Computers & Industrial Engineering 155 (2021) 107141

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

An optimal Best-Worst prioritization method under a 2-tuple linguistic environment in decision making

Álvaro Labella [a,*], Bapi Dutta [b], Luis Martínez [a]

[a] University of Jaén, Department of Computer Science, Spain
[b] The Logistics Institute–Asia Pacific, National University of Singapore, Singapore

ARTICLE INFO

Keywords:
Best-Worst method
2-tuple linguistic model
Multi-criteria group decision making

ABSTRACT

Multi-criteria group decision making (MCGDM) deals with decision makers who evaluate alternatives over several criteria. MCGDM problems evolve in tandem with the progress of our society. Such progress has given rise to the large-scale group decision making (LS-GDM) problems in which hundreds of decision makers may participate in the decision process and new challenges to face such as groups' formation and polarization opinions. Most real world MCGDM problems present changing contexts with uncertainty that cannot be modeled by numerical values. Under these circumstances, the use of linguistic variables and computing with words (CW) processes have provided successfully results. Concretely, the 2-tuple linguistic computational model stands out because its precise linguistic computations and high interpretability. On the other hand, pairwise comparison is a widely used elicitation technique in MCGDM, but a large number of comparisons might lead inconsistent decision makers' preferences. The Best-Worst method (BWM) reduces the number of pairwise comparisons and the inconsistency in decision makers' opinions. Several BWM approaches have been proposed to manage linguistic information but none of them take advantage of the 2-tuple linguistic computational process based on the CW approach, which would allow to obtain precise and understandable results. This paper aims to present an extended 2-tuple BWM to reduce the number of pairwise comparisons in MCGDM problems and model the uncertainty associated with them to accomplish accuracy computations and obtaining interpretable results. Moreover, we apply our proposal to LS-GDM scenarios in which polarization opinions and sub-groups identification, ignored from any of BWM proposals, are considered. Finally, the new model is applied to several illustrative MCGDM problems.

1. Introduction

Decision making (DM) is a common daily task for human beings. Either because of the increasing complexity of the decisions or the need to consider different points of view for making the decision, *group decision making* (GDM) has become a fundamental activity in our society. GDM problems are defined as decision situations in which several decision makers, $\{e_1,\ldots,e_s\}$, with their own attitudes try to reach a common solution to a problem consisting of two or more possible solutions or alternatives, $\{x_1,\ldots,x_m\}$. Decision makers may evaluate the alternatives according to different and usually conflicting criteria, $\{C_1,\ldots,C_n\}$, giving rise to *multi-criteria group decision making* (MCGDM). MCGDM is constantly evolving and the apparition of technological advances such as social networks (Sueur, Deneubourg, & Petit, 2012) and big data (Dumbill, 2013) or social needs such as e-markets (Büyüközkan, 2004) has resulted in a new type of MCGDM problems, so-called *large-scale group decision making* (LS-GDM) problems, in which hundreds or thousands of decision makers may be involved in the decision process (Labella, Liu, Rodríguez, & Martínez, 2018; Liu, Fan, & Zhang, 2016; Rodríguez, Labella, Tré, & Martínez, 2018; Xu, Du, & Chen, 2015). LS-GDM problems present novel challenges to face respect to the classical GDM problems such as the decision makers' polarization opinions (Dong, Zhou, & Martínez, 2019). Polarization in opinions is quite common in LS-GDM and such polarized views have to be identified in order to understand the proper dynamics of groups with a large number of stakeholders/participants (Abrams, Wetherell, Cochrane, Hogg, & Turner, 1990; Hogg, Turner, & Davidson, 1990). There might not have any unanimous agreement/view towards an alternative but there might have multiple sub-groups and each sub-group agreed unanimously on a ranking of the set of alternatives.

* Corresponding author.
E-mail addresses: alabella@ujaen.es (Á. Labella), bapi.iitr@gmail.com (B. Dutta), martin@ujaen.es (L. Martínez).

https://doi.org/10.1016/j.cie.2021.107141
Received 15 June 2020; Received in revised form 21 December 2020; Accepted 19 January 2021
Available online 2 February 2021
0360-8352/

Most real-world MCGDM problems present inherent uncertain provoked by changing contexts and complexity. Therefore, the decision makers have to deal with incomplete and vague information when they express their preferences, which may result a really hard task. Under these circumstances, the decision makers prefer to use linguistic assessments, which have been successfully modeled (Martínez & Herrera, 2012) by linguistic variables (Zadeh, 1975). The use of linguistic information requires *computing with words* (CW) processes (Zadeh, 1996; Zadeh, 2013a, 2013b), in which words (in a natural or artificial language) and not number are manipulated to obtain the solution for the decision problem, emulating the human beings' reasoning process in which, from linguistic premises, the results are also represented in a linguistic way. According to the latter, Yager introduced in Yager (2004) a CW scheme in which points out the importance of the translation and retranslation processes in CW. The former translates the linguistic inputs into a machine-manipulate format based on fuzzy tools in which the computations are carried out and the latter converts the results into linguistic information to facilitate their comprehension. Many fuzzy based linguistic modelling approaches have been developed for CW (Herrera, Alonso, Chiclana, & Herrera-Viedma, 2009; Martínez & Herrera, 2012; Martínez, Ruan, & Herrera, 2010) but the 2-tuple linguistic representation model stands out because its simplicity, interpretability and its capability to carry out precise linguistic computations without any kind of approximation because of the use of the symbolic translation (Rodríguez & Martínez, 2013).

On the other hand, the decision makers' preferences elicitation is a fundamental pillar in any DM problem. *Pairwise comparison* method proposed by Thurstone (Thurstone, 1927) is a widely used technique for the decision makers' preferences elicitation, which has been applied in popular MCGDM methods such as the Analytic Hierarchy Process (AHP) (Saaty, 1990). However, it is impossible to neglect the inconsistency in pairwise comparison matrices (Forman & Selly, 2001), which might be the cause of wrong results. Inconsistency in decision makers' opinions often takes place when there are a large number of criteria or alternatives, since the number of pairwise comparisons increases and, consequently, the complexity of solving MCGDM problems. Keeping in mind the latter limitation, Rezaei proposed in Rezaei (2015) a new MCGDM method to obtain the prioritization of the objects (alternatives and/or criteria) in a DM problem, so-called *Best-Worst method* (BWM), which uses fewer comparison data by establishing a new structure of making pairwise comparisons. This new structure consists of comparing only the best and the worst element of the problem, both selected by the decision makers, with the rest of them. In this way, it is not necessary to compare all the elements with each other and the inherent inconsistency of the pairwise comparisons is reduced when the number of criteria or alternatives is too large. Further, several optimization models (Brunelli & Rezaei, 2019; Rezaei, 2015, 2016) have been put forwarded in the literature to derive the priority weights from the reference comparisons of the Best-Worst objects with the rest of objects. However, there is no guideline and comparison analysis of these approaches to infer which one is the best on the aspect of preserving the decision maker's given preference.

The BWM proposed by Rezaei (2015) considers pairwise comparison matrices in which the human assessments are provided by crisp numbers (1–9 scale-based). However, real-world MCGDM problems present vague and uncertain information that cannot be modeled by discrete values. In this realm, several proposals have been put forward by the researchers to accommodate different types of uncertainty within the framework of BWM (Aboutorab, Saberi, Asadabadi, Hussain, & Chang, 2018; Ali & Rashid, 2019; Guo & Zhao, 2017; Mi & Liao, 2019; Pamučar, Petrović, & Ćirović, 2018; Ren, 2018). In the same vein, Guo and Zhao introduced the *Fuzzy BWM* in Guo and Zhao (2017), an extension of the BWM to the fuzzy environment to model the inherent uncertainty in many real-life decision scenarios. This proposal replaces the discrete pairwise comparison matrices with fuzzy comparing judgements represented by linguistic expressions, which facilitate the decision makers' elicitation. Afterwards, linguistic terms are transformed into triangular fuzzy numbers (TFNs) (Zadeh, 1965) by obtaining more reliable decision results under uncertainty environments (Martínez, Ruan, Herrera, Herrera-Viedma, & Wang, 2009; Sánchez, 2007). However, Fuzzy BWM provides results represented by fuzzy numbers, far from the initial linguistic information provided by the decision makers and difficult to understand. In a similar way, Pamučar et al. presented recently a new approach for the BWM based on interval-valued fuzzy-rough numbers (Pamučar et al., 2018). In this proposal, decision makers also provide their preferences by using linguistic premises to model uncertainty in pairwise comparisons but, in this occasion, the results are represented by interval-valued fuzzy-rough numbers. As in the previous proposal, the results are again far away from the initial preferences provided by the decision makers and they are still hard to interpret.

Therefore, taking account the previous premises, our contribution consists of a new extension of BWM under a CW (Mendel et al., 2010; Zadeh, 1996; Zadeh, 2013a, 2013b) paradigm based on the 2-tuple linguistic representation model. In our BWM proposal, decision makers provide their preferences by using fuzzy linguistic pairwise comparisons close to their way of thinking. Subsequently, the results are also provided by linguistic information represented by 2-tuple linguistic values and the same domain that decision makers have used to provide their preferences, by facilitating their understanding. Furthermore, the 2-tuple linguistic BWM is based on the BWM prioritization approach that best preserves initial decision makers' preferences. However, although the deviation among the initial decision makers' preferences and derived priority is minimal, they cannot evaluate how much their initial linguistic preferences have been preserved from the results. For this reason, we also propose a retranslation process in which priority results are transformed into a preference linguistic matrix that uses the same linguistic domain that decision makers have used to provide their assessments. In this way, decision makers can also evaluate easily the deviation among their preferences and the results and, in turn, it increases the understanding of the latter. Additionally, in order to evaluate the consistency of the decision makers' preferences and avoid wrong results, a consistency ratio is proposed. In addition, note that all the BWM proposals ignore the new challenges related to the LS-GDM problems and obtain a unique preference ranking by minimizing the deviation of the given preference information from the group of decision makers, by ignoring the apparition of sub-groups and opinions polarization, which is quite common in this kind of problems. Thus, our proposal also tries to come up with the multiple views and opinions polarization in LS-GDM. Finally, in order to show all the novelties and advantages of our proposal, several illustrative MCGDM problems are introduced and solved. To sum up, our study aims to:

- Analyze and compare different prioritization approaches for BWM and select for our proposal the one that preserves as much as possible the initial decision makers' preferences.
- Propose a new extension of the BWM, so-called 2-tuple BWM, which follows a CW approach by obtaining from fuzzy linguistic pairwise comparisons, linguistic results represented by 2-tuple linguistic values easily understandable and precise. Additionally, in order to facilitate the decision makers' understanding of the preservation of the given linguistic preferences, the deviation from the original opinions is also translated into linguistic values.
- Propose a consistency ratio for the 2-tuple BWM proposal considering the average cognitive behaviour of the experts in order to detect the inconsistency in their preferences and avoid incorrect solutions.
- Apply the so-called 2-tuple BWM to LS-GDM problems in which the different attitudes and polarization of the opinions of the decision makers are taken into account to find out the proper dynamics of the group, facing one of the most relevant challenges in this type of problems.

To do this, the paper is structured as follows: Section 2 reviews some basic concepts about the BWM and the 2-tuple linguistic model in order to facilitate the understanding of the proposal. Section 3 introduces different approaches to compute the prioritization of the elements in a DM process. Section 4 presents the extension of the BWM for linguistic information based on the 2-tuple linguistic model and introduces a novel consistency measure. Section 5 extends the proposal to LS-GDM problems and deals with the apparition of sub-groups and opinions polarization. Section 6 shows different practical examples in order to show the performance and usefulness of the proposal for classical DM and LS-GDM problems. Finally, in Section 7 some concluding remarks are pointed out.

2. Preliminaries

This section reviews several concepts related to BWM and 2-tuple linguistic model in order to understand easily this contribution.

2.1. Best-Worst method

BWM (Rezaei, 2015) is a MCGDM method that derives the criteria weights by reducing the number of pairwise comparisons among criteria. To do this, the best criterion C_B and the worst criterion C_W are predetermined by the experts and compared with the rest of the criteria. These comparisons are named as *reference comparison*. The steps that compose the BWM for the criteria weights computations are:

- **Step 1**: Determine a set of decision criteria.
- **Step 2**: Select the best criterion C_B and the worst criterion C_W. In case that there are several best and worst criteria, these can be selected arbitrary.
- **Step 3**: Make pairwise comparison among C_B and the rest of the criteria, by obtaining the Best to Others (BO) vector, $BO = \{a_{B1}, a_{B2}, \ldots a_{Bn}\}$, where B_j denotes the preference degree of C_B over the criterion C_j and $a_{Bj} \geqslant 1, j = 1, 2, \ldots n, \ j \neq B$.
- **Step 4**: Make pairwise comparison among C_W and the rest of the criteria, by obtaining the Worst to Others (WO) vector, $WO = \{a_{1W}, a_{2W}, \ldots a_{nW}\}$, where a_{jW} denotes the preference degree of the criterion C_j over C_W and $a_{jW} \geqslant 1, j = 1, 2, \ldots n, \ j \neq B \ or \ W$.
- **Step 5**: Compute the weights of criteria by optimization models. For each reference comparison, the optimal weights of criteria satisfy $w_B/w_j = a_{Bj}$ and $w_j/w_W = a_{jW}$. Therefore, the maximum absolute differences $|w_B/w_j - a_{Bj}|$ and $|w_j/w_W - a_{jW}|$ should be minimized. Different prioritization approaches for BWM have been introduced in the literature, some of them have introduced in Section 3.

Note that Rezaei also introduced in Rezaei (2015) a consistency ratio for evaluating the consistency of the pairwise comparisons based on numerical scales. According to Rezaei proposal, a comparison is fully consistent when $a_{Bj} \times a_{jW} = a_{BW}$ for all j, where a_{Bj}, a_{jW} and a_{BW} are respectively the preference of the best criterion over the criterion j, the preference of criterion j over the worst criterion, and the preference of the best criterion over the worst criterion. To indicate how consistent a comparison is, a *consistency ratio* is calculated as follows:

$$Consistency\ Ratio = \frac{\varepsilon^*}{Consistency\ Index} \quad (1)$$

where ε^* represents the maximum absolute difference among the optimal weights and the reference comparisons and *Consistency Index* is a numerical value obtained from a_{BW} (see (Rezaei, 2015) for further details).

Lately, an extension to the fuzzy environment of the BWM was proposed by Guo and Zhao (2017). The biggest difference among the fuzzy extension and the original proposal is the use of fuzzy linguistic information in the reference comparisons. Experts provide their assessments by using linguistic expressions that are transformed subsequently into TFNs, (l, m, u) (see Guo & Zhao, 2017 for further detail).

2.2. 2-tuple Linguistic Model

There are several linguistic computational models based on the fuzzy linguistic approach to accomplish linguistic computations. However, in terms of interpretability and precision, the 2-tuple linguistic model proposed by Martínez, Rodríguez, and Herrera (2015), Herrera and Martínez (2000) stands out from the rest of symbolic fuzzy based models (Rodríguez, Labella, & Martínez, 2016). This linguistic model, based on the concept of symbolic translation, represents the information by means of a 2-tuple, (s_i, α), where s_i is a linguistic label belonging to a predefined linguistic term set $S = \{s_1, s_2 \ldots s_g\}$ and $\alpha \in [-0.5, 0.5)$ a numerical value that represents the translation of the fuzzy membership function that represents the closest term s_i, if s_i does not match exactly the computed linguistic information (see Fig. 1). The value of α is defined as:

$$\alpha = \begin{cases} [-0.5, 0.5) & if \ s_i \in \{s_1, s_2, \ldots, s_{g-1}\} \\ [0, 0.5) & if \ s_i = s_0 \\ [-0.5, 0] & if \ s_i = s_g \end{cases}$$

The symbolic computation on linguistic terms in S returns a value $\beta \in [0, g]$, which is transformed into an equivalent 2-tuple linguistic value, (s_i, α) by means of the function Δ_S defined as follows:

Definition 1. (*Martínez et al., 2015*) Let $S = \{s_0, \ldots, s_g\}$ be a set of linguistic terms and $\overline{S}$ *the 2-tuple set associated with* S defined as $\overline{S} = S \times [-0.5, 0.5)$. The function $\Delta_S : [0, g] \rightarrow \overline{S}$ is given by:

$$\Delta_S(\beta) = (s_i, \alpha), \ with \begin{cases} i = round(\beta) \\ \alpha = \beta - i \end{cases}$$

with $round(\cdot)$ the function that assigns to β the closest integer number $i \in \{0, \ldots, g\}$ to β. Therefore, a 2-tuple linguistic value (s_i, α) can be represented by its equivalent numerical value β in the interval of granularity of S, $[0, g]$.

Proposition 1. *Let* $S = \{s_0, \ldots s_g\}$ *be a linguistic term set and* $(s_i, \alpha) \in \overline{S}$ *be a 2-tuple linguistic value. There is a function,* Δ^{-1}*:*

$$\Delta^{-1} : \overline{S} \rightarrow [0, g]$$
$$\Delta_S^{-1}(s_i, \alpha) = \alpha + i = \beta$$

Remark 1. From Definition 1 and Preposition 1, the transformation of a linguistic term $s_i \in S$ into a 2-tuple linguistic value in $\overline{S}$ is carried out by adding a zero as a symbolic translation to the linguistic term:

$$s_i \in S \rightarrow (s_i, 0) \in \overline{S}$$

Furthermore, the 2-tuple linguistic model has associated a

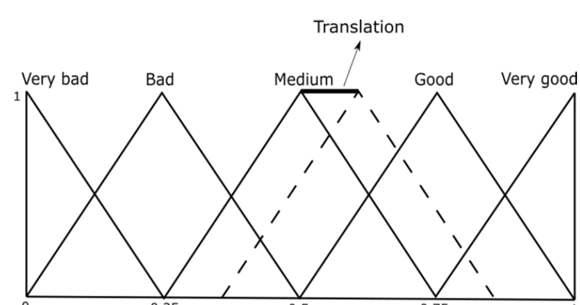


Fig. 1. Symbolic translation.

computational model (Martínez et al., 2015).

In later works (Tai & Chen, 2009), the 2-tuple linguistic model was extended in order to interpret values from the scale $[0,1]$ rather than conventional scale $[0,g_1]$ corresponding to the linguistic term set $S_1 = \{s_0^1, s_1^1, \dots s_{g_1}^1\}$. In such cases any value $\beta \in [0,1]$ can be mapped to the 2-tuple linguistic terms set $\overline{S}_1$ via the mapping $\Delta_1 : \left[0,1\right] \rightarrow \overline{S}_1 = S_1 \times \left[-\frac{1}{2g_1}, \frac{1}{2g_1}\right)$.

Definition 2. (Tai & Chen, 2009) A value β whose value belongs to interval [0, 1] will be obtained after aggregating the result of evaluation using the linguistic variable set S_1. Then the symbolic translation process is applied to translate β into a 2-tuple linguistic variable. The generalized translation function (Δ_1) can be represented as

$$\Delta_1\left(\beta\right) = \left(s_i, \alpha\right) \text{ where} \begin{cases} s_i & i = round(\beta \cdot g_1) \\ \alpha = \beta - \dfrac{i}{g_1} & \alpha \in \left[-\dfrac{1}{2g_1}, \dfrac{1}{2g}\right) \end{cases}$$

Clearly the value of the symbolic translation depends on the cardinality of the linguistic term set S_1. On the other hand, the 2-tuple representation from the 2-tuple information (s_i, α) could be translated into equivalent numerical value $\beta \in [0,1]$ with the help of re-translation function $\Delta_1^{-1} : \overline{S}_1 \rightarrow [0,1]$.

Definition 3. (Tai & Chen, 2009) A reverse equation Δ_1^{-1} is necessary to return an equivalent numerical value $\beta \in [0,1] \subset \Re$ from a 2-tuple linguistic variable (s_i, α). According to the concept of symbolic translation, an equivalent numerical value β is computed as

$$\Delta_1^{-1}\left(s_i, \alpha\right) = \beta = \frac{i}{g_1} + \alpha.$$

3. Prioritization approaches for Best-Worst method

One of the key issues associated with the BWM is the deriving the priority of the objects from the pairwise comparisons of the best and worst objects. Consequently, our proposal has to deal with this issue too. There are several approaches that allow us to obtain such priority but, they may provide different results. For this reason, it is necessary to analyse all these approaches and select properly the most suitable one for our proposal.

3.1. Prioritization process

To describe the prioritization process, we provide its mathematical formalization as follows. Let $X = (x_1, x_2, \dots, x_n)$ be the set of objects. We are going to denote the best and worst objects of X as x_B and x_W, respectively. Clearly, the indexes $B, W \in \{1, 2, \dots, n\}$. Next, the decision maker compares the best and worst objects with the rest of the objects in a pairwise fashion by using the numeric scale '1–9', where '1' represents the objects x_i and x_j are equally important and '9' represents that the object x_i is extremely more important than x_j. The decision maker's pairwise preference could be summarized in the following matrix:

$$D = \begin{array}{c} \\ x_1 \\ x_2 \\ \vdots \\ x_B \\ \vdots \\ x_n \end{array} \begin{array}{c} \begin{array}{cccccccc} x_1 & x_2 & \cdots & x_B & \cdots & x_W & \cdots & x_n \end{array} \\ \begin{pmatrix} - & - & \cdots & - & \cdots & a_{1W} & \cdots & - \\ - & - & \cdots & - & \cdots & a_{2W} & \cdots & - \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ a_{B1} & a_{B2} & \cdots & - & \cdots & a_{BW} & \cdots & a_{Bn} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ - & - & \cdots & - & \cdots & a_{nW} & \cdots & - \end{pmatrix} \end{array}$$

To determine the priority vector from the above pairwise comparison matrix different optimization models have been put forwarded in the literature. We describe them briefly in the following subsections.

3.1.1. Non-linear optimization models

In this approach, the priority weight vector $w = (w_1, w_2, \dots, w_n)$ is deriving from the pairwise preference matrix based on the principle of the minimization of the maximized deviation from the given preferences and that can be put in the following optimization model (Rezaei, 2015):

$$(\mathbf{M-1})$$

$$\begin{array}{ll} \min\max\limits_{j} & \left\{ \left|\dfrac{w_B}{w_j} - a_{Bj}\right|, \left|\dfrac{w_j}{w_W} - a_{jW}\right| \right\} \\ \text{s.t.} & \begin{cases} \sum\limits_{j=1}^{n} w_j = 1, \\ w_j \geqslant 0, \ \text{forall } j = 1, 2, \dots, n. \end{cases} \end{array} \tag{2}$$

Further, **(M-1)** can be transformed into the equivalent optimization model

$$(\mathbf{M-2})$$

$$\begin{array}{ll} \min & \xi \\ \text{s.t.} & \begin{cases} \left|\dfrac{w_B}{w_j} - a_{Bj}\right| \leqslant \xi \\ \left|\dfrac{w_j}{w_W} - a_{jW}\right| \leqslant \xi \\ \sum\limits_{j=1}^{n} w_j = 1, \\ w_j \geqslant 0, \ \text{forall } j = 1, 2, \dots, n. \end{cases} \end{array} \tag{3}$$

Solving the non-linear optimization model **(M-2)**, we can obtain optimal priority vector $w^* = (w_1^*, w_2^*, \dots, w_n^*)$.

3.1.2. Interval weight estimation approach

Rezaei (2016) argued that in case of inconsistency with more than three objects the optimization model **(M-2)** might have multiple optimal solutions. To handle this issue, he proposed the idea of interval weight estimation. But Rezaei (2016), did not provide any theoretical proof or evidence to support the existence of multiple optimal solutions.

In this approach, interval weight is derived from the pairwise comparison matrix following a two phases algorithm. In the first phase, **(M-1)** is solved to find the optimal value ξ^*, while the second phase involved finding the range of each weight component with ξ^* by solving the flowing optimization problem

$$(\mathbf{M-3})$$
$$\text{optimize} \quad w_j$$
$$\text{s.t.} \begin{cases} |\frac{w_B}{w_j} - a_{Bj}| \leqslant \xi^* \\ |\frac{w_j}{w_W} - a_{jW}| \leqslant \xi^* \\ \sum_{j=1}^{n} w_j = 1, \\ w_j \geqslant 0, \ \text{forall } j = 1,2,\ldots,n. \end{cases} \tag{4}$$

By solving the model **(M-3)** for maximum and minimum of w_j, we can find the optimal interval priority weight of the j-th object and similarly for the rest of the objects. Then, one can employ a probabilistic interval prioritization method to rank the objects and the centre value of the interval is considered as a representative of the weight of the object.

3.1.3. Relaxed linear model

Rezaei (2016) proposed a linear model to compute the approximate priority weight of the objects without giving any proper justification/reason. The approximate priority weights w^* is obtained by solving the following linear optimization model:

$$(\mathbf{M-4})$$
$$\min \quad \xi^L$$
$$\text{s.t.} \begin{cases} w_B - a_{Bj}w_j \leqslant \xi^L \\ w_B - a_{Bj}w_j \geqslant -\xi^L \\ w_j - a_{jW}w_W \leqslant \xi^L \\ w_j - a_{jW}w_W \geqslant -\xi^L \\ \sum_{j=1}^{n} w_j = 1, \\ w_j \geqslant 0, \ \text{forall } j = 1,2,\ldots,n. \end{cases} \tag{5}$$

The optimal value ξ^{L*} obtained by solving the model **(M-4)**, considered as an indicator of the consistency of the given preference.

3.1.4. Multiplicative linear model

Brunelli and Rezaei (2019) proposed a new metric, inspired from the abelian group theory of the pairwise comparison and claimed that the metric is more suitable with the multiplicative context of BWM than absolute value metric, which is more suitable for the additive framework. Using the *logarithmic* transformation $v_i = \ln w_i, b_{Bj} = \ln a_{Bj}$ and $b_{jW} = \ln a_{Bj}$, they propose the following mathematical model:

$$(\mathbf{M-5})$$
$$\min \quad \xi$$
$$\text{s.t.} \begin{cases} b_{Bj} - (v_B - v_j) = c_{Bj} - d_{Bj} \\ c_{Bj} + d_{Bj} \leqslant \xi \\ b_{jW} - (v_j - v_W) = e_{jW} - f_{jW} \\ e_{jW} + f_{jW} \leqslant \xi \\ \sum_{j=1}^{n} v_j = 0, \\ c_{Bj}, d_{Bj}, e_{jW}, f_{jW} \geqslant 0 \end{cases} \tag{6}$$

Once we found the optimal solution of the model **(M-5)** in terms of the $v^* = (v_1^*, v_2^*, \ldots, v_n^*)$, the transformation $w_j = e^{v_j} (j = 1,2,\ldots,n)$ is used to obtain the original weight followed by a normalization process.

3.2. Selection of the prioritization method

Previously, we have mentioned the four prioritization methods to find the weights from the reference comparisons with a numerical scale. It is quite reasonable that the different prioritization methods may yield different results. Therefore, there is a need for appropriate selection criteria of the BWM prioritization methods. Among the suite of the criteria used to measure the performance of the prioritization methods, Euclidean distance (ED) is popular and widely used (Dong, Hong, Xu, & Yu, 2011).

The key idea behind the ED based measure is that it computes the difference between the given pairwise comparisons and the derived priority weight obtained from the prioritization method. The Euclidean distance-based measure can be defined as follows:

$$ED = \sqrt{\sum_{j=1}^{n} \left(\frac{w_B}{w_j} - a_{Bj}\right)^2 + \left(\frac{w_j}{w_W} - a_{jW}\right)^2} \tag{7}$$

Based on the ED criteria, we attempt to evaluate the performance of the priority derivation models of the BWM. For that purpose, we consider the example illustrated by Rezaei (2016). A buyer of a car needs to prioritize five criteria for the selecting car: quality (C_1), price (C_2), comfort (C_3), safety (C_4) and style (C_5). For the buyer, price is the most important criteria, i.e., the best criteria while style is the less important criteria i.e., the worst criteria. The buyer provides the pairwise comparisons of the Best-Worst criteria with the others and that are summarized in the following pairwise comparison matrix:

$$P = \begin{array}{c} \\ C_1 \\ C_B \\ C_3 \\ C_4 \\ C_W \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_B & C_3 & C_4 & C_W \end{array} \\ \begin{pmatrix} - & - & - & - & 4 \\ 2 & 1 & 4 & 3 & 8 \\ - & - & - & - & 2 \\ - & - & - & - & 3 \\ - & - & - & - & 1 \end{pmatrix} \end{array}$$

Note that the given reference comparison P is not fully consistent. In the fully consistent case, it has been found that all the prioritization models produce the same weight vector. For this reason, we have considered the not fully consistent reference comparison to compare the performance of the prioritization methods. Now, the above-mentioned prioritization models are employed to find the priority weights from pairwise comparisons P and the results are summarized in Table 1 with the ED criteria.

Based on the ED measure criteria, it is evident from the Table 1 that the original non-linear model **(M-2)** of deriving priority weights has the least difference from the originally given preference while the priority weights derived from the multiplicative linear model **(M-5)** deviates significantly from the original preference. Note that the given pairwise comparisons are almost fully consistent (when $a_{B4} = 2, a_{4W} = 4$ or $a_{B4} = 4, a_{4W} = 2$) and that might be the reason for the obtained priorities by all the methods are very close. To find the performance of prioritization methods under different level of inconsistency, we consider 9 pairwise comparisons matrices that are derived from the original preference matrix P by setting $a_{B4} = 2$ and $a_{4W} \in \{1,2,\ldots,9\}$. Afterwards, all prioritization methods are employed to derive the priority weights in each case and subsequently, ED measures are computed. The performance of the prioritization methods concerning the ED measures is depicted in Fig. 2. It has been observed that the non-linear model **(M-2)** produces the least deviation in all the considered inconsistent cases. Although, the performance of the interval weight estimation model **(M-3)** is very close to **(M-2)**, there is a substantial difference. The prioritization processes **(M-4)** and **(M-5)** yield priority weights that deviate significantly from

Table 1
Comparison of the prioritization methods.

prioritization model	w_1	w_2	w_3	w_4	w_5	ED
(M-2)	0.222499	0.451444	0.112463	0.158174	0.0554198	**0.087262**
(M-3)	0.221711	0.451634	0.113061	0.158241	0.055443	0.088095
(M-4)	0.229508	0.448087	0.114754	0.153005	0.0546448	0.186440
(M-5)	0.219722	0.457040	0.109861	0.158447	0.054931	0.263977

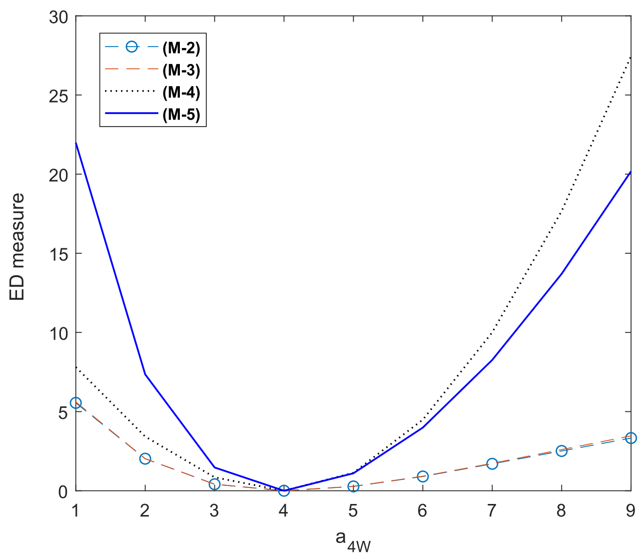


Fig. 2. ED measures for variation in evaluation.

the original pairwise comparisons and produce unreliable results. In light of the previous analysis, we are in the opinion that the non-linear prioritization method **(M-2)** produces more reliable results for the not fully consistent pairwise comparisons. Therefore, in this study, we adopt the non-linear prioritization method **(M-2)** to find weights of the objects.

Remark 2. For solving **(M-2)-(M-5)**, we have coded optimization models in *JuMP*[1] (Julia for Mathematical Programming) and called different solvers to find the optimal solutions. Specifically, the non-linear optimization models are solved by calling *IPOPT*[2] solver (Interior Point OPTimizer) while the linear optimization models have been solved by calling *CPLEX* solver.

4. Linguistic Best-Worst method based on 2-tuple model

This section introduces the extension of BWM based on the 2-tuple linguistic model. First, the linguistic scale used by the decision makers to provide their preferences is defined in Section 4.1. The formalization of the Best-Worst pairwise comparison matrix is then provided in Section 4.2. The prioritization process to obtain the weights from linguistic information and their corresponding transformation into linguistic values is described in Sections 4.3. Afterwards, Section 4.4 introduces a re-translation function to calculate the deviation among the given preferences and the derived priority. Finally, the consistency ratio is defined in Sections 4.5 and 4.6.

4.1. Linguistic scale

In many real-life DM scenarios, a decision maker needs to compare the objects associated with the complex concepts and the preference of the decision maker's involves subjectivity. In such scenarios, the quantification of the decision maker's preference with a precise numeric scale is quite difficult. As the linguistic description is easily understood by human beings even when the context is abstract and complex, it is more convenient for the decision maker to express his/her preference in linguistic terms. In this view, we use the fuzzy linguistic scale for the BWM (Lootsma, 1980; Van Laarhoven & Pedrycz, 1983) (see Table 2): $S^{BWM} = \{ l_1 =$ Equally Importance (EI), $l_2 =$ Weekly Important (WI), $l_3 =$ Fairly Important (FI), $l_4 =$ Very Important (VI), $l_5 =$ Absolutely Important (AI) $\}$.

Based on the linguistic term set S^{BWM}, the decision-maker could be able to provide the preference of the pairwise comparisons of the best and worst objects with the rest of the objects. The preference of the best object, x_B over any object x_j can take a value of linguistic term from the set S^{BWM}.

Remark 3. Note that we have considered five linguistic labels that are represented by the integers from $[1,5]$ instead of $[0,5]$. Therefore, we re-write the translation and re-translation functions as follows: $\Delta : [1,5] \rightarrow S^{BWM} \times [-0.5, 0.5)$ and $\Delta^{-1} : S^{BWM} \times [-0.5, 0.5) \rightarrow [1,5]$.

4.2. Formation of Best-Worst comparison matrix

Once the best and worst objects have been identified from the set of objects under consideration by the decision maker, the key issue in the decision process boils down to accumulate the preference information regarding the pairwise comparisons of the best and worst objects with the rest of the objects from the decision maker.

Given the linguistic term set, S^{BWM} with an associated 2-tuple computational model, the decision maker first compares the best object x_B with the rest of the objects $X \backslash \{x_B\}$ and expresses his/her assessments via a single linguistic term that are subsequently transformed into 2-tuple linguistic values (see Remark 1). The same has been done for the worst objects x_W. Decision maker's preference information can be summarized in the following Linguistic Best-Worst Pairwise Comparison Matrix (LBWPCM) with 2-tuple entries:

Table 2
Linguistic scale with membership functions.

Linguistic terms	Membership function
Equally Importance (EI)	$(1,1,1)$
Weekly Important (WI)	$(2/3,1,3/2)$
Fairly Important (FI)	$(3/2,2,5/2)$
Very Important (VI)	$(5/2,3,7/2)$
Absolutely Important (AI)	$(7/2,4,9/2)$

[1] http://www.juliaopt.org/JuMP.jl/v0.13/.
[2] https://www.coin-or.org/Ipopt/documentation/.

$$L = \begin{array}{c} \\ x_1 \\ x_2 \\ \vdots \\ x_B \\ \vdots \\ x_n \end{array} \begin{array}{c} \begin{array}{ccccccccc} x_1 & x_2 & \cdots & x_B & \cdots & x_W & \cdots & x_n \end{array} \\ \left(\begin{array}{ccccccccc} - & - & \cdots & - & \cdots & (d_{1W}, \alpha_{1W}) & \cdots & - \\ - & - & \cdots & - & \cdots & (d_{2W}, \alpha_{2W}) & \cdots & - \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ (d_{B1}, \alpha_{B1}) & (d_{B2}, \alpha_{B2}) & \cdots & - & \cdots & (d_{BW}, \alpha_{BW}) & \cdots & (d_{Bn}, \alpha_{Bn}) \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ - & - & \cdots & - & \cdots & (d_{nW}, \alpha_{nW}) & \cdots & - \end{array} \right) \end{array}$$

where (d_{Bj}, α_{Bj}) is the linguistic preference of the decision maker against the comparison of the x_B with x_j and (d_{jW}, α_{jW}) denotes the linguistic preference for the comparison of x_W with x_j. It is evident that $d_{Bj}, d_{jw} \in S^{BWM}$ and $\alpha_{Bj}, \alpha_{jW} \in [-0.5, 0.5)$.

4.3. Finding the priority weights

After the decision maker has completed the preference elicitation tasks by providing LBWPCM, we attempt to find the priority weights, which fit the given linguistic preference information optimally. Now, the optimal priority weights $w = (w_1, w_2, \ldots, w_n)$ would be driven in such a way that the given preference for each object j should be preserved as much as possible. In other words, the optimal priority weights should satisfy the qualities $\frac{w_B}{w_j} = \Delta^{-1}\left(\left(d_{Bj}, \alpha_{Bj}\right)\right)$ and $\frac{w_j}{w_W} = \Delta^{-1}\left(\left(d_{jW}, \alpha_{jW}\right)\right)$ to some extent. This principle can be translated into a min–max optimization problem, where our objective is to minimize the maximum absolute difference $\left|\frac{w_B}{w_j} - \Delta^{-1}\left(\left(d_{Bj}, \alpha_{Bj}\right)\right)\right|$ and $\left|\frac{w_j}{w_W} - \Delta^{-1}\left(\left(d_{jW}, \alpha_{jW}\right)\right)\right|$ for all j. Therefore, the optimal priority weights from the LBWPCM can be obtained by solving the non-linear optimization problem

$$(\mathbf{M-6})$$

$$\begin{array}{ll} \min\max_j & \left\{ \left|\frac{w_B}{w_j} - \Delta^{-1}\left(\left(d_{Bj}, \alpha_{Bj}\right)\right)\right|, \left|\frac{w_j}{w_W} - \Delta^{-1}\left(\left(d_{jW}, \alpha_{jW}\right)\right)\right| \right\} \\ \text{s.t.} & \begin{cases} \sum_{j=1}^{n} w_j = 1, \\ w_j \geqslant 0, \ \text{forall } j = 1, 2, \ldots, n. \end{cases} \end{array} \tag{8}$$

Finally, it can be transformed into minimization problem with non-linear objectives as follows:

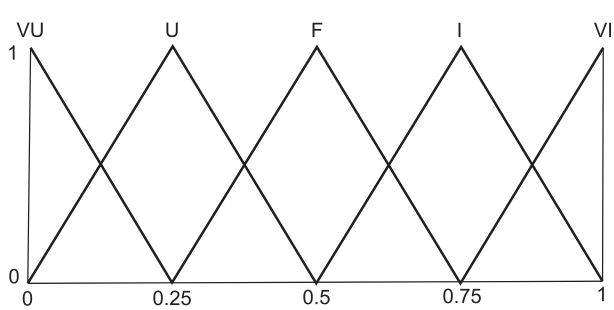
$$(\mathbf{M-7})$$

$$\begin{array}{ll} \min & \xi \\ \text{s.t.} & \begin{cases} \sum_{j=1}^{n} w_j = 1, \\ \left|\frac{w_B}{w_j} - \Delta^{-1}\left(\left(d_{Bj}, \alpha_{Bj}\right)\right)\right| \leqslant \xi, \\ \left|\frac{w_j}{w_W} - \Delta^{-1}\left(\left(d_{jW}, \alpha_{jW}\right)\right)\right| \leqslant \xi, \\ w_j \geqslant 0, \ \text{forall } j = 1, 2, \ldots, n. \end{cases} \end{array} \tag{9}$$

Solving the optimization model **(M-7)**, we can obtain the optimal priority weights $w^* = (w_1^*, \ldots., w_n^*)$ and ξ^*.

So far, we obtain numerical values to represent the weights. However, the weights should be easily interpretable for the decision maker to understand the priority results in a natural language. In spite of a fuzzy BWM approach was proposed in Guo and Zhao (2017) in which decision makers provided their preferences by means of linguistic assessments, the resulting weights were represented by numerical values or fuzzy numbers, very far from the way of decision makers express their opinions. In this vein, we introduce a linguistic term set (see Fig. 3), $S_1 = \{s_0 = \text{Very Unimportant (VU)}, s_1 = \text{Unimportant (U)}, s_2^1 = \text{Fair (F)}, s_3^1 = \text{Important (I)}, s_4^1 = \text{Very Important (VI)}\}$ and maps it to $[0, 1]$ via the retranslation step Δ_1^{-1} (see Def. 3). Further, we are able to interpret any value from the interval $\beta \in [0, 1]$ linguistically with help of translation operator $(\Delta_1(\cdot))$ (see Def. 2) as a priority weight of a particularly object under consideration. In this way, the numerical weights can be transformed into linguistic information.

Definition 4. Let $w^* = (w_1^*, \ldots., w_n^*)$ be the optimal priority weights (numeric) derived from **(M-7)**. Then the corresponding 2-tuple linguistic optimal priority weights $l_{w^*} = (l_{w_1^*}, l_{w_2^*}, \ldots, l_{w_n^*})$ such that $l_{w_j} \in \overline{S}_1 = S_1 \times \left[-\frac{1}{2g_1}, \frac{1}{2g_1}\right) = S_1 \times [-0.125, 0.125)$ can be obtained from numeric weight w^* as follows:



Linguistic terms	membership function
Very Unimportant (VU)	(0, 0, 0.25)
Unimportant (U)	(0, 0.25, 0.5)
Fair (F)	(0.25, 0.5, 0.75)
Important (I)	(0.5, 0.75, 1)
Very Important (VI)	(0.75, 1, 1)

Fig. 3. Linguistic scale for weights.

$$l_{w^*} = \Delta_1(w*) = \left(\Delta_1(w_1^*), \Delta_1(w_2^*), \ldots.., \Delta_1(w_n*)\right) \quad (10)$$

Remark 4. Note that the adaptation of the non-linear model to find the priority weights comes from the fact in Section 3.2 that the non-linear model produces the most reliable results than other comparative prioritization models.

Remark 5. Note that we have used a linguistic term set S_1 with cardinality $g_1 = 5$ in our proposal and consequently, the symbolic translation of the 2-tuple linguistic weights is between $[-0.125, 0.125)$ according to Eq. 2. However, the decision-maker could use any linguistic term set as long as its cardinality follows the psychologist recommendation that less than 5 terms being not sufficiently informative and more than 9 terms being too much for a proper understanding of their differences (Miller, 1956).

4.4. Re-translation function to analyse deviation from original opinions

By means of the prioritization process, we obtain the priority weights of the reference objects $w = (w_1, w_2, \ldots, w_n)$ in the numerical scale $[0, 1]$ by preserving as much as possible the initial decision makers' preferences (see Section 3). Afterwards, they are subsequently transformed into the 2-tuple linguistic values $l_w = (l_{w_1}, l_{w_2}, \ldots, l_{w_n})$ (see Section 4.3). However, decision makers cannot evaluate how much their initial linguistic preferences have been preserved in the derived priority. To provide this information to the decision makers and facilitate the understanding over the prioritization weights, we apply a re-translation process that transforms the derived numerical prioritization weights into a LBWPCM. The resulting Best-Worst comparison preference employs the same linguistic scale used by the decision maker to give their assessments thus, he/she can evaluate easily the deviation among his/her preferences and the derived priority. For this purpose, we introduce a translation function, T such that

$$\overline{a}_{Bj} = T\left(w_B \big/ w_j\right) = \begin{cases} \frac{w_B}{w_j} & \text{if} \quad 1 \leqslant \frac{w_B}{w_j} \leqslant 5 \\ 1 & \text{if} \quad \frac{w_B}{w_j} < 1 \\ 5 & \text{if} \quad \frac{w_B}{w_j} > 5 \end{cases} \quad (11)$$

$$\overline{a}_{jW} = T\left(w_j \big/ w_W\right) = \begin{cases} \frac{w_j}{w_W} & \text{if } 1 \leqslant \frac{w_j}{w_W} \leqslant 5 \\ 1 & \text{if} \quad \frac{w_j}{w_W} < 1 \\ 5 & \text{if} \quad \frac{w_j}{w_W} > 5 \end{cases} \quad (12)$$

The translation function T allows us to restore the original linguistic scale from the priority weights. By employing the original linguistic translation function $T(\cdot)$ on $\overline{a}_{Bj}$ and $\overline{a}_{jW}$ for all j, we can construct the LBWPCM $\overline{L}$, from the derived priority weight $w = (w_1, w_2, \ldots, w_n)$. Therefore, the decision maker could easily compare the derived results $\overline{L}$ and given linguistic preference L.

4.5. Consistency ratio for linguistic Best Worst Method

Even though, it is expected that the decision maker will provide a fully consistent preference, the bounded rational behaviour of the decision maker yields inconsistency in the preference (Tversky & Kahneman, 1992; Wang, Wang, & Martínez, 2017). Measuring such inconsistency in the given preferences of decision maker's is necessary for making a reliable choice. In this view, Rezaei (2015) introduces the notion of consistency ratio (CR) that indicates how much the given preference is consistent. In order to check the consistency degree of LBWPCM, we extend the notion of CR in the 2-tuple linguistic environment by providing the definition of consistency and analogous concepts for LBWPCM as follows:

Definition 5. A given LBWPCM is said to be consistent if

$$\Delta^{-1}((d_{Bj}, \alpha_{Bj})) \times \Delta^{-1}((d_{jW}, \alpha_{jW})) = \Delta^{-1}(d_{BW}, \alpha_{BW}) \text{ forall } j \in \{1, 2, \ldots, n\}. \quad (13)$$

When the condition Eq. (13) does not hold for at least one j, we will say that the given LBWPCM is not fully consistent or inconsistent. In this case, we compute the consistency ratio to measure the consistency degree.

In inconsistent scenarios, the violation of the Eq. (13) resulted the satisfaction of the inequalities $\Delta^{-1}((d_{Bj}, \alpha_{Bj})) \times \Delta^{-1}((d_{jW}, \alpha_{jW})) \lesseqgtr \Delta^{-1}(d_{BW}, \alpha_{BW})$ for at least one j and that decreases the consistency degree of the LBWPCM. The possible highest value of the inequality occurs when $\Delta^{-1}((d_{Bj}, \alpha_{Bj}))$ and $\Delta^{-1}((d_{jW}, \alpha_{jW}))$ are equal to $\Delta^{-1}(d_{BW}, \alpha_{BW})$. Introducing a deviation variable, the inequity could be expressed as the following equality:

$$(\Delta^{-1}((d_{Bj}, \alpha_{Bj})) - \xi) \times (\Delta^{-1}((d_{jW}, \alpha_{jW})) - \xi) = (\Delta^{-1}(d_{BW}, \alpha_{BW}) + \xi) \quad (14)$$

When $\Delta^{-1}((d_{Bj}, \alpha_{Bj}) = \Delta^{-1}((d_{jW}, \alpha_{jW})) = \Delta^{-1}(d_{BW}, \alpha_{BW})$, the Eq. (14) transformed into

$$\xi^2 - (1 + 2\Delta^{-1}(d_{BW}, \alpha_{BW}))\xi + (\Delta^{-1}(d_{BW}, \alpha_{BW})))^2 - \Delta^{-1}\left(d_{BW}, \alpha_{BW}\right) = 0 \quad (15)$$

As $\Delta^{-1}(d_{BW}, \alpha_{BW}) \in [1, 9]$, we can find the maximal possible ξ by solving the Eq. (15). Further, one can use this maximum values of ξ as a consistency index and the corresponding consistency ratio of the LBWPCM will obtain as follows:

$$CR = \frac{\xi^*}{Consistency\ Index} \quad (16)$$

where ξ^* is obtained from the optimization model **(M-6)**.

4.6. Random consistency measure

It has been observed that the maximum inconsistency in a given Best-Worst preference occurs when $a_{Bj} = a_{jW} = a_{BM}$ and the consistency ratio is the relative measure of the inconsistency concerning the maximum inconsistency in $[0, 1]$ scale, where 0 indicates the perfect consistent scenario while 1 represents perfectly inconsistency scenario. However, such a notion of consistency is derived from the pessimistic view which may deviate significantly from decision makers' common cognitive behaviour. Further, there are no guidelines about the acceptable level of consistency.

Note that once decision maker fixed the best and worst objects with the ratio of the best to worst, the probability of occurring the maximal inconsistency depends not only the ratio a_{BW} but also on the number of objects under the consideration. For example, if there are 6 reference objects with $a_{BW} = l_6$, then the probability of the occurring the maximum inconsistency is $\frac{1}{6^6}$. When $a_{BW} = l_8$, the probability of occurring maximum inconsistency for the same scenario becomes $\frac{1}{8^6}$. Thus, the occurrence of the maximum inconsistency is a quite rare event. Due to this fact, we are going to introduce the notion of random consistency index (RCI) of the LBWPCM based on the average cognitive behaviour of the decision maker.

From a set of n reference objects, the decision maker first chooses the best and worst objects and selects a linguistic label to provide the preference for the best to worst object $a_{BW} = l_k$. The inconsistency in preference information may occur due to bounded rational behaviour in the

Table 3
Average random consistency index.

n	$RACI(n, a_{BW})$							
	2	3	4	5	6	7	8	9
3	0.18361	0.37665	0.55037	0.75272	0.95964	1.18496	1.40357	1.64334
4	0.29530	0.54800	0.81162	1.09939	1.40172	1.71159	2.05341	2.40471
5	0.36089	0.64679	0.96388	1.30009	1.64452	2.04431	2.42486	2.848
6	0.40466	0.71746	1.06277	1.45598	1.84730	2.27373	2.71134	3.14877
7	0.43129	0.76699	1.14422	1.55603	1.98077	2.43942	2.90589	3.41465
8	0.44958	0.80825	1.20708	1.64420	2.09304	2.58759	3.07247	3.56621
9	0.77678	1.37058	2.00741	2.62732	3.27161	3.93522	4.67623	5.33078

rest of $2n-4$ judgements. As $a_{BW} = l_k$, the rational decision maker will use the linguistic labels $\{l_1, l_2, \ldots, l_k\}$ to fill up the rest of $2n-4$ preferences. With these choices, there are k^{2n-4} possible configurations of the decision maker's preferences consisting of both consistent and inconsistent preferences. Decision maker's preferences on the referenced objects will match with one of the configurations. We define the average consistency index as the mean of consistency of all possible configuration of preferences. However, when $n \geqslant 5$, the number of configurations k^{2n-4} becomes very large and finding average consistency index becomes a computationally tedious task. Keeping this fact in mind, we introduce the notion of *random average consistency index* (RACI), which defined as an average value of consistency ξ that are generated by selecting N configurations of the LBWPCMs for a given n and a_{BW} at random, i.e.

$$RACI\left(n, a_{BW}\right) = \frac{1}{N}\sum_{k=1}^{N}\xi^{(k)}\left(n, a_{BW}\right) \tag{17}$$

where $\xi^{(k)}(n, a_{BW})$ is consistency of the k-th sample LBWPCM with best to worst preference a_{BW}. The consistency $\xi^{(k)}(n, a_{BW})$ is obtained by solving the optimization model $(\mathbf{M}-\mathbf{7})$. Clearly, $RACI(n, a_{BW})$ can be viewed as an average cognitive inconsistency of the decision maker associated with LBWPCM. By setting up $N = 10,000$, we have computed $RACI(n, a_{BW})$ for different values of n and a_{BW} and the results are reported in the Table 3. From Table 3, it is evident that average cognitive behaviour of the decision makers $RACI(n, a_{BW})$ deviates significantly from the decision maker's extreme attitude, i.e. completely pessimistic view given by the decision maker. Moreover, Table 3 also depicts the phenomenon associated with human cognitive behaviour that with the increase of the number of choices inconsistency in the judgments gradually increases.

Based on the notion of $RACI(n, a_{BW})$, we re-define the consistency ratio for a given LBWPCM as follows:

$$CR\left(n, a_{BW}\right) = \frac{\xi(n, a_{BW})}{RACI(n, a_{BW})} \tag{18}$$

The new consistency ratio, defined by Eq. (18), can be viewed as a decision maker's cognitive deviation from the average rational decision makers' behaviour. The RACI index not only depends on the best to worst ratio a_{BW} but also on the number of the objects under consideration when the linguistic scale is fixed. It is the point of departure of our proposed RACI index from the Rezai's proposal of consistency. Although one may feel that any value just below the RACI may be sufficient to guarantee a reasonable consistency, practically that is not sufficient. Thus, we need to set a threshold to ensure reasonable consistency in the given reference comparison. It is needless to say that threshold with a value close to zero could ensure nearly-perfect consistency. But such conservative value may enforce the decision maker to behave near-perfect way, and that could add cognitive bias. As there are a very few nearly-consistent reference comparisons for a given scale with a fixed best to worst ratio, this could be very high cognitively demanding to the decision maker. Thus, a value very close to zero may be too restrictive while a larger value more than 0.5 would not be able to maintain a reasonable consistency. The primary random experiment with 5 reference objects and $1-9$ scale indicates that the probability of generating a random reference comparison with consistency threshold 0.2 is near 0.0007 while for the threshold 0.35 it is about 0.0096. Further, in both of the cases, the transitivity of the preferences are preserved. Based on these facts, we set $CR(n, a_{BW}) \leqslant 0.35$ as a consistency threshold, which is not very restrictive and guarantee sufficient consistency to generate reasonable results.

5. Optimal group formation with linguistic best-worst preferences

Classically, GDM framework has been adopted to obtain priority on a set of objects in BWMs, where a unique preference ranking is obtained by minimizing the deviation of the given preference information from the group of decision makers (Safarzadeh, Khansefid, & Rasti-Barzoki, 2018). However, for instance, in LS-GDM problems, polarization in opinions and the identification of such polarized views need to be analyzed to understand the proper dynamics of groups with a large number of stakeholders/participants properly (Abrams et al., 1990; Hogg et al., 1990).

In this section, we are interested in identifying the formation of all sub-groups of decision makers based on the coherence of their opinions. In particular, we aim to partition the group of decision makers into different sub-groups by minimizing the maximized deviation of the preference information given by each of the sub-groups of decision makers. This kind of settings will allow us to tackle the complex LS-GDM problems by relaxing the uniqueness of the preference ranking and allowing the partition among the different sub-groups. The critical distinction of our approach from the existing GDM based on the BWM is that our approach tries to come up with the multiple views according to the cohesion of each sub-group. In contrast, the existing methods attempt to brute-force the formation of a single ranking from group preferences.

To describe our approach mathematically, we introduce a few notations in the following. Let $\{e_1, e_2, \ldots, e_t\}$ be a group of decision makers. Each decision maker is characterized by his/her attitude, knowledge and expertise. Decision makers provide their linguistic preference over the objects using the linguistic term set S^{BWM}. The preference of the k-th decision maker is summarized in the linguistic best-worst pair-wise comparison matrix, in short,

$$LBWPCM_k = \begin{array}{c} \\ x_B \\ x_W \end{array} \begin{array}{c} \begin{array}{ccccccc} x_1 & x_2 & \cdots & x_B & \cdots & x_W & \cdots & x_n \end{array} \\ \begin{pmatrix} r^k_{B1} & r^k_{B2} & \cdots & r^k_{BB} & \cdots & r^k_{BW} & \cdots & r^k_{Bn} \\ r^k_{W1} & r^k_{W2} & \cdots & r^K_{WB} & \cdots & r^k_{WB} & \cdots & r^k_{Wn} \end{pmatrix} \end{array}$$

where, $r_{Bj}^k = (d_{Bj}^k, \alpha_{Bj}^k)$ with $d_{Bj}^k \in S^{BWM}$ and $\alpha_{Bj}^k \in [-0.5, 0.5)$.

We intend to divide the group of decision makers into p disjoint sub-groups by minimizing the sum of maximized deviation of the given preference information by each sub-groups. In this sequel, we introduce binary variables to capture the association of the decision makers to the sub-groups as follows:

$$y_{kg} = \begin{cases} 1 & \text{if } k-\text{th decision maker belongs to } g-\text{th sub}-\text{group} \\ 0 & \text{otherwise} \end{cases} \tag{19}$$

Our interest lies in finding the association of the decision variables y_{kg}, $(k = 1, 2, \ldots, t, g = 1, 2, \ldots, p)$ with the numeric priority vector of the each sub-group $w^g = (w_1^g, w_2^g, \ldots, w_n^g), (g = 1, 2, \ldots, p)$ corresponding to the 2-tuple linguistic weight vector $l_{w_g} = (l_{w_1^g}, l_{w_2^g}, \ldots, l_{w_n^g})(g = 1, 2, \ldots, p)$ by minimizing the sum of maximized deviation of the each sub-groups, which can be written as the following optimization problem:

$(\mathbf{M-8})$

$$\min \sum_{g=1}^{p} \sum_{k=1}^{t} y_{kg} \max_j \left\{ \left|\frac{w_B^g}{w_j^g} - \Delta^{-1}\left(\left(d_{Bj}^k, \alpha_{Bj}^k\right)\right)\right|, \left|\frac{w_j^g}{w_W^g} - \Delta^{-1}\left(\left(d_{jW}^k, \alpha_{jW}^k\right)\right)\right| \right\}$$

$$\text{s.t.} \begin{cases} \sum_{g=1}^{p} y_{kg} = 1, k = 1, 2, \ldots, t \\ \sum_{k=1}^{t} y_{kg} \geqslant 1, g = 1, 2, \ldots, p \\ \sum_{j=1}^{n} w_j^g = 1, g = 1, 2, \ldots, p \\ w_j^g \geqslant 0, \text{ forall } j = 1, 2, \ldots, n, g = 1, 2, \ldots, p. \end{cases} \tag{20}$$

It is to be noted that in the optimization model **(M-8)**, the first and second set of constraints are required to form sub-groups of the decision makers. The first set of constraints dictated that each decision maker will be associated with exactly one sub-group, while the second set of constraints guaranteed that each sub-group would contain at least one decision maker. In the aim of simplifying the optimizing model **(M-8)**, we set

$$\xi_{kg} = \max_j \left\{ \left|\frac{w_B^g}{w_j^g} - \Delta^{-1}\left(\left(d_{Bj}^k, \alpha_{Bj}^k\right)\right)\right|, \left|\frac{w_j^g}{w_W^g} - \Delta^{-1}\left(\left(d_{jW}^k, \alpha_{jW}^k\right)\right)\right| \right\} k = 1, 2, \ldots, t, g = 1, 2, \ldots, p$$

With this transformation, the optimization model **M-8** can be written as follows:

$(\mathbf{M-9})$

$$\min \sum_{g=1}^{p} \sum_{k=1}^{t} y_{kg} \xi_{kg}$$

$$\text{s.t.} \begin{cases} \sum_{g=1}^{p} y_{kg} = 1, \ k = 1, 2, \ldots, t \\ \sum_{k=1}^{t} y_{kg} \geqslant 1, \ g = 1, 2, \ldots, p \\ \left|\frac{w_B^g}{w_j^g} - \Delta^{-1}\left(\left(d_{Bj}^k, \alpha_{Bj}^k\right)\right)\right| \leqslant \xi_{kg} \ j = 1, 2, \ldots, n, k = 1, 2, \ldots, t, g = 1, 2, \ldots, p \\ \left|\frac{w_j^g}{w_W^g} - \Delta^{-1}\left(\left(d_{jW}^k, \alpha_{jW}^k\right)\right)\right| \leqslant \xi_{kg} \ j = 1, 2, \ldots, n, k = 1, 2, \ldots, t, g = 1, 2, \ldots, p \\ \sum_{j=1}^{n} w_j^g = 1, \ g = 1, 2, \ldots, p \\ w_j^g \geqslant 0, \ \text{forall } j = 1, 2, \ldots, n, g = 1, 2, \ldots, p. \end{cases} \tag{21}$$

By solving the non-linear mixed-integer programming model, we can obtain the composition of each sub-groups with the priority weights, which will allow us to preference over the alternatives. Specifically, the part of the solution $\{y_{kg}^*\}$ gives us the association of the decision makers to different sub-groups and $w^{g*} = (w_1^{g*}, \ldots, w_n^{g*})$ provides us the priority weights of the g-th sub-group. The corresponding 2-tuple linguistic weight vectors can obtained as follows: $l_{w^{g*}} = \Delta_1(w^{g*}) = (\Delta_1(w_1^{g*}), \Delta_1(w_2^{g*}), \ldots, \Delta_1(w_n^{g*}))$.

Solving the mixed-integer non-linear programming (MINLP) model **(M-8)** for a large group (more than 30 decision makers (Chen & Liu, 2006)) is quite a daunting task. However, there are effective global optimization algorithms that have been widely used in solving industrial problems with a few hundreds of variables. Their limited scalability restricted us to apply it in LS-GDM problems with thousands of variables. To this end, we use the recently developed solver JUNIPER (Kröger, Coffrin, Hijazi, & Nagarajan, 2018) to obtain a solution of the MINLP model **(M-9)**. The key advantage of the JUNIPER over other MINLP solvers is that it can find quickly high-quality feasible solutions without guarantees of global optimality (Kröger et al., 2018).

The optimal value $\sum_{g=1}^{p}\sum_{k=1}^{t}y_{kg}^{*}\xi_{kg}^{*}$ indicates the total deviation of all the sub-groups preference from the original opinions of the decision maker under the optimal group partition configuration. This can be termed as a total inconsistency of the for the group configuration. The quantity $\sum_{k=1}^{t}y_{kg}^{*}\xi_{kg}^{*}$ denotes the total deviation of the members of the g-th sub-group original opinions from the derived g-th sub-group priority weights $(w_1^{g*}, w_2^{g*}, \ldots, w_n^{g*})$. We will refer $\sum_{k=1}^{t}y_{kg}^{*}\xi_{kg}^{*}$ as the g-th sub-group inconsistency. The optimal value $\sum_{g=1}^{p}\sum_{k=1}^{t}y_{kg}^{*}\xi_{kg}^{*} = 0$ suggests that the members of the respective sub-groups provide fully consistent opinions. For a better understanding of the decision makers' on the aspect that how the derived preference deviate from their original linguistic preference, we can translate the priority weights $(w_1^{g*}, w_2^{g*}, \ldots, w_n^{g*})$ into linguistic preference matrix according to methodology described in Section 4.4 and present to the members of the g-th sub-group.

As consistency is vital to make reliable decision outcomes, the decision makers need to provide linguistic preference with reasonable consistency. To make sure that it is reasonable to measure the consistency of each decision maker via Eq. (16) before the commencement of the group formation process. Different sub-groups might have different levels of inconsistency. From the optimal decisions $\{\xi_{kg}^{*}\}$, we can define the RACI of the g-th sub-groups as follows:

$$CR^{g} = \max\left\{\frac{\xi_{kg}^{*}}{RACI(n, a_{BW})}\forall\, k \text{ with } y_{kg} = 1\right\}.$$

Further, the inconsistency of the group can be defined from the inconsistency measures of the sub-groups as follows: $CR = \max\{CR^{g} : g = 1, 2, \ldots, p\}$.

It is relatively straightforward that for a fixed set of decision makers, there exists a lot of different configuration of the group, which can be detected by varying the index of the number of sub-groups p. We could use the inconstancy indexes, defined above to evaluate the different configurations of the group to find the most appropriate configuration.

Remark 6. When $p = 1$, i.e., there is no partition among the group of decision makers, the group priority weight-driven optimization model **(M-8)** translates into the best-worst GDM model proposed by Safarzadeh et al. (2018) under the assumption that each decision maker's opinion is equally important in the formation of the group opinion.

Remark 7. For a fixed set of decision makers (t), we may interested to check different configurations of the group by varying the number of sub-groups (i.e., p). With the increment of a single sub-group, we add t more binary variables (y_{kg}) to our GDM model **(M-8)** and that increase the complexity.

6. Illustrative example

In order to show the usefulness and validity of the novel version of BWM based on the 2-tuple linguistic model, this section introduces two different examples. The former describes a simple real DM problem in which just a single expert participates in the decision process. The latter introduces an LS-GDM problem in which a large number of experts participate in the decision process. Both examples will be solved with the proposed BWM, by obtaining the weights of the criteria and, to conclude, the ranking of the alternatives.

Table 4
Best criterion comparisons.

The best criterion	C_1	C_2	C_3	C_4
C_3	WI	FI	–	AI

Table 5
Worst criterion comparisons.

The worst criterion	C_4
C_1	FI
C_2	WI
C_3	AI
C_4	–

Table 6
Weights obtained by different approaches.

Weights	Fuzzy BWM	2-tuple BWM
w_1	(0.2378, 0.2875, 0.3344)	(U,0.01)
w_2	(0.1496, 0.1721, 0.2103)	(U,−0.08)
w_3	(0.3878, 0.4206, 0.4477)	(F, −0.02)
w_4	(0.1117, 0.1177, 0.1290)	(VU,0.09)

6.1. Multi-criteria decision making example

Let us suppose a travel agency wants to acquire a set of cars for their business. After initial screening they have shortlisted four cars, say $X = \{x_1, x_2, x_3, x_4\}$, as a possible options for the acquisition, which are evaluated over a set of four criteria $C = \{C_1 : Style, C_2 : Power, C_3 : Price, C_4 : Color\}$.

To prioritize the criteria according to the linguistic BWM, the decision maker is required to identify the best and worst criteria from C considering the company's key needs. Based on such needs, the decision maker found that the best criterion is C_2 and C_4 is the worst one. Once the best and worst criteria have been identified, the decision maker makes pairwise comparisons among C_B and C_W with the rest of the criteria. In order to facilitate the elicitation of the preferences, the decision maker expresses his/her assessments making use of the linguistic scale introduced in Section 4.1, The comparisons are shown in Tables 4 and 5:

6.1.1. 2-tuple BWM resolution

First, the linguistic Best-Worst preference of the decision maker is transformed into 2-tuple linguistic Best-Worst preference matrix as follows:

$$L = \begin{matrix} & C_1 & C_2 & C_3 & C_4 \\ \begin{matrix} C_3 \\ C_4 \end{matrix} & \multicolumn{4}{c}{\begin{pmatrix} (l_2,0) & (l_3,0) & (l_1,0) & (l_5,0) \\ (l_3,0) & (l_2,0) & (l_5,0) & (l_1,0) \end{pmatrix}} \end{matrix}$$

Now, we employ the model **(M-7)** to derive the priority weights from L and obtain 2-tuple linguistic priority weights $l_w^{*} = ((U, 0.01), (U, -0.08), (F, -0.02), (VU, 0.09))$ with $\xi^{*} = 0.171584$. From Eq. (18), it is found that the consistency ratio for the given linguistic preferences is $CR(4,5) = 0.1561$, which indicates that the given preference information is consistent enough to find a reasonable result. The 2-tuple linguistic weight is easily understandable to the decision maker as a linguistic description is much easier to interpret than a numeric one.

Model **(M-7)** obtains priority weights by preserving as much as possible the decision maker's initial preferences (see Section 3). However, in order to the buyer also knows the deviation of his/her initial preferences regarding the resulting weights, we further transform the derived numeric priority weights i.e., $\Delta_1^{-1}(l_w^{*})$ into linguistic Best-Worst preference information by utilizing Eqs. (11) and (12) as follows:

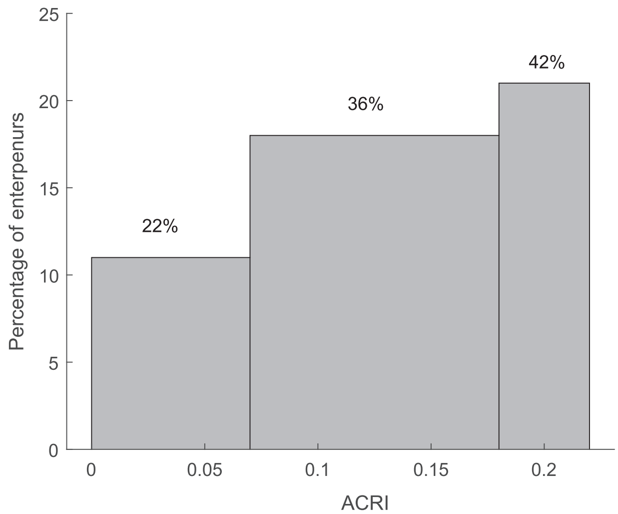


Fig. 4. Distribution of the entrepreneurs' RACI.

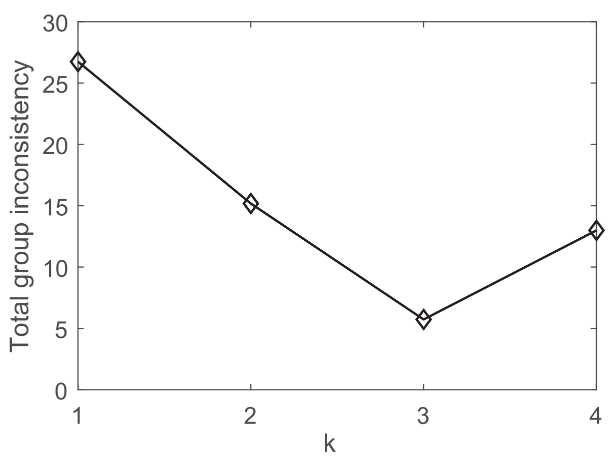


Fig. 5. The value of the objectives (total group inconsistencies) obtained by JUNIPER for different values of k={1,2,3,4}.

$$\bar{L} = \begin{array}{c} \\ C_3 \\ C_4 \end{array} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \\ \end{array}\begin{pmatrix} (l_2,-0.17) & (l_3,-0.17) & (l_1,0) & (l_5,0) \\ (l_3,-0.17) & (l_2,-0.17) & (l_5,0) & (l_1,0) \end{pmatrix}$$

As the prioritization weights are translated into the given preference information format employing 2-tuple linguistic values, which keeps the initial linguistic terms used by the buyer to provide their assessments, by means of the comparison between L and $\bar{L}$, the buyer can easily check how his/her given information is preserved by the derived priority weights. Such comparison, in turn, facilitates the interpretation of the weights.

6.1.2. Comparative analysis with Fuzzy Best-Worst method

To compare the 2-tuple BWM with the fuzzy BWM, we use the latter proposal to compute the fuzzy weights by considering the same MCGDM example introduced in the previous section. Table 6 shows that both the approaches produced similar weights of the criteria. However, the weights of the 2-tuple BWM are expressed in a 2-tuple linguistic form and by the same linguistic domain that decision makers have used to provide their preferences, which makes easier to interpret the results than with a representation based on fuzzy numbers. In addition to interpretation, the 2-tuple BWM method guarantees precise results thanks to the use of symbolic translation, which allows to carry out computations in a continuous domain without any kind of approximation. Finally, the prioritization approach used in 2-tuple BWM to obtain the weights also guarantees to achieve a solution with a minimal deviation in the initial experts' preferences and decision makers can evaluate such a deviation from the transformation of the weights into a preference matrix that, in turn, makes it even easier to understand the results.

6.2. Large-scale group decision making problem

Let us suppose a group of 50 entrepreneurs who have decided to make a donation to help the development of their hometown. Such donation is intended for the construction of a sustainable building, being four the possible options, $X = \{x_1 : hospital, x_2 : shoppingcenter, x_3 : school, x_4 : vocationaltrainingcenter\}$. To make this complex decision, the entrepreneurs consider the follow aspects to make comparisons among the possible options:

- *Investment*: the total amount of money needed to carry out the construction of the building.
- *Opinion of the inhabitants*: the inhabitants of the city will be asked about their preferences.
- *Place*: depending of the building, the construction will be carried out in different places of the city. The aim is to avoid harming citizens as little as possible during the construction of the building.
- *Socio-economic impact*: it is intended that the new building leads to a social and economic benefit for the city.

The entrepreneurs are asked to select the best and worst investment options and to provide preferences over best to others and worst to others by using the linguistic term set S^{BW}. For sake of space, the selection of the best and worst options and their comparisons with the rest of the options for each entrepreneur are available in https://sinbad2.ujaen.es/sites/default/files/2020-12/LS-GDM_preferences.pdf.. We aim to identify the formation of the consistent sub-groups from the given best-worst preference information.

First, we check the consistency of each entrepreneur's preference. For this purpose, we computed the relative consistency as defined in Eq. (18). The distribution of relative consistency has been depicted in Fig. 4. There is reasonable consistency in the given opinions by the entrepreneurs as the maximum CR is less than 0.34.

We intend to analyze the compositions of the groups for different values of the k, the number of sub-groups. Specifically, we vary k in the set $\{1,2,3,4\}$. For each of the values of k, we solve the model **(M-9)** by using JUNIPER solvers to find the compositions of the groups. Specifically, we coded the optimization model **(M-9)** in JuMp version 0.18 and invoked the JUNIPER solver to find the results to **(M-9)** reported in this paper. Note that in JUNIPER solver configuration, the NLP solver Ipopt is used for solving the continuous relaxation sub-problems and the MIP solver Cbc is used in the feasibility pump heuristic. The variation of the objectives for the different values of the k has been depicted in Fig. 5. In the trivial case, i.e., $k = 1$, the inconsistency in the group is quite high in comparison to other cases and that indicates a larger degree of disagreement in the group. In fact, there is a significant drop in objectives from $k = 1$ to $k = 2$ and this fact indicates the presence of more than one group among the decision makers as there is sufficient conflict among the decision makers opinions.

Now, we are going to look into the compositions of the different group structures. For the different values of $k \in \{1,2,3,4\}$, we report configurations (Fig. 6) of the different groups with each sub-groups total inconsistencies and cardinality in Table 7. We label the sub-groups as G_1, G_2, G_3 and G_4. For $k = 2$, the total inconsistency has reduced significantly. The entrepreneurs is divided into two sub-groups G_1 and G_2 with cardinality 29 and 21, respectively. The total internal inconsistency against the subgroups G_1 and G_2 are 9.99 and 5.19, respectively. Further, average inconsistency level for the sub-group G_1, i.e. total inconsistencies/cardinality of the sub-group 0.58 and for G_2, 0.72 are

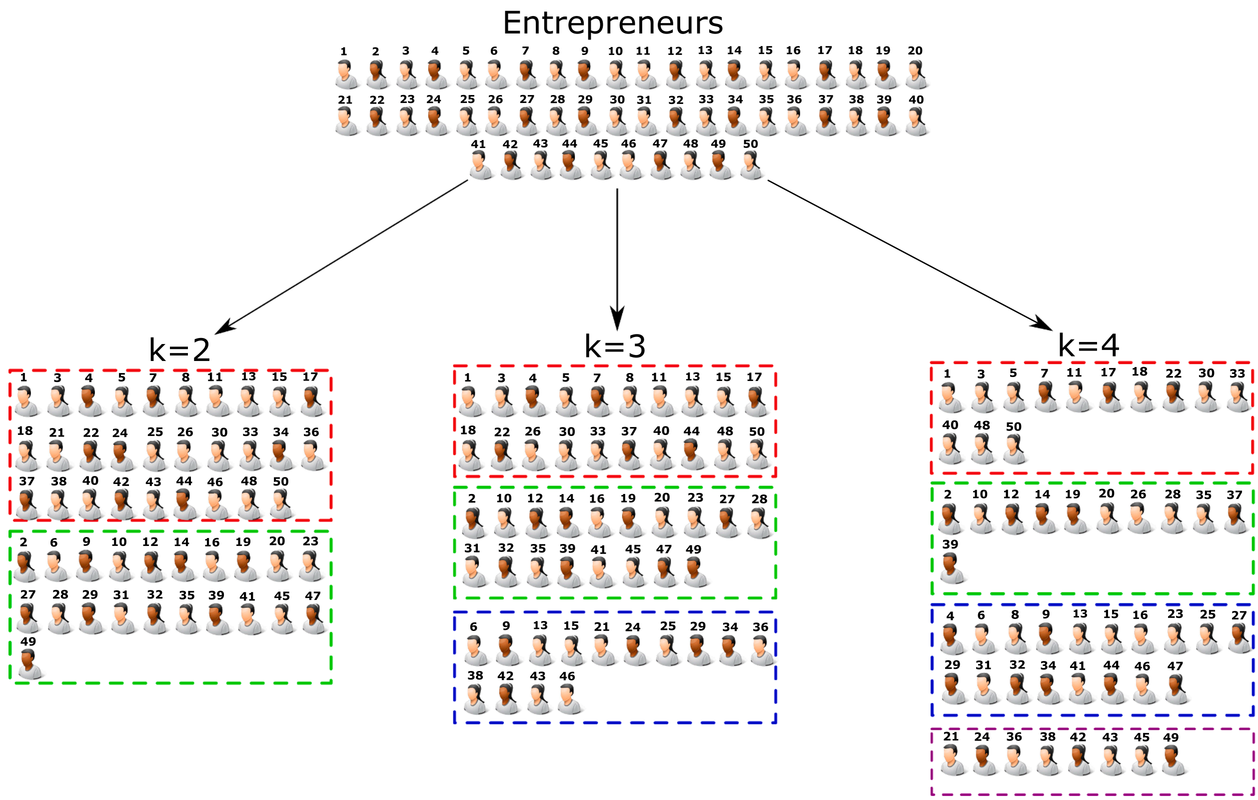
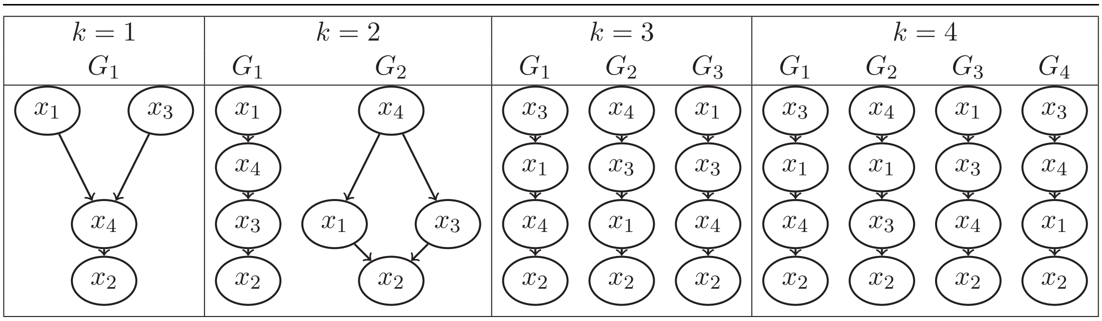


Fig. 6. Group configuration for different values of *k*.

Table 7
Inconsistencies and cardinality of the groups.

	G_1		G_2		G_3		G_4	
k	Inconsistency	Cardinality	Inconsistency	Cardinality	Inconsistency	Cardinality	Inconsistency	Cardinality
1	26.74	50	–	–	–	–	–	–
2	9.99	29	5.19	21	–	-	–	–
3	1.70	18	2.80	18	1.23	14	–	–
4	1.30	13	2.31	13	7.65	18	1.74	8

Table 8
Ordering of the alternatives for the different configuration of groups.



considerably decreased from the single group scenario. When $k = 3$, the total inconsistency further drops almost 64% from the case $k = 2$ and entrepreneurs are almost equally distributed among three sub-groups $\{G_1, G_2, G_3\}$. The internal inconsistency of the sub-groups reduced significantly along with the average internal inconsistency. The sub-group G_3 has the average inconsistency 0.09 while G_3 has the highest average inconsistency level 0.27. For $k = 4$, the total inconsistency increased and reached just below the level of $k = 2$. In fact, the internal coherence of the sub-groups decreases. In a nutshell, all the evidence support that it is reasonable to partition the group into at least two sub-groups of the decision makers.

Let us now analyze the priorities of the alternatives exhibited by the different configurations of the group of entrepreneurs. For the $k = 1$, we obtain the 2-tuple linguistic priority weights $l_w^* = ((U, 0.0761), (U, -0.1196), (F, 0.0761), (U, -0.0326))$. Further the corresponding numeric weight $\Delta_1^{-1}(l_w^*)$ can be transformed into linguistic best-worst preference information by utilizing Eqs. (11) and (12) to understand the deviation from original preference of the decision makers as follows:

$$\bar{L}^{k=1} = \begin{matrix} & x_1 & x_2 & x_3 & x_4 \\ x_1/x_3 & (l_1,0) & (l_3,-0.4992) & (l_1,0) & (l_1,0.19) \\ x_2 & (l_3,-0.4992) & (l_1,0) & (l_3,-0.4992) & (l_2,-0.3328) \end{matrix}$$

It is evident to the entrepreneurs that group decision deviates significantly from their original opinions. Further, we obtain partial order of the alternatives from the priorities w^* that has been reported in the Table 8 against the column $k = 1$, where there is a tie between x_1 and x_3 for the choice of most suitable option. When $k = 2$, we obtain the 2-tuple linguistic priority weights for the two sub-groups G_1 and G_2 as follows: $l_w^{g_1 *}$=((F,−0.0455), (VU,0.0909), (U,0.0227), (U,−0.0682)) and $l_w^{g_2 *}$=((U,-0.05), (VU,0.01), (U,-0.05), (U,-0.0975), (F,0)). Translating the corresponding numeric priority vectors $\Delta_1^{-1}(l_w^{g_1 *})$ and $\Delta_1^{-1}(l_w^{g_2 *})$ into decision maker's linguistic best-worst preference format, we have

$$\bar{L}^{g_1} = \begin{matrix} & x_1 & x_2 & x_3 & x_4 \\ x_1 & (l_1,0) & (l_5,0) & (l_2,-0.333) & (l_3,-0.5) \\ x_2 & (l_5,0) & (l_1,0) & (l_3,0) & (l_2,0) \end{matrix}$$

$$\bar{L}^{g_2} = \begin{matrix} & x_1 & x_2 & x_3 & x_4 \\ x_4 & (l_3,-0.5) & (l_5,0) & (l_3,-0.5) & (l_1,0) \\ x_2 & (l_2,0) & (l_1,0) & (l_2,0) & (l_5,0) \end{matrix}$$

From $\bar{L}^{g_1}$ and $\bar{L}^{g_2}$, we found that the agreement among entrepreneurs of the respective sub-groups improved. Further, from the priority weights, we could clearly identify the best and worst alternatives for the respective sub-groups and reported in Table 8 against the column $k = 2$. Setting up a hospital (x_1) is the best option among the members of the sub-group G_1, while members of G_2 preferred vocational training centre (x_4) over other options. Clearly, there is a polarization among the sub-groups G_1 and G_2, where members of G_1 think that better healthcare facility is the key determinant factor for the sustainable development of the community while members of G_2 feel that preparing youth for the jobs through vocational training is the most factor to the sustainable development of the community. Similarly, we obtain the ordering of the possible community development for the values of $k = 3$ and $k = 4$ and report it in Table 8. The ranking results suggest the division of the decision makers' into different homogeneous sub-groups.

7. Conclusions

In this study, we have investigated the classical prioritization method, Best-Worst, in the light of linguistic fuzzy information by adopting 2-tuple linguistic computational model, which enhanced the interpretability of the linguistic fuzzy approach with the concept of symbolic translation. For the facilitation of the reference comparisons for the best and worst objects with other objects, we have defined a linguistic term set which has utilized by decision makers to complete the linguistic Best-Worst pairwise comparison matrix. The priority weight of the objects from the linguistic Best-Worst pairwise comparison matrix has been obtained by solving the non-linear optimization model because of its superiority in comparisons to the other prioritization methods that have been established through numerical experiments. To present the priority weights into decision maker's understandable format, i.e., in terms of linguistic information, we have proposed a re-translation function that allows decision maker to understand and interpret easily the priority of the objects. To address the consistency and reliability issue of the given linguistic Best-Worst preference, we have introduced the notion of a new consistency ratio based on the idea of how far the decision maker cognitively deviates from the average cognitive behaviour of the decision maker. Afterwards, we have investigated the optimal configuration of the group based on the coherence of the decision makers opinions, given via linguistic best-worst preference and formulated a non-linear mixed integer programming problem to identify such configurations of the group, where each sub-groups agreed unanimously on a ranking order of the objects. With the help of didactic examples, we have illustrated the working and demonstrated the applicability of the proposed methodology.

The key advantages of the proposed methodology can be pointed out as follows: (1) the adaptation of 2-tuple computational model and the proposed re-translation of the weight in the form linguistic information equivalent to the decision maker original opinion enhance the accuracy and interpretability of the linguistic BWM; (2) the proposed consistency ratio offers a better understating of the measure inconsistency in terms of average cognitive behaviour than the original pessimistic approach of measuring consistency; (3) the proposed group configuration identifying approach is capable of detecting multiple views within the group according to the cohesion of the subgroups.

CRediT authorship contribution statement

Álvaro Labella: Conceptualization, Methodology, Validation, Formal analysis, Investigation, Writing - original draft, Writing - review & editing. **Bapi Dutta:** Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Writing - original draft, Writing - review & editing. **Luis Martínez:** Conceptualization, Methodology, Validation, Formal analysis, Investigation, Writing - original draft, Writing - review & editing, Supervision, Funding acquisition.

References

Aboutorab, H., Saberi, M., Asadabadi, M. R., Hussain, O., & Chang, E. (2018). Zbwm: The z-number extension of best worst method and its application for supplier development. *Expert Systems with Applications, 107*, 115–125.

Abrams, D., Wetherell, M., Cochrane, S., Hogg, M. A., & Turner, J. C. (1990). Knowing what to think by knowing who you are: Self-categorization and the nature of norm formation, conformity and group polarization. *British Journal of Social Psychology, 29*, 97–119.

Ali, A., & Rashid, T. (2019). Hesitant fuzzy best-worst multi-criteria decision-making method and its applications. *International Journal of Intelligent Systems, 34*, 1953–1967.

Brunelli, M., & Rezaei, J. (2019). A multiplicative best–worst method for multi-criteria decision making. *Operations Research Letters, 47*, 12–15.

Büyüközkan, G. (2004). Multi-criteria decision making for e-marketplace selection. *Internet Research, 14*, 139–154.

Chen, X., & Liu, R. (2006). Improved clustering algorithm and its application in complex huge group decision-making. *Systems Engineering and Electronics, 28*, 1695–1699.

Dong, Y., Hong, W.-C., Xu, Y., & Yu, S. (2011). Selecting the individual numerical scale and prioritization method in the analytic hierarchy process: A 2-tuple fuzzy linguistic approach. *IEEE Transactions on Fuzzy Systems, 19*, 13–25.

Dong, Q., Zhou, X., & Martínez, L. (2019). A hybrid group decision making framework for achieving agreed solutions based on stable opinions. *Information Sciences, 490*, 227–243.

Dumbill, E. (2013). A revolution that will transform how we live, work, and think: An interview with the authors of big data. *Big Data, 1*, 73–77.

Forman, E. H., & Selly, M. A. (2001). *Decision by objectives: How to convince others that you are right.* World Scientific.

Guo, S., & Zhao, H. (2017). Fuzzy best-worst multi-criteria decision-making method and its applications. *Knowledge-Based Systems, 121*, 23–31.

Herrera, F., Alonso, S., Chiclana, F., & Herrera-Viedma, E. (2009). Computing with words in decision making: foundations, trends and prospects. *Fuzzy Optimization and Decision Making, 8*, 337–364.

Herrera, F., & Martínez, L. (2000). A 2-tuple fuzzy linguistic representation model for computing with words. *IEEE Transactions on Fuzzy Systems, 8*, 746–752.

Hogg, M. A., Turner, J. C., & Davidson, B. (1990). Polarized norms and social frames of reference: A test of the self-categorization theory of group polarization. *Basic and Applied Social Psychology, 11*, 77–100.

Kröger, O., Coffrin, C., Hijazi, H., & Nagarajan, H. (2018). Juniper: An open-source nonlinear branch-and-bound solver in Julia. In *International conference on the integration of constraint programming, artificial intelligence, and operations research* (pp. 377–386). Springer.

Labella, Á., Liu, Y., Rodríguez, R., & Martínez, L. (2018). Analyzing the performance of classical consensus models in large scale group decision making: A comparative study. *Applied Soft Computing, 67*, 677–690.

Liu, Y., Fan, Z., & Zhang, X. (2016). A method for large group decision-making based on evaluation information provided by participators from multiple groups. *Information Fusion, 29*, 132–141.

Lootsma, F. A. (1980). Saaty's priority theory and the nomination of a senior professor in operations research. *European Journal of Operational Research, 4*, 380–388.

Martínez, L., & Herrera, F. (2012). An overview of the 2-tuple linguistic model for computing with words in decision making: extensions, applications and challenges. *Information Sciences, 207*, 1–18.

Martínez, L., Rodríguez, R. M., & Herrera, F. (2015). *The 2-tuple Linguistic Model*. Springer International Publishing.

Martínez, L., Ruan, D., & Herrera, F. (2010). Computing with words in decision support systems: an overview on models and applications. *International Journal of Computational Intelligence Systems, 3*, 382–395.

Martínez, L., Ruan, D., Herrera, F., Herrera-Viedma, E., & Wang, P. P. (2009). Linguistic decision making. *Tools and Applications*.

Mendel, J. M., Zadeh, L. A., Trillas, E., Yager, R., Lawry, J., Hagras, H., & Guadarrama, S. (2010). What computing with words means to me [discussion forum]. *IEEE Computational Intelligence Magazine, 5*, 20–26.

Mi, X., & Liao, H. (2019). An integrated approach to multiple criteria decision making based on the average solution and normalized weights of criteria deduced by the hesitant fuzzy best worst method. *Computers & Industrial Engineering, 133*, 83–94.

Miller, G. A. (1956). The magical number seven, plus or minus two: Some limits on our capacity for processing information. *Psychological Review, 63*, 81.

Pamučar, D., Petrović, I., & Ćirović, G. (2018). Modification of the best–worst and mabac methods: A novel approach based on interval-valued fuzzy-rough numbers. *Expert Systems with Applications, 91*, 89–106.

Ren, J. (2018). Selection of sustainable prime mover for combined cooling, heat, and power technologies under uncertainties: An interval multicriteria decision-making approach. *International Journal of Energy Research, 42*, 2655–2669.

Rezaei, J. (2015). Best-worst multi-criteria decision-making method. *Omega, 53*, 49–57.

Rezaei, J. (2016). Best-worst multi-criteria decision-making method: Some properties and a linear model. *Omega, 64*, 126–130.

Rodríguez, R. M., Labella, A., & Martínez, L. (2016). An overview on fuzzy modelling of complex linguistic preferences in decision making. *International Journal of Computational Intelligence Systems, 9*, 81–94.

Rodríguez, R. M., Labella, Á., Tré, G. D., & Martínez, L. (2018). A large scale consensus reaching process managing group hesitation. *Knowledge-Based Systems, 159*, 86–97.

Rodríguez, R. M., & Martínez, L. (2013). An analysis of symbolic linguistic computing models in decision making. *International Journal of General Systems, 42*, 121–136.

Saaty, T. L. (1990). How to make a decision: The analytic hierarchy process. *European Journal of Operational Research, 48*, 9–26.

Safarzadeh, S., Khansefid, S., & Rasti-Barzoki, M. (2018). A group multi-criteria decision-making based on best-worst method. *Computers & Industrial Engineering, 126*, 111–121.

Sánchez, J. (2007). *Modelos para la combinación de preferencias en toma de decisiones: herramientas y aplicaciones*. Spain: University of Granada.

Sueur, C., Deneubourg, J., & Petit, O. (2012). From social network (centralized vs. decentralized) to collective decision-making (unshared vs. shared consensus). *PLoS One, 7*, e32566.

Tai, W., & Chen, C. (2009). A new evaluation model for intellectual capital based on computing with linguistic variable. *Expert Systems with Applications, 36*, 3483–3488.

Thurstone, L. L. (1927). A law of comparative judgment. *Psychological Review, 34*, 273.

Tversky, A., & Kahneman, D. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty, 5*, 297–323.

Van Laarhoven, P. J., & Pedrycz, W. (1983). A fuzzy extension of saaty's priority theory. *Fuzzy Sets and Systems, 11*, 229–241.

Wang, L., Wang, Y., & Martínez, L. (2017). A group decision method based on prospect theory for emergency situations. *Information Sciences, 418*, 119–135.

Xu, X., Du, Z., & Chen, X. (2015). Consensus model for multi-criteria large-group emergency decision making considering non-cooperative behaviors and minority opinions. *Decision Support Systems, 79*, 150–160.

Yager, R. R. (2004). On the retranslation process in Zadeh's paradigm of computing with words. *IEEE Transactions on Systems, Man and Cybernetics, Part B (Cybernetics), 34*, 1184–1195.

Zadeh, L. A. (1965). Fuzzy sets. *Information and Control, 8*, 338–353.

Zadeh, L. A. (1975). The concept of a linguistic variable and its application to approximate reasoning, part i, ii, iii. *Information Sciences 8,8,9, 199–249*(301–357), 43–80.

Zadeh, L. A. (1996). Fuzzy logic = computing with words. *IEEE Transactions on Fuzzy Systems, 4*, 103–111.

Zadeh, L. A. (2013a). Computing with words in Information/Intelligent systems: Foundations. *Physica, 33*.

Zadeh, L. A. (2013b). Computing with words in Information/Intelligent systems: Applications. *Physica* (volume 34).