\Delta t \in \mathbb{R}$, Continuous time step; $h \in \mathbb{R}$, Discrete time step; $\alpha \in \mathbb{R}$, Adams-Bashforth constant; $\beta \in \mathbb{R}$, Adams-Moulton constant; $\mathbf{I} \in \mathbb{R}^m \times m$, Identity matrix; T , Transpose operator; $\lambda \in \mathbb{R}$, Levenberg damping factor; $\hat{\mathbf{x}}_{1^{(k+1)}} \in \mathbb{R}^m$, $\hat{\mathbf{x}}_{2^{(k+1)}} \in \mathbb{R}^m$, Guess values of the Load-Flow state vector; $\epsilon \in \mathbb{R}$, Security factor; $\sigma_1 \in \mathbb{R}$, $\sigma_2 \in \mathbb{R}$, Damping coefficients; $h_{min} \in \mathbb{R}$, Minimum step size; $h_{max} \in \mathbb{R}$, Maximum step size; $\rho \in \mathbb{R}$, Constant for adapting the initial step size; $\gamma \in \mathbb{R}$, Loading factor; $P_{G_i} \in \mathbb{R}$, Active power generation at i^{th} bus; $Q_{G_i} \in \mathbb{R}$, Reactive power generation at i^{th} bus; $P_{D_i} \in \mathbb{R}$, Active power demand at i^{th} bus; $Q_{D_i} \in \mathbb{R}$, Reactive power demand at i^{th} bus; $P_{0_i} \in \mathbb{R}$, Active power consumption at rated voltage at i^{th} bus; $Q_{0_i} \in \mathbb{R}$, Reactive power consumption at rated voltage at i^{th} bus; $V_{0_i} \in \mathbb{R}$, Rated voltage at i^{th} bus; $a_{0_i} \in \mathbb{R}$, Parameter to model the constant impedance load; $b_{0_i} \in \mathbb{R}$, Parameter to model the constant impedance load; $a_{1_i} \in \mathbb{R}$, Parameter to model the constant current load; $b_{1_i} \in \mathbb{R}$, Parameter to model the constant current load; $a_{2_i} \in \mathbb{R}$, Parameter to model the constant power load; $b_{2_i} \in \mathbb{R}$, Parameter to model the constant power load

these techniques, NR is considered as the most standard one.¹⁻³ NR normally converges in few iterations in most cases; however, its convergence becomes slower when the system is large. Moreover, NR has local convergence; therefore, the initial guess point must be carefully taken, especially in large-scale systems. On the other hand, FD has been developed to improve the convergence speed of NR; however, it normally employs many more iterations, and its robustness is poorer. Apart from the aforementioned standard LF method, other methods have been proposed for the LF solution (see, for example, Stott³ and references therein).

For convenience, we have classified the LF problems into the following three categories:

- Well-conditioned cases solution for the LF problem. This solution is reachable using a flat initial guess point (ie, all PQ buses' voltage magnitudes equal to 1, and all bus voltage angles equal to 0) and a standard LF solver.
- Ill-conditioned cases solution for the LF problem, but the standard LF solvers fail when a flat initial guess point is considered.
- Unsolvable case: The solution of the LF does not exist. Typically, it occurs when the loading level of the system is higher than the maximum loadability point.⁶

Ill-conditioned cases are usually characterized by large R/X ratios and/or its loading level is near to the maximum loadability point. On the other hand, the size of the system is not an explicit reason of ill conditioning; however, large-scale systems normally tends to be ill conditioned easier than do small systems. In this paper, we have focused on the large-scale ill-conditioned systems. These systems are especially difficult to solve because they are normally ill conditioned and require very efficient LF techniques able to manage large vector and matrix computations.

Standard LF solvers do not have good performance in ill-conditioned systems⁷; hence, the standard NR and FD often fail in these cases. Different robust LF solvers have been developed to solve the ill-conditioned cases.⁸⁻¹⁰ The robust LF solvers can be broadly classified into second-order LF solvers, LF solvers based on the continuous Newton method, and LF solvers based on the Levenberg iteration.

Second-order LF solvers truncate the Taylor series expansion at second-order terms. The Iwamoto method is considered the most-referenced second-order LF solver.⁸ It calculates an optimal multiplier of each iteration, which is used to modify the corrector vector of the LF problem and to avoid the divergence. This multiplier is computed as the result of an optimization problem, which aims to minimize the least squares function. Despite its outstanding robustness features, the Iwamoto method suffers from very slow convergence. Moreover, it is a Newton-like based method; therefore, it becomes slower while the system size is increasing. A huge variety of second-order LF solvers, based on the Iwamoto principle, have been proposed in the literature.¹¹⁻¹⁶

The continuous Newton was first proposed by Hetzler¹⁷ and later adapted for LF problem by Milano.¹⁰ In this reference, the author proposed two novel methodologies to solve the LF problem based on numerical methods: the so-called simple robust method (SRM), which is based on the forward Euler methodology and the fourth-order Runge-Kutta LF method (RK4). These methods were compared with the standard NR, FD, and the Iwamoto method. Both proposed methodologies proved to be more robust than the standard LF solvers and are much more efficient than the Iwamoto method. Nevertheless, the computational time and the number of iterations are still too high; furthermore, the number of iterations grows logarithmically with the convergence threshold.

An LF solver based on the Levenberg-Marquardt (LM) method has been recently presented by Milano,¹⁸ which is a very promising technique for ill-conditioned cases. Several improvements of the standard LM method have been proposed by Pourbagher and Derakhshandeh.^{19,20}

Second-order and Levenberg methods are suitable for small- and medium-scale ill-conditioned cases^{8,19,20}; nevertheless, their convergence becomes very slow in large-scale systems. Moreover, they may fail in large-scale ill-conditioned systems if the initial guess point is not good enough. On the other hand, the methods based on the continuous Newton method show fast convergence, and they are effective in large-scale systems.

Following the continuous Newton principle, any numerical method can be adapted to solve the LF problem, since the LF can be viewed as a set of autonomous ordinary differential equations (ODEs). However, only the aforementioned SRM and RK4 have been tested on the LF solution. In this paper, we try to fill this gap by proposing various LF solvers based on the Adams-Bashforth formulation. Several Adams-Bashforth methods are proposed and tested using various test systems by comparing their performance with those of other well-known LF methods.

The rest of this paper is structured as follows. The LF formulation and continuous Newton are outlined in Section 2. In Section 3, the Adams-Bashforth numerical methods are described and adapted for the LF problem. The simulations carried out and the main results obtained are reported in Section 4. Finally, the main conclusions are extracted in Section 5.

2 | LOAD FLOW FORMULATION

The nodal voltages magnitudes and angles are considered as the main results of the LF solution. These values are obtained by iteratively solving the set of active and reactive nodal power mismatches²¹:

$$\Delta P_i = P^{\text{sch}} - \sum_{j=1}^n |V_i| |V_j| |Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j), \quad (1)$$

$$\Delta Q_i = Q^{\text{sch}} - \sum_{j=1}^n |V_i| |V_j| |Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j), \quad (2)$$

where ΔP_i and ΔQ_i are the active and reactive power mismatches at bus i , respectively; P^{sch} and Q^{sch} are the injected active and reactive power at bus i , respectively; $V_i \angle \delta_i$ are the complex voltage at bus i ; $Y_{ij} \angle \theta_{ij}$ is the ij th element of admittance matrix; and n is the total number of buses. Voltage magnitudes at PQ buses and voltage angles at PV and PQ buses are the unknowns of the LF. Therefore, the LF unknown vector can be defined for convenience:

$$\mathbf{x} = \{\delta_{\text{PV}} \cup \delta_{\text{PQ}} \cup \mathbf{V}_{\text{PQ}}\}, \quad (3)$$

where δ_{PV} and δ_{PQ} are the voltage angle vector of PV and PQ buses, respectively. \mathbf{V}_{PQ} is the voltage magnitude vector of PQ buses. With the aim to simplify the notation, a simpler version of Equations 1 and 2 will be used onwards:

$$\mathbf{g}(\mathbf{x}) = 0. \quad (4)$$

It is compulsory to establish some condition to finalize the iterative process; normally it ends when the maximum power mismatch 1 and 2 becomes lower than a specified threshold:

$$\|\mathbf{g}(\mathbf{x}^{(k)})\|_{\infty} \leq \epsilon, \quad (5)$$

where ϵ is the convergence tolerance. The iterative process also ends when the number of iterations surpasses a limit k_{max} ; then, it is said that LF has failed. As it was mentioned in Section 1, NR can be considered as the most standard LF method. A generic k th iteration of the NR method in the LF can be calculated as follows:

$$\begin{aligned} \Delta \mathbf{x}^{(k)} &= -\mathbf{J}_{\mathbf{x}^{(k)}}^{-1} \mathbf{g}(\mathbf{x}^{(k)}), \\ \mathbf{x}^{(k+1)} &= \mathbf{x}^{(k)} + \Delta \mathbf{x}^{(k)}, \end{aligned} \quad (6)$$

where $\mathbf{J}_{\mathbf{x}}$ is the Jacobian matrix of the system, which is the first-order partial derivatives of Equation 5 with respect to unknown vector \mathbf{x} . Therefore, the LF is solved by iterative computing Equation 6 until the convergence criteria 4 is met or the maximum number of iterations k_{max} is surpassed.

2.1 | Brief description of the continuous Newton method

Because the methodology proposed in this paper is based on the continuous Newton principle, a brief description of the continuous Newton method in the LF is illustrated in this subsection. First, let us consider the following set of ODE:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}). \quad (7)$$

The simplest method of numerical integrating Equation 7 is the well-known explicit Euler method, whose generic i th step is run, as follows:

$$\begin{aligned} \Delta \mathbf{x}^{(i)} &= \Delta t \mathbf{f}(\mathbf{x}^{(i)}), \\ \mathbf{x}^{(i+1)} &= \mathbf{x}^{(i)} + \Delta \mathbf{x}^{(i)}, \end{aligned} \quad (8)$$

where Δt is a given time step. A further view allows us to establish an analogy between the generic k th iteration of the NR in the LF 6 and the generic i th step of the explicit Euler method 8 if the following is considered:

$$\mathbf{f}(\mathbf{x}) = -\mathbf{J}_{\mathbf{x}}^{-1}\mathbf{g}(\mathbf{x}). \quad (9)$$

Therefore, the LF can be seen as a set of ODE. It is just needed to change the function $\mathbf{f}(\mathbf{x})$ by Equation 9; the time step Δt is replaced by the step size h and the i th step by the k th iteration. Hence, the generic k th iteration of the SRM proposed by Milano¹⁰ is simply defined as follows:

$$\begin{aligned} \Delta \mathbf{x}^{(k)} &= h \left(-\mathbf{J}_{\mathbf{x}^{(k)}}^{-1} \mathbf{g}(\mathbf{x}^{(k)}) \right), \\ \mathbf{x}^{(k+1)} &= \mathbf{x}^{(k)} + \Delta \mathbf{x}^{(k)}. \end{aligned} \quad (10)$$

Other numerical methods apart from explicit Euler method can be employed to integrate Equation 9. In the work of Milano,¹⁰ the RK4 has been employed. In this paper, we propose to use other well-known numerical methods called the Adams-Bashforth methods. It is worth to mention that the Iwamoto method⁸ can be seen as the variable step SRM method,¹⁰ where the optimal multiplier is substituted by the step size h . In the work of Milano,¹⁰ it was demonstrated that the continuous Newton method for LF resolution is asymptotically stable. The reachability of the solution depends on the starting point, which must be within the region of attraction.

3 | PROPOSED LF SOLVERS BASED ON THE ADAMS-BASHFORTH FORMULA

The Adams-Bashforth is a family of explicit numerical methods.²²⁻²⁴ They belong to the family of linear multistep numerical methods, since they need the information of several previous points to compute the following step. The generic formulation of an s th-order Adams-Bashforth method is defined as follows:

$$\Delta \mathbf{x}^{(i)} = \Delta t \left[\alpha_1 \mathbf{f}(\mathbf{x}^{(i)}) + \alpha_2 \mathbf{f}(\mathbf{x}^{(i-1)}) + \dots + \alpha_s \mathbf{f}(\mathbf{x}^{(i-s+1)}) \right], \quad (11)$$

where α are real coefficients. As can be seen, the following point $\mathbf{x}^{(i+1)}$ is computed by pondering the information of the current point $\mathbf{x}^{(i)}$ and $s - 1$ previous points. This procedure is depicted in Figure 1 for $s = 2$.

Depending on the value of s , several Adams-Bashforth methods are proposed. It is important to know that the first-order Adams-Bashforth method ($s = 1$) is the previously described explicit Euler method 8. An important topic of the Adams-Bashforth methods is that they need to be initialized. At the beginning of the procedure, the methods lack information about any previous point; therefore, it is required to provide $s - 1$ previous points through other numerical methods (we assume that the initial guess point is taken as $\mathbf{x}^{(-s+1)}$). Therefore, a generic $(s - 1)$ th numerical method (O^{s-1}) must be used at the beginning of the process. Hence, the general procedure of the Adams-Bashforth methods can be summarized as follows:

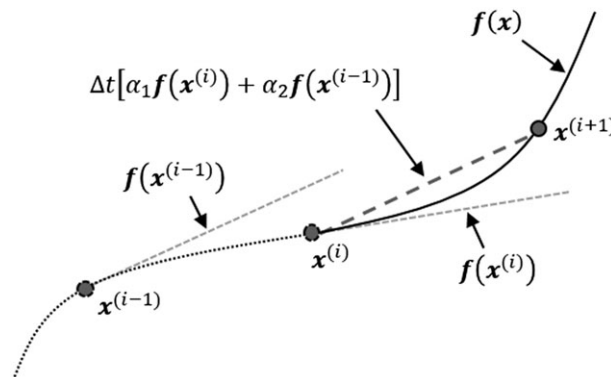


FIGURE 1 Sketch of the Adams-Bashforth method

At $i = 0$,

$$\mathbf{x}^{(-s+1)},$$

$$O^{s-1} \rightarrow \mathbf{x}^{(0)}, \mathbf{x}^{(-1)}, \dots, \mathbf{x}^{(-s+2)}. \quad (12)$$

For $i \geq 1$,

$$\Delta \mathbf{x}^{(i)} = \Delta t \left[\alpha_1 \mathbf{f}(\mathbf{x}^{(i)}) + \alpha_2 \mathbf{f}(\mathbf{x}^{(i-1)}) + \dots + \alpha_s \mathbf{f}(\mathbf{x}^{(i-s+1)}) \right],$$

$$\mathbf{x}^{(i+1)} = \mathbf{x}^{(i)} + \Delta \mathbf{x}^{(i)}. \quad (13)$$

The most logical way to initialize an s th-order Adams-Bashforth method can be achieved by carrying out an $(s - 1)$ th-order Runge-Kutta method.²² Nevertheless, specifically constructed Runge-Kutta methods can be used for initializing them.²⁵

In this paper, we exploit the idea exposed by Milano.¹⁰ It is established that given the analogy between the LF and a set of ODE presented by Hetzler,¹⁷ any well-assessed numerical method can be used in the LF resolution. Therefore, employing the Adams-Bashforth methods in the LF should be approachable following the basic principles presented by Milano¹⁰ and in Section 2.1.

Therefore, the Adams-Bashforth methods can be adapted for the LF problem, as follows:

$$\Delta \mathbf{x}^{(k)} = h \left[\alpha_1 \mathbf{f}(\mathbf{x}^{(k)}) + \alpha_2 \mathbf{f}(\mathbf{x}^{(k-1)}) + \dots + \alpha_s \mathbf{f}(\mathbf{x}^{(k-s+1)}) \right],$$

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \Delta \mathbf{x}^{(k)}, \quad (14)$$

where $\mathbf{f}(\mathbf{x})$ was already defined in Equation 9. Depending on the order of the method, expression 14 changed according to the previous points required and the value of the coefficients α . In this paper, we consider the second-order Adams-Bashforth method (AB2), since the first-order Adams-Bashforth formula is the explicit Euler method, which was already studied for the LF solution by Milano.¹⁰ It is worth to mention that the higher-order Adams-Bashforth methods can be easily adapted for the LF solution following the procedure indicated in this paper.

A generic k th iteration of the proposed AB2 in the LF can be defined as follows:

$$\Delta \mathbf{x}^{(k)} = h \left[-1/2 \mathbf{f}(\mathbf{x}^{(k-1)}) + 3/2 \mathbf{f}(\mathbf{x}^{(k)}) \right],$$

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \Delta \mathbf{x}^{(k)}. \quad (15)$$

3.1 | Adams-Bashforth-Moulton methods

The Adams-Bashforth methods are explicit numerical methods; therefore, the function value at the following point $\mathbf{f}(\mathbf{x}^{(k+1)})$ is not required to calculate the following point $\mathbf{x}^{(k+1)}$ explicitly. On the other hand, the implicit methods is another well-known family of numerical methods. Oppositely, these methods require to evaluate the function at the following point $(\mathbf{x}^{(k+1)})$ before it is calculated. The implicit version of the Adams-Bashforth formulas is the so-called Adams-Moulton methods.²² A general expression of the Adams-Moulton methods in the LF can be defined as follows:

$$\Delta \mathbf{x}^{(k)} = h \left[\beta_1 \mathbf{f}(\mathbf{x}^{(k+1)}) + \beta_2 \mathbf{f}(\mathbf{x}^{(k)}) + \dots + \beta_{s+1} \mathbf{f}(\mathbf{x}^{(k-s)}) \right], \quad (16)$$

where β are real coefficients similar to α coefficients for the Adams-Bashforth formulas. Frequently, an iterative technique is employed to address the problem of computing $\mathbf{f}(\mathbf{x}^{(k+1)})$ without knowing the value of $\mathbf{x}^{(k+1)}$; however, this technique is totally inefficient since it needs to nest two iterative techniques into the algorithm. Alternatively, an

explicit method can be employed instead; this methodology is known as predictor-corrector.²⁶ Therefore, an explicit method is used to compute an approximate value of the following point ($\widehat{\mathbf{x}}^{(k+1)}$); then, the implicit method is run to correct it. When an explicit Adams-Bashforth method is used as predictor and an implicit Adams-Moulton method is used as corrector, the approach is called Adams-Bashforth-Moulton.²⁷⁻²⁹ The simplest Adams-Bashforth-Moulton method employs the explicit Euler method as predictor and the implicit trapezoidal rule as corrector. This methodology is called the Heun method²⁴ or one-step Adams-Bashforth-Moulton method.²⁸ A generic k th iteration of the Heun method in the LF is summarized as follows:

$$\begin{aligned}\widehat{\mathbf{x}}^{(k+1)} &= \mathbf{x}^{(k)} + h\mathbf{f}\left(\mathbf{x}^{(k)}\right), \\ \Delta\mathbf{x}^{(k)} &= h/2\left[\mathbf{f}\left(\mathbf{x}^{(k)}\right) + \mathbf{f}\left(\widehat{\mathbf{x}}^{(k+1)}\right)\right], \\ \mathbf{x}^{(k+1)} &= \mathbf{x}^{(k)} + \Delta\mathbf{x}^{(k)}.\end{aligned}\quad (17)$$

3.2 | The proposed Jacobian adjusted Adams-Bashforth method

This method is inspired from the Jacobian adjusted LM method proposed by Lagace et al.^{30,31} We propose to take the advantage of this philosophy. It computes an intermediate step of each iteration, calculates the Jacobian matrix at this intermediate point, and uses this information to update the unknown vector. In this paper, we have translated this idea to the proposed AB2, combining a Levenberg step and a Newton step in the same iteration. Anyway, a generic k th iteration of the proposed Jacobian adjusted Adams-Bashforth (JAB) method is defined as follows:

$$\begin{aligned}\widehat{\mathbf{x}}^{(k)} &= \mathbf{x}^{(k)} + \mathbf{f}\left(\mathbf{x}^{(k)}\right)/2, \\ \mathcal{L}^{(k)} &= \left[\mathbf{J}_{\widehat{\mathbf{x}}^{(k)}}^T \mathbf{J}_{\widehat{\mathbf{x}}^{(k)}} + \lambda \mathbf{I}\right]^{-1} \mathbf{J}_{\widehat{\mathbf{x}}^{(k)}}^T \mathbf{g}\left(\mathbf{x}^{(k)}\right), \\ \mathbf{x}^{(k+1)} &= \mathbf{x}^{(k)} + h\left[-1/2\mathcal{L}^{(k)} + 3/2\mathbf{f}\left(\mathbf{x}^{(k)}\right)\right],\end{aligned}\quad (18)$$

where \mathbf{I} is the identity matrix and \mathbf{T} is the transpose operator. The parameter λ is crucial for the straightforward convergence of the method. In this paper, it is updated in each iteration using¹⁹

$$\lambda = \left\|\mathbf{g}\left(\mathbf{x}^{(k)}\right)\right\|_2^{1.3}. \quad (19)$$

3.3 | Proposing an adaptive scheme for the proposed approaches

Choosing the step size is a hard task in the LF solvers based on the continuous Newton method; therefore, using an adaptive step size is usually the best choice. In this paper, we propose a novel methodology to update the step size of the proposed methods based on the Richardson extrapolation.^{32,33} To achieve that, two guess values of the unknown vector $\widehat{\mathbf{x}}_1^{(k+1)}$ and $\widehat{\mathbf{x}}_2^{(k+1)}$ are computed in each iteration using the explicit Euler method with $h/2$ and $h/4$, respectively. Then, this information is used to calculate ζ , as follows:

$$\zeta = \left\|\frac{\widehat{\mathbf{x}}_1^{(k+1)} - \widehat{\mathbf{x}}_2^{(k+1)}}{h}\right\|_\infty. \quad (20)$$

The step size is then updated in each iteration according to the following heuristic rule¹⁰:

$$\begin{aligned} \text{if } \zeta > \varepsilon, \text{ then } h &\leftarrow \max\{\sigma_1 h, h_{\min}\}, \\ \text{if } \zeta \leq \varepsilon, \text{ then } h &\leftarrow \min\{\sigma_2 h, h_{\max}\}, \end{aligned} \quad (21)$$

where ε is a security factor, σ_1 and σ_2 are damping coefficients, and h_{\min} and h_{\max} are the minimum and maximum step sizes, respectively. Rule 21 tries to identify when the step size h can be increased in order to accelerate the convergence. Oppositely, the step size is reduced with the aim to avoid the divergence when $\zeta > \varepsilon$.

Choosing the initial step size properly is very important for the performance of the proposed approaches. The value of $\|\mathbf{g}(\mathbf{x}^{(0)})\|_{\infty}$ can be taken as an index of the goodness of the initial guess point used. Hence, if the initial guess point is not good enough, the initial step size must be reduced consequently. In this paper, the following simple rule is considered for choosing the initial step size:

$$\begin{aligned} \text{if } \|\mathbf{g}(\mathbf{x}^{(0)})\|_{\infty} > \rho, \text{ then } h &\leftarrow h_{\min}, \\ \text{if } \|\mathbf{g}(\mathbf{x}^{(0)})\|_{\infty} \leq \rho, \text{ then } h &\leftarrow 1, \end{aligned} \quad (22)$$

where ρ is a parameter that must be defined at the beginning of the algorithm. With the aim of completeness, the whole AB2 procedure for LF problem is summarized in the flowchart in Figure 2.

From Figure 2, it can be noticed that the flowchart of the proposed methodology is very similar to that from the standard NR solver. It just needs to remove the blocks corresponding to initialization of the method and substitute

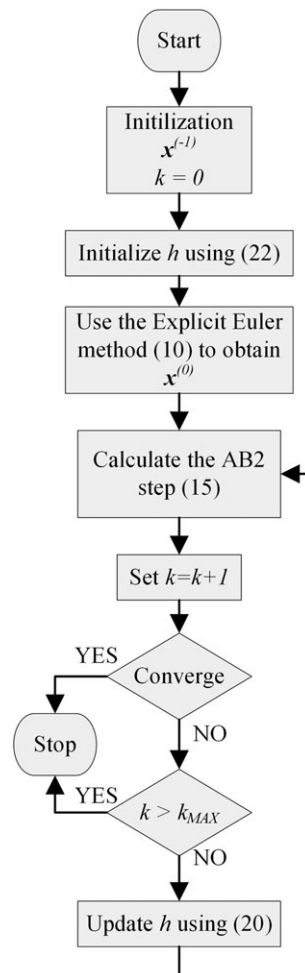


FIGURE 2 Proposed AB2 flowchart. AB2 indicates second-order Adams-Bashforth method

expression 15 with the standard NR step 6. Regarding the Heun and JAB methods, the Euler initialization block should be removed and expression 15 must be substituted by Equations 17 and 18, respectively. In the case of JAB, the parameter λ is also updated in each iteration using Equation 19.

On the other hand, the convergence of the proposed methods can be easily accelerated by switching to the standard NR when the convergence has reached a determined threshold. This measure was already proposed by Milano¹⁰ and applied to the SRM and the RK4. In this paper, we have considered this scheme to the proposed AB2 and JAB, so, at the first iterations, the procedure runs according to that shown in Figure 2; then, when the value of $\|g(x^{(k)})\|_\infty$ is smaller than 10^{-1} , the algorithm switches to the NR. With the aim to provide a straightforward comparison, this mechanism has been also used in the SRM and the RK4.

It is worth to notice that the most used constant power load model has been used in this paper; nevertheless, other load models can be easily accommodated to the proposed methodology (see Appendix A).

All parameters defined to adapt the step size can be chosen freely based on heuristic criteria and personal experiences. Nevertheless, there are some guidelines to properly tune them. First, the value of σ_1 should be slightly smaller than 1; oppositely, σ_2 should be tuned higher than 1 so that the step size is reduced by the effect of σ_1 and increased by the effect of σ_2 . On the other hand, the values of the minimum and maximum step sizes should be fixed smaller and higher than 1, respectively; furthermore, the value of h_{\max} should not be much bigger than the unity in order to ensure the robustness features of the proposed methods. Finally, the value of ρ can be difficult to tune. It mainly depends on the region of attraction shown by the system analyzed, which essentially depends on the condition of the case studied. Based on our experience, the value of ρ should always be higher than 10^2 . Nevertheless, rule 22 can be omitted, and tune the initial step size always to be equal to h_{\min} . This measure frequently provides good robustness; however, the efficiency of the proposed methods can be seriously affected. Table 1 reports the value of the parameters employed in the simulations of the presented paper.

4 | RESULTS AND DISCUSSION

The performance of proposed LF methods is validated using the following three systems:

- The IEEE 300-bus test system,³⁴ which is naturally well conditioned; therefore, we have considered to intentionally deteriorate its condition by multiplying the real part of the branch impedance by 2 and modifying the initial guess, adding a real error factor equal to 0.2 to the initial voltage magnitudes at PQ buses. This case can be considered as a medium-scale system.
- Polish system that consists of 3375 buses and 4161 lines in the winter 2007-2008 evening peak (3375-bus). The details of this system are available in MATPOWER.³⁵ This case can be considered as a large-scale system.
- Portion of the European transmission system that consists of 13 659 buses and 20 467 lines (13 659-bus). This system is available for download in MATPOWER,³⁵ and further details can be found in Jozs et al³⁶ and Fliscounakis et al.³⁷ This case can be considered as a large-scale or very-large-scale system.

All simulations have been carried out using a personal computer Intel Core i5-7500 3.4 GHz using the software package MATPOWER 6.0.³⁸ In all cases, the convergence tolerance has been taken as $\epsilon = 10^{-4}$.

The generator's reactive power limits are considered in the proposed methods. Based on the common procedure used in case of limits violation, the calculated reactive power of each generator is checked during the LF solution to determine whether it is within the limits or not. If there is a violation, then it is set equal to the limit violated and the bus voltage magnitude is released, where the connected bus is converted to PQ bus.

TABLE 1 Parameters employed in simulations

| | Value |
|------------|-----------------|
| ϵ | 0.003 |
| σ_1 | 0.95 |
| σ_2 | 1.05 |
| h_{\min} | 0.3 |
| h_{\max} | 1.2 |
| ρ | 1×10^2 |

With the aim to check the performance of the proposed methods, the following LF solvers are compared:

- Standard NR method;
- The Iwamoto method⁸;
- Two robust methods based on the continuous Newton principle (SRM and RK4)¹⁰;
- Several LF methods based on the Levenberg iteration:
- Standard LM method.¹⁸ The adaptive scheme 19 has been used for this method.
- High-order LM (HLM) method.¹⁹
- Nonmonotone line search with corrected LM method (NM-LM).²⁰
- Jacobian adjusted LM (LMJ) method.^{30,31} The adaptive scheme 19 has been used for this solver.
- Proposed LF method based on the AB2 method (AB2).
- Proposed LF method based on the Heun method.
- Proposed Jacobian adjusted Adams-Bashforth-Levenberg approach (JAB).

The total computation time and required iterations of different LF methods used to solve the three studied systems with a flat initial guess point are given in Table 2.

Based on the results obtained, several conclusions may be remarked:

- The proposed methods have been able to converge in all studied systems, while the remainder methods faced different convergence difficulties.
- The methods based on the Levenberg iteration have mainly failed because of an excessive number of iterations.
- Both HLM and NM-LM failed because of the computational burden in the 13 659-system. It is worth to mention that HLM and NM-LM need to invert a matrix versus another one, instead of inverting a matrix versus a vector-like NR method. Therefore, the computational burden is unacceptable in the biggest system.
- Both the Iwamoto method and SRM converged in the 13 659-bus system; however, they have been attracted to a wrong solution (solution provided by the standard NR; using the initial guess provided by the base cases has been considered as the correct solution).
- Among all proposed methods, JAB gives the best performance in a number of iterations. However, the proposed AB2 has been more efficient in the 300-bus and 3375-bus systems.

TABLE 2 Total computation time and required iterations number for solving the studied systems using different LF methods

| Method | Case | | | | | |
|----------------------|-------------------|----------|-------------------|----------|-------------------|----------|
| | 300-Bus System | | 3375-Bus System | | 13 659-Bus System | |
| | Time, s | No. Iter | Time, s | No. Iter | Time, s | No. Iter |
| Standard NR | Diverge | — | Diverge | — | Diverge | — |
| Iwamoto ⁸ | Fail ^a | — | Fail ^a | — | Fail ^b | — |
| LM ¹⁸ | Fail ^a | — | Fail ^a | — | Fail ^a | — |
| HLM ¹⁹ | Fail ^a | — | Fail ^a | — | Fail ^c | — |
| NM-LM ²⁰ | Fail ^a | — | Fail ^a | — | Fail ^c | — |
| LMJ ^{30,31} | Fail ^a | — | Fail ^a | — | Fail ^a | — |
| SRM ¹⁰ | Diverge | — | Fail ^a | — | Fail ^b | — |
| RK4 ¹⁰ | 0.161 | 17 | Diverge | — | 6.302 | 18 |
| AB2 | 0.082 | 28 | 0.717 | 31 | 2.132 | 22 |
| Heun | 0.193 | 42 | 1.774 | 43 | 2.365 | 22 |
| JAB | 0.103 | 23 | 0.945 | 24 | 1.519 | 8 |

^aIt did not converge in 50 iterations.

^bIt did not reach the correct solution.

^cFail for computational burden.

- The RK4 has failed in the 3375-bus system, whereas the proposed methods have successfully converged. This is because the flat initial guess is far away from the solution. In the case of the RK4, the flat initial guess has lain outside of the region of attraction.
- All proposed methods employed many iterations; nevertheless, they have been very efficient. On the other hand, the AB2 and JAB have showed a high degree of efficiency in the 300-bus system, even better than the RK4. The reason for this is that the RK4 needs to factorize a matrix up to four times per iteration, whereas the proposed methods need to factorize it one time (AB2) or two times (Heun and JAB).
- The proposed methods employed many iterations in the 300-bus and 3375-bus systems. This is due to the adaptation of the initial step size using rule 22. This did not happen in the 13 659-bus case, which is reflected in a smaller number of iterations.

The ability of the proposed methodologies for solving the LF for heavy loading conditions has been explored in the studied 3375-bus system (the base case is practically the maximum loadability situation in the 13 659-bus system). To do that, the active power load at PV buses and the reactive and active power loads at PQ buses have been multiplied by a real factor γ . Table 3 shows the total number of iterations of the proposed approaches for solving the LF in the 3375-bus system for different loading levels.

The proposed approaches have been able to solve the LF in the 3375-bus system even for heavy loading conditions, employing almost the same number of iterations in all situations.

4.1 | Scalability of the proposed methods

The suitability of the proposed approaches has been analyzed in several medium-scale and large-scale systems. As it was already commented, the proposed JAB is suitable for very-large-scale systems, employing a relatively small CPU time for solving a system with more than 10 000 buses. This size may correspond to a large interconnected system (eg, the European transmission system). On the other hand, in a medium-scale system, the proposed AB2 has showed a high degree of efficiency. Figure 3 shows a comparison of the CPU time employed by the proposed methods in the different studied cases.

TABLE 3 Required iterations number for solving the 3375-bus system for different loading conditions

| γ | Method | | |
|----------|---------|---------|---------|
| | AB2 | Heun | JAB |
| 1.05 | 31 | 43 | 24 |
| 1.07 | 31 | 43 | 24 |
| 1.10 | 31 | 45 | 25 |
| 1.12 | 32 | 45 | 25 |
| 1.15 | 32 | 45 | 25 |
| 1.16 | Diverge | Diverge | Diverge |

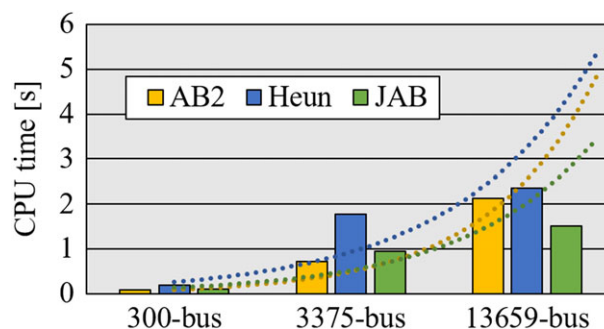


FIGURE 3 Comparison of the CPU time employed by the proposed approaches for solving the load flow in the studied cases. AB2 indicates second-order Adams-Bashforth method; JAB, Jacobian adjusted Adams-Bashforth

TABLE 4 Total computation time and required iterations number for solving several small-scale systems using different LF methods

| Method | Case | | | | | |
|--------|---------------|----------|---------------|----------|----------------|----------|
| | 30-Bus System | | 57-Bus System | | 118-Bus System | |
| | Time, s | No. Iter | Time, s | No. Iter | Time, s | No. Iter |
| AB2 | 0.011 | 9 | 0.018 | 11 | 0.043 | 14 |
| Heun | 0.035 | 14 | 0.047 | 16 | 0.062 | 18 |
| JAB | 0.009 | 4 | 0.011 | 4 | 0.021 | 6 |

From Figure 3, one can conclude that the proposed JAB is more scalable than the other proposed approaches. If we follow its trend line, it is easy to check how it grows slower than other lines.

The proposed methods are very suitable for large-scale and very-large-scale systems, since they are able to get a very good balance between robustness and efficiency. Nevertheless, with the aim to provide a complete analysis, Table 4 shows the number of iterations and CPU time employed by the proposed techniques in the small-scale IEEE 30-bus, 57-bus, and 118-bus test systems. As can be seen, the proposed methods employ a reasonable number of iterations and CPU time even in small-scale systems.

5 | CONCLUSION

In this paper, several LF methods based on the Adams-Bashforth formulation have been proposed. The proposed methods have been tested using several medium-scale and large-scale ill-conditioned systems (300-bus, 3375-bus, and 13 659-bus). In the studied systems, the flat initial guess point is not good enough; hence, most of the well-known LF methods present different convergence difficulties.

An adaptive step size mechanism based on the Richardson extrapolation has been presented. This mechanism is very effective in the studied cases, being able to identify when the step size must be reduced and when it can be increased. The initial step size is crucial for the robustness of the proposed methods. It especially depends on the goodness of the initial guess point used. A simple but effective technique for choosing the initial step size has been also developed.

The proposed methods have turned out to be very effective for solving the studied cases. The proposed AB2, Heun, and JAB methods have converged, while other LF methods have faced different convergence issues. Moreover, the proposed methods have showed a high degree of efficiency, occasionally improving the CPU time employed by the RK4 for solving the studied cases. This is mainly due to the number of matrix factorizations during the iterative procedure; while the RK4 needs to factorize up to four matrices in each iteration, the proposed methods only require 1 (AB2) or 2 (Heun and JAB) factorizations. Anyway, the superiority of the proposed methodologies with respect of the RK4 has been remarked in the 3375-bus case. In this case, the RK4 has failed while the proposed methods have successfully converged. This demonstrates that, occasionally, the proposed techniques may be more robust than the RK4.

The scalability of the proposed approaches has been analyzed, concluding that the proposed JAB would be more scalable than the remainder proposed methods to systems with more than 10 000 buses. On the other hand, the performance of the proposed methodologies for solving the LF in small-scale systems has been tested. The proposed methods have employed a reasonable number of iterations for solving the LF in several standard small-scale test systems. Therefore, it can be easily concluded that, despite that the main feature of the proposed methodologies is solving the LF in large-scale ill-conditioned systems efficiently, they can be effectively adapted for systems with smaller sizes like radial distribution systems.

The future work will focus on studying the effectiveness of other Adams-Bashforth and predictor-corrector numerical methods in the LF problem.

ORCID

Francisco Jurado  <https://orcid.org/0000-0001-8122-7415>

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APPENDIX A

Considering the constant current and constant impedance loads in the proposed methods

In this paper, the most-common constant power load model has been employed. However, the loads can be also modeled as constant current, constant impedance, or polynomial combination of the three models, as follows:

$$P_i^{\text{sch}}(V_i) = P_{G_i} - P_{D_i}(V_i) = P_{G_i} - P_{0_i} \left[a_{0_i} \left(\frac{V_i}{V_{0_i}} \right)^2 + a_{1_i} \left(\frac{V_i}{V_{0_i}} \right) + a_{2_i} \right], \quad (23)$$

$$Q_i^{\text{sch}}(V_i) = Q_{G_i} - Q_{D_i}(V_i) = Q_{G_i} - Q_{0_i} \left[b_{0_i} \left(\frac{V_i}{V_{0_i}} \right)^2 + b_{1_i} \left(\frac{V_i}{V_{0_i}} \right) + b_{2_i} \right], \quad (24)$$

where P_{G_i} and Q_{G_i} are the active and reactive power generation at bus i , respectively. P_{D_i} and Q_{D_i} are the active and reactive power demand at bus i , respectively. P_{0_i} and Q_{0_i} are the active and reactive power consumption at rated voltage V_{0_i} at bus i , respectively. Parameters (a_{0_i}, b_{0_i}) , (a_{1_i}, b_{1_i}) , and (a_{2_i}, b_{2_i}) are used to model the constant impedance, constant current, and constant power loads, respectively.

Models 23 and 24 can be easily incorporated in the proposed methods by substituting these expressions in Equations 1 and 2.