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HERRAMIENTAS Y UTILIDADES SOFTWARE DE APOYO A LA TOMA DE DECISIÓN LINGÜÍSTICA Y A PROCESOS DE CONSENSO BAJO INCERTIDUMBRE

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Herramientas y Utilidades Software de Apoyo a la Toma de Decisión Lingüística y a Procesos de Consenso Bajo Incertidumbre

Memoria de Tesis presentada por

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para optar al grado de Doctor en Informática

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La memoria titulada *Herramientas y Utilidades Software de Apoyo a la Toma de Decisión Lingüística y a Procesos de Consenso Bajo Incertidumbre*, que presenta D. Francisco Javier Estrella Liébana para optar al grado de doctor, ha sido realizada dentro del Programa de Doctorado en Tecnologías de la Información y la Comunicación de la Universidad de Jaén bajo la dirección del Dr. D. Luis Martínez López y de la Dra. Dña. Macarena Espinilla Estévez. Para su evaluación, esta memoria se presenta como un conjunto de trabajos publicados, acogiéndose y ajustándose a lo establecido en el punto 3 del artículo 23 del *Reglamento de los Estudios de Doctorado de la Universidad de Jaén*, aprobado en febrero de 2012.

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Capítulo 1

Introducción

El presente capítulo realiza una introducción general a la memoria de tesis doctoral titulada: *Herramientas y Utilidades Software de Apoyo a la Toma de Decisión Lingüística y a Procesos de Consenso Bajo Incertidumbre*. En él, queda expuesta la motivación que impulsa la investigación, así como los objetivos que nos hemos planteado fruto de ella y, finalmente, la estructura que sigue la memoria de investigación.

1.1. Motivación

Los seres humanos, nos vemos expuestos continuamente a diferentes situaciones en las que debemos decantarnos por una opción o conjunto de opciones para resolver un determinado problema. Idealmente, la selección del conjunto de opciones óptimo se realizará tras valorar cada una de las opciones consideradas en base a un conjunto de criterios. Sin embargo, no siempre es posible llevar a cabo un proceso de valoración racional y objetivo, ya que es habitual que existan múltiples factores externos y subjetivos que afecten al problema [4, 60]. Debido a ello, el estudio de los procesos de decisión, del cual se ocupa la teoría de la decisión [31, 120], ha sido continuo objeto de investigación durante las últimas décadas.

De modo general, podemos definir la *Toma de Decisión* (TD) como el proceso mediante el cual seleccionamos la mejor alternativa o conjunto de alternativas de entre un conjunto dado [20, 107]. Los problemas de TD pueden llevarse a cabo en diferentes contextos o ambientes de decisión, pudiendo encontrase problemas definidos en los tres siguientes contextos [31, 120]:

1. *Certidumbre*: Si todos los elementos que intervienen en el mismo son conocidos con exactitud.
2. *Riesgo*: Si al menos uno de los elementos que interviene en el problema depende del azar.
3. *Incertidumbre*: Si al menos uno de los elementos que interviene en el problema no es conocido con exactitud, disponiendo de una información incompleta, vaga o imprecisa.

El ambiente de decisión en el que se define el problema de TD determina el modo de modelar la información del mismo. Así, en los problemas de TD definidos en ambientes de *certidumbre* la información se modela mediante valores numéricos exactos, mientras que en los problemas definidos en ambientes de *riesgo* la información se modela mediante valores probabilísticos. En los problemas de TD definidos en ambientes de *incertidumbre*, dado que la incertidumbre tiene carácter no probabilístico al estar relacionada con la información involucrada en el problema, es habitual el uso de términos lingüísticos [90]. La lógica difusa y el enfoque lingüístico difuso brindan un conjunto de herramientas para modelar y gestionar la incertidumbre por medio de variables lingüísticas [141], que proporciona una mejora en la flexibilidad y fiabilidad de los modelos de decisión bajo incertidumbre. El uso de variables lingüísticas en los procesos de TD o *Toma de Decisión Lingüística* (TDL) ha proporcionado buenos resultados al tratar la incertidumbre de los expertos que proporcionan la información sobre las alternativas en diferentes ámbitos como evaluación sensorial [83,85,86,112], servicios electrónicos inteligentes [81,84,89], diagnósticos clínicos [24] o teoría de sistemas [13, 101] entre otros [3, 6, 8–10, 35, 39, 40, 50, 51, 54–59, 118, 119, 134, 139].

En la actualidad es posible disponer de herramientas software con las que llevar a cabo procesos de TD tanto en ambientes de certidumbre como de riesgo [62,138]. Sin embargo, no existe ninguna herramienta software que permita resolver problemas definidos en ambientes de incertidumbre mediante TDL y que pueda ser adaptada para dar soporte a aplicaciones concretas de un modo simple y automatizado. Esto obliga a abordar la resolución de los problemas de TDL mediante métodos tradicionales tales como hojas de cálculo, formularios o documentos de texto, que convierten el proceso en una tarea lenta y propensa a errores, y que frena la difusión de los resultados teóricos y su aplicación en los problemas del mundo real.

Otra problemática a considerar en esta memoria aparece en los problemas de *Toma de Decisión en Grupo* (TDG) [5]. Usualmente, para resolver un problema de TD se realiza una selección de alternativas en la cual, las diferentes valoraciones de los expertos son agregadas a fin de obtener una valoración colectiva para cada alternativa y posteriormente, en función de dichas valoraciones, se selecciona la alternativa o conjunto de alternativas que constituyen la solución del problema.

Si bien un proceso de selección de alternativas es suficiente en muchos casos, en un problema de TDG, en el que se deben considerar las preferencias de varios expertos, puede ser recomendable alcanzar un acuerdo previo entre el grupo para que, la solución obtenida, sea satisfactoria para todos los expertos que toman parte en el problema. Una posible opción para ello es emplear *procesos de alcance de consenso* o *procesos de consenso*, los cuales son procesos iterativos donde se busca acercar las opiniones de todos los expertos en sucesivas rondas para incrementar el nivel de acuerdo en el grupo antes de obtener una solución al problema de TD [14, 88, 113].

Los procesos de consenso para problemas de TDG bajo incertidumbre han adquirido gran interés investigador en los últimos años, pudiéndose encontrar en la literatura un gran número de propuestas de modelos de este tipo [11, 45, 52, 53, 56, 72, 91, 99, 100, 113, 128, 131]. Sin embargo, la abundancia de modelos disponibles, la ausencia de un marco que los caracterice, y la falta de herramientas software que permitan analizar los modelos de procesos de consenso así como estimar cual será su comportamiento, hace que sea difícil y complejo seleccionar el modelo más apropiado para llevar a cabo un proceso de consenso en un problema de TDG concreto.

Nos encontramos por tanto con que, de modo similar a la situación existente en TDL, aún contando con numerosos resultados teóricos que hacen posible llevar a cabo procesos de consenso en problemas de TDG, la ausencia de herramientas software adecuadas para su análisis dificulta la utilización de los resultados obtenidos. Por tanto, la necesidad de herramientas para aplicar la TDL en problemas reales y para analizar los procesos de consenso en TDG, motiva la investigación realizada en esta memoria, en la cual nos hemos centrado en superar estos retos:

- La falta de herramientas software para TDL obstaculiza su aplicación en los problemas del mundo real. Por tanto, sería necesario disponer de una herramienta software genérica para TDL que permita aplicar los actuales resultados teóricos en la resolución de problemas reales.
- La multitud de modelos de procesos de consenso hace complejo decidir cual utilizar en un problema de TDG. Así, es necesario contar con herramientas software que permitan analizar los procesos de consenso para facilitar la obtención de soluciones adecuadas en problemas de TDG.

1.2. Objetivos

Partiendo de la motivación y consideraciones planteadas en la sección previa, el propósito de esta investigación se centra en la propuesta de desarrollar herramientas software para la TDL y los procesos de consenso para TDG bajo incertidumbre, de gran valor para el análisis, la investigación y la aplicación de los resultados obtenidos en los problemas de TD del mundo real.

En base a este propósito nos planteamos los siguientes objetivos:

1. Estudiar modelos de TDL y su aplicación en diferentes ámbitos, así como analizar los conceptos y criterios para enfocar la resolución de los problemas de TDL con independencia del marco de decisión donde se encuentren definidos.

2. Desarrollar una herramienta software para TDL basada en un esquema flexible unificado de resolución, que permita tratar con problemas de TD bajo incertidumbre definidos en marcos lingüísticos y complejos. La herramienta debe permitir adaptar e implementar de forma sencilla los modelos de resolución de TDL, con el fin de integrar nuevos modelos a los iniciales y así permitir el desarrollo de aplicaciones basadas en TDL para problemas del mundo real.

3. Aplicar el software propuesto en la resolución de problemas del mundo real basados en TD, para agilizar y automatizar los procesos involucrados en dichos problemas, en los cuales el cálculo y modelado de la información resulta tedioso y susceptible a errores.

4. Desarrollar una herramienta software para simular procesos de consenso en problemas de TDG bajo incertidumbre que permita analizar el comportamiento de dichos procesos. La herramienta deberá facilitar la realización de estudios comparativos entre diferentes modelos de procesos de consenso atendiendo a sus características. Además, la herramienta deberá ser sencilla de extender con nuevos modelos de procesos de consenso.

1.3. Estructura

Para alcanzar los objetivos planteados en la sección anterior, y según lo establecido en el artículo 23, punto 3, de la normativa vigente para los Estudios de Doctorado en la Universidad de Jaén, correspondiente al programa establecido en el RD. 99/2011, esta memoria de investigación será presentada como un conjunto de artículos publicados por el doctorando.

Dichas publicaciones constituyen el núcleo de la tesis y corresponden a tres artículos científicos publicados en Revistas Internacionales indexadas por la base de datos *JCR* (Journal Citation Reports), producida por *ISI* (Institute for Scientific Information), junto a un artículo que se encuentra sometido a revisión en una Revista Internacional también indexada por JCR en el momento de finalización de esta memoria. Por tanto, la memoria se compone de un total de cuatro publicaciones, tres de ellas publicadas en revistas de reconocido prestigio.

A continuación, hacemos una breve descripción de la estructura de esta memoria:

- **Capítulo 1**: Presenta una introducción general mediante la cual conocer la problemática de investigación que se trata en esta memoria de investigación, así como las propuestas y objetivos que se persiguen en ella.
- **Capítulo 2**: Revisa los conceptos teóricos que empleamos en nuestras propuestas. En primer lugar se aborda la TDL para, seguidamente, realizar una revisión de los modelos utilizados en la resolución de TDL en esta memoria, el modelo de representación y el modelo computacional basado en 2-tupla lingüística y las diferentes extensiones propuestas a partir de este modelo que permiten llevar a cabo la resolución de problemas de decisión definidos en marcos de decisión complejos. Por último, se introducen los conceptos básicos relacionados con los procesos de consenso para TDG junto con una clasificación para procesos de este tipo.
- **Capítulo 3**: Resume las propuestas que conforman la memoria de investigación a través de las publicaciones donde han sido publicadas. Para cada una de ellas, una breve discusión de los resultados obtenidos es proporcionada.
- **Capítulo 4**: Constituye el núcleo de la tesis doctoral, conteniendo un compendio de las publicaciones obtenidas como resultado de la investigación realizada. Para cada una de las publicaciones, se indican los índices de calidad donde las propuestas han sido publicadas.
- **Capítulo 5**: Expone las conclusiones finales extraídas de esta investigación, propuestas para futuros trabajos y la información relacionada con el software registrado y las publicaciones derivadas de la presente investigación.
- **Apéndice A**: Presenta un sumario en inglés de la memoria de investigación que introduce la motivación y objetivos de la misma, resume las propuestas que

la conforman, y expone las conclusiones finales extraídas de la investigación realizada, junto con las propuestas para trabajos futuros y el conjunto de resultados obtenidos fruto de esta investigación.

Capítulo 2

Conceptos Teóricos y Antecedentes

En este capítulo revisamos los conceptos teóricos en los que se apoya la investigación realizada en esta memoria. Para ello, en primer lugar introducimos los conceptos básicos sobre los problemas de toma de decisión lingüísticos y los procesos de computación con palabras que son llevados a cabo en la resolución de dichos problemas. Introducidos estos conceptos, llevamos a cabo una revisión del modelo de representación y del modelo de computación lingüístico 2-tupla, así como de las diferentes extensiones propuestas para este modelo que permiten su uso en marcos de decisión complejos, los cuales implementaremos en una herramienta software para la toma de decisión lingüística, que utilizaremos para resolver problemas del mundo real. Por último, realizamos una breve descripción de los procesos de consenso en problemas de toma de decisión en grupo, así como una clasificación de estos procesos basada en las características de los mismos, que emplearemos en nuestra propuesta de una herramienta software para el análisis de los procesos de consenso en toma de decisión en grupo bajo incertidumbre.

2.1. Toma de Decisión Lingüística

En esta sección se introducen brevemente diferentes conceptos básicos de la toma de decisión para, posteriormente, realizar una revisión de la toma de decisión lingüística.

2.1.1. Toma de Decisión

Podemos definir la *Toma de Decisión* (TD) como el proceso mediante el cual seleccionamos la mejor alternativa o conjunto de alternativas de entre un conjunto dado [20,107]. La TD es empleada por los individuos para tomar decisiones difíciles y complejas [23, 120]. Para ello es necesario contar con métodos y modelos que permitan representar cada problema y analizar las diferentes alternativas.

Haciendo uso de la teoría de la decisión [31,120] es posible clasificar un problema de TD en función de sus elementos básicos, atendiendo a:

1. El número de criterios a valorar [36, 37, 64, 80, 82]:

    - *Monocriterio*: Para cada alternativa existe un único criterio a evaluar, correspondiéndose la valoración de la alternativa con el valor dado a este criterio. Frecuentemente, la solución al problema se corresponde con aquella alternativa que presenta una mejor valoración para este criterio.
    - *Multicriterio*: Existen dos o más criterios a evaluar para cada alternativa. La valoración de la alternativa se corresponde con el valor combinado de las valoraciones dadas a todos los criterios. Normalmente, la solución al problema se corresponde con aquella alternativa que presenta una mejor valoración considerando todos los criterios.

2. El número de expertos que participan en el proceso [49, 53, 54, 56, 68, 106, 122, 133, 135]:

- *Monoexperto*: La evaluación es realizada por un único experto. La valoración de cada alternativa se corresponde con la opinión dada por el experto. La solución al problema viene dada por la combinación de las valoraciones dadas por el experto para las diversas alternativas.
- *Multiexperto o grupo*: La evaluación es realizada por dos o más expertos. La valoración de cada alternativa se corresponde con el valor combinado de las evaluaciones dadas por todos los expertos para la misma. La solución al problema se corresponde con aquella alternativa que presenta una mejor valoración considerando las evaluaciones de todos los expertos.

3. El ambiente de decisión en el que se toman las decisiones [31, 120]:

- *Ambiente de certidumbre*: Un problema de toma de decisión está definido en un ambiente de certidumbre cuando son conocidos con exactitud todos los elementos y estados que intervienen en el mismo.
- *Ambiente de riesgo*: Un problema de decisión está definido en un ambiente de riesgo cuando al menos uno de los elementos o estados que intervienen en el problema depende del azar y debe ser modelado mediante modelos probabilísticos.
- *Ambiente de incertidumbre*: Un problema de decisión está definido en un ambiente de incertidumbre cuando no son conocidos con exactitud todos los elementos o estados que intervienen en el problema, disponiendo de información incompleta, vaga o imprecisa sobre las distintas alternativas que implica que sean valoradas de forma cualitativa.

El conjunto de alternativas que constituye la solución de un problema de TD se obtiene tras llevar a cabo un proceso de selección de alternativas, el cual se divide en dos fases [110] (véase Figura 2.1):

1. *Fase de agregación*. Las preferencias de los expertos sobre cada uno de los criterios que caracterizan a las alternativas son combinadas empleando un operador de agregación para obtener una valoración global de cada alternativa.

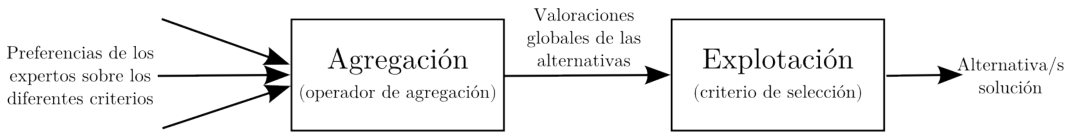

Figura 2.1: Proceso de selección de alternativas.

2. *Fase de explotación.* Se selecciona el conjunto de alternativas que constituyen la solución al problema en base a las valoraciones globales de las alternativas.

Uno de los aspectos fundamentales al abordar un problema de TD es el modelado de las preferencias, proceso mediante el cual se lleva a cabo la definición de una representación concreta con la cual los expertos expresan sus valoraciones en el problema [111]. En el proceso de modelado de las preferencias se deben considerar aspectos como el conocimiento de los expertos acerca del problema, sus experiencias o sus creencias, así como la naturaleza de las alternativas o los criterios que las definen.

2.1.2. Computación con Palabras en Toma de Decisión

En los problemas de TD definidos en ambientes de incertidumbre es habitual que los expertos se sientan más cómodos al valorar las alternativas empleando términos lingüísticos cercanos al modo en el que se expresan habitualmente [90]. El enfoque lingüístico difuso [141], el cual se basa en la teoría de conjuntos difusos [140], permite modelar la incertidumbre de la información involucrada en el problema mediante variables lingüísticas [141]. Así, la mayor expresividad permitida por las palabras hace que las variables lingüísticas sean menos precisas que las variables numéricas y que, consecuentemente, se adecuen mejor a la valoración de alternativas en ambientes de incertidumbre.

Una variable lingüística se caracteriza por su valor *sintáctico* o *etiqueta* y su valor *semántico* o *significado*, siendo la *etiqueta* una palabra o frase perteneciente a un conjunto de términos lingüísticos, y su *significado* un subconjunto difuso en un universo de discurso [141] (véase Figura 2.2). Formalmente, una variable lingüística se define del siguiente modo [141]:

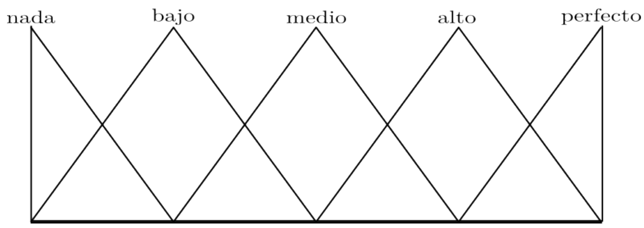

Figura 2.2: Conjunto de términos lingüísticos con sus etiquetas y semántica.

Definición 1 *Una variable lingüística está caracterizada por una quíntupla de elementos, $(H, T(H), U, G, M)$, en la que:*

- H *es el nombre de la variable.*
- $T(H)$ *es el conjunto de valores lingüísticos o etiquetas lingüísticas.*
- U *es el universo de discurso de la variable.*
- G *es una regla sintáctica (que normalmente toma forma de gramática) para generar los valores de* $T(H)$*.*
- M *es una regla semántica que asocia a cada elemento de* $T(H)$ *su significado. Para cada valor* $L \in T(H)$*,* $M(L)$ *es un subconjunto difuso de* U*.*

El uso de variables lingüísticas en un problema de TD o *Toma de Decisión Lingüística* (TDL), hace necesario llevar a cabo procesos de *computación con palabras* (CWW, Computing With Words) como la comparación, la negación o la agregación de variables lingüísticas. En los procesos de CWW, los cálculos son realizados sobre palabras o frases dadas en lenguaje natural a fin de obtener resultados en el dominio de expresión lingüístico original [90, 92, 137, 142].

De modo general, un proceso de CWW se lleva a cabo empleando un esquema genérico de tres fases (véase Figura 2.3):

1. *Traslación.* Transformación de las variables lingüísticas de entrada al tipo concreto de variables con el que se va a trabajar.

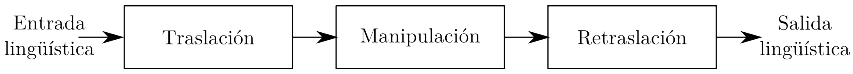

Figura 2.3: Proceso general de computación con palabras.

2. *Manipulación.* Operar sobre las valoraciones obtenidas en la fase anterior para la obtención de las valoraciones globales para cada alternativa.

3. *Retraslación.* Transformación de las valoraciones globales a variables lingüísticas en el dominio de expresión lingüístico empleado para la salida, el cual no tiene por qué ser el mismo que el dominio original.

La TDL ha sido aplicada con éxito en múltiples problemas del mundo real. Como consecuencia de ello, en la literatura se han propuesto diferentes modelos de computación lingüísticos para llevar a cabo los procesos de CWW en problemas de TDL, siendo los modelos más utilizados en la actualidad los siguientes [90]:

- *Modelo basado en funciones de pertenencia* [24]. En este modelo se realizan operaciones directamente sobre los funciones de pertenencia de los términos lingüísticos usando el principio de extensión [30]. Los resultados obtenidos son números difusos que, si bien son precisos, resultan difíciles de interpretar. Para mejorar la interpretabilidad de los resultados puede ser realizado un proceso de aproximación en el cual, el número difuso es transformado en un término lingüístico [24, 137]. Sin embargo, esta transformación implica perder información y precisión en los resultados.

- *Modelo basado en conjuntos difusos tipo-2* [93]. En este modelo se emplean conjuntos difusos tipo-2 para modelar las valoraciones lingüísticas [93]. Los conjuntos difusos tipo-2 son conjuntos difusos en los que la función de pertenencia es definida mediante un número difuso [141]. Los resultados obtenidos son conjuntos difusos tipo-2, siendo necesario llevar a cabo un proceso de aproximación de los mismos para poder ser interpretados de forma sencilla. En este proceso de aproximación, al igual que ocurre en el modelo basado en funciones de pertenencia, se produce pérdida de información sobre los resultados.

- *Modelos simbólicos* [26]. En estos modelos las operaciones se realizan sobre los índices de las etiquetas lingüísticas, las cuales tienen un orden definido. Operar sobre los índices permite simplificar los procesos de CWW así como obtener resultados fáciles de interpretar. Algunos de los modelos simbólicos existentes en la literatura son los siguientes:

  - *Modelo basado en escalas ordinales y operadores max-min* [136]. En este modelo la información se representa mediante el enfoque lingüístico difuso pero se impone un orden lineal en el conjunto de términos. Si bien los resultados obtenidos son términos lingüísticos fáciles de interpretar, al no imponerse ninguna restricción en la forma de las funciones de pertenencia de estos, podría producirse pérdida de información.

  - *Modelo basado en combinaciones convexas* [25]. Este modelo es una extensión del modelo anterior. El modelo realiza la agregación de los términos lingüísticos mediante la combinación convexa de sus índices. Los resultados obtenidos son valores reales en el intervalo $[0, g]$, siendo $g + 1$ el número de términos lingüísticos del dominio de expresión lingüístico. Al obtener como resultado números reales, es necesario llevar a cabo un proceso de aproximación para obtener una solución en el dominio de expresión lingüístico, lo cual implica que se pueda perder información.

  - *Modelo basado en la representación lingüística 2-tupla* [46, 47]. En este modelo las operaciones se realizan sobre información lingüística expresada mediante 2-tupla, permitiendo trabajar en un dominio de expresión lingüístico, pero tratándolo como un universo continuo. Los resultados obtenidos son valores lingüísticos 2-tupla, no produciéndose pérdida de información en el proceso de CWW.

De entre todos los modelos computacionales lingüísticos estudiados, el modelo lingüístico 2-tupla es el único que permite obtener resultados lingüísticos sin perdida de información [108]. Además, al considerar el dominio de expresión lingüístico como un universo continuo, el modelo ha sido extendido para manejar contextos de decisión complejos como heterogéneos, multigranulares o no balanceados [42, 43, 48].

Las propuestas para TDL que realizamos en esta memoria de investigación utilizan el modelo lingüístico 2-tupla y sus extensiones por lo que, en las siguientes secciones, se revisa este modelo y se presentan sus extensiones para marcos de decisión complejos.

2.2. Modelo Lingüístico 2-tupla

El modelo lingüístico 2-tupla es un modelo lingüístico simbólico [26] que permite llevar a cabo los procesos de CWW sin pérdida de información en dominios de expresión lingüísticos con una *granularidad* o número de etiquetas impar, que se encuentran simétrica y uniformemente distribuidos, y en los que la etiqueta central representa el valor de la indiferencia.

En las siguientes secciones se revisa el modelo de representación y el modelo de computación 2-tupla lingüístico, así como sus extensiones para marcos de decisión complejos de diferente tipo.

2.2.1. Modelo de Representación Lingüístico 2-tupla

El modelo de representación lingüístico 2-tupla se basa en el concepto de traslación simbólica [46].

Definición 2 *Sea* $S = \{s_0, ..., s_g\}$ *un conjunto de términos lingüísticos y* $\beta \in [0, g]$ *un valor en el intervalo de granularidad de* S*. La traslación simbólica de un término lingüístico* s_i *es un número valorado en el intervalo* $[-.5, .5)$ *que expresa la diferencia de información entre una cantidad de información expresada por el valor* $\beta \in [0, g]$ *obtenido en una operación simbólica y el valor entero más próximo,* $i \in \{0, ..., g\}$*, que indica el índice de la etiqueta lingüística* (s_i) *más cercana en* S*.*

En este modelo, la representación de la información se realiza mediante un par de valores denominado *2-tupla*, (s_i, α), donde $s_i \in S$ es el índice de la etiqueta lingüística y $\alpha \in [-.5, .5)$ la traslación simbólica de la misma (véase Figura 2.4).

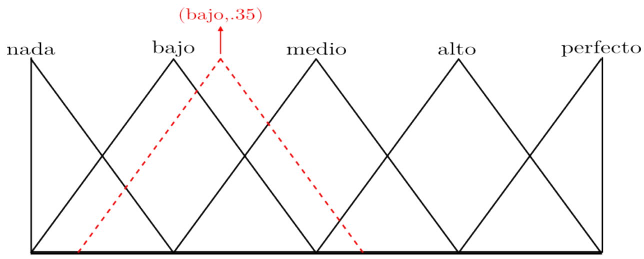

Figura 2.4: Representación 2-tupla lingüística.

El modelo define un conjunto de funciones que permite operar sobre 2-tupla sin perdida de información.

Definición 3 *Sea $S = \{s_0, ..., s_g\}$ un conjunto de términos lingüísticos y $\beta \in [0, g]$ un valor que representa el resultado de una operación simbólica. El conjunto de 2-tupla asociado a S es definido como $\langle S \rangle = S \times [-0.5, 0.5)$ y la función $\Delta_S : [0, g] \longrightarrow \langle S \rangle$ es dada por:*

$$\Delta_S(\beta) = (s_i, \alpha), \quad con \quad \begin{cases} i = & round\,(\beta), \\ \alpha = \beta - i, \end{cases}$$

donde $round(\cdot)$ es un operador de redondeo, s_i es la etiqueta con índice más cercano a β y α es el valor de la traslación simbólica.

Definición 4 *Sea $S = \{s_0, \ldots, s_g\}$ un conjunto de términos lingüísticos y (s_i, α) una 2-tupla. Se define la función Δ_S^{-1}, tal que aplicada sobre una 2-tupla (s_i, α) devuelve su valor numérico $\beta \in [0, g]$.*

$$\Delta_S^{-1} : \langle S \rangle \longrightarrow [0, g]$$

$$\Delta_S^{-1}(s_i, \alpha) = i + \alpha = \beta$$

Comentario 1 La conversión de un término lingüístico en una 2-tupla lingüística consiste en añadir el valor cero como su traslación simbólica [46]:

$$s_i \in S \rightarrow (s_i, 0) \in \langle S \rangle$$

2.2.2. Modelo de Computación Lingüístico 2-tupla

El modelo de representación lingüístico 2-tupla define un modelo computacional asociado en el que se definen las siguientes operaciones [46]:

1. *Operador de comparación de 2-tupla.* La comparación de información lingüística representada por medio de 2-tupla se realiza de acuerdo a un orden lexicográfico. Consideremos dos 2-tupla (s_k, α_1) y (s_l, α_2) que representan cantidades de información:

   - Si $k < l$, entonces (s_k, α_1) es menor que (s_l, α_2).
   - Si $k = l$, entonces
     - Si $\alpha_1 = \alpha_2$, entonces (s_k, α_1) y (s_l, α_2) representan la misma información.
     - Si $\alpha_1 < \alpha_2$, entonces (s_k, α_1) es menor que (s_l, α_2).
     - Si $\alpha_1 > \alpha_2$, entonces (s_k, α_1) es mayor que (s_l, α_2).

2. *Operador de negación de 2-tupla.* El operador de negación sobre una 2-tupla se define como:

   $$Neg(s_i, \alpha) = \Delta_S(g - (\Delta_S^{-1}(s_i, \alpha))),$$

   siendo $g + 1$ la cardinalidad del conjunto de etiquetas S.

3. *Operador de agregación de 2-tupla.* La agregación de información consiste en obtener un valor que combine un conjunto de valores. En la literatura, podemos encontrar numerosos operadores de agregación que nos permiten combinar la información de acuerdo a distintos criterios. Cualquiera de estos operadores puede ser fácilmente extendido para trabajar con 2-tupla usando funciones Δ_S y Δ_S^{-1}, que transforman valores numéricos en 2-tupla y viceversa sin pérdida de información. Un operador de agregación definido para trabajar con 2-tupla toma como valores de entrada 2-tupla y su resultado es otra 2-tupla [46,79,102].

2.2.3. Extensiones del Modelo Lingüístico 2-tupla para Marcos de Decisión Complejos

El modelo lingüístico 2-tupla revisado en la Sección 2.2, permite llevar a cabo la resolución de problemas de TDL definidos en marcos de decisión lingüísticos en los que existe un único dominio de expresión lingüístico. Sin embargo, los problemas de TDL pueden definirse en marcos de decisión complejos en los que sea necesario disponer de más de un dominio de expresión lingüístico (marcos lingüísticos multigranulares), de dominios de expresión de diversa naturaleza (marcos heterogéneos), o de dominios de expresión lingüísticos no simétricos ni uniformemente distribuidos (marcos lingüísticos no balanceados), para poder modelar toda la información del problema de TD.

En la literatura han sido presentadas diferentes extensiones del modelo lingüístico 2-tupla que permiten llevar a cabo los procesos de CWW en marcos de decisión complejos como los mencionados anteriormente. En nuestra propuesta de una herramienta software para TDL, haremos uso de estas extensiones para tratar con los problemas definidos en marcos de decisión complejos. Por ello, en las siguientes secciones se lleva a cabo una breve revisión de las mismas.

2.2.3.1. Marcos Lingüísticos Multigranulares

En los problemas de TD multicriterio o multiexperto, puede ocurrir que, debido a la incertidumbre de la información, las valoraciones estén expresadas en múltiples dominios lingüísticos de diferente granularidad. Este tipo de situaciones de decisión definen un marco de decisión lingüístico multigranular [19,61]. Han sido propuestas tres extensiones basadas en el modelo lingüístico 2-tupla para problemas de TD definidos en marcos de este tipo [34,41,48], las cuales se revisan a continuación.

A - Enfoque de fusión para gestionar información lingüística multigranular

Esta extensión fue presentada en [41] y proporciona un marco multigranular lingüístico completamente flexible. La extensión no impone ninguna limitación relacionada con la granularidad de los dominios lingüísticos, así como a la forma de la función de pertenencia de cada uno de los términos lingüísticos. Gracias a ello, cualquier dominio de representación lingüístico puede ser empleado para recoger las valoraciones de los expertos.

El proceso de valoración de esta extensión se realiza del siguiente modo (véase Figura 2.5):

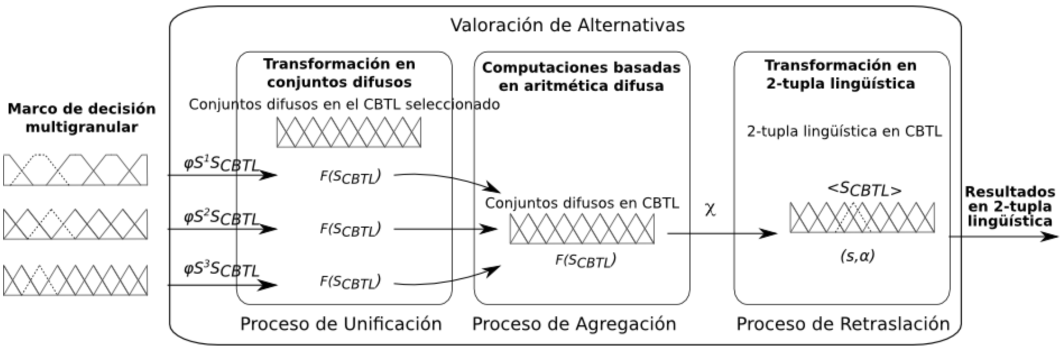

Figura 2.5: Proceso de valoración en el enfoque de fusión para gestionar información lingüística multigranular.

1. *Proceso de unificación.* La información multigranular es unificada en conjuntos difusos en un dominio lingüístico específico denominado *Conjunto Básico de Términos Lingüísticos* (CBTL), $S_{CBTL} = \{s_i, i = 0, \ldots, g\}$. El CBTL puede ser cualquiera de los dominios lingüísticos definidos en el marco de decisión, siempre y cuando pueda representar valores lingüísticos 2-tupla. No obstante, lo más indicado es que el CBTL sea el dominio lingüístico que permita conservar el máximo de información posible [41]. Un término lingüístico $s_l^k \in S^k$, tal que $S^k = \{s_l, l = 0, \ldots, g_k\}$ y $g_k \leq g$, es unificado en un conjunto difuso en

el CBTL usando la función de transformación $\varphi_{S^k S_{CBTL}} : S^k \to F(S_{CBTL})$:

$$\varphi_{S^k S_{CBTL}}(s_l^k) = \sum_{i=0}^{g}(s_i/\gamma_i) \tag{2.1}$$

siendo $\gamma_i = \max_y \min\{\mu_{s_l}(y), \mu_{s_i}(y),\, i = 0, \ldots, g\}$.

2. *Proceso de agregación.* La información expresada en múltiples conjuntos de términos lingüísticos ha sido unificada en conjuntos difusos en el CBTL. Por tanto, en este proceso las computaciones son realizadas directamente en los conjuntos difusos empleando aritmética difusa [30].

3. *Proceso de retraslación.* En este proceso, los resultados expresados en conjuntos difusos, $F(S_{CBTL})$, son transformados en valores lingüísticos 2-tupla en el CBTL empleando la función $\chi : F(S_{CBTL}) \to \langle S_{CBTL} \rangle$:

$$\chi\left(\{(s_0,\gamma_0),(s_1,\gamma_1),\ldots,(s_g,\gamma_g)\}\right) = \Delta_S\left(\frac{\sum_{i=0}^{g} i\,\gamma_i}{\sum_{i=0}^{g}\gamma_i}\right) = (s,\alpha) \in \langle S_{CBTL}\rangle. \tag{2.2}$$

Las transformaciones realizadas sobre conjuntos difusos pueden ocasionar que se pierda información, por lo que los resultados obtenidos en el proceso de valoración pueden ser inexactos.

B - Jerarquías lingüísticas

Las jerarquías lingüísticas fueron propuestas en [48] para superar las limitación de la extensión previa en cuanto a su falta de precisión [41].

Una *Jerarquía Lingüística* (JL) se define como la unión de todos los niveles que la conforman $l(t, n(t))$. Cada nivel t, se corresponde con un conjunto de términos lingüísticos denotado como $S^{n(t)} = \{s_0^{n(t)}, ..., s_{n(t)-1}^{n(t)}\}$, con la granularidad $n(t)$.

$$JL = \bigcup_t l(t, n(t)) \tag{2.3}$$

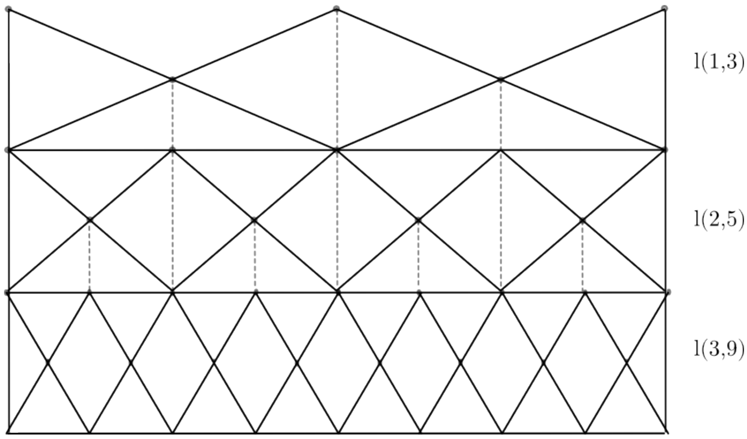

Figura 2.6: Jerarquía lingüística con tres niveles.

En una JL cada nivel debe ser un conjunto de términos lingüísticos simétrico y uniformemente distribuido, debiendo soportar el nivel $t+1$ los puntos modales del nivel t (véase Figura 2.6). Para ello, la granularidad del nivel $t+1$ es obtenida a partir de la granularidad de su predecesor como:

$$n(t+1) = (2 \cdot n(t)) - 1 \tag{2.4}$$

El proceso de valoración en esta extensión se realiza de la siguiente forma (véase Figura 2.7):

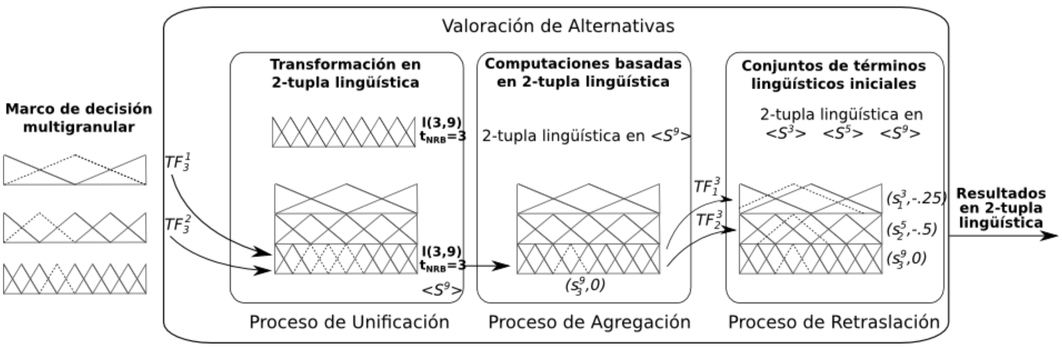

Figura 2.7: Proceso de valoración en una jerarquía lingüística.

1. *Proceso de unificación.* La extensión define la función de transformación $TF_{t'}^{t} : \langle S^{n(t)} \rangle \to \langle S^{n(t')} \rangle$. Esta función permite transformar términos lingüísticos entre diferentes niveles de la JL de un modo preciso:

$$TF_{t'}^{t}(s_i^{n(t)}, \alpha) = \Delta_S \left(\frac{\Delta_S^{-1}(s_i^{n(t)}, \alpha) \cdot (n(t') - 1)}{n(t) - 1} \right) \tag{2.5}$$

La información lingüística es unificada en un único nivel de la JL, el cual se denomina *Nivel de Representación Básico* (NRB) y se nota como t_{NRB}. La unificación es llevada a cabo empleando la Ecuación (2.5) con $TF_{t_{NRB}}^{t}$.

2. *Proceso de agregación.* La información es unificada en valores lingüísticos 2-tupla en el nivel t_{NRB}. Por tanto, el modelo computacional lingüístico 2-tupla es empleado para realizar los procesos de CWW y obtener resultados lingüísticos en $S^{(n(t_{NRB}))}$ [46].

3. *Proceso de retraslación.* Los resultados están expresados en el nivel t_{NRB} mediante valores lingüísticos 2-tupla. Estos resultados pueden ser expresados de un modo preciso en cualquier conjunto de términos lingüísticos definido en el marco de decisión del problema por medio de la función de transformación definida en la Ecuación (2.5) con $TF_{t}^{t_{NRB}}$.

C - Jerarquías lingüísticas extendidas

Esta extensión fue propuesta en [34] para ofrecer más flexibilidad en marcos lingüísticos multigranulares que la extensión basada en JL [48], manteniendo la precisión en los procesos de CWW.

Una *Jerarquía Lingüística Extendida* (JLE) no impone ninguna restricción en cuanto a la granularidad de cada conjunto de términos lingüísticos incluido en ella, añadiendo un último nivel, t^*, para soportar todos los puntos modales de los conjuntos de términos lingüísticos iniciales (véase Figura 2.8).

La granularidad del nivel t^*, se calcula usando la granularidad de los conjuntos de términos lingüísticos iniciales:

$$n(t^*) = mcm\,(n(1) - 1, n(2) - 1, \ldots, n(m) - 1) + 1, \tag{2.6}$$

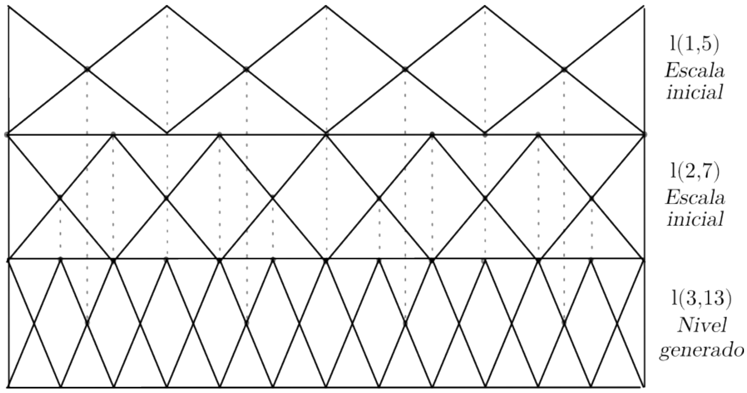

Figura 2.8: Jerarquía lingüística extendida con 2 niveles iniciales.

donde mcm es el *mínimo común múltiplo* y m el número de conjuntos de términos lingüísticos definidos en el marco de decisión. Por tanto, una JLE es la unión de los niveles originales y el nuevo nivel generado (véase Figura 2.8).

$$JLE = \left(\bigcup_{t=m}^{t=1} l(t, n(t)) \right) \bigcup l(t^*, n(t^*)) \tag{2.7}$$

El proceso de valoración en esta extensión es similar al de las JL, tomando en consideración que el nivel de unificación en este caso debe ser t^* para asegurar que los procesos de CWW sean llevados a cabo sin perder información(véase Figura 2.9).

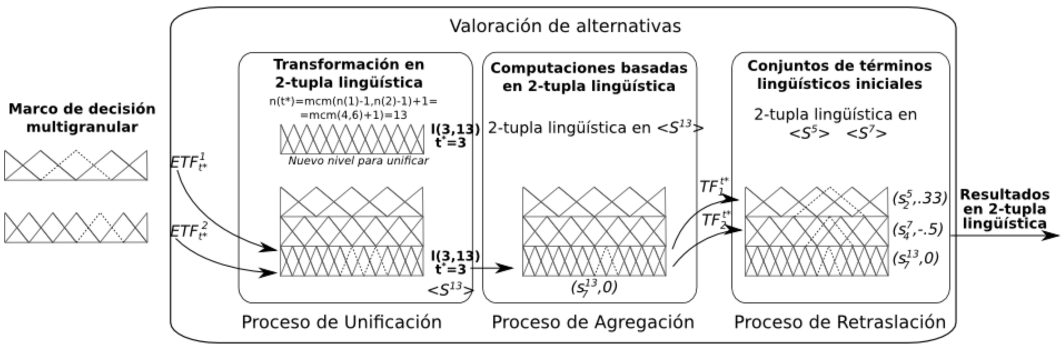

Figura 2.9: Proceso de valoración en una jerarquía lingüística extendida.

1. *Proceso de unificación.* La información expresada en múltiples conjuntos de términos lingüísticos es unificada sin pérdida de información en el nivel de unificación t^*, usando la función de transformación $ETF^t_{t^*} : \langle S^{n(t)} \rangle \to \langle S^{n(t^*)} \rangle$:

$$ETF^t_{t*} = \Delta_S \left(\frac{\Delta_S^{-1}(s_i^{n(t)}, \alpha) \cdot (n(t^*) - 1)}{n(t) - 1} \right) \tag{2.8}$$

2. *Proceso de agregación.* La información unificada está representada en valores lingüísticos 2-tupla en el nivel de unificación t^*. Por ello, los procesos de CWW son llevados a cabo empleando el modelo lingüístico computacional 2-tupla [46], obteniendo resultados lingüísticos 2-tupla en $\langle S^{n(t^*)} \rangle$.

3. *Proceso de retraslación.* En este proceso, los resultados lingüísticos 2-tupla son expresados en cada conjunto de términos lingüísticos original de un modo preciso mediante la función de transformación $ETF_t^{t^*}$, la cual se define como:

$$ETF_t^{t*} = \Delta_S \left(\frac{\Delta_S^{-1}(s_i^{n(t^*)}, \alpha) \cdot (n(t) - 1)}{n(t^*) - 1} \right) \tag{2.9}$$

2.2.3.2. Marcos Heterogéneos

De forma similar que en el marco anterior, en ciertos problemas de TD los expertos pueden preferir expresar sus opiniones en diferentes dominios de expresión en función de su conocimiento y experiencia, como dominios numéricos, intervalares o lingüísticos, considerando la imprecisión e incertidumbre de la información relacionada así como la naturaleza de los criterios a valorar. En estas situaciones, el problema de decisión se define en un marco de decisión heterogéneo.

En [49] fue presentada una extensión para tratar con información expresada en dominios de expresión de diferente tipo: numéricos, intervalares y lingüísticos. La extensión se basa en el enfoque de fusión para la gestión de información lingüística multigranular revisado en la Sección 2.2.3.1-A, y define dos nuevas funciones de transformación para transformar en conjuntos difusos valores numéricos e intervalares.

- Enfoque de fusión para gestionar información heterogénea

El proceso de valoración con esta extensión es llevado a cabo del siguiente modo (véase Figura 2.10):

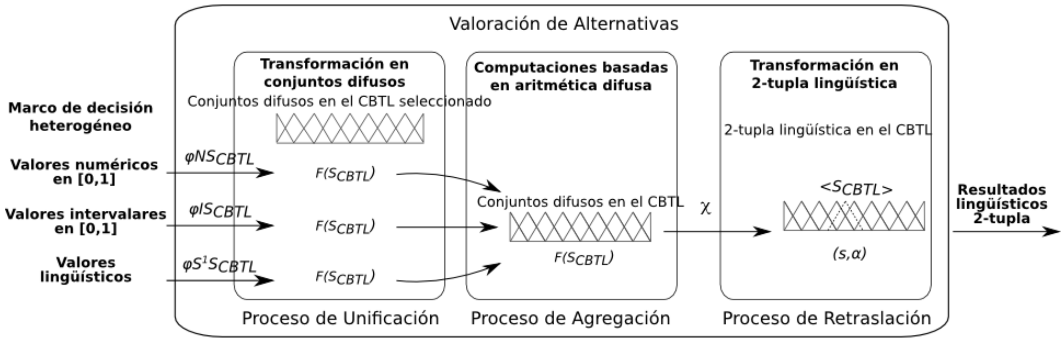

Figura 2.10: Proceso de valoración en el enfoque de fusión para gestionar información heterogénea.

1. *Proceso de unificación.* La información heterogénea es unificada en un dominio lingüístico seleccionado denominado *Conjunto Básico de Términos Lingüísticos* (CBTL), $S_{CBTL} = \{s_i, i = 0, \ldots, g\}$, el cual debe ser el dominio lingüístico que permita conservar la mayor información posible [41]. En el proceso de unificación, cada valoración debe ser transformada mediante la función de transformación correspondiente al dominio de expresión en el que se ha definido:

   (a) *Dominio numérico.* Cuando $v \in [0, 1]$, se aplica la función de transformación para valores numéricos $\varphi_{NS_{CBTL}} : [0, 1] \to F(S_{CBTL})$:

   $$\varphi_{NS_{CBTL}}(v) = \sum_{i=0}^{g}(s_i/\gamma_i) \tag{2.10}$$

   donde $\gamma_i = \mu_{s_i}(v) \in [0, 1]$ es el grado de pertenencia de v a $s_i \in S_{CBTL}$.

   (b) *Dominio intervalar.* Dado un valor intervalar $I = [\underline{i}, \overline{i}]$ en $[0, 1]$, para llevar a cabo su transformación se asume que tiene una representación inspirada en los conjuntos difusos [75] como sigue:

   $$\mu_I(x) = \begin{cases} 0 & si\ x < \underline{i} \\ 1 & si\ \underline{i} \leq x \leq \overline{i} \\ 0 & si\ \overline{i} < x \end{cases} \tag{2.11}$$

   donde $x \in [0, 1]$.

   Cuando $v \in P([0, 1])$, se aplica la función de transformación para valores intervalares $\varphi_{IS_{CBTL}} : P([0, 1]) \to F(S_{CBTL})$, la cual se apoya en la representación definida en la Ecuación (2.11) y se define como:

$$\varphi_{IS_{CBTL}}(v) = \sum_{i=0}^{g}(s_i/\gamma_i) \tag{2.12}$$

donde $\gamma_l^i = \max_y \min\{\mu_I(y), \mu_{s_l}(y)\}$, con $l = \{0, \ldots, g\}$, siendo $\mu_I(\cdot)$ y $\mu_{s_i}(\cdot)$ funciones de pertenencia asociadas con el intervalo $I \in P([0,1])$ y el término $s_l \in S_{CBTL}$, respectivamente.

(c) *Dominio lingüístico.* Cuando $v \in S^k$, de modo que $S^k = \{s_0^k, \ldots, s_{g_k}^k\}$ u $g_k < g$, se aplica la función de transformación para valores lingüísticos definida en la Ecuación (2.1), $\varphi_{S^k S_{CBTL}}$.

2. *Proceso de agregación.* De igual modo que en el enfoque de fusión para gestionar información lingüística multigranular, las computaciones son realizadas directamente en los conjuntos difusos empleando aritmética difusa [30].

3. *Proceso de retraslación.* Los resultados agregados están expresados en conjuntos difusos en el CBTL, $F(S_{CBTL})$. En este proceso, los conjuntos difusos son transformados en valores lingüísticos 2-tupla en el CBTL empleando la función de transformación χ, definida en la Ecuación (2.2).

Al igual que en el enfoque de fusión para gestionar información lingüística multigranular, las transformaciones realizadas en conjuntos difusos pueden ocasionar la pérdida de información.

2.2.3.3. Marcos Lingüísticos No Balanceados

Los marcos de decisión lingüísticos no balanceados son necesarios en aquellos casos en los que es necesario valorar con una mayor granularidad en un lado del conjunto de términos lingüísticos.

- Metodología lingüística difusa para tratar con conjuntos de términos lingüísticos no balanceados

En [42] fue propuesta una metodología lingüística difusa basada en JL [48] para tratar con conjuntos de términos lingüísticos no balanceados. La metodología proporciona un algoritmo que permite representar la semántica de un conjunto de términos lingüísticos no balanceado mediante una JL, y una función booleana $Puente(S)$, que es empleada en los procesos de CWW (véase Figura 2.11).

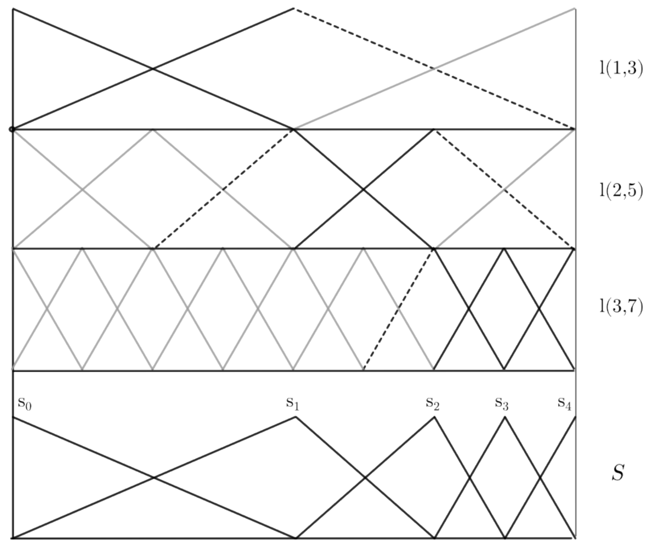

Figura 2.11: Conjunto de términos lingüístico no balanceado.

El algoritmo genera la semántica para un conjunto de términos lingüísticos no balanceado $\mathcal{S}$ a partir de una JL, $JL(\langle\mathcal{S}\rangle) = \{(s_{I(i)}^{G(i)}, \alpha),\ i = \{0, ..., g\},\ s_{I(i)}^{G(i)} \in \mathcal{S}\}$, siendo $I(i)$ la función que asigna el índice de la etiqueta que representa su semántica en la JL y $G(i)$ la función que asigna a cada etiqueta la granularidad del nivel en que está representada.

El proceso de valoración con la metodología es como se indica a continuación (véase Figura 2.12):

1. *Proceso de unificación.* La semántica del conjunto de términos lingüístico no balanceado $JL(\langle\mathcal{S}\rangle)$ pertenece a los diferentes niveles de la JL. Por ello, la

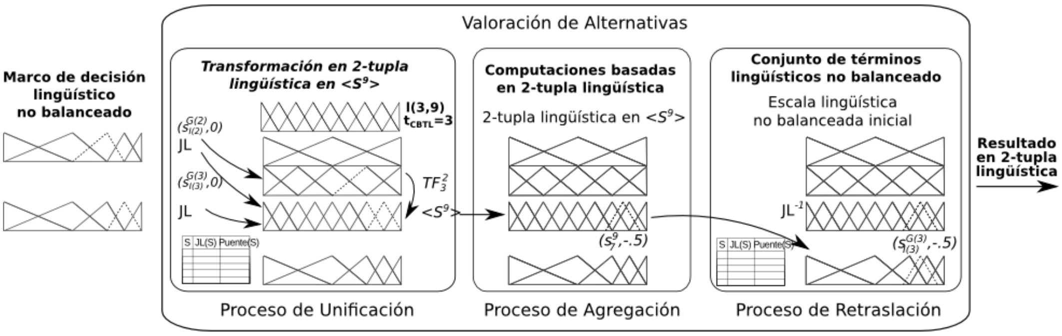

Figura 2.12: Proceso de valoración con la metodología lingüística difusa para tratar con conjuntos de términos lingüísticos no balanceados.

información expresada en el conjunto de términos lingüístico no balanceado es unificada al nivel de unificación t_{NRB}. Para realizar la unificación, en primer lugar cada término no balanceado es transformado a un valor lingüístico 2-tupla en su respectivo nivel de la JL por medio de la función de transformación $\mathcal{JL} : \langle \mathcal{S} \rangle \rightarrow JL(\langle \mathcal{S} \rangle)$:

$$\forall (s_i, \alpha) \in (\mathcal{S} \times [-0.5, 0.5)) \Longrightarrow \mathcal{JL} : (s_i, \alpha) = (s_{I(i)}^{G(i)}, \alpha). \tag{2.13}$$

En segundo lugar, los términos lingüísticos expresados en diferentes niveles de la JL son unificados en el nivel de unificación t_{NRB}, empleando la función de transformación definida en la Ecuación (2.5), $TF_{t_{NRB}}^{t'}$ con $l(t', G(i))$.

2. *Proceso de agregación.* La información unificada está representada en valores lingüísticos 2-tupla en el nivel t_{NRB}. Por ello, es empleado el modelo computacional lingüístico 2-tupla [46] para llevar a cabo los procesos de CWW, obteniéndose como resultado valores lingüísticos 2-tupla expresados en $\langle S^{(n(t_{NRB}))} \rangle$.

3. *Proceso de retraslación.* En este proceso, los resultados lingüísticos 2-tupla expresados en el nivel t_{NRB}, son transformados al conjunto de términos lingüístico no balanceado por medio de la función de transformación definida en la Ecuación (2.14). Esta función de transformación está definida en base a casos [42] y está basada en la satisfacción de condiciones impuestas en $JL(\langle \mathcal{S} \rangle)$

y la función booleana $Puente(S)$.

$$\mathcal{JL}^{-1} : \langle S^{t_{NRB}} \rangle \rightarrow \langle \mathcal{S} \rangle \tag{2.14}$$

2.3. Procesos de Consenso

Como se ha mencionado en la Sección 2.1.1, en un problema de TD el objetivo es seleccionar la mejor alternativa o conjunto de alternativas de entre un conjunto dado. En ocasiones, esta selección de alternativas debe realizarse atendiendo a las opiniones de varios individuos o expertos, definiéndose un problema de Toma de Decisión en Grupo (TDG). Un problema de TDG puede ser caracterizado como un problema de decisión en el que [65]:

1. Existen n alternativas, $X = \{x_1, \ldots, x_n\}$.
2. Existen m individuos o expertos, $E = \{e_1, \ldots, e_m\}$.
3. Cada experto, e_i, expresará sus valoraciones u opiniones sobre el conjunto de alternativas, X.
4. La solución del problema es obtenida en función de las valoraciones u opiniones expresadas por todos los expertos, E, sobre todas las alternativas, X.

Para recoger las opiniones de los expertos, es necesario emplear alguna estructura de preferencias como vectores de utilidad [11], órdenes de preferencia [116] o *relación de preferencia difusa* [96,99,117], siendo esta última la estructura más comúnmente empleada en TDG bajo incertidumbre. Una relación de preferencia es una matriz $n \times n$ del modo:

$$P_i = \begin{pmatrix} - & \ldots & p_i^{1n} \\ \vdots & \ddots & \vdots \\ p_i^{n1} & \ldots & - \end{pmatrix}$$

donde cada valoración, p_i^{lk}, mide el grado de preferencia de la alternativa x_l sobre la alternativa x_k según el experto e_i.

En un problema de TDG, si las opiniones iniciales de los expertos son muy distantes, puede ser poco probable obtener una solución que resulte satisfactoria para todos ellos. Los *procesos de alcance de consenso* o *procesos de consenso*, surgen como una fase adicional iterativa en el proceso de resolución de los problemas de TDG, mediante la cual se busca que los expertos aproximen sus opiniones en caso de que las discrepancias existentes entre los mismos imposibilite alcanzar un acuerdo [14, 88, 113]. Así, el objetivo de un proceso de consenso es encontrar una solución consensuada entre todos los expertos, entendiéndose el consenso como un *estado de acuerdo mutuo entre los miembros de un grupo, en el cual todas las motivaciones de los expertos han sido consideradas para alcanzar la satisfacción del grupo* [113].

De forma clásica, el consenso se ha definido como un acuerdo unánime entre todos los expertos, en el cual, todos ellos concuerdan completamente en cuanto a la valoración de las alternativas [74]. Si bien, este tipo de consenso puede ser conveniente e inclusive necesario en determinados problemas de TDG, como puede ser en el caso del veredicto de un jurado [7], en la mayoría de situaciones donde es necesario llevar a cabo un proceso de consenso, basta con que exista un acuerdo entre la mayoría de expertos respecto a las principales alternativas [14, 88]. Dichas situaciones permiten emplear enfoques más flexibles que midan el grado de acuerdo existente entre los expertos, el cual podría ser completo o a parcial.

Basándose en la teoría de conjuntos difusos [140], Kacprzyk introdujo la noción de *soft consensus* [65], el cual se basa en el concepto de mayoría difusa [65]. Se puede considerar que existe un estado de *soft consensus* cuando *la mayor parte de los expertos están de acuerdo en casi todas las opciones relevantes* [66, 67]. Al considerar el consenso desde este punto de vista, disponemos de un enfoque flexible, que resulta más que suficiente en la mayoría de los problemas de TDG, habiéndose demostrado como un enfoque adecuado con excelentes resultados en gran cantidad de problemas de diversas áreas [67, 69, 72].

El éxito del *soft consensus* en el consenso se fundamenta en que no es un concepto rígido que únicamente se da si existe un acuerdo completo entre los expertos, si no que viene dado por el grado de acuerdo entre el grupo, grado que índica como de próximas están las opiniones individuales de los expertos entre sí. Para medir el consenso en función del grado de acuerdo existente, es necesario definir medidas de consenso que permitan calcular el grado actual de acuerdo en el grupo a partir de las opiniones individuales de los expertos. En la literatura, ha sido propuesto un amplio abanico de medidas de consenso [5, 44, 67, 69, 114], las cuales podemos clasificar en dos categorías:

1. *Basadas en la distancia a la preferencia colectiva* [5,44,114]. El grado de consenso se mide en base a la distancia existente entre las preferencias individuales de cada uno de los expertos, P_i, respecto a la preferencia colectiva, P_c, la cual representa una medida de la opinión global del grupo calculada a partir de la agregación de las preferencias individuales, $P_c = \phi\{P_1, \ldots, P_m\}$.
2. *Basadas en la distancia entre las preferencias de los expertos* [67,69]. El grado de consenso se mide en función de los grados de similitud, $sim(P_i, P_j)$, entre las preferencias de cada par de expertos del grupo, $(e_i, e_j), i < j$, los cuales son agregados para obtener el grado de consenso, $\phi\{sim(P_1, P_2), \ldots, sim(P_{m-1}, P_m)\}$.

Para que un proceso de consenso pueda ser llevado a cabo con éxito, es importante que el grupo asuma ciertas condiciones previamente a iniciar el proceso [88]:

- Cada miembro del grupo debe entender el proceso que se va a usar para alcanzar el acuerdo, debiendo clarificarse cualquier posible duda antes de comenzar.
- Los expertos deben estar dispuestos a colaborar, modificando sus valoraciones en caso necesario.

Para coordinar el proceso y supervisar a los expertos, en varios procesos de consenso suele participar un moderador humano que guía la discusión entre el grupo [88]. El moderador se encarga de evaluar en cada ronda el consenso alcanzado así como de informar a los expertos sobre las modificaciones que deben realizar para poder llegar a un acuerdo.

De modo general, los procesos de consenso se realizan siguiendo tres fases:

1. *Medida del consenso*: Las valoraciones de los expertos son agregadas empleando alguna de las medidas de consenso mencionadas anteriormente a fin de calcular el grado de consenso existente.

2. *Control del consenso*: El grado de consenso es comparado con un umbral de consenso, μ, definido previamente. Si el grado de consenso alcanzado es superior a este umbral se procede a realizar la selección de alternativas, en caso contrario será necesario realizar una ronda adicional. Para evitar que se lleven a cabo un número excesivo de rondas en caso de que no sea posible alcanzar el umbral de consenso fijado, suele ser establecido un número máximo de rondas permitidas.

3. *Progreso del consenso*: Si el grado de consenso alcanzado en una ronda es inferior al umbral establecido, se deberá aplicar un mecanismo que permita aumentar el acuerdo en la ronda posterior. Usualmente, el mecanismo variará en función de si en el proceso de consenso participa un moderador o no. Si existe la figura del moderador en el proceso, este identificará las valoraciones que están dificultando el consenso y recomendará a los expertos que hagan las modificaciones necesarias. Si en el proceso no interviene un moderador, será llevada a cabo una actualización automática de la información existente, no siendo necesario que los expertos participen nuevamente en el proceso.

2.3.1. Clasificación de Procesos de Consenso

Como se indica en la sección previa, en los últimos años los procesos de consenso para TDG han sido ampliamente estudiados, pudiéndose encontrar un gran número de modelos de procesos de consenso diferentes en la literatura. La existencia de múltiples modelos unido al variado comportamiento de estos dificulta la selección del más adecuado para un problema de TDG. Por dicho motivo, es conveniente realizar una categorización de los modelos de procesos de consenso en base a sus características para así facilitar su estudio y análisis posterior.

Para realizar su clasificación nos basamos en dos criterios:

- *Mecanismo de progreso del consenso.* Podemos clasificar los modelos entre aquellos en los que interviene un moderador que genera recomendaciones a los expertos, y los que realizan una actualización automática de la información existente.

- *Tipo de medida de consenso empleada.* También es posible realizar la clasificación de los modelos atendiendo a si estos emplean medidas de consenso basadas en la distancia a la preferencia colectiva o entre las preferencias de los expertos.

Tomando en consideración los criterios descritos es posible clasificar los modelos de procesos de consenso en función de sus características (véase Figura 2.13):

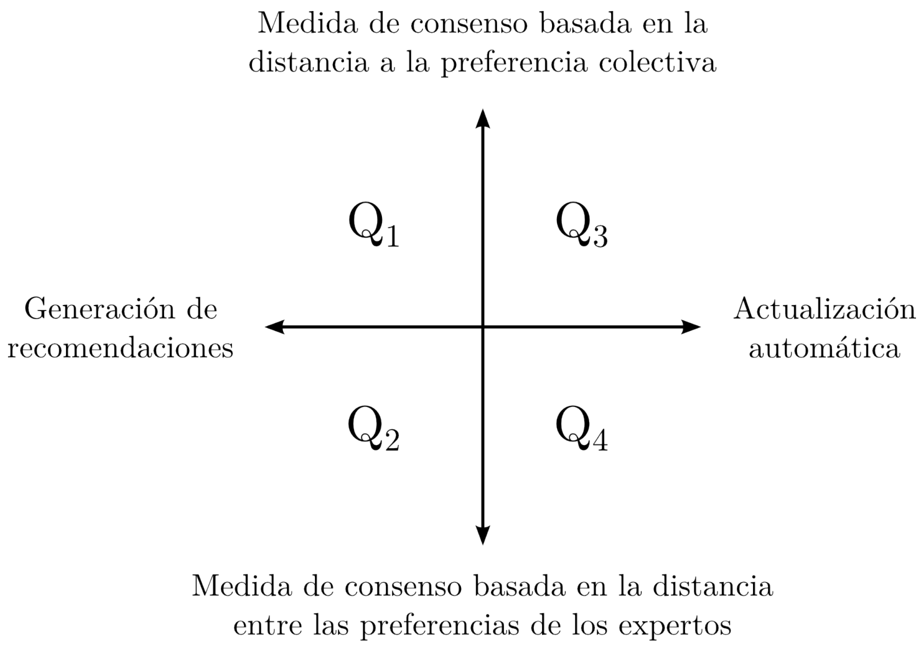

Figura 2.13: Clasificación de modelos de procesos de consenso.

(Q_1) Modelos de consenso con generación de recomendaciones y medidas de consenso basadas en la distancia a la preferencia colectiva [11,12,22,29,53,63,99,100].

(Q_2) Modelos de consenso con generación de recomendaciones y medidas de consenso basadas en la distancia entre las preferencias de los expertos [1, 15, 16, 21, 32, 33, 52, 56, 70–72, 91, 103].

(Q_3) Modelos de consenso con actualizaciones automáticas y medidas de consenso basadas en la distancia a la preferencia colectiva [6, 17, 28, 38, 77, 124–132, 144].

(Q_4) Modelos de consenso con actualizaciones automáticas y medidas de consenso basadas en la distancia entre las preferencias de los expertos [18, 97, 98, 143].

A partir de la clasificación es posible categorizar de forma sencilla un modelo en función de sus características. La clasificación de un modelo nos permite estimar su comportamiento general y nos facilitará la realización de estudios comparativos entre diferentes modelos.

Capítulo 3

Discusión de los Resultados

El presente capítulo resume las propuestas que conforman esta memoria de investigación, llevando a cabo para cada una de ellas una breve discusión de los resultados obtenidos.

El capítulo se estructura en base a cuatro propuestas:

1. *Software para Toma de Decisión Lingüística basado en el Modelo Lingüístico 2-tupla y sus Extensiones: FLINTSTONES.*
2. *Aplicaciones de FLINTSTONES basadas en Toma de Decisión Lingüística.* Esta propuesta esta formada a su vez por dos propuestas:
   a) *Evaluación sensorial del Aceite de Oliva Virgen.*
   b) *Proceso de Selección de Empresas para un Parque Tecnológico.*
3. *Software para Procesos de Consenso: AFRYCA.*

3.1. Software para Toma de Decisión Lingüística basado en el Modelo Lingüístico 2-tupla y sus Extensiones: FLINTSTONES

Como vimos en el Capítulo 2, el modelo lingüístico 2-tupla así como sus extensiones han sido aplicados con éxito para la resolución de diversos problemas de TD del mundo real definidos en marcos de decisión lingüísticos y complejos [35, 39, 87]. Aún así, nos encontramos con que no existen actualmente herramientas software que permitan llevar a cabo la resolución de problemas de TDL empleando estos modelos.

Ante la ausencia de soluciones software de este tipo, se propone el desarrollo de una suite para TDL basada en el modelo lingüístico 2-tupla y sus extensiones llamada *FLINTSTONES* (Fuzzy LINguisTic deciSion TOols eNhancEment Suite)[1], la cual permite abordar la resolución de problemas de TD definidos en marcos de decisión lingüísticos y complejos bajo un enfoque unificado que automatiza y simplifica su proceso de resolución. Para dar a conocer FLINTSTONES y fomentar su uso, se ha desarrollado una página web[2] desde la que es posible descargar la herramienta así como acceder a múltiples recursos relacionados con la suite.

Dado que FLINTSTONES implementa el modelo lingüístico 2-tupla y sus extensiones para resolver problemas de TDL en diferentes marcos de decisión, hemos propuesto e implementado en la suite un algoritmo capaz de seleccionar entre el modelo lingüístico 2-tupla y sus extensiones, el adecuado a un problema en función del marco de decisión en que ha sido definido.

Para desarrollar una herramienta software con estas características, ha sido necesario realizar en primer lugar un estudio del modelo lingüístico 2-tupla así como de sus diferentes extensiones para marcos de decisión complejos, el cual ha quedado

[1] Registro general de la propiedad intelectual en la comunidad autónoma de Andalucía. Número de asiento registral 04/2014/10539

[2] http://sinbad2.ujaen.es/flintstones

recogido en el Capítulo 2. Como resultado de este estudio, se propone un esquema flexible unificado para la resolución de los problemas de TDL que permite:

1. Adaptar los modelos generales para la resolución de TDL a dicho esquema.

2. Acoplar nuevos modelos de resolución de TDL a dicho esquema.

La utilización de un esquema de resolución flexible unificado hace necesario diseñar la arquitectura de la suite poniendo especial énfasis en dos aspectos claves:

- *Alta cohesión funcional*: Las diferentes partes funcionales deben ser independientes del resto de modo que la incorporación de funcionalidad adicional o la modificación de la existente pueda llevarse a cabo sin que el resto de elementos se vean afectados.

- *Reusabilidad*: La funcionalidad debe ser genérica para permitir su reutilización.

Para diseñar una suite que maximice los anteriores aspectos se ha empleado una arquitectura basada en componentes, la cual permite que las diferentes partes funcionales que forman la suite queden encapsuladas en componentes. En el diseño de FLINTSTONES se han empleado multitud de componentes, los cuales pueden ser empleados de forma independiente para proporcionar una funcionalidad o bien, ser utilizados por otros componentes para implementar nuevas funcionalidades.

La propuesta queda recogida en el siguiente artículo (ver Sección 4.1):

F. J. Estrella, M. Espinilla, F. Herrera, L. Martínez, FLINTSTONES: A fuzzy linguistic decision tools enhancement suite based on the 2-tuple linguistic model and extensions. Information Sciences, vol. 280, pp. 152-170, 2014.

3.2. Aplicaciones de FLINTSTONES basadas en Toma de Decisión Lingüística

A pesar de que en la literatura existe una amplia variedad de propuestas para resolver problemas de TD mediante TDL y múltiples aplicaciones donde los modelos de TDL pueden ser utilizados, no es habitual encontrar herramientas software que las implementen y que permitan emplearlas de una forma sencilla y automatizada en diferentes ámbitos. Esto, usualmente se debe a alguno de los siguientes motivos:

1. La adaptación de las herramientas software para TD existentes no es factible, ya sea porque su licencia no lo permite, o bien porque su diseño interno no puede ser adaptado.
2. El desarrollo a medida de una herramienta software es un proceso costoso, no siendo viable acometer su desarrollo en muchas ocasiones.

Al emplear en FLINTSTONES un esquema flexible unificado que subyace en una arquitectura basada en componentes enfocada a la reusabilidad funcional, se facilita el desarrollo de herramientas software específicas que simplifiquen y automatice la resolución de problemas en diferentes ámbitos.

En esta sección se recoge la propuesta de dos herramientas software específicas, construidas a partir de la base de componentes que conforman FLINTSTONES, y que permiten agilizar la resolución de dos problemas concretos del mundo real. La primera de estas herramientas está centrada en el ámbito de la evaluación sensorial y la segunda, en los procesos de selección de empresas para un parque tecnológico.

Con estas propuestas, se pretende demostrar la idoneidad de la suite para la creación de aplicaciones basadas en modelos de TDL, así como obtener herramientas software de utilidad en problemas del mundo real para los que no existe actualmente ninguna disponible.

3.2.1. Evaluación Sensorial del Aceite de Oliva Virgen

Los procesos de Evaluación Sensorial (ES) [2, 27, 83, 112, 115] comprenden un conjunto de técnicas que intentan describir y medir las propiedades sensoriales que presentan los productos para proporcionar información útil de los mismos [76]. Una de las principales dificultades con las que se encuentra la ES es la naturaleza de la información que se evalúa, ya que se trata de evaluaciones realizadas mediante la información que se obtiene a través de los sentidos humanos. Dicha información es fuertemente subjetiva, vaga y con un alto grado de incertidumbre por lo que, consecuentemente, es difícil de valorar de forma precisa.

En esta propuesta nos centramos en el proceso de ES del Aceite de Oliva Virgen (AOV), el cual se realiza de forma estandarizada siguiendo las directrices fijadas por el Consejo Oleícola Internacional (COI)[3]. El proceso de ES para el AOV permite establecer una clasificación comercial del mismo en sus diferentes categorías comerciales a partir de las percepciones proporcionadas por un grupo de expertos. Dicha clasificación resulta de vital interés para su comercialización al tener influencia directa en su precio de venta.

La ES de un AOV es llevaba a cabo por diferentes expertos que individualmente miden la intensidad de diferentes atributos sensoriales, lo cual ofrece una información con un alto grado de incertidumbre. Siguiendo las directrices fijadas por el COI, esta incertidumbre es tratada mediante un tratamiento probabilístico en el que se emplea la mediana de las valoraciones. Sin embargo, dicha incertidumbre no tiene carácter probabilístico, si no que está asociada a las percepciones de los expertos al valorar los atributos sensoriales. Este proceso de evaluación puede ser visto como un problema de TD multicriterio multiexperto definido en un ambiente de incertidumbre, donde es necesario valorar cada uno de los atributos sensoriales y agregar dichas valoraciones para obtener una valoración global con la que realizar la clasificación comercial de una muestra dada de AOV. Por tanto, el conjunto de métodos que brinda la TD es una herramienta de gran utilidad para modelar el proceso de ES del AOV.

[3]http://www.internationaloliveoil.org

El uso del modelo lingüístico 2-tupla y sus extensiones permite tratar convenientemente con información de este tipo, habiendo demostrado proporcionar buenos resultados en ámbitos similares [35, 39, 87]. En [85], fue propuesto un modelo lingüístico de ES para el AOV en el que se emplea un conjunto de términos lingüístico no balanceado [42] para valorar las intensidades de los atributos sensoriales de una muestra de AOV, así como un proceso de valoración donde se clasifican las muestras evaluadas en función de las valoraciones lingüísticas.

La investigación presentada en esta propuesta nace de la validación del modelo de ES para el AOV presentado en [85], la cual ha sido llevada a cabo bajo el proyecto de investigación *Desarrollo de Sistemas Instrumentales de Análisis y Modelización mediante la Lógica Difusa aplicados a la Caracterización Sensorial del Aceite de Oliva Virgen Extra*[4]. Al realizar la validación del modelo fue puesto de manifiesto que el mismo realizaba una clasificación errónea de algunas muestras de AOV que se encontraban dudosas entre dos categorías. Tras realizar un análisis de las muestras de AOV clasificadas incorrectamente por el modelo, se determinó que el conjunto de términos lingüísticos no balanceado empleado carecía de una cantidad de etiquetas lingüísticas apropiada para realizar una clasificación correcta de un AOV según la normativa establecida por el COI.

Nos centramos por tanto, en la propuesta de un nuevo modelo de ES para el AOV derivado del modelo inicial presentado en [85]. Nuestra modelo propone el uso de un nuevo conjunto de términos lingüístico no balanceado [42] que permite recoger de forma adecuada las intensidades de los atributos sensoriales y así, clasificar correctamente las muestras de AOV dudosas entre dos categorías.

Para simplificar y automatizar el proceso empleando el nuevo modelo, proponemos una herramienta software basada en FLINTSTONES que implementa el nuevo modelo lingüístico de ES para el AOV y que permite, a partir de las valoraciones expresadas por los expertos, tal y como ocurre en los procesos de ES reales, obtener la clasificación comercial de una muestra de AOV siguiendo las directrices estable-

[4]Financiado por la Consejería de economía, innovación y ciencia. Proyectos I+D+I. Línea específica del olivar y aceite de oliva. Ref.: AGR-6487

cidas por el COI. Además, esta herramienta software es capaz de emplear diferentes conjuntos de términos lingüísticos no balanceados para el proceso de ES del AOV, no quedando vinculada a un conjunto de términos lingüísticos no balanceado concreto.

La propuesta queda recogida en el siguiente artículo (ver Sección 4.2):

F. J. Estrella, M. Espinilla, L. Martínez, Fuzzy Linguistic Olive Oil Sensory Evaluation Model based on Unbalanced Linguistic Scales. Journal of Multiple-valued Logic and Soft Computing, vol. 22, pp. 501-520, 2014.

3.2.2. Proceso de Selección de Empresas para un Parque Tecnológico

Un parque tecnológico es un centro de alta innovación implantado e impulsado desde la universidad para fortalecer la colaboración entre universidad, industria y gobierno [105]. Mediante esta colaboración se busca potenciar el avance tecnológico, realizar la transferencia del conocimiento generado a la sociedad y fomentar el desarrollo económico [105].

Un parque tecnológico proporciona importantes ventajas competitivas a las empresas asentadas en él, al facilitarles una ubicación estratégica y el acceso a recursos financieros. Por ello, son muchas las empresas que tratan de establecerse dentro de los parques tecnológicos. Sin embargo, debido a la limitada extensión de un parque tecnológico y a las restricciones de innovación necesarias para formar parte de ellos, es necesario realizar un proceso de selección con el que elegir las empresas adecuadas.

En este proceso de selección se deben analizar múltiples criterios de una amplia gama de disciplinas, criterios que además son evaluados con un alto grado de incertidumbre. Esto provoca que el proceso de selección sea altamente complejo, encontrándose definido en un contexto heterogéneo [49] en el que además, los expertos proveerán información vaga e imprecisa fruto de sus dudas al valorar criterios que presentan un alto grado de incertidumbre. Como vimos en el Capítulo 2, el uso del enfoque lingüístico difuso ha proporcionado buenos resultados al tratar con la incertidumbre relacionada con la vaguedad de las valoraciones.

Recientemente, ha sido propuesto el concepto de *Conjuntos de Términos Lingüísticos Difusos Dudosos* (CTLDD) [109] los cuales, mediante el uso de expresiones lingüísticas comparativas próximas a las usadas por los humanos al expresar sus opiniones, buscan ayudar a los expertos que dudan entre varias valoraciones.

Para realizar el proceso de selección de la empresa adecuada para formar parte de un parque tecnológico, podría emplearse cualquiera de los distintos modelos de decisión multicriterio existentes en la literatura como AHP [73], TOPSIS [78, 94, 123], VIKOR [95], ELECTRE o PROMETHEE [104]. Sin embargo, tras revisar la literatura existente, nos encontramos con que actualmente, ninguno de ellos es capaz de tratar directamente con el tipo de información e incertidumbre mencionados. En vista de ello, proponemos la definición de un proceso de selección basado en un nuevo modelo TOPSIS difuso, capaz de tratar con problemas de decisión definidos en marcos heterogéneos en los que pueden emplearse valores numéricos, términos lingüísticos y expresiones lingüísticas comparativas basadas en CTLDD.

Para agilizar el proceso de selección, proponemos el desarrollo de una herramienta software basada en FLINTSTONES que implementa el nuevo modelo y permite llevar a cabo todas las fases del proceso de selección, desde la elicitación de las valoraciones hasta el proceso de selección, incluyendo además un análisis sensitivo [121] de los resultados obtenidos.

Esta herramienta se complementa con la propuesta de una aplicación *rica de internet* o *Rich Internet Application* (RIA) desarrollada con la base de componentes de FLINTSTONES y denominada *Flintstones Gathering Cloud* (FGC). FGC facilita la recogida de las valoraciones realizadas por los expertos mediante el uso de tecnologías web, permitiendo llevar a cabo el proceso de forma remota y distribuida.

La propuesta queda recogida en el artículo sometido a revisión (ver Sección 4.3):

F. J. Estrella, S. Çevik, R. Rodríguez, B. Öztayşi, L. Martínez, C. Kahraman, Selecting firms for University Technoparks: A Hesitant Linguistic Fuzzy TOPSIS model, Computers & Industrial Engineering, sometido a revisión, 2015.

3.3. Software para Procesos de Consenso: AFRYCA

Como se mencionó en el Capítulo 2, en los problemas de TDG puede ser necesario alcanzar, previamente a la selección de alternativas, un acuerdo entre los expertos para que la solución obtenida sea considerada válida por todos ellos. Los procesos de consenso surgen como una fase adicional iterativa de los procesos de TDG con objeto de encontrar una solución que sea lo suficientemente satisfactoria para el conjunto de expertos involucrados en el problema de TDG [14, 88, 113].

En los últimos años, los procesos de consenso han pasado a ocupar un rol destacado dentro de la TDG bajo incertidumbre, habiendo sido propuestos en la literatura una amplia variedad de modelos diferentes para llevar a cabo los procesos de consenso en dichos contextos [11, 45, 52, 53, 56, 72, 91, 99, 100, 113, 128, 131]. Cada uno de estos modelos se vale de diferentes mecanismos para lograr alcanzar la solución más satisfactoria para el conjunto de expertos que participan en el problema de TDG [14, 88, 113].

El amplio número de modelos existente y sus variados funcionamientos y características, hace que sea difícil seleccionar el más adecuado para alcanzar una solución de consenso en un problema de TDG concreto. En vista de ello proponemos un framework para la simulación de los procesos de consenso con diferentes modelos al que hemos denominado *AFRYCA* (A FRamework for the analYsis of Consensus Approaches)[5].

El framework propuesto permite analizar el comportamiento de los modelos de proceso de consenso en diferentes problemas de TDG, así como llevar a cabo estudios comparativos a partir de la clasificación realizada en la Sección 2.3.1). Al igual que FLINTSTONES, presentado en la Sección 3.1, AFRYCA se ha desarrollado empleando una arquitectura de componentes que permite que la integración de nuevos modelos pueda ser llevada a cabo sin necesidad de modificar la funcionalidad existente.

[5] http://sinbad2.ujaen.es/afryca

Para demostrar la utilidad de AFRYCA, su propuesta se acompaña de un estudio experimental en el que se han simulado varios procesos de consenso empleando diferentes modelos.

La propuesta queda recogida en el siguiente artículo (Sección 4.1):

I. Palomares, F. J. Estrella, L. Martínez, F. Herrera, Consensus under a Fuzzy Context: Taxonomy, Analysis Framework AFRYCA and Experimental Case of Study. Information Fusion, vol. 20, issue November 2014, pp. 252-271, 2014.

Capítulo 4

Publicaciones

En virtud de lo establecido en el artículo 23, punto 3, de la normativa vigente para los Estudios de Doctorado de la Universidad de Jaén, correspondiente al programa establecido en el RD. 99/2011, en este capítulo se presentan las publicaciones que componen el núcleo de la presente tesis doctoral.

Dichas publicaciones se corresponden a tres artículos científicos publicados en Revistas Internacionales indexadas por la base de datos JCR (*Journal Citation Reports*), producida por ISI (*Institute for Scientific Information*), además de un artículo sometido a revisión en una Revista Internacional también indexada por JCR en el momento de finalización de esta memoria.

4.1. FLINTSTONES: A fuzzy linguistic decision tools enhancement suite based on the 2-tuple linguistic model and extensions

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FLINTSTONES: A fuzzy linguistic decision tools enhancement suite based on the 2-tuple linguistic model and extensions

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ABSTRACT

Uncertainty in real world decision making problems not always has probabilistic nature, in such cases the use of linguistic information to model and manage such an uncertainty has given good results. The adoption of linguistic information implies the accomplishment of processes of computing with words to solve linguistic decision making problems. In the specialized literature, several computational models can be found to carry out such processes. However, there is a shortage of software tools that develop and implement these computational models. The 2-tuple linguistic model has been widely used to operate with linguistic information in decision problems due to the fact that provides linguistic results that are accurate and easy to understand for human beings. Furthermore, another advantage of the 2-tuple linguistic model is the existence of different extensions to accomplish processes of computing with words in complex decision frameworks. Due to these reasons, in this paper a fuzzy linguistic decision tools enhancement suite so-called *Flintstones* is proposed to solve linguistic decision making problems based on the 2-tuple linguistic model and its extensions. Additionally, the *Flintstones* website is also presented, this website has been deployed and includes a repository of case studies and datasets for different linguistic decision making problems. Finally, a case study solved by *Flintstones* is illustrated in order to show its performance, usefulness and effectiveness.

1. Introduction

Decision making processes are one of the most frequent mankind activities in daily life. In order to solve decision making problems, usually, human beings, experts, provide either their knowledge about a set of different alternatives in a given activity to make a decision by means of reasoning processes [2,14,29,34,48,49,58]. Generally, the modeling of such knowledge by linguistic information in decision making is motivated because these situations are defined under uncertainty that has a non-probabilistic nature. In such cases, experts feel more comfortable providing their knowledge by using terms close to human beings cognitive model. Fuzzy logic and fuzzy linguistic approach provide tools to model and manage such a uncertainty by means of linguistic variables [67], improving the flexibility and offering reliability of the decision models in different fields [12,16,33,61].

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Fig. 1. CWW paradigm.

The use of linguistic information involves the need to operate with linguistic variables. Computing with Words (CWW) is a paradigm based on a procedure that emulates human cognitive processes to make reasoning processes and decisions in environments of uncertainty and imprecision [68]. In this paradigm the objects of computation are words or sentences from a natural language and results are also expressed in a linguistic expression domain that, usually, corresponds to the initial linguistic domain. To do so, a computational scheme (see Fig. 1), which includes a translation phase and a retranslation phase, has been defined in such paradigm [18,40,42,64].

The linguistic preference modeling in decision making can be managed by means of CWW processes. However, there are some decision situations that define complex frameworks in which to carry out CWW processes could be not enough. These complex frameworks are briefly detailed below:

- *Heterogeneous frameworks*: Decision problems where each expert may express his/her assessments in different expression domains, depending on the level of knowledge, experience or the nature of criteria that characterized the set of alternatives. Therefore, the assessments are expressed with non-homogeneous information such as, numerical, interval or linguistic [26,32,46,50].
- *Multi-granular linguistic frameworks*: Decision problems with multiple experts or multiple criteria in which appear linguistic information assessed in multiple linguistic term sets with different granularity. Therefore, the assessments of the problem are represented in multiple linguistic scales [5,13,21,25,28].
- *Unbalanced linguistic frameworks*: Decision problems in which it is necessary to assess preferences with a greater granularity on one side of the linguistic scale regarding the another one. Hence, linguistic terms of the scale are neither uniformly nor symmetrically distributed. Therefore, experts express their assessments in an unbalanced linguistic scale [1,4,22,57].

Different linguistic computational models for decision making have been introduced in the literature [8–10,59,63]. However, the 2-tuple linguistic model [23,24] has been compared with them and it has been showed as the most appropriate model in linguistic decision making, considering the computing with words paradigm [24,52]. The main advantage of the 2-tuple linguistic model is its computational model that offers linguistic results in the original linguistic domain in a precise way.

Furthermore, the 2-tuple linguistic model has been extended to perform processes of CWW in complex decision frameworks [13,21,22,25,26,39] and have been successfully applied in different fields such as sustainable energy [15], recommender systems [51], sensory evaluation [16,38], personnel selection [30], quality of service [17], performance appraisal [12], vendor selection problem [3], soft consensus [27,47,69] or software project selection [71]. Given that the 2-tuple linguistic model and its extensions keep the CWW scheme showed in Fig. 1, together with its own features and extensions make of it a flexible and adaptable model to solve decision making problems in all type of decision frameworks.

Notwithstanding there are many linguistic computational models and a lot of applications solved by using them, there is a lack of software tools to solve linguistic decision problems carrying out CWW processes. In [36] was proposed *Decider*, a linguistic decision support system that develops and implements a fuzzy multicriteria group decision making method. *Decider* can deal with complex decision frameworks and has been applied to different evaluation problems [33,35,55,70]. To do so, *Decider* uses a method that unifies the information into triangular fuzzy numbers, which are aggregated to obtain a closeness coefficient for each alternative. This method considers the distance measure between the fuzzy group assessment of each alternative and both a group ideal solution and a group negative ideal solution. The weakness of *Decider* is that the proposed method provides closeness coefficients expressed in the unit interval and, therefore, cannot be considered inside of the CWW paradigm (see Fig. 1). Due to this fact, the proposed method lacks the retranslation phase and computed closeness coefficients cannot be easily interpreted.

In [6] was proposed *jFuzzyLogic*[1] that is an open source Java library which offers a fuzzy inference system. Although the library has been extended to handle decision problems with linguistic information by means the linguistic 2-tuple model and some extensions, this library is far from being a complete tool focused on solving decision problems with linguistic and complex frameworks.

Beyond the scope of linguistic decision making, three interesting software tools can be found. First, *DECERNS* (Decision Evaluation in Complex Risk Network Systems) [66] that is a web-based spatial decision support system for multi-criteria analysis of a wide range of spatially-distributed alternatives. Second, the *decision deck project* [41] that offers open source software tools which develops multicriteria decision aid techniques to support complex decision aid processes.

[1] https://salty.unice.fr/wiki/salty-public/Deliverables.

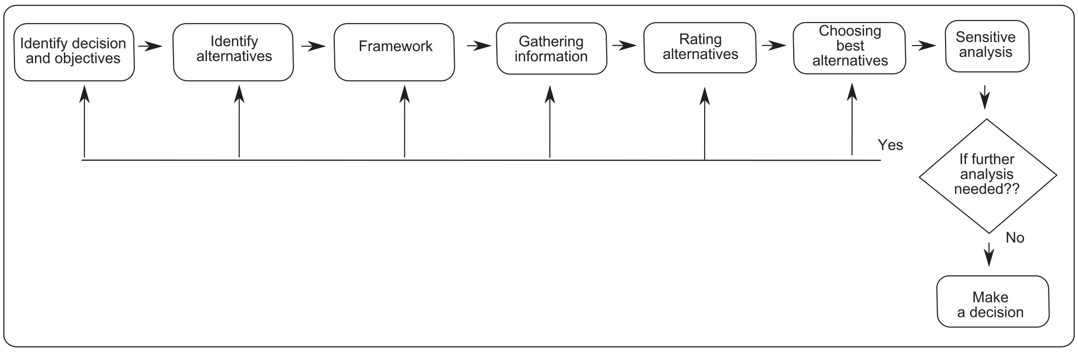


Fig. 2. Decision resolution scheme.

Finally, *LINGO*[2] is a comprehensive tool designed to help build and solving linear, nonlinear, and integer optimization models,

In spite of different software tools to deal with decision and linguistic decision problems, it is clear that there is a lack of software tools within the CWW paradigm to solve these decision problems. The aim of this paper is to present a decision software suite called *Fuzzy LINguisTic deciSion TOols eNhancEment Suite* (*Flintstones*),[3] based on the 2-tuple linguistic model and its extensions in order to solve decision problems defined in linguistic and complex frameworks, offering linguistic results that facilitate their understandability. Furthermore, not only this paper introduces a linguistic decision software but also presents the *Flintstones* website in which different releases can be download together with a repository of case studies and datasets for different decision making problems with linguistic and complex frameworks that can be solved by using *Flintstones*. In order to show its performance, usefulness and effectiveness, a case study defined in a multi-granular linguistic framework is solved step by step by *Flintstones*.

The paper is structured as: Section 2 provides a revision about the decision scheme as well as the foundations of the 2-tuple linguistic model and the different extensions based on this model for CWW in complex frameworks. Section 3 presents the linguistic decision method implemented and developed by *Flintstones* as well as its architecture and the technologies used. In Section 4, the website of the proposed software suite that includes a repository of case studies and datasets is presented as well as the resolution of a case study with *Flintstones*. Finally, in Section 5, conclusions and future works are pointed out.

2. Preliminaries

In this section, we first provide a brief revision of a general decision scheme and the use in it of linguistic information that will be adapted by *Flintstones*. We then provide a review of the foundations of the 2-tuple linguistic model and its extensions to solve linguistic decision making problems with linguistic and complex frameworks.

2.1. Decision scheme

In [7] was proposed a common decision resolution scheme that has been adapted or extended according to the needs of the decision situations [20,49,54]. The common decision resolution scheme consists of following eight phases [7] (see Fig. 2):

- *Identify decision and objectives*.
- *Identify alternatives*.
- *Framework:* The structure and elements of the decision problem are defined: experts, criteria, etc.
- *Gathering information:* The information provided by experts is collected, according to the defined framework.
- *Rating alternatives:* The gathered information provided by experts is aggregated to obtain a collective value for each alternative. Therefore, in this phase, it is necessary to carry out a solving process in order to compute the collective assessments for the set of alternatives, using appropriate aggregation operators. Other authors call this phase *aggregation phase* [54].
- *Choosing the alternative/s:* Normally, the highest collective assessment corresponds to the best alternative [31] that is selected to solve the decision making. To do so, it may use a choice function that assigns a choice degree for each alternative [19]. This phase is also called *exploitation phase* [54].

[2] http://www.lindo.com/.
[3] http://serezade.ujaen.es/flintstones/.

- *Sensitive analysis*: The information computed is analyzed. If the information is not good enough to make a decision, it is necessary to return to the previous phases in order to make a depth analysis.
- *Make a decision*: The information obtained from the previous decision analysis can be used to make a decision.

The common decision resolution scheme may include linguistic information to model and manage decision situations under non-probabilistic uncertainty. This fact implies the need to operate with linguistic terms in order to compute linguistic assessments for alternatives, according to the CWW scheme (Fig. 1).

In line with our aims in this paper, the 2-tuple linguistic model and its extensions will be implemented by *Flintstones* due to the fact that provide adequate computational models to deal with linguistic information in decision problems defined in linguistic and complex frameworks.

2.2. 2-Tuple linguistic representation model and its extensions

In this section, the representation and computational models for 2-tuple values are reviewed. A brief revision of extensions of the 2-tuple linguistic model to carry out CWW processes in complex frameworks is then provided.

2.2.1. 2-Tuple linguistic representation model

This model was presented in [23] to avoid the loss of information and improve the precision in processes of CWW when the linguistic term set has an odd value of granularity, being triangular-shaped, symmetrical and uniformly distributed its membership functions.

The 2-tuple linguistic model represents the information by means of a pair of values (s, α), where s is a linguistic term with syntax and semantics, and α is a numerical value that represents the *symbolic translation*. Let $S = \{s_0, \dots, s_g\}$ be a linguistic term set and $\beta \in [0, g]$ a numerical value in its interval of granularity.

Definition 1 [23]. The symbolic translation is a numerical value assessed in $[-0.5, 0.5)$ that supports the difference of information between a counting of information β assessed in the interval of granularity $[0, g]$ of the term set S and the closest value in $S = \{s_0, \dots, s_g\}$ which indicates the index of the closest linguistic term in S.

This model defines a set of functions to facilitate the computational processes with 2-tuple linguistic values [23].

Definition 2 [23]. Let $S = \{s_0, \dots, s_g\}$ be a set of linguistic terms. The 2-tuple set associated with S is defined as $\langle S \rangle = S \times [-0.5, 0.5)$. The function $\Delta_S : [0, g] \longrightarrow \langle S \rangle$, is defined by:

$$\Delta_S(\beta) = (s_i, \alpha), \quad \text{with} \quad \begin{cases} i = round(\beta), \\ \alpha = \beta - i, \end{cases} \tag{1}$$

where $round(\cdot)$ assigns to β the integer number $i \in \{0, 1, \dots, g\}$, closest to β.

Proposition 1. *Let $S = \{s_0, \dots, s_g\}$ be a linguistic term set and (s_i, α) be a 2-tuple linguistic value. There is always a function Δ_S^{-1} such that from a 2-tuple linguistic value, it returns its equivalent numerical value $\beta \in [0, g]$ as $\Delta_S^{-1}(s_i, \alpha) = i + \alpha$.*

Remark 1. It is obvious that the conversion of a linguistic term into 2-tuple linguistic value consists of adding a value 0 as symbolic translation.

2.2.2. 2-Tuple linguistic computational model

The 2-tuple linguistic representation model has a linguistic computational model associated based on Δ_S^{-1} and Δ_S in order to accomplish CWW processes in a precise way:

- *Comparison of 2-tuple linguistic values*. The comparison of linguistic information represented by 2-tuple linguistic values is carried out according to an ordinary lexicographic order.
  Let (s_k, α_1) and (s_l, α_2) be two 2-tuple linguistic values, each one representing a counting of information.
  - If $k < l$, then $(s_k, \alpha_1) < (s_l, \alpha_2)$.
  - If $k = l$, then
    1. If $\alpha_1 = \alpha_2$, then (s_k, α_1) and (s_l, α_2) represent the same information.
    2. If $\alpha_1 < \alpha_2$, then $(s_k, \alpha_1) < (s_l, \alpha_2)$.
    3. If $\alpha_1 > \alpha_2$, then $(s_k, \alpha_1) > (s_l, \alpha_2)$.

- *Negation operator of a 2-tuple linguistic value*. The negation operator over a 2-tuple linguistic value is defined as: $Neg(s_i, \alpha) = \Delta_S(g - (\Delta_S^{-1}(s_i, \alpha)))$, being $g + 1$ the cardinality of S.
- *2-Tuple linguistic aggregation operators*. The 2-tuple linguistic aggregation operator consists of obtaining a value that summarizes a set of 2-tuple linguistic values. Therefore, the result of an aggregation process of a set of 2-tuple linguistic values must be a 2-tuple linguistic value. Several 2-tuple linguistic aggregation operators have been proposed in the literature [23,44,45,60,62,65].

2.3. 2-Tuple linguistic model extensions for linguistic complex frameworks

Decision making situations under uncertainty can define linguistic complex frameworks (multi-granular linguistic, heterogeneous, unbalanced linguistic) that need more than just a linguistic domain to model all information involved in the decision problem. In such contexts, the extensions of the 2-tuple linguistic model can perform processes of CWW in these complex frameworks, obtaining satisfactory results in linguistic decision problems.

These extensions follow the CWW paradigm and share a common process for rating alternatives in the decision scheme in a proper way (see Fig. 3). A further detailed overview of these extensions can be found in [39]. To facilitate the understanding of the decision solving methods in *Flintstones*, in the following subsections are briefly reviewed the different 2-tuple linguistic extensions designed to deal with complex decision frameworks.

2.3.1. Multi-granular linguistic frameworks

Usually, in decision situations with multiple criteria or several experts, the preferences are expressed in multiple linguistic term sets with different granularity, considering the imprecision and uncertainty of the related information. These decision situations define a multi-granular linguistic framework [5,28] and require an adequate solving process to manage such frameworks. Three extensions based on the 2-tuple linguistic model have been proposed to deal with multiple linguistic scales [13,21,25] that are reviewed below.

2.3.1.1. Fusion approach for managing multi-granular linguistic information. This extension was presented in [21] and provides a total flexible linguistic framework because it does not impose any limitation related with the granularity of each linguistic term set as well as the shape of the fuzzy membership functions of each linguistic term.

The description of this extension in the rating process is the following one (see Fig. 4):

1. *Unification process*. The multigranular information is unified into a specific linguistic domain called *Basic Linguistic Term Set* (BLTS) and noted as $S_{BLTS} = \{s_i, i = 0, \ldots, g\}$, which is selected with the aim of keeping as much knowledge as possible (see [21]). This BLTS might be a linguistic term set fixed in the framework with the condition that this set can be represented by a 2-tuple linguistic value. A linguistic term $s_l^k \in S^k$, such that $S^k = \{s_l, l = 0, \ldots, g_k\}$ and $g_k \leqslant g$, is unified into fuzzy sets in the BLTS by using the transformation function $\varphi_{S^k S_{BLTS}} : S^k \to F(S_{BLTS})$ defined as:

$$\varphi_{S^k S_{BLTS}}(s_l^k) = \sum_{i=0}^{g} (s_i/\gamma_i), \tag{2}$$

where $\gamma_i = \max_y \min\{\mu_{s_l}(y), \mu_{s_i}(y), i = 0, \ldots, g\}$.

2. *Aggregation process*. The information expressed in multiple linguistic scales has been unified into fuzzy sets in the BLTS. Therefore, in this process, the computations are directly carried out on fuzzy sets by using the fuzzy arithmetic [11].
3. *Retranslation process*. In this process, the results expressed into fuzzy sets, $F(S_{BLTS})$, are transformed into 2-tuple linguistic values in the BLTS by the function $\chi : F(S_{BLTS}) \to \langle S_{BLTS} \rangle$ that is defined as:

$$\chi\Big(\{(s_0, \gamma_0), (s_1, \gamma_1), \ldots, (s_g, \gamma_g)\}\Big) = \Delta_S\left(\frac{\sum_{i=0}^{g} i\gamma_i}{\sum_{i=0}^{g} \gamma_i}\right) = (s, \alpha) \in \langle S_{BLTS} \rangle. \tag{3}$$

It is noteworthy that the transformations in which fuzzy sets are involved can imply a lack of information. Therefore, the 2-tuple linguistic results obtained in the rating process can be inaccurate.

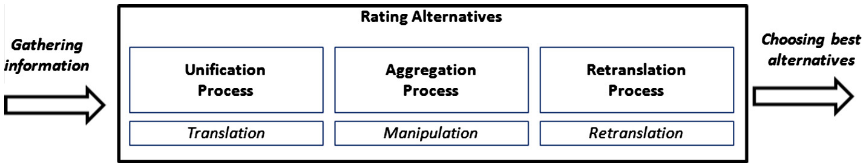


Fig. 3. Common schema in rating process of extensions based on the 2-tuple linguistic model.

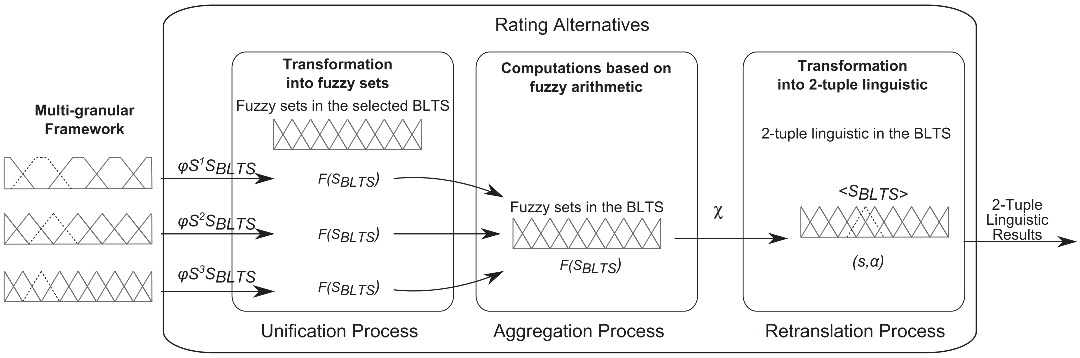


Fig. 4. Rating process in fusion approach for managing multi-granular linguistic information.

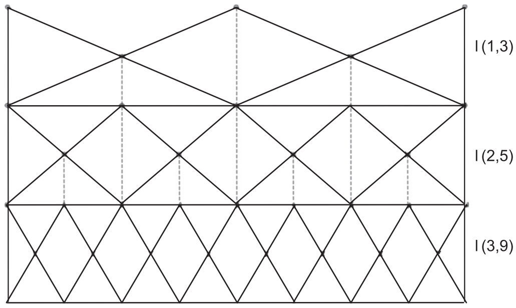


Fig. 5. Linguistic hierarchy with 3 levels.

2.3.1.2. Linguistic hierarchies. Linguistic hierarchies was proposed in [25] to overcome drawbacks related to the accuracy and the expression domain of the linguistic results of the previous extension [21].

A linguistic hierarchy is defined by the union of its levels $l(t, n(t))$. Each level t corresponds to a linguistic term set denoted as $S^{n(t)} = \{s_0^{n(t)}, \ldots, s_{n(t)-1}^{n(t)}\}$ with a granularity of uncertainty of $n(t)$.

$$LH = \bigcup_t l(t, n(t)). \tag{4}$$

In order to ensure the accomplishment of the CWW processes without loss of information, each level must be a linguistic term set symmetrically and uniformly distributed and the level $t+1$ must keep the former modal points of the level t (see Fig. 5). To do so, the granularity of the level $t+1$ is obtained from the granularity of its predecessor as:

$$n(t+1) = (2 \cdot n(t)) - 1. \tag{5}$$

The rating process of this extension to operate with information expressed in multiple linguistic scales is described as follows (see Fig. 6):

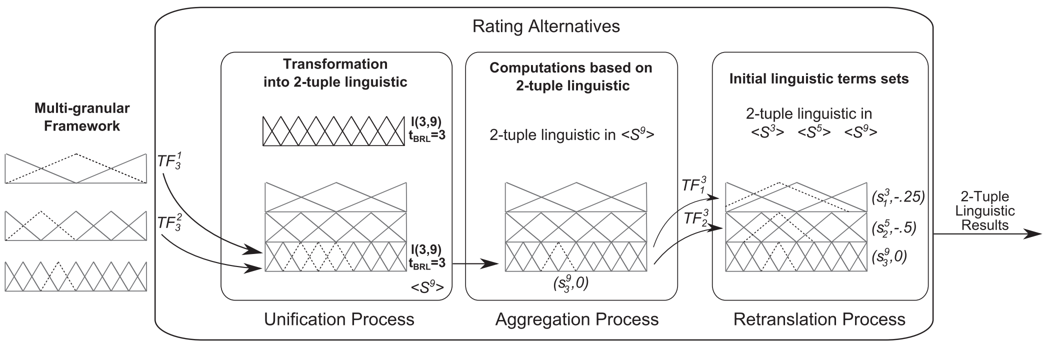


Fig. 6. Rating process in linguistic hierarchies.

1. *Unification process*. This extension defines the transformation function $TF^{t}_{t'} : \langle S^{n(t)}\rangle \rightarrow \langle S^{n(t')}\rangle$, designed for transforming linguistic terms from different levels in the *LH* in an accurate way as follows:

$$TF^{t}_{t'}(s_i^{n(t)}, \alpha) = \Delta_S\left(\frac{\Delta_S^{-1}(s_i^{n(t)}, \alpha)\cdot(n(t')-1)}{n(t)-1}\right). \tag{6}$$

The linguistic information is unified into a single level of the *LH*, called *Basic Representation Level* and noted as t_{BRL}. This unification is carried out by means of Eq. (6) with $TF^{t}_{t_{BRL}}$.
2. *Aggregation process*. The information is expressed in 2-tuple linguistic values in the level t_{BRL}. Therefore, the 2-tuple linguistic computation model is carried out to performance CWW processes and to obtain linguistic results in $S^{(n(t_{BRL}))}$ [23].
3. *Retranslation process*. The 2-tuple linguistic results have been expressed in the unified level. These results can be expressed in each initial linguistic term set defined in the framework in a precise way by means of the transformation function defined by Eq. (6) with $TF^{t_{BRL}}_{t}$.

2.3.1.3. Extended linguistic hierarchies. This extension was proposed in [13] to offer more flexibility in the multi-granular linguistic framework than the extension based on LH [25], keeping accuracy in the CWW processes.

An extended linguistic hierarchy (ELH) does not impose any rule about the granularity of each linguistic term set that is included in the hierarchy and adds a last level that is noted as t^* to keep all the former modal points of the original linguistic scales (see Fig. 7).

The new level t^*, keeps all the information in CWW processes and its granularity is computing using the granularities of the initial linguistic term sets as follows:

$$n(t^*) = lcm(n(1)-1, n(2)-1, \ldots, n(m)-1) + 1, \tag{7}$$

being *lcm* the *least common multiple* and *m* the number of initial linguistic scales. Therefore, an extended linguistic hierarchy is the union of original levels and the new generated level (see Fig. 7).

$$ELH = \left(\bigcup_{t=m}^{t=1} l(t, n(t))\right) \bigcup l(t^*, n(t^*)). \tag{8}$$

The rating process in this extension is similar to the extension dealing with LH, taking into account that the unified level in this case must be t^* (see Fig. 8) in order to ensure the CWW processes without loss of information.

1. *Unification process*. The information expressed in multiple linguistic scales is unified in the new level t^*, using the transformation function $ETF^{t}_{t^*} : \langle S^{n(t)}\rangle \rightarrow \langle S^{n(t^*)}\rangle$ without loss of information and defined by:

$$ETF^{t}_{t*} = \Delta_S\left(\frac{\Delta_S^{-1}(s_i^{n(t)}, \alpha)\cdot(n(t^*)-1)}{n(t)-1}\right). \tag{9}$$

2. *Aggregation process*. The unified information has been represented in 2-tuple linguistic values in the new level t^*. So, the processes of CWW are carried out by using the 2-tuple linguistic computational model [23], obtaining 2-tuple linguistic results in $\langle S^{n(t^*)}\rangle$.
3. *Retranslation process*. In this process, the 2-tuple linguistic results are expressed in each original linguistic term set in a precise way by means of the transformation function $ETF^{t^*}_{t}$ given by:

$$ETF^{t*}_{t} = \Delta_S\left(\frac{\Delta_S^{-1}(s_i^{n(t^*)}, \alpha)\cdot(n(t)-1)}{n(t^*)-1}\right). \tag{10}$$

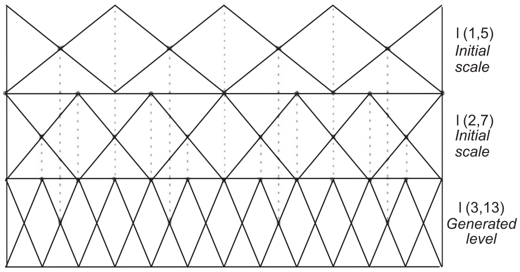


Fig. 7. Extended linguistic hierarchies with two initial levels.

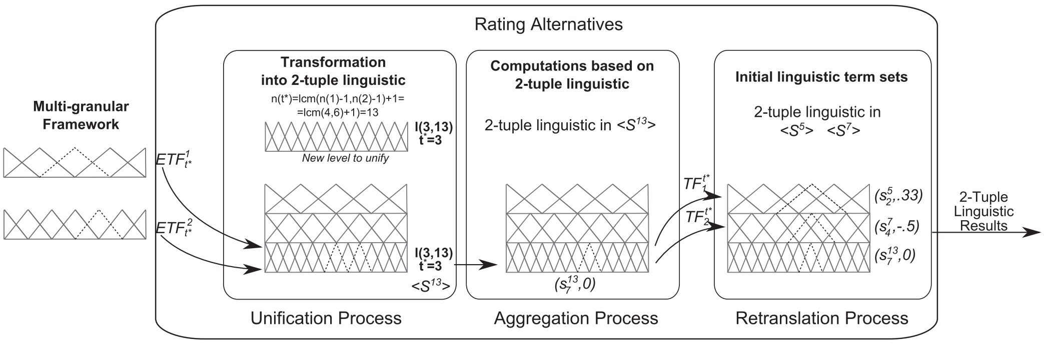


Fig. 8. Rating process in extended linguistic hierarchies.

2.3.2. Heterogeneous frameworks

Similarly to the previous framework, sometimes in decision making situations, experts with diverse background usually may express their preferences in different expression domains such as: numerical domain, interval-valued domain and linguistic domains, considering the imprecision and uncertainty of the related information as well as the nature of assessed criteria. Therefore, these situations define a heterogeneous framework that requires an adequate process to solve the decision problem.

2.3.2.1. Fusion approach for managing heterogeneous information. In [26] was presented an extension to deal with information expressed in different expression domains: numerical domain, interval-valued domain and any linguistic term set. This extension shares the operation of the fusion approach for managing multi-granular linguistic information (reviewed in Section 2.3.1.1), defining two new transformation functions to transform numerical values and interval values into fuzzy sets. Therefore, this extension provides a total flexible framework in which experts can express their preferences by means of different expression domains. The rating process with this extension to deal with heterogeneous frameworks is described as follows (see Fig. 9):

1. *Unification process*. The heterogeneous information is unified into a selected linguistic domain called *Basic Linguistic Term Set* (BLTS) and noted as $S_{BLTS} = \{s_i, i = 0, \ldots, g\}$ that is chosen with the aim of keeping as much knowledge as possible (see [26]). Hence, each assessment is transformed by using an adequate transformation function, according to its expression domain:
   (a) *Numerical domain*. When $v \in [0,1]$, a numerical transformation function $\varphi_{NS_{BLTS}} : ([0,1]) \to F(S_{BLTS})$ is applied by:

$$\varphi_{NS_{BLTS}}(v) = \sum_{i=0}^{g} (s_i/\gamma_i), \tag{11}$$

   where $\gamma_i = \mu_{s_i}(v) \in [0,1]$ is the membership degree of v to $s_i \in S_{BLTS}$.
   (b) *Interval domain*. When $v \in P([0,1])$, an interval transformation function $\varphi_{IS_{BLTS}} : P([0,1]) \to F(S_{BLTS})$, is applied by:

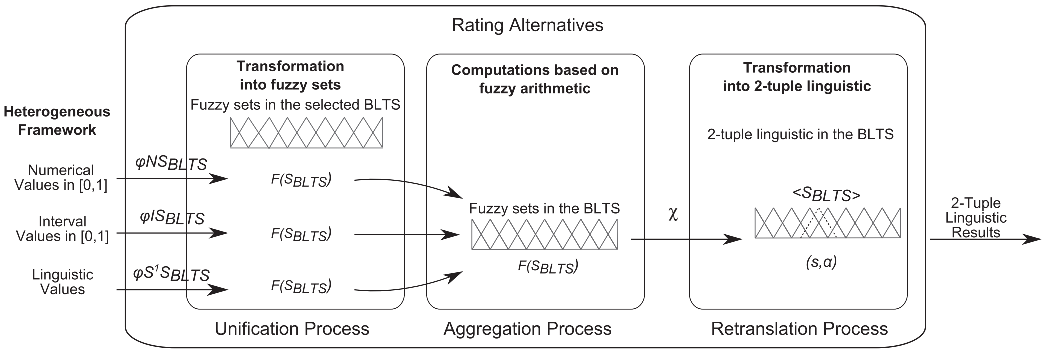


Fig. 9. Rating process in fusion approach for managing heterogeneous information.

$$\varphi_{IS_{BLTS}}(v) = \sum_{i=0}^{g}(s_i/\gamma_i), \tag{12}$$

where $\gamma_l^i = \max_y \min\{\mu_I(y), \mu_{s_l}(y)\}$, with $l = \{0, \ldots, g\}$, being $\mu_I(\cdot)$ and $\mu_{s_l}(\cdot)$ membership functions associated with the interval $I \in P([0,1])$ and the term $s_l \in S_{BLTS}$, respectively.

(c) *Linguistic domain*. When $v \in S^k$, such that $S^k = \{s_0^k, \ldots, s_{g_k}^k\}$ and $g_k < g$, the linguistic transformation function $\varphi_{S^k S_{BLTS}}$ is applied. This function was defined in Eq. (2).

2. *Aggregation process*. In the same way as the fusion approach for managing multi-granular linguistic information, the computations are directly carried out in fuzzy sets by using the fuzzy arithmetic [11].
3. *Retranslation process*. The aggregated results have been expressed in fuzzy sets in the BLTS, $F(S_{BLTS})$. In this process, these results are transformed into 2-tuple linguistic values by the function χ that was defined in Eq. (3). Therefore, the aggregated fuzzy sets are transformed into 2-tuple linguistic values in the BLTS.

As fusion approach for managing multi-granular linguistic information, the fusion approach for managing heterogeneous information can imply a lack of information in the transformations in which fuzzy set are involved.

2.3.3. Unbalanced linguistic frameworks

The unbalanced linguistic frameworks appear in decision situations under uncertainty when is necessary to assess preferences with a greater granularity on one side of the linguistic scale regarding the another one.

2.3.3.1. Fuzzy linguistic methodology to deal with unbalanced linguistic term sets. In [22] was presented a methodology based on LH [25] that provides an algorithm to represent the semantics of an unbalanced linguistic scale as well as a boolean function $Brid(S)$, which is used in CWW processes (see Fig. 10).

This algorithm builds the semantics for an unbalanced linguistic term set $\mathcal{S}$ with a $LH, LH(\langle\mathcal{S}\rangle) = \{(s_{I(i)}^{G(i)}, \alpha), i = \{0, \ldots, g\}, s_{I(i)}^{G(i)} \in \mathcal{S}\}$, being $I(i)$ the function that assigns the index of the label that represents its semantics in the *LH* and $G(i)$ the function that assigns to each label the granularity of the level in which it is represented.

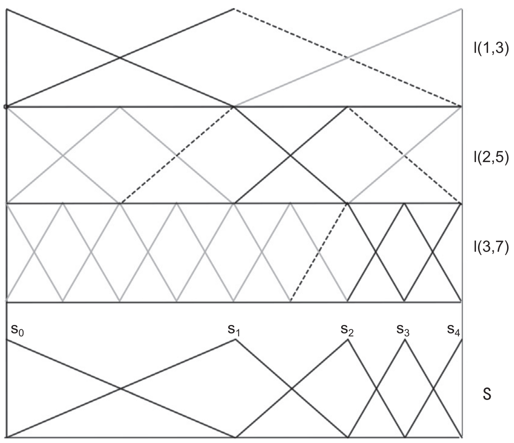
The description of this methodology in the rating process is the following (see Fig. 11):

1. *Unification process*. The semantics of the unbalanced linguistic term set $LH(\langle\mathcal{S}\rangle)$ belongs to different levels of the LH. In this process, the information expressed in the unbalanced linguistic scale is unified at a unified level t_{BRL}. To do so, first, each unbalanced term is transformed into 2-tuple linguistic value in its respective level of the LH by means the transformation function $\mathcal{LH} : \langle\mathcal{S}\rangle \to LH(\langle\mathcal{S}\rangle)$ defined as:

$$\forall(s_i, \alpha) \in (\mathcal{S} \times [-0.5, 0.5)) \Rightarrow \mathcal{LH} : (s_i, \alpha) = (s_{I(i)}^{G(i)}, \alpha). \tag{13}$$

Second, linguistic terms expressed in different levels of the LH are unified into the unified level t_{BRL}, by using the transformation function $TF_{t_{BRL}}^{t'}$ that was defined in Eq. (6) with $l(t', G(i))$.

2. *Aggregation process*. In this step, the 2-tuple linguistic computation model is performed [23], obtaining 2-tuple linguistic results expressed in $\langle S^{(n(t_{BRL}))}\rangle$.



S	$LH(S)$	$Brid(S)$
s_0 = *Poor*	$s_{I(0)}^{G(0)} = s_0^3$	False
s_1 = *Average*	$s_{I(1)}^{G(1)} = s_1^3$ or $s_{I(1)}^{G(1)} = s_2^5$	True
s_2 = *Good*	$s_{I(2)}^{G(2)} = s_3^5$ or $s_{I(2)}^{G(2)} = s_6^9$	True
s_3 = *Very Good*	$s_{I(3)}^{G(3)} = s_7^9$	False
s_4 = *Excellent*	$s_{I(4)}^{G(4)} = s_8^9$	False

Fig. 10. Unbalanced linguistic scale and the table with the related information.

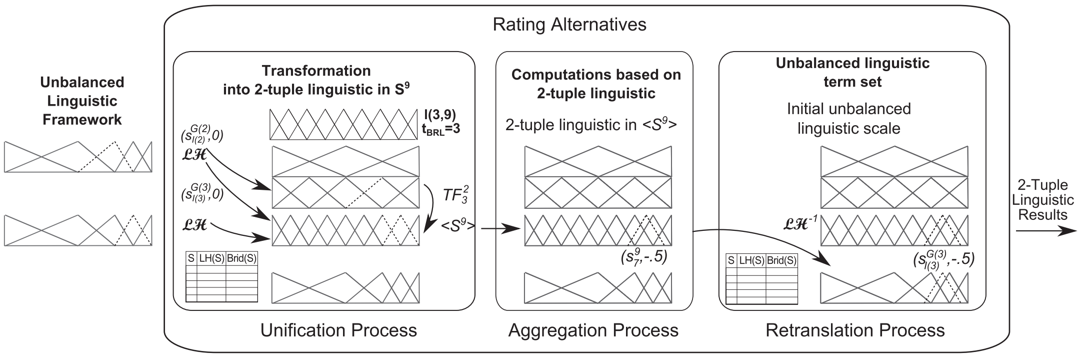


Fig. 11. Rating process with the fuzzy linguistic methodology to deal with unbalanced linguistic term sets.

3. *Retranslation process*. In this process, 2-tuple linguistic results expressed in the unified level are transformed in the unbalanced linguistic scale by means of the transformation function showed in Eq. (14). This transformation function was defined by cases in [22] and is based on the satisfaction of conditions imposed on $LH(\langle\mathcal{S}\rangle)$ and the boolean function $Brid(S)$.

$$\mathcal{LH}^{-1} : \langle S^{t_{BRL}} \rangle \rightarrow \langle \mathcal{S} \rangle. \tag{14}$$

Therefore, 2-tuple linguistic results are expressed in the initial unbalanced linguistic scale.

3. FLINTSTONES: a fuzzy linguistic decision tools enhancement suite

Once the basics have been reviewed, this section introduces a novel decision tools suite called *Flintstones* to solve decision making problems under uncertainty by using the 2-tuple linguistic model and its extensions. To do so, we first present the linguistic decision method that is implemented and developed by *Flintstones*. We then present the architecture and technologies of the software suite.

3.1. Linguistic decision method for Flintstones based on the 2-tuple linguistic model and its extensions

Our aim is to develop a software suite of tools to solve linguistic decision making problems based on the 2-tuple linguistic model and its extensions, dealing with linguistic and complex frameworks. Hence, it is necessary to propose a specific linguistic decision solving method that includes the selection of a solving process based on the 2-tuple linguistic model, according to the framework in which the decision problem is defined.

Such a general linguistic decision scheme is presented in Fig. 12 and it has been adapted from the common decision scheme that was showed in Fig. 2. Following, the proposed decision method is described in further detail, showing the operation of each phase in *Flintstones*:

3.1.1. Framework

This phase defines the framework that includes the elements involved in the decision making problem:

- A finite set of alternatives $X = \{x_1, \ldots, x_h\}$.
- A set of criteria $C = \{c_1, \ldots, c_n\}$ that might be grouped.
- A set of experts $E = \{e_1, \ldots, e_m\}$ that could also be grouped. The set of experts will provide the assessments of the decision problem.

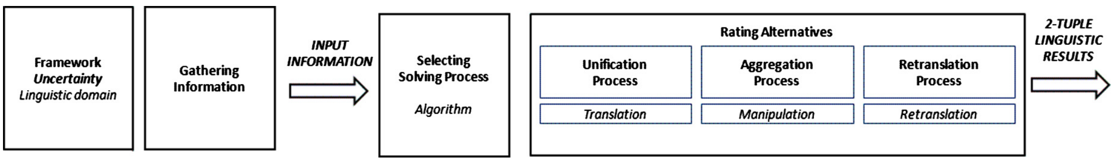


Fig. 12. Linguistic decision method based on the 2-tuple linguistic model and its extensions.

- The set of expression domains $\mathcal{F}$ in which the assessments provided by experts will be expressed, allowing the definition of the following domains of expression: numerical domain (*num*), interval domain (*int*), linguistic domain (*lin*), and an unbalanced linguistic domain defined in a linguistic hierarchy (*linUnb*).
- Fix expression domains that will be used by experts, according to the uncertainty and the nature of criteria as well as the background of each expert.

3.1.2. Gathering information

In this phase, each expert e_k, provides the assessments by means of assessment vectors

$$U^k = (v_{ij}^k : i = 1 \ldots n,\ j = 1, \ldots, m : v_{ij}^k \in \mathcal{F}).$$

The assessment v_{ij}^k, provided by each expert e_k, for each criterion c_i of each alternative x_j, is expressed in the allocated expression domain in the framework $\mathcal{F}$.

3.1.3. Selecting a solving process

A decision solving process based on the 2-tuple linguistic representation model should be selected to carry out the rating process in the next phase. The selection of the suitable solving process is based on expression domains defined in the framework and the way to obtain the linguistic results. In a decision making problem with a single scale in which its linguistic terms are uniform and symmetrically distributed, the *2-tuple linguistic computational model* will be carried out in the rating process. However, if the decision problem facing a linguistic complex context, the algorithm selects *the fusion approach for managing heterogeneous information* when the information involved in the problem is expressed with non-homogeneous information. In the decision situations in which the information is expressed in an unbalanced linguistic scale built from a linguistic hierarchy, the *fuzzy linguistic methodology to deal with unbalanced linguistic term sets* will be chosen.

There are three extensions to deal with multi-granular linguistic contexts. On the one hand, *the fusion approach* always can be applied due to the fact that it offers a total flexible framework. However, this extension can provide linguistic results with a lack of accuracy. For this reason, when another extension to deal with multi-granular linguistic context can be applied, this will be selected in order to ensure accurate linguistic results. On the other hand, the extension based on *extended linguistic hierarchies* is a generalization of the extension based on *linguistic hierarchies* that includes a new level in the hierarchy to unify the multi-granular information and accomplish the CWW processes. In order to reduce the computations, the linguistic hierarchies will be selected when the multi-granular linguistic context allows its application.

Therefore to select the right solving process it has been developed an algorithm that requires the following input information about the framework:

- *edNum* $\in \{True, False\}$ indicates if a numerical expression domain *num* was defined in $\mathcal{F}$.
- *edInt* $\in \{True, False\}$ establishes if an interval expression domain *int* was defined in $\mathcal{F}$.
- *edLinUnb* $\in \{True, False\}$ determines if an unbalanced linguistic domain defined with a *linguistic hierarchy linUnb* was fixed in $\mathcal{F}$.
- *tamEdLinLis* $\in \mathbb{N}^+$ defines the number of linguistic scales established in the set of expression domains $\mathcal{F}$.
- *edLin* $= \{card, 2T\}$ describes a linguistic domain *lin* fixed in the framework that is characterized by two values. First, *card* $\in \mathbb{N}^+$ that indicates the cardinality of the linguistic domain and, the second value, $2T \in \{True, False\}$ that establishes if the linguistic domain can be represented by 2-tuple linguistic values, i.e., the linguistic term set has an odd value of granularity and whose membership functions are triangular-shaped, symmetrically and uniformly distributed in the unit interval.
- *edLinList* $= \{edLin_i; i = 1, \ldots, tamEdLinLis\}$ is a vector of *edLin* that provides information about the linguistic scales established in $\mathcal{F}$.

Algorithm 1 illustrates the proposed procedure to select the suitable solving process for the decision making problem. The algorithm selects the $id \in \{1, 2, 3, 4, 5, 6\}$ that identifies the solving process according to the following references:

1. *2-Tuple linguistic computational model.*
2. *Fusion approach for managing multi-granular linguistic information.*
3. *Linguistic hierarchies.*
4. *Extended linguistic hierarchies.*
5. *Fusion approach for managing heterogeneous information.*
6. *Fuzzy linguistic methodology to deal with unbalanced linguistic term sets.*

Algorithm 1. Algorithm to select the suitable solving process

```
Require: edNum, edInt, edLinUnb, tamEdLinLis, edLinList
Ensure: id
if (edLin[1].2T = true) and (tamEdLinLis = 1) then
  return 1
else if (edNum = true) or (edInt = true) then
  return 5
else if (edLinUnb = true)
  return 6
else
  edLinListShortCard ⇐ short (edLinList,edLinList.card)
  i ⇐ 1
  while i < tamEdLinLis do
    if (edLinListShortCard.edLin[i].2T = false) then
      return 2
    else if (edLinListShortCard[i+1].card ≠ ((edLinListShortCard[i].card)-1)·2+1) then
      return 4
    else
      i ⇐ i + 1
    end if
  end while
  return 3
end if
```

3.1.4. Rating alternatives

The aim of this phase is to obtain a 2-tuple linguistic global assessment for each alternative that is easily interpreted. Taking into account the previous phase, a linguistic assessment for each alternative is computed, using the selected solving process that allows to manage the information expressed in the decision framework. Each solving process follows the common scheme with the following three processes (see Fig. 12):

1. *Unification process*. Decision making problems under uncertainty can be defined in linguistic complex frameworks (multi-granular linguistic, heterogeneous or unbalanced linguistic). As was reviewed in Section 2.3, to manage these frameworks, the extensions based on the 2-tuple linguistic model represent the gathered information using different expression domains. Therefore, the first process is to represent the gathered information into a unified domain.
2. *Aggregation process*. In this second process, the unified information is aggregated selecting aggregation operators for the unified domain in order to obtain a global assessment for each alternative that summarizes its gathered information.
3. *Retranslation process*. A retranslation process is needed to express the global assessment for each alternative in a linguistic expression domain that can be easily interpreted by experts, keeping the CWW scheme (see Fig. 1).

3.2. Architecture and technologies

It is important to show the architecture of *Flintstones* and the technologies used in the software suite of tools. *Flintstones* has been developed using the Rich Client Platform (RCP)[4] that provides a framework to build desktop applications, client applications, with rich functionality.

The main value of RCP is that allows to quickly develop professional applications with native look-and-feel on multiple platforms. Another advantage of RCP is that its components have a high quality and are actively maintained.

Four modules have been built in *Flintstones* (see Fig. 13) in order to separate the decision scheme, the solving processes, the set of aggregation operators and a series of interface representation elements. The details of each type of basic module are described below.

- *Libraries* provide structures and procedures with the aim of supporting the resolution of the decision problem. These libraries include elements such as: experts, criteria, alternatives, assessments and expression domains.
- *Graphical User Interface* (GUI) that allows users to interact with the software suite of tools.
- *Methods* that develop the 2-tuple linguistic computational model and its extensions in order to solve decision making problems with linguistic and complex contexts.

[4] http://www.eclipse.org/home/categories/rcp.php.

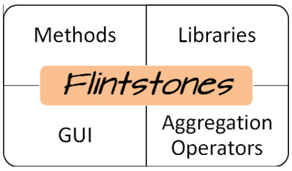


Fig. 13. Modules in *Flintstones*.

- *Operators* implement the set of aggregation operators that can be used to aggregate the information involved in the decision problem. This module includes the most popular aggregation operators: *maximum, minimum, median, arithmetic mean, weighted average and ordered weighted average*.

Flintstones offers several advantages based on its architecture and technologies. The strongest points of the software suite are:

- *Flintstones* has been developed with an RCP based on Java. Therefore, this suite of tools can be used on any machine with Java Virtual Machine (JVM), independently of its operating system.
- The software suite is divided in four separated modules. Due to this fact, it is possible to upgrade the software suite just by making changes in a particular module.
- The structure of *Flintstones* is ready to include new aggregation operators as well as new solving processes in a fast and simple way. As a result, it reduces programming task because it offers the *Libraries module* and *Graphical User Interface module* that includes a full structure, simplifying the integration of new aggregation operators and new extensions based on the 2-tuple linguistic model.

4. *Flintstones* website: case studies and datasets

The development of a software suite of tools is very important but it is not enough if users cannot use it to verify its performance with real datasets in order to make comparisons with either their own proposals or problems. Therefore, we have

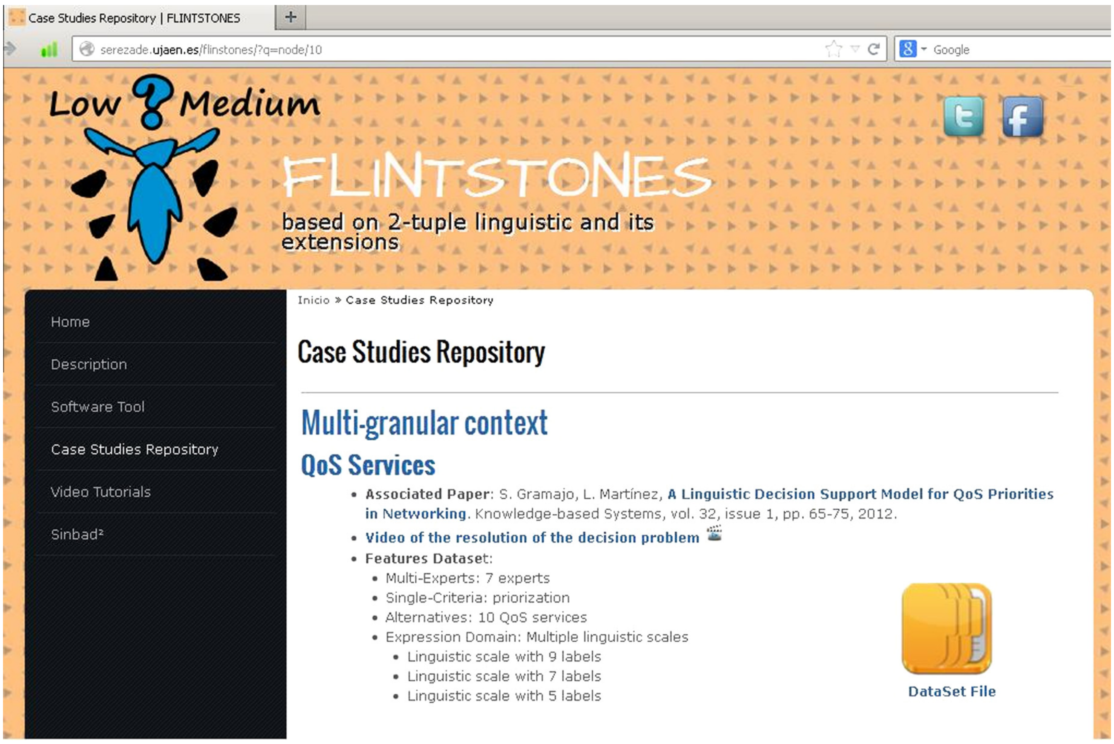


Fig. 14. *Flintstones* website.

Table 1
Repository of case studies and datasets.

Framework	Extension	Application	Year
Multi-granular linguistic	Fusion for multi-granular information	Decision making [13]	2011
	Linguistic hierarchies	Decision making [25]	2001
		Sensory evaluation [38]	2008
	Extended linguistic hierarchies	Decision making [13]	2011
		Quality of services in networking [17]	2012
Heterogeneous	Fusion for heterogeneous information	360-Degree performance appraisal [12]	2013
		Sustainable energy evaluation [15]	2012
		ERP evaluation processes [56]	2009
Unbalanced linguistic	Methodology to deal with unbalanced scales	Sensory evaluation [37]	2009
		Decision making [22]	2008

not only developed *Flintstones* as a decision software suite but also we have deployed the *Flintstones* website[5] that includes a repository of case studies and their corresponding datasets as well as different interesting sections. In this section, we present the *Flintstones* website and a case study is solved in detail by using *Flintstones*.

4.1. Flintstones website

The website has been designed with the aim to publish all *Flintstones* released versions and a repository of case studies with real datasets that can be solved with the software suite of tools. Furthermore, different interesting sections with theoretical foundations as well as video tutorials about the software suite of tools can be found in the website. Each section of the website is described briefly below:

- *Description*. In order to provide the theoretical foundations of *Flintstones*, the main theoretical concepts are briefly introduced in this section of the website. Examples of these concepts are *Computing with words paradigm* or the *2-tuple linguistic representation model*.
- *Software tool*. All *Flintstones* released versions will be available in the website. The current version, v1.0, has all the functionality to create, manage and solve decision making problems with linguistic and complex frameworks. In order to run *Flintstones*, it is only necessary to download and unpack the *zip* file and execute the *flintstones.jar* file. This file can be used on any machine with the JVM, independently of the operating system. The software tool is licensed under the terms of the GNU General Public License.[6]
- *Analysis and design*. Technical aspects related to the analysis and design of the suite are provided in order to offer a complete view of the internal structure of the suite. So, this website section shows the architecture of the suite as well as the package and class diagrams.
- *Case studies repository*. Case studies of decision making problems are available in the website, which are categorized by the type of framework (multi-granular linguistic framework, heterogeneous framework and unbalanced linguistic framework). Each case study is associated with its datasets for *Flintstones* that includes the definition of the framework and the set of assessments provided by experts. Furthermore, each case study is associated with the research paper in which the use of the 2-tuple linguistic representation model or any of its extensions has been successfully applied to it. The repository is alive, new datasets will be included, Table 1 shows a summary of the case studies which are currently incorporated in the repository.
- *Video tutorials*. A set of video tutorials that illustrate the functionality of *Flintstones* are showed in this section. Each basic functionality has been briefly described and has been illustrated in a video tutorial, which can be directly reproduced from the website.

4.2. On the use of Flintstones for a case study

To facilitate the understanding of *Flintstones*, we describe a decision making problem for installing an Enterprise Resource Planning (ERP) that is defined in a multi-granular linguistic framework. Therefore, *Flintstones* will select and run the linguistic decision method proposed in Section 3.1.

4.2.1. Framework

Let us suppose a company which plans to invest a sum of money in the best ERP among four possible alternatives $X = \{x_1, \ldots, x_4\}$ that were selected by departments involved in the process. The final decision lies on a group of four experts $E = \{e_1, \ldots, e_4\}$ that must evaluate the alternatives according to four benefit criteria $C = \{c_1, \ldots, c_4\}$, which are respectively:

[5] http://serezade.ujaen.es/flintstones/.
[6] http://www.gnu.org.

standard degree, *interrelation with other subsystems*, *degree of modularity* and *learning curve for users*. Due to the fact that the decision problem implies imprecision and uncertainty that has not probabilistic nature, the set of criteria will be evaluated in a linguistic domain.

In this case study, the group of experts have different knowledge degree about the set of criteria. Therefore, it seems suitable that experts can express their preferences in two different linguistic term sets based on their own knowledge. A linguistic term set with 5 labels for experts with a less knowledge degree is defined as well as a linguistic term set with 7 labels for experts with a bit more knowledge degree. Both linguistic term sets are distributed symmetrically and uniformly around the central label. The elements involved in framework are included in the datasets of this case study for *Flintstones*, i.e., the group of four experts, the set of four criteria and the set of alternatives as well as the linguistic expression domains and their allocation (see Fig. 15).

4.2.2. Gathering information

The case studies of the repository are associated to datasets which include all assessments (see Fig. 14). In this phase, the assessments provided by experts for each criterion for each alternative are provided, according to expression domains fixed in the framework. The assessment provided by the expert e_1, for the alternative x_2 about the criterion c_2 that are expressed in the linguistic term set with 7 labels is shadowed in Fig. 16.

4.2.3. Selecting a solving process

Here, *Flintstones* carries out Algorithm 1 to select the suitable solving process for the decision making problem, taking into account the expression domains established in the framework. The input information and the selection algorithm is as follows: $edNum = False, edInt = False, edLinUnb = False, tamEdLinLis = 2$ and $edLinList = ((5, True), (7, True))$.

Our case study is defined in a linguistic complex context, particularly in a multi-granular linguistic framework. According to the proposed algorithm, the value 3 is returned that corresponds to the suitable solving process based on *extended linguistic hierarchies*. It is noteworthy that the extension based on *fusion approach for managing multi-granular linguistic information* could also be applied to solve our case study (see Fig. 17).

However, as it was reviewed in Section 2.3, the methodologies based on *fusion of information* can provide linguistic results with loss of information. Therefore, the selected suitable solving process is based on *extended linguistic hierarchies* due to the fact that this extension provides linguistic accuracy results. It is remarkable that the algorithm selects the suitable solving process and allows also use other solving process available to compare results.

Finally, the extension to deal with multi-granular linguistic frameworks called *linguistic hierarchies* cannot be applied because this extension requires a condition of granularity (see Eq. (5)) that the framework does not fulfill.

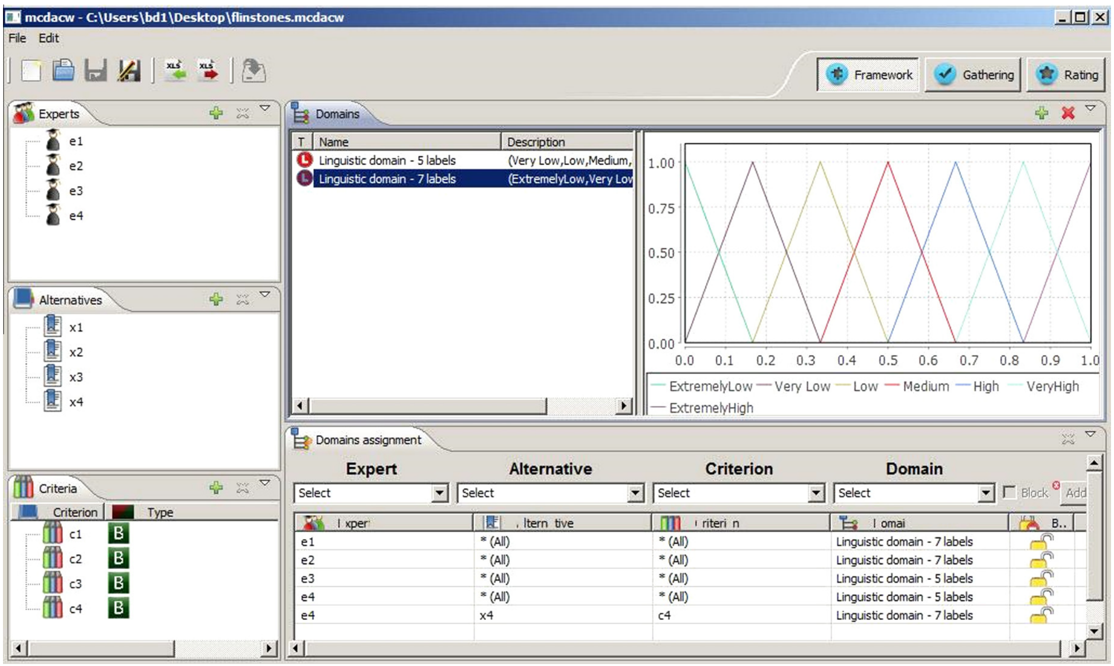


Fig. 15. Framework of ERP case study.

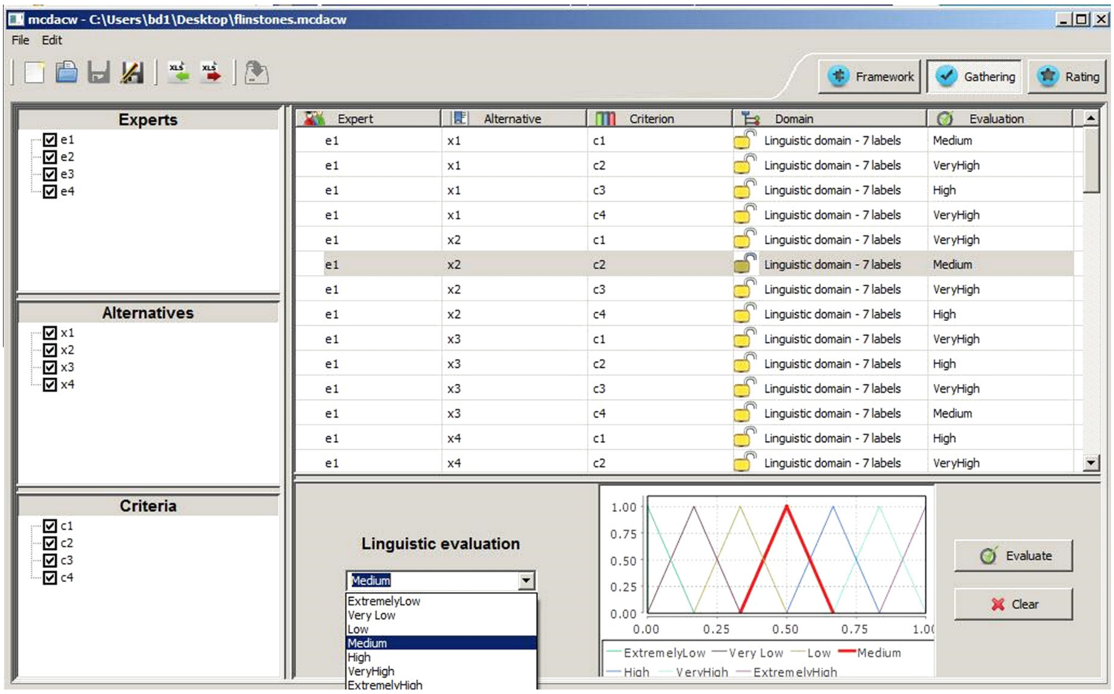


Fig. 16. Gathered information.

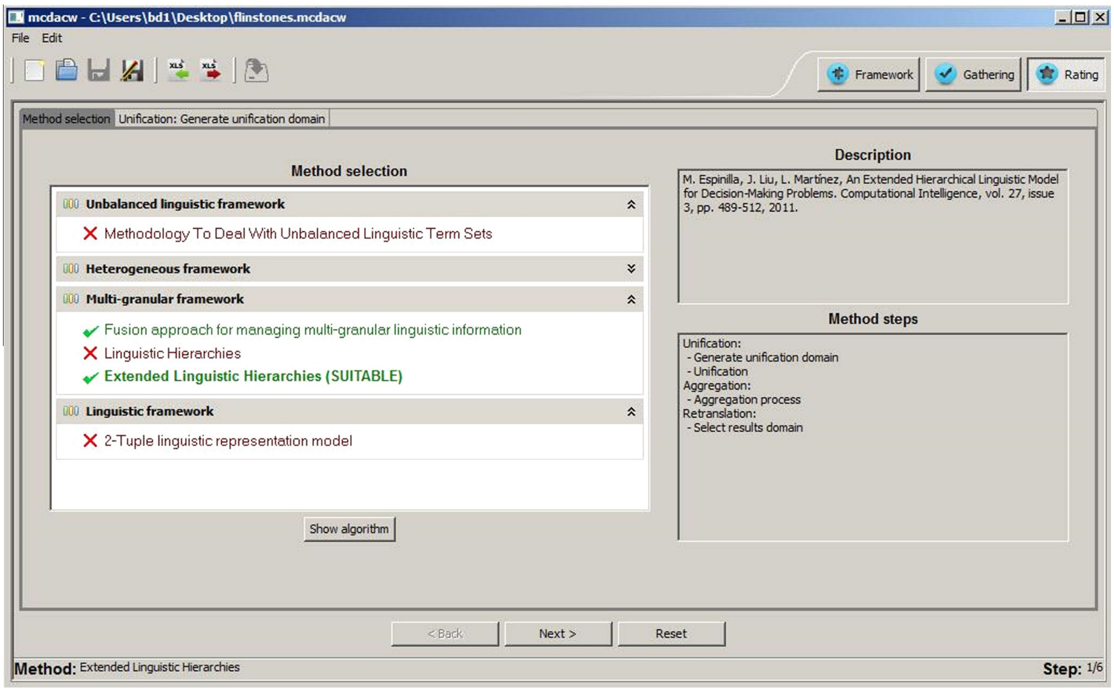


Fig. 17. Selection the suitable solving process.

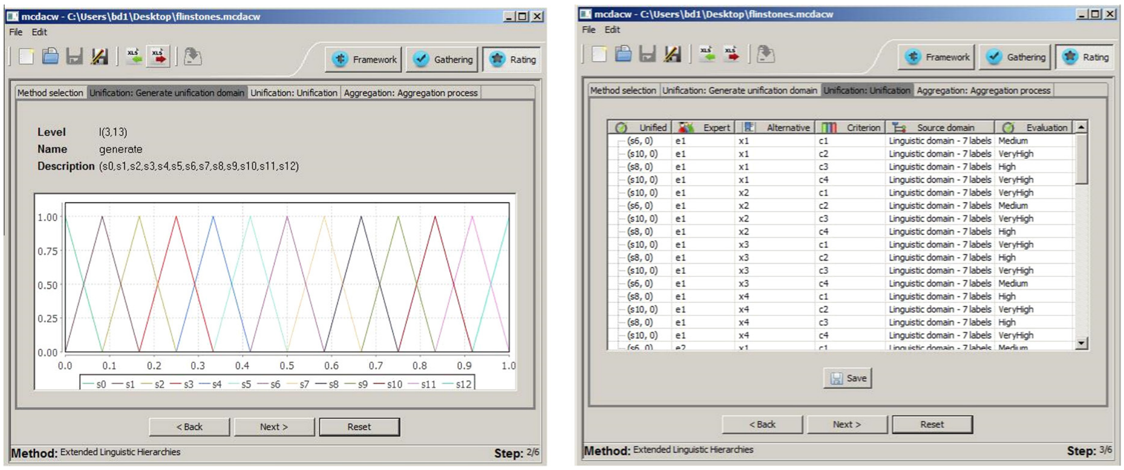


Fig. 18. (a) New generated level and (b) unified information.

4.2.4. Rating alternatives

In this phase, the rating process with the selected solving process based on *extended linguistic hierarchies* is carried out. The processes of the rating alternatives to compute the linguistic assessment for each alternative are described below:

1. *Unification process.* According to the granularities of the initial linguistic term sets defined in the framework, 5 labels and 7 labels, a new linguistic term set with 13 labels is generated (see Fig. 18a), according to Eq. (7): $n(t^*) = lcm(n(1) - 1, n(2) - 1) + 1 = lcm(4, 6) + 1 = 12 + 1 = 13$. The gathered information is then unified into the new generated level (see Fig. 18b).
2. *Aggregation process.* In this process, the information is aggregated selecting the set of aggregation operators for 2-tuple linguistic values (see Fig. 19a).
   In this case study, on the one hand, the *2-tuple linguistic weighted average operator* is selected to aggregate the preferences provided by experts for each criterion with the following weighting vector $w_e = (0.2, 0.3, 0.3, 0.2)$. On the other hand, the *2-tuple linguistic arithmetic mean operator* is used to compute a collective value for each alternative, aggregating its collective assessments. The computed global assessment for each alternative is expressed in the unified level t^* that corresponds to $\langle S^{13} \rangle$.
3. *Retranslation process.* In order to provide 2-tuple linguistic results that are easily understandable for experts, the computed linguistic results are expressed on each initial linguistic term set defined in the decision framework. In this case study, the computed results expressed in S^{13} can be transformed into the two initial scales, S^5 and S^7 (see Fig. 19b). It is noteworthy that these transformations are carried out without loss of information.

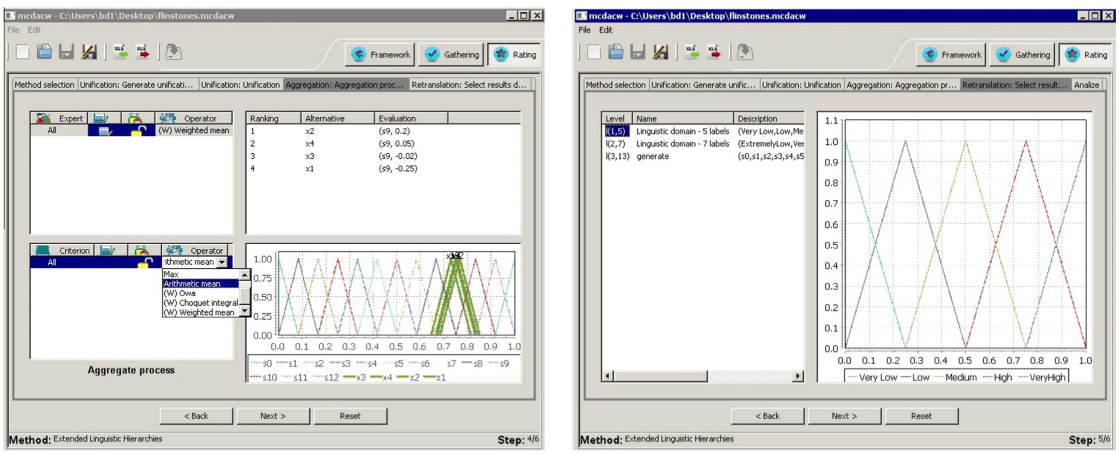


Fig. 19. (a) Aggregation process and (b) retranslation process.

5. Conclusions and future works

There is a perceived need of software tools for solving linguistic decision making problems. The use of the 2-tuple linguistic model and its extensions shows a clear gap to fulfill. Despite the wide range of successful applications in different fields, there is not yet developed any software tool in order to deal with this linguistic model. This paper has presented a software suite so-called *Flintstones* that implements the 2-tuple linguistic model to solve linguistic decision making problems under uncertainty and its extensions to deal with linguistic complex frameworks such as multi-granular linguistic frameworks, heterogeneous frameworks and unbalanced linguistic frameworks. Furthermore, this paper has presented the *Flintstones* website that includes a repository of case studies and datasets for different linguistic decision making problems in order to validate the performance of the software suite with real datasets and to make comparisons with either other proposals or problems. Finally, the description to solve a decision making problem for installing an ERP by *Flintstones* is illustrated.

Our future works are addressed to extend *Flintstones* and its website in different ways. The first way is the development of new extensions based on the 2-tuple linguistic representation model that could be proposed in the future as well as other different methodologies under uncertainty as hesitant linguistic fuzzy sets [53] or type-2 fuzzy sets [43] within the CWW paradigm. The second way is to increase the repository of case studies on the website with the wide range of applications in different fields in which the 2-tuple linguistic representation model and its extensions have provided satisfactory results. Finally, to point out the diffusion of *Flintstones* and its website in order to increase the number of aggregation operators developed by others authors in order to analyze and test the results with them.

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4.2. Fuzzy Linguistic Olive Oil Sensory Evaluation Model based on Unbalanced Linguistic Scales

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Fuzzy Linguistic Olive Oil Sensory Evaluation Model based on Unbalanced Linguistic Scales

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Sensory evaluation is a process that involves knowledge acquired via human senses. Generally, sensory evaluation processes are defined in unbalanced contexts because these are focused on one side of the scale used to collect sensory information. The olive oil sensory evaluation is defined in this kind of context to establish the quality of the olive oil, being the quality a key factor in its marketing. The international olive council established three quality categories and a quantitative method based on statistical analysis to classify a olive oil sample. The perceptions in sensory evaluation processes involves imprecision and uncertainty that has a non-probabilistic nature. Therefore, the use of the fuzzy linguistic approach for modeling and managing can provide successful results in such processes. This paper presents a fuzzy linguistic sensory evaluation model that uses an unbalanced linguistic scale in order to classify olive oil and its validation. The validation process has involved taster panels that choose and validate the unbalanced linguistic scale to measure the intensity of each sensory feature and classify olive oil, using an unbalanced linguistic scale. Finally, a software prototype that was developed to carry out the classification of olive oil samples with the fuzzy linguistic sensory evaluation model is also presented.

Keywords: Sensory evaluation, fuzzy linguistic approach, unbalanced linguistic scale, olive oil, linguistic information, perceptions

1 INTRODUCTION

Sensory evaluation is an evaluation discipline that is carried out to evoke, measure, analyze, and interpret reactions of the sensory features of

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products [1]. This evaluation discipline has an important impact on many industrial areas such as comestibles, cosmetic and textile by its broad use for determining the quality of end products, solving conflicts between customers and producers, developing new products, and exploiting new markets adapted to the consumer's preference [2–4].

The sensory information is perceived by the human senses of *sight, smell, taste, touch and hearing* and always implies uncertainty and imprecision. In whatever process of sensory evaluation is necessary to establish a panel with a number of trained or untrained individuals which provides the sensory information of products in a scale [5].

Often, sensory features need to be assessed with greater distinction on one side of the scale than on the another side [6]. For example, when a company carries out a consumer test to study the satisfaction of its product, the company is focused on obtaining the degree of satisfaction consumer: *completely satisfied*, *very satisfied*, *slightly satisfied*. However, if consumers are dissatisfied, generally, the company is not interested in knowing at what level. This kind of sensory evaluation process is defined in an unbalanced context [7, 8].

This paper focuses on a sensory evaluation process of olive oil defined in an unbalanced context. The quality of a olive oil sample is established by its sensory profile in which each sensory attribute (positive or negative) is measured by a trained tasters panel in order to classify the olive oil among one of three quality categories established: *Virgin extra*, *Virgin* and *Lampant*. The quality category is a key factor in its marketing because *an excellent quality implies a higher price in the market*. So, International Olive Council (IOC)* defined an official procedure to assess the sensory features of olive oil and the methodology based on statistical analysis for its classification [9].

Usually, the statistical procedure is used to model and manage sensory evaluation processes [10]. However, the sensory information implies uncertainty and imprecision that has a non-probabilistic nature [11]. Researches have shown that the fuzzy linguistic approach [12] and the fuzzy sets theory [13] are useful tools to model and manage consistently the uncertainty and vagueness presented in sensory evaluation processes of many products [11,14,15] like mango drink [16], tea [17], coffee [18] sausages [19] or Indian yogurt [20].

Therefore, the procedure fixed by the IOC to assess the sensory evaluation features and obtain the classification of olive oil is becoming to be a hot topic of discussion and debate because manages inconsistently the uncertainty and vagueness presented in the sensory evaluation process of olive oil. This fact involves some issues as a high level of training of tasters to provide accurate

* http://www.internationaloliveoil.org/

sensory information that implies a long-term training and, sometimes, frustrated tasters or wrong classifications.

Previously, we proposed a fuzzy linguistic sensory evaluation model [6] to establish the quality category of a olive oil sample through the definition of an unbalanced linguistic scale [7,8] that uses the fuzzy linguistic approach to model and manage consistently such an uncertainty, providing a solution to the mentioned issues.

The aim of this paper is to evaluate and validate the fuzzy linguistic sensory evaluation model applied to olive oil with two taster panels. To do so, first it was evaluated what was the most adequate unbalanced linguistic scale to assess the intensity with which tasters perceive sensory features (negatives and positives). It was then validated the classification obtained by the fuzzy linguistic sensory evaluation model with the unbalanced linguistic, comparing with the category provided by IOC methodology. To do so, it was carried out a sensory evaluation case study for a set of olive oil samples, belonging to different categories.

Furthermore, due to the fact that the fuzzy linguistic sensory evaluation model uses the computational model for unbalanced linguistic terms set in order to classify the olive oil, in this paper is presented a software prototype to carry out sensory evaluation processes of olive oil with the fuzzy linguistic sensory evaluation model, using an unbalanced linguistic scale [6] in an automatic way.

The paper is organized as follows. Section 2 reviews the methodology for the sensory evaluation of olive oil established by International Olive Council. Section 3 introduces in short the fuzzy unbalanced linguistic sensory evaluation model that will be used in the following sections. Section 4 shows the validation process of the fuzzy linguistic sensory evaluation model applied to olive oil as well as an illustrative sensory evaluation process. Section 5 presents the software prototype that is developed to support the fuzzy linguistic sensory evaluation model. Finally, in Section 6, conclusions are drawn.

2 CLASSICAL SENSORY EVALUATION METHODOLOGY FOR OLIVE OIL

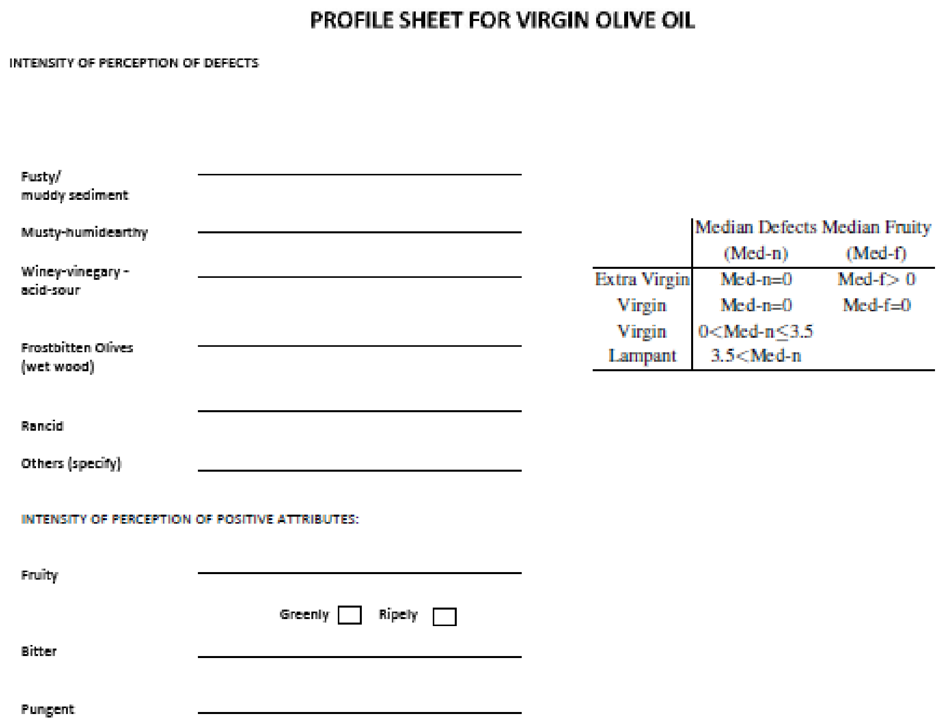
The sensory evaluation methodology of olive oil is regulated by the International Olive Council (IOC) with several guidelines and instructions concerning the tasting of olive oil [9]. In order to understand our proposal, here it is reviewed some concepts about this methodology.

Three different quality levels are distinguished for the olive oil from a sensory point of view by an official tasting panel.

- *Extra virgin* is the category with the highest quality, olive oils under this label are free of defects and fruity flavor (positive attribute) is perceived in these.
- *Virgin* is the category in which olive oils have slightly negative sensory features or have not fruity flavor. These olive oils are suitable for consumption.
- *Lampant*, olive oils under this label present significant sensory negative features, being unpalatable.

Each taster on the panel smells and then tastes the olive oil under consideration. Following, they assess the intensity of each sensory feature (negatives and positives) on a 10-cm scale in the profile sheet provided (see Figure 1).

The panel leader collects the profile sheets and reviews the intensities assigned to the different sensory attributes. The category of olive oil is established, comparing the median value of the defect (median of the defect perceived with the greatest intensity) and the median for the fruity attribute with the reference ranges that are shown in Figure 1. It is noteworthy that these medians are expressed by a real numbers and the value of the robust coefficient of variation must not be greater than 20%.


PROFILE SHEET FOR VIRGIN OLIVE OIL

INTENSITY OF PERCEPTION OF DEFECTS

Fusty/ muddy sediment

Musty-humidearthy

Winey-vinegary - acid-sour

Frostbitten Olives (wet wood)

Rancid

Others (specify)

INTENSITY OF PERCEPTION OF POSITIVE ATTRIBUTES:

Fruity

Greenly Ripely

Bitter

Pungent

	Median Defects (Med-n)	Median Fruity (Med-f)
Extra Virgin	Med-n=0	Med-f> 0
Virgin	Med-n=0	Med-f=0
Virgin	0<Med-n≤3.5	
Lampant	3.5<Med-n	

FIGURE 1
IOC profile sheet and reference ranges

A detailed description about the procedure, the number of tasters as well as the basic vocabulary, test glasses and test booth can be found in [21–23].

3 LINGUISTIC BACKGROUND

Our proposal is based on the use of unbalanced linguistic information for the sensory evaluation that needs processes of computing with words (CW) [8]. Therefore, in this section are reviewed different concepts regarding the managing of linguistic information that are used in the fuzzy linguistic olive oil sensory evaluation model.

3.1 Fuzzy Linguistic Approach

As it was pointed out, sensory information implies uncertainty, vagueness and imprecision and the use of the Fuzzy Linguistic Approach [12] has provided successful results modelling and managing this type of information in some different fields such as sustainable energy [24], personnel selection [25], performance appraisal [26], vendor selection problem [27], estate appraisal [28].

The fuzzy linguistic approach represents such a information as linguistic values by means of linguistic variables and the semantics of the terms are given by fuzzy numbers defined in the [0,1] interval, which are usually described by membership functions [12].

The use of linguistic information implies process of CW, there are different linguistic computational model to operate with linguistic information [29]. Due to the use of an unbalanced terms set in our proposal, we use the 2-tuples linguistic model to accomplish such as computations because it facilitates the managing of this type of information [8].

The linguistic 2-tuple representation model is based on the concept of *symbolic translation* [30] and represents the linguistic information when the linguistic term sets are symmetrical and uniformly distributed through a 2-tuple (s, α), where $s \in S = \{s_0, \ldots, s_g\}$ is a linguistic term and α is a numerical value that represents the symbolic translation [30]. Thereby, being $\beta \in [0, g]$ the value generated by a symbolic aggregation operation, we can assign a 2-tuple (s, α) that expresses the equivalent information of that given by β.

Definition 1. [30]. Let $S = \{s_0, \ldots, s_g\}$ be a set of linguistic terms. The *2-tuple set associated with* S is defined as $\langle S\rangle = S \times [-0.5, 0.5)$ and the function $\Delta_S : [0, g] \longrightarrow \langle S\rangle$ is given by,

$$\Delta_S(\beta) = (s_i, \alpha), \quad \text{with} \quad \begin{cases} i = \text{round}(\beta), \\ \alpha = \beta - i, \end{cases} \tag{1}$$

where *round* assigns to β the integer number $i \in \{0, 1, \ldots, g\}$ closest to β.

Proposition 1. *Let $S = \{s_0, \ldots, s_g\}$ be a linguistic term set and (s_i, α) be a linguistic 2-tuple. There is always a function Δ_S^{-1} such that, from a linguistic 2-tuple, it returns its equivalent numerical value $\beta \in [0, g]$.*

Remark 1. *From definition 1, it is obvious that the conversion of a linguistic term into a linguistic 2-tuple consist of adding a value 0 as symbolic translation: $s_i \in S \Longrightarrow (s_i, 0) \in \langle S \rangle$.*

The linguistic 2-tuple representation model has a linguistic computing model associated that accomplishes CW processes in a precise way. Different aggregation operators have been proposed for linguistic 2-tuple [30–32]. In our proposal, it is used the median aggregation operator for linguistic 2-tuple, since the IOC methodology computes collective sensory intensities based on the calculation of the median [9].

Definition 2. [6]. Let $((s_1, \alpha), \ldots, (s_n, \alpha)) \in \langle S \rangle^n$ be a vector of linguistic 2-tuples. The *2-tuple Median operator* is the function $Med : \langle S \rangle^n \longrightarrow \langle S \rangle$ defined by:

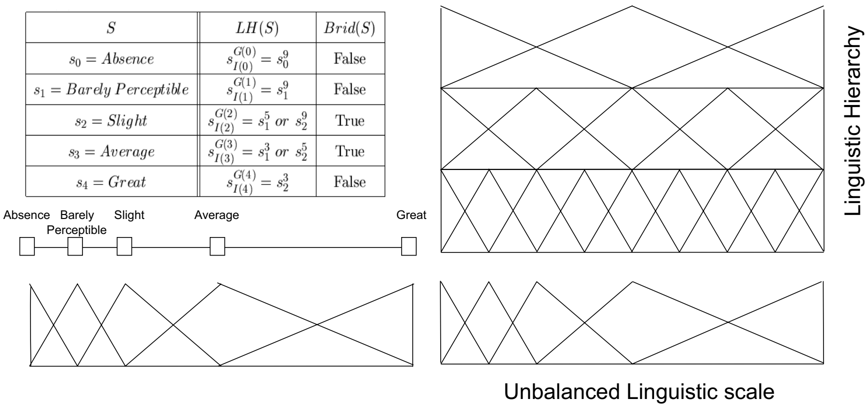
$$
\begin{array}{ll}
\textit{if } n \textit{ is odd} & Med\big((s_1, \alpha), ..., (s_n, \alpha)\big) = (s_i, \alpha) \\
\textit{if } n \textit{ is even} & Med\big((s_1, \alpha), ..., (s_n, \alpha)\big) = \Delta_S\left(\frac{\Delta_S^{-1}(s_{i-1},\alpha)+\Delta_S^{-1}(s_i,\alpha)}{2}\right)
\end{array}
$$

where (s_i, α) is the $round(\frac{n+1}{2})$-th largest element of $\langle S \rangle^n$.

3.2 Unbalanced Linguistic Information

In [7] was developed a methodology to obtain a semantic representation algorithm for unbalanced linguistic term sets and carry out processes of CW in a precise way.

First, it defines an algorithm to build the semantics for an unbalanced linguistic term sets by using a *linguistic hierarchy* [33] that is a set of levels, where each level is a linguistic term set with different granularity from the remaining of levels of the hierarchy that are ordered according to their granularity. Each level is denoted as $\mathbf{l(t, n(t))}$, t indicates the level of the hierarchy and *n(t)* the granularity of the linguistic term set of the level t. Given a LH, $S^{n(t)}$ denotes the linguistic term set of LH corresponding to the level t of LH with a granularity of uncertainty of $n(t)$: $S^{n(t)} = \{s_0^{n(t)}, ..., s_{n(t)-1}^{n(t)}\}$. In [33] was defined a transformation function between labels from different levels to carry out processes of CW without loss of information that has been defined as $TF_{t'}^{t} : l(t, n(t)) \longrightarrow l(t', n(t'))$.



S	$LH(S)$	$Brid(S)$
$s_0 = Absence$	$s^{G(0)}_{I(0)} = s^9_0$	False
$s_1 = Barely\ Perceptible$	$s^{G(1)}_{I(1)} = s^9_1$	False
$s_2 = Slight$	$s^{G(2)}_{I(2)} = s^5_1$ or s^9_2	True
$s_3 = Average$	$s^{G(3)}_{I(3)} = s^3_1$ or s^5_2	True
$s_4 = Great$	$s^{G(4)}_{I(4)} = s^3_2$	False

FIGURE 2
Unbalanced linguistic scale

The algorithm proposed in [7] returns a *Hierarchical semantic representation,* $LH(\mathcal{S})$, for an unbalanced linguistic terms set $\mathcal{S} = \{s_i,\ i = 0, ..., g\}$ and obtains its representation in the LH. Furthermore, the algorithm builds a structure as the table illustrated in Figure 2 with information that supports the computations with the unbalanced terms set.

The semantic obtained $LH(\mathcal{S}) = s^{G(i)}_{I(i)},\ i = 0, ..., g$, it is such that $\forall s_i \in \mathcal{S}\ \exists l(t, n(t)) \in LH$ that contains a label $s^{n(t)}_k \in S^{n(t)}$. Therefore, $I(i) = k$ and $G(i) = n(t)$, being I and G functions that assign to each label $s_i \in \mathcal{S}$ the index of the label that represents it in the linguistic hierarchy and the granularity of label set of linguistic hierarchy in which it is represented, respectively.

Second, the methodology defines a computational model for unbalanced linguistic term sets based on the 2-tuple computational model and the LH to accomplish the processes of CW without loss of information. To do so, two unbalanced linguistic transformation functions converts an unbalanced linguistic term $s_i \in \mathcal{S}$ into a linguistic term in the LH $s^{n(t)}_k \in LH = \bigcup_t l(t, n(t))$ and vice versa.

1. $\mathfrak{LH}$: It is a transformation function that associates with each unbalanced linguistic 2-tuple (s_i, α), $s_i \in \mathcal{S}$, its respective linguistic 2-tuple in LH $(s^{n(t)}_k, \alpha)$, $s^{n(t)}_k \in LH$, it is defined as $\mathfrak{LH} : (\mathcal{S} \times [0.5, -0.5)) \rightarrow (LH \times [0.5, -0.5))$, such that, $\forall (s_i, \alpha_i) \in (\mathcal{S} \times [0.5, -0.5)) \Longrightarrow \mathfrak{LH}(s_i, \alpha_i) = (s^{G(i)}_{I(i)}, \alpha_i)$.
2. $\mathfrak{LH}^{-1}$: Transformation function that associates with each linguistic 2-tuple expressed in LH its respective unbalanced linguistic 2-tuple in $\mathcal{S}$, it is defined as $\mathfrak{LH}^{-1} : (LH \times [0.5, -0.5)) \rightarrow (\mathcal{S} \times [0.5, -0.5))$, being t a level of LH, then it was defined by cases (see [7]).

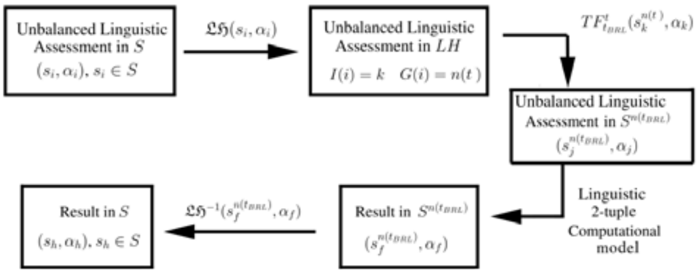


FIGURE 3
Unbalanced Linguistic Computational Model

The semantics of the unbalanced terms set, $LH(\mathcal{S})$, belong to different levels of the LH so, it is not possible to operate directly with these terms. Therefore, it is necessary to transform these terms expressed in different levels into an unique level of the LH, called *Basic Representation Level* and noted as t_{BRL}, by using the transformation function, $TF^t_{t_{BRL}}$. Once the information has been conducted into one expression domain, $\langle S^{n(t_{BRL})}\rangle$, it can be accomplished processes of CW based on the 2-tuple computational model without loss of information. The results obtained by processes of CW can be expressed in linguistic 2-tuple in the unbalanced terms set, $\langle\mathcal{S}\rangle$, by using the transformation functions $TF^{t_{BRL}}_t$ and $\mathfrak{LH}^{-1}$.

A graphical example of the computational model for an unbalanced terms set is illustrated in Figure 3. A detailed description about the algorithm to build the semantics for an unbalanced linguistic term sets and the computational model to accomplish the processes of CW can be found in [7].

4 FUZZY UNBALANCED LINGUISTIC MODEL FOR OLIVE OIL SENSORY

Before carrying out a validation process of the olive oil sensory evaluation process, here it is described the fuzzy unbalanced linguistic sensory evaluation model [6] used later on.

The fuzzy linguistic sensory evaluation model manages the uncertainty involve in the tasters' perceptions, using the fuzzy linguistic approach. It is noteworthy that the linguistic aggregation operator to compute the collective intensity for each sensory attribute as well as the reference ranges of intensities to classify the olive oil samples are equivalent to the quantitative methodology proposed by IOC.

The fuzzy linguistic sensory evaluation model with an unbalanced linguistic terms set consists of phases [6] illustrated in Figure 4 and further detailed bellow:

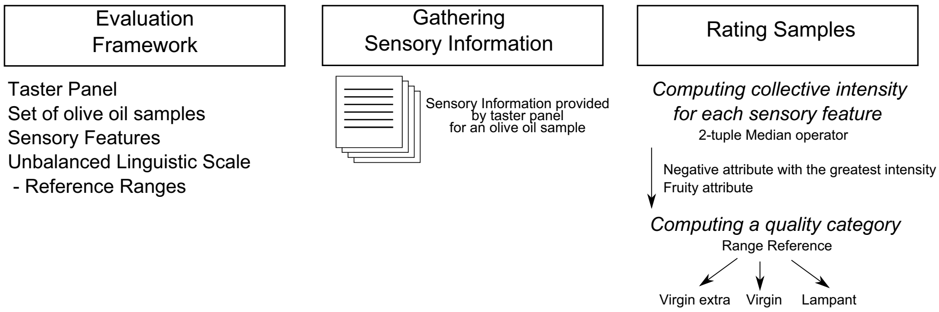


FIGURE 4
Fuzzy Linguistic Sensory Evaluation Model

4.1 Evaluation Framework

It defines the structure of the sensory evaluation process: the set of tasters, $E = \{e_1, ..., e_n\}$, the set of olive oil samples that will be evaluated, $X = \{x_1, ..., x_m\}$, the set of sensory features, $F = \{f_1, ..., f_h\}$, and, finally, the unbalanced linguistic scale in which tasters' perceptions will be expressed, $\mathcal{S} = \{s_i,\ i = 0, ..., g\}$.

In order to define this scale, it is necessary to set its number of terms, its syntaxis and its distribution. The semantic of each term is provided by the *Hierarchical semantic representation, $LH(\mathcal{S})$*, using the algorithm proposed in [7]. Finally, in the evaluation framework, it is necessary to transform the reference ranges to classify the olive oil sample into linguistic 2-tuples in the unbalanced linguistic scale.

4.2 Gathering Sensory Information

Once the framework has been defined to evaluate the set of olive oil samples, the sensory information must be provided by the taster panel. In a profile sheet with the unbalanced linguistic scale, $\mathcal{S}$, fixed in the evaluation framework, each taster e_i, provides the intensity perceived about each sensory characteristic f_k, of the olive oil sample x_j, by means of an utility vector: $U_i = \{u^i_{11}, ..., u^i_{1h}, ..., u^i_{m1}, ..., u^i_{mh}\}$

4.3 Rating Samples

This phase computes a collective intensity for each sensory attribute in order to classify each olive oil sample, according to the perceived intensities. Therefore, it is necessary to accomplish CW processes for unbalanced linguistic terms set [7, 8]. According to the computational scheme reviewed in Figure 3, tasters preferences are transformed into linguistic 2-tuples and then into linguistic 2-tuples in $S^{n(t_{BRL})}$ by means of $\mathfrak{LH}$ and $TF^t_{t_{BRL}}$.

Once the sensory information has been conducted into one expression domain, $\langle S^{n(t_{BRL})}\rangle$, it is applied a two-step aggregation process to compute the quality category for each olive oil sample.

1. *Computing collective intensity for each sensory feature*: first, it is computed a collective linguistic 2-tuple, (u_{jk}, α), for each sensory feature f_k, of the olive oil sample x_j, using the aggregation operator *2-tuple Median operator* (see Definition 2) on the assessments provided by tasters represented in t_{BRL}: $(u_{jk}, \alpha) = Med((u^1_{jk}, \alpha_1), \ldots, (u^n_{jk}, \alpha_n))$.
2. *Computing a quality category for each olive oil sample*: the final aim of the rating process is classified each olive oil sample among one of three quality categories established: *virgin extra*, *virgin* and *lampant*. To do so, the collective intensities are transformed into linguistic 2-tuples in the unbalanced terms set, $\langle \mathcal{S}\rangle$, by using the transformation functions $TF_t^{t_{BRL}}$ and $\mathfrak{LH}^{-1}$, then taking into account the reference ranges [6] as well as the collective intensity of the fruity attribute and the collective intensity of the defect perceived with the greatest intensity, each olive oil sample is classified.

5 PROCESS OF VALIDATION

Here, it is presented the validation of the fuzzy linguistic sensory evaluation model based on fuzzy linguistic approach with an adequate unbalanced linguistic scale to carry out the classification of olive oil. The validation process is conducted in two phases. The aim of the first phase is to analyze and validate a suitable unbalanced linguistic scale that will be used by tasters to assess the sensory features of olive oil. The goal of the second phase is to validate the classification obtained with the fuzzy linguistic sensory evaluation model, carrying out a sensory evaluation case study. Finally, in this section, it is presented an illustrative example of the sensory evaluation case study in which an olive oil sample is classified, using the fuzzy unbalanced linguistic sensory evaluation model.

In the validation process are involved two accredited taster panels. The first panel was accredited in 2008 and the second panel in 2010. So, in this process are implied two panel leaders and 18 tasters (7 women and 11 men between 22 and 65 years old) that was selected, trained and monitored by the panel leader in accordance with their skills in distinguishing among similar samples.

5.1 An Adequate Unbalanced Linguistic Scale

The aim of this phase is to obtain a suitable unbalanced linguistic scale to measure tasters' perceptions, using linguistic terms.

The scale initially proposed to use in the fuzzy linguistic evaluation sensory model for taster panels was illustrated in Figure 2. During several meetings, taster panels were analyzing the scale and provided the following considerations:

- An unbalanced linguistic scale is a good option in the sensory evaluation process of olive oil because the left side of the scale marks the difference between an olive oil classified as *virgin extra*, *virgin* and *lampant* and this side measures the absence of a sensory attribute as well as significant and soft intensities. The another side, right side, measures strong intensities. Therefore, it is necessary a higher distinction on the left side of the scale than on the right side.
- The number of labels of the initial unbalanced linguistic scale, 5 labels, is insufficient to measure tasters' perceptions because olive oil samples can be classified incorrectly when these samples are doubtful between two categories.

In order to avoid incorrect classifications due to the scale used to measure tasters' perceptions, the taster panels proposed to include two new labels, one on each side of the scale. Therefore, the new unbalanced linguistic scale was proposed with 7 labels to measure adequate tasters' perceptions in order to correctly classify doubtful olive oil samples between two categories. The syntaxis provided by taster panel for this suitable unbalanced linguistic scale was the following: $\mathcal{S} = \{$*Absence, Almost Negligible, Very Soft, Soft, Average, Large, Extreme*$\}$.

Once tasters proposed the suitable unbalanced linguistic scale, it was necessary to compute the semantic of each linguistic term. Therefore, it was used the algorithm to build the semantics for an unbalanced linguistic terms set, using a LH [7]. The *Hierarchical semantic representation*, $LH(\mathcal{S})$ for the unbalanced linguistic terms set $\mathcal{S}$, is illustrated in Figure 5.

5.2 Validation of the Classification provided by the Fuzzy Linguistic Sensory Model

Here, the aim is to validate the classification provided by the fuzzy linguistic sensory evaluation model for olive oil samples, using the unbalanced linguistic scale validated previously by taster panels.

To do so, a sensory evaluation case study for a set of olive oil samples, belonging to different categories, was carried out. In this study, it was necessary to define a set of sensory profiles, select a set of samples according these profiles and verify them. Afterward, the set of olive oil samples were classified by the fuzzy linguistic sensory evaluation model in order to compare classifications and validate the fuzzy linguistic sensory evaluation model for

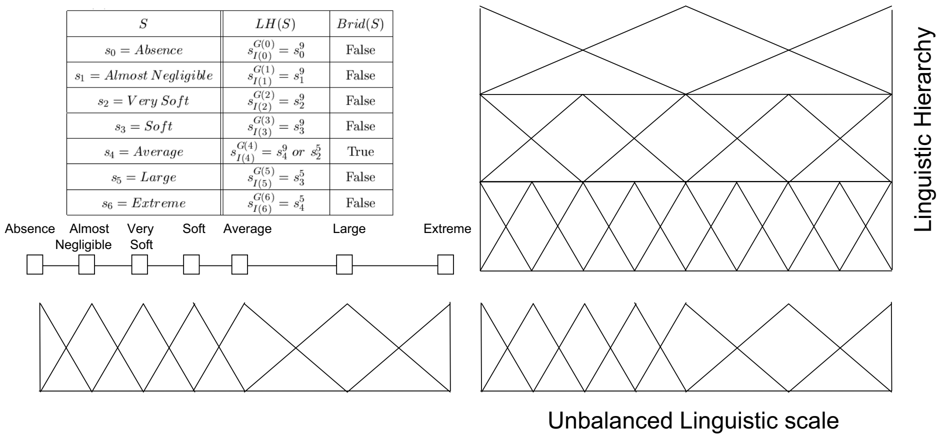


FIGURE 5
Hierarchical semantic representation of the Unbalanced Linguistic Scale

olive oil samples. Figure 6 illustrates the validation process scheme and its steps are described in detail below.

The set of olive oil samples was established with different sensory profiles that includes samples clearly classified as one category and samples doubtful between two categories (see Figure 6). So, seven sensory profiles were defined and three olive oil samples were searched for each sensory profile (21 olive oil samples) by two panel leaders during the 2012 olive campaign in the province of Jaén (Spain).

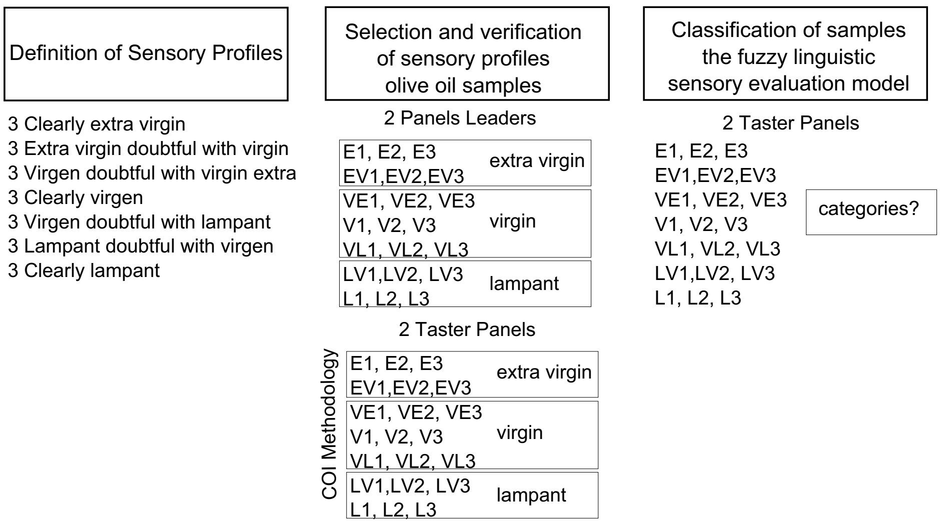
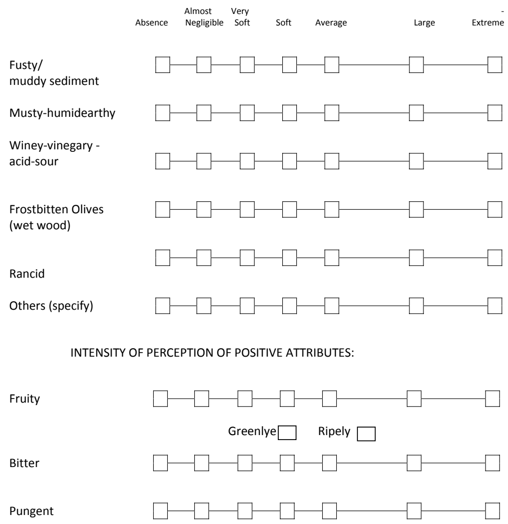
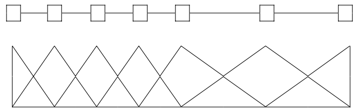


FIGURE 6
Scheme of the validation process of fuzzy linguistic sensory evaluation model





	Median Defects (Med-n)	Median Fruity (Med-f)
Extra Virgin	Med-n = (Absence,0)	Med-f > (Absence,0)
Virgin	Med-n = (Absence,0)	Med-f = (Absence,0)
Virgin	(Absence,0) < Med-n Med-n <= (Soft,0)	
Lampant	(Soft,0) < Med-n	

FIGURE 7
Proposed profile sheet and reference ranges

A key factor was to verify the correct selection of olive oil samples by two panel leaders, according to the defined sensory profiles. To do so, each sample was classified using the IOC method, reviewed in the section 2, by two accredited taster panels in order to detect misclassifications and to avoid its negative impact in the validation process of the fuzzy linguistic sensory evaluation model, using the unbalanced linguistic scale.

Once sensory profile of each olive oil sample of the set was verified, the set of olive oil samples was classified with the fuzzy linguistic sensory evaluation model, reviewed in the section 4 with the unbalanced linguistic scale validated previously. To do so, the profile sheet to collect the sensory information by taster panels was defined. Furthermore, it was necessary to transform the reference ranges proposed by IOC methodology into linguistic 2-tuple on the unbalanced linguistic scale to classify olive oil samples. The profile sheet and reference ranges based on the new unbalanced linguistic scale are shown in Figure 7.

During one month, two taster panels were trained in the fuzzy linguistic sensory evaluation model, using the profile sheet with the suitable unbalanced linguistic scale. Once the panel was trained, the set of 21 olive oil samples was evaluated by taster panel with the fuzzy linguistic sensory evaluation model, using the unbalanced linguistic scale.

For each olive oil sample of the set, the category provided by the fuzzy linguistic sensory evaluation model with the validated unbalanced linguistic scale, matched with the category computed by IOC methodology and the expert opinion provided by two panel leaders.

Therefore, the results of the sensory evaluation case study were very satisfactory. The suitable unbalanced linguistic scale offers more flexibility to express perceptions and models and manages consistently the uncertainty and vagueness presented in sensory evaluation processes. Furthermore, the fuzzy unbalanced linguistic sensory evaluation model does not require a high level of precision, providing the same classification that the IOC methodology and the expert opinion of two panel leaders. Therefore, the fuzzy linguistic sensory evaluation model can imply a mid-term training of the taster panel.

5.3 An illustrative sensory evaluation process

In order to clarify the fuzzy linguistic sensory evaluation model for olive oil with the unbalanced linguistic scale, an example of the sensory evaluation case study is shown.

Evaluation Framework

The evaluation framework included a panel of eight tasters, $E = \{e_1, \ldots, e_8\}$, who assessed an olive oil sample, $X = \{x_1\}$="*VE1*" that corresponded with a sample classified as *virgin* for the IOC methodology but doubtful with *extra virgin*. This olive oil sample was characterized by a set of sensory attributes $f = \{f_1, \ldots, f_9\}$, using the suitable unbalanced linguistic scale $\mathcal{S}$, illustrated in Figure 7.

Gathering Sensory Information

The taster panel provided their intensities for each sensory feature (negative and positive) of the olive oil sample "*VE1*" by filling the profile sheet (see Figure 7). The provided intensities by olive oil sample "*VE1*" are shown in Table 1.

	Negative features						**Positive features**		
Features Taster	f_1 **Fusty**	f_2 **Musty**	f_3 **Winey**	f_4 **Frostbitten**	f_5 **Rancid**	f_6 **Others**	f_7 **Fruity**	f_8 **Bitter**	f_9 **Pungent**
e_1	Absen.	Absen.	Absen.	Absen.	Absen.	Absen.	Alm. Neg.	Alm. Neg.	Alm. Neg.
e_2	Absen.	Absen.	Absen.	Absen.	Absen.	Absen.	V. Soft	V. Soft	V. Soft
e_3	Absen.	Absen.	Absen.	Absen.	Absen.	Absen.	V. Soft	Alm. Neg.	Alm. Neg.
e_4	Absen.	Absen.	Absen.	Absen.	Absen.	Absen.	V. Soft	V. Soft	V. Soft
e_5	Alm. Neg.	Absen.	Absen.	Absen.	Absen.	Absen.	V. Soft	V. Soft	V. Soft
e_6	Alm. Neg.	Absen.	Absen.	Absen.	Absen.	Absen.	V. Soft	V. Soft	V. Soft
e_7	Alm. Neg.	Absen.	Absen.	Absen.	Absen.	Absen.	V. Soft	V. Soft	V. Soft
e_8	V. Soft	Absen.	Absen.	Absen.	Absen.	Absen.	V. Soft	Alm. Neg.	Alm. Neg.

TABLE 1
Sensory information about the olive oil sample: "*VE1*"

	Negative features						Positive features		
Features	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9
Taster	**Fusty**	**Musty**	**Winey**	**Frostbitten**	**Rancid**	**Others**	**Fruity**	**Bitter**	**Pungent**
e_1	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_1^9, 0)$	$(s_1^9, 0)$	$(s_1^9, 0)$
e_2	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_2^9, 0)$	$(s_2^9, 0)$	$(s_2^9, 0)$
e_3	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_2^9, 0)$	$(s_1^9, 0)$	$(s_1^9, 0)$
e_4	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_2^9, 0)$	$(s_2^9, 0)$	$(s_2^9, 0)$
e_5	$(s_1^9, 0)$	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_2^9, 0)$	$(s_2^9, 0)$	$(s_2^9, 0)$
e_6	$(s_1^9, 0)$	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_2^9, 0)$	$(s_2^9, 0)$	$(s_2^9, 0)$
e_7	$(s_1^9, 0)$	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_2^9, 0)$	$(s_2^9, 0)$	$(s_2^9, 0)$
e_8	$(s_2^9, 0)$	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_2^9, 0)$	$(s_1^9, 0)$	$(s_1^9, 0)$
Median	$(s_1, -0.5)$	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_0^9, 0)$	$(s_2^9, 0)$	$(s_2^9, 0)$	$(s_2^9, 0)$

TABLE 2
Sensory information about the olive oil sample: "*VE1*"

Rating Samples

The sensory information provided by the tasters was transformed into linguistic 2-tuples, using the *Remark 1*. Then, these linguistic 2-tuples were transformed into the corresponding level of the LH by means of $\mathfrak{LH}$.

$$\mathfrak{LH}(Absence, 0) = \mathfrak{LH}(s_0, 0) = (s_0^{n(3)}, 0) = (s_0^9, 0)$$
$$\mathfrak{LH}(Almost\ Negligible, 0) = \mathfrak{LH}(s_1, 0) = (s_1^{n(3)}, 0) = (s_1^9, 0)$$
$$\mathfrak{LH}(VerySoft, 0) = \mathfrak{LH}(s_2, 0) = (s_2^{n(3)}, 0) = (s_2^9, 0)$$

Computing collective intensity for each sensory feature. The collective intensity for each sensory feature was computed, using the *2-tuple Median operator* (see Definition 2). The collective intensity for each sensory feature are shown in Table 2. An example of the computation of the collective intensity for the *Fusty* attribute is presented bellow.

$$Med\big((s_0^9, 0), (s_0^9, 0), (s_0^9, 0), (s_0^9, 0), (s_1^9, 0), (s_1^9, 0), (s_1^9, 0), (s_2^9, 0)\big) =$$
$$= \Delta_S\big(\tfrac{\Delta_S^{-1}(s_0^9,0)+\Delta_S^{-1}(s_1^9,0)}{2}\big) = \Delta_S(\tfrac{0+1}{2}) = \Delta_S(0.5) = (s_1^9, -0.5)$$
$$\mathfrak{LH}^{-1}(s_1^9, -0.5) = (s_1, -0.5) = (Almost\ Negligible, -0.5)$$

Computing a quality category for the olive oil sample. Finally, taking into account the reference ranges shown in Figure 7 as well as the median for the fruity attribute, *(Very Soft,0)*, and the median of the defect perceived with the greatest intensity which corresponds to *Fusty* sensory feature, *(Almost Negligible,-0.5)*, the olive oil sample "*VE1*" was classified as *Virgin*.

6 SOFTWARE PROTOTYPE

To accomplish the validation process, a software prototype was developed to carry out the sensory evaluation case study, using the fuzzy linguistic sensory evaluation model with the suitable unbalanced linguistic scale.

The functionality of the software prototype is based on the phases of the fuzzy linguistic sensory evaluation model, reviewed in the section 3. The phases of the fuzzy linguistic sensory evaluation model are carried out by an user with *panel leader* role. The functionality of the software prototype is presented bellow, evaluating the olive oil sample: "*EV1*".

6.1 Evaluation Framework

In this phase, firstly, the unbalanced linguistic scale utilized to assess the intensity of sensory attributes is defined by introducing the number of the linguistic terms, 7, their distribution and their syntaxis. The software prototype executes the algorithm to build the semantic for an unbalanced linguistic terms set, using a LH (see Figure 8).

The elements involved in the sensory evaluation case study are included in the software prototype, i.e, the panel of eight tasters, positives and negatives attributes, and the olive oil sample. Finally, the unbalanced linguistic scale is assigned as expression domain for the taster panel (see Figure 9).

6.2 Gathering Sensory Information

In this phase, the sensory information about sensory attributes expressed in the unbalanced linguistic scale is included in the software prototype (see Figure 10).

6.3 Rating Sample

The classification of the olive oil sample is carried out according to the median of the negative attributes perceived with the greatest intensity and the

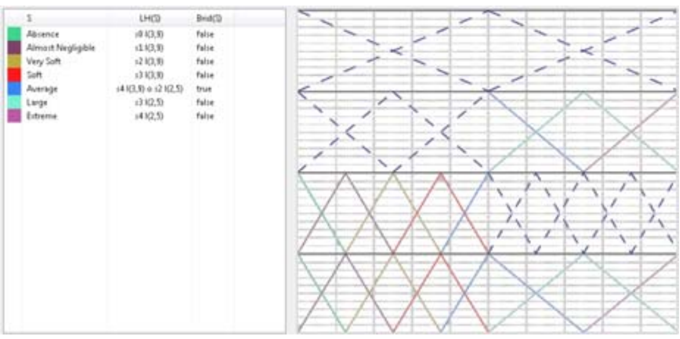


FIGURE 8
Hierarchical semantic representation of the Unbalanced Linguistic Scale

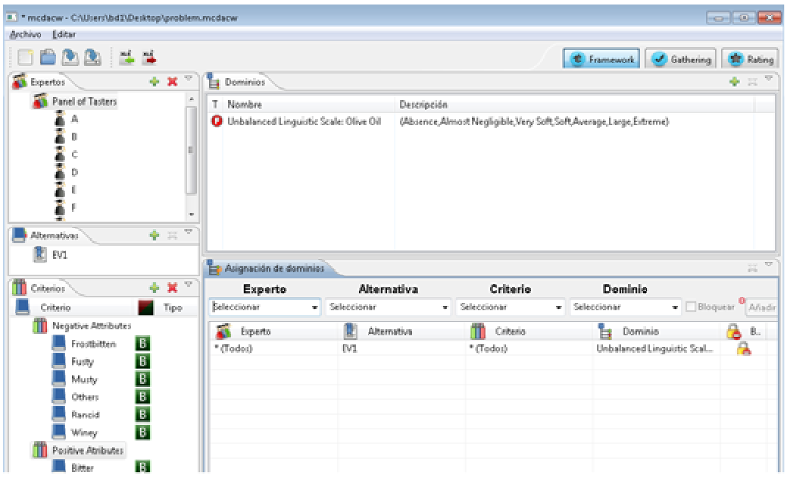


FIGURE 9
Evaluation framework

median of the fruity attribute. The software prototype accomplishes directly processes of CW without loss of information, dealing with the LH [8, 33] and linguistic 2-tuples [30]. The software prototype computes the median for the negative attribute and median for the fruity attribute (see Figure 11).

The main advantage of this software prototype is the automation of complex processes as: building of the semantic for an unbalanced linguistic terms set using a LH and processes of computing with words.

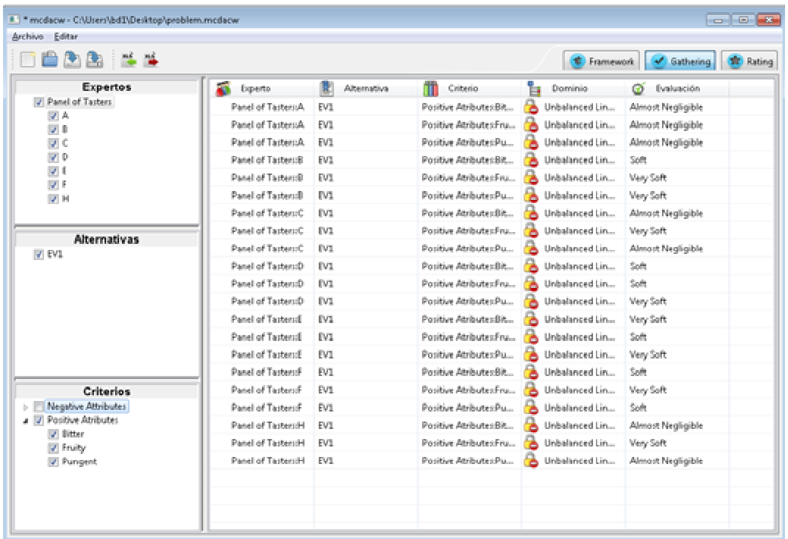


FIGURE 10
Gathering Information

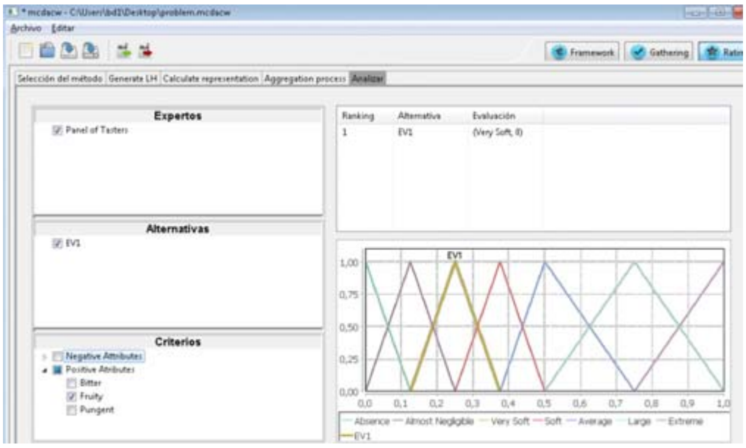


FIGURE 11
Rating Sample

7 CONCLUSIONS

Sensory evaluation processes imply uncertainty and vagueness and, generally, are defined in unbalanced contexts. In our previous research, we proposed a fuzzy linguistic sensory evaluation model [6] based on the fuzzy linguistic approach in order to provide a suitable model to deal with such uncertainty in unbalanced sensory evaluation process. In this paper, we have focused on the sensory evaluation process of olive oil, which lies in unbalanced context, in order to obtain the quality category, using the fuzzy linguistic sensory evaluation model. In this paper, we have validated with two taster panels an adequate unbalanced linguistic scale to assess sensory features of the olive oil as well as the classification obtained with the fuzzy linguistic sensory evaluation model, carrying out a sensory evaluation case study. In this paper, we have demonstrated that the fuzzy linguistic sensory evaluation model applied to olive oil provides more flexibility to express the perceptions, offering the same classification that the official method, using the validated unbalanced linguistic scale. Finally, we have presented the software prototype that we have developed to conduct the sensory evaluation case study, following the fuzzy linguistic sensory evaluation model. This software prototype offers an useful tool to carry out such process in a an automatic, easy and fast way.

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4.3. Selecting firms for University Technoparks: A Hesitant Linguistic Fuzzy TOPSIS model

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Abstract: A technopark is an innovation center created to enhance the collaboration between the university and firms. Due to its benefits many firms would like to be in it. Therefore, it is usually necessary a complex selection process that implies multiple conflicting criteria assessed under uncertain circumstances because of the imprecision, hesitancy or vagueness shown by the experts involved. This paper proposes a fuzzy TOPSIS Multi-Criteria Decision Making (MCDM) method to cope with previous uncertainties by using fuzzy modelling and hesitant fuzzy linguistic term sets that will facilitate the experts elicitation of information in order to obtain accurate, reliable and robust results in the selection process. Eventually this model will be implemented within FLINTSTONES software to support the selection process in a case study.

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1. Introduction

A technopark (technology park) is an innovation center established at a university campus to enhance the collaboration between the university, industry and government. It aims at creating a high technologic and economic development and providing knowledge to the society [29]. A technopark can be also called *university research park, science park, technocity or science and technology park.* A technopark is a different organization from the common high-technology business districts, because it is more organized, planned and a university takes part in it.

The first university technopark in the world started in the early 1950s near Stanford University. Other well-known technoparks in USA are the

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University Research Park at University of Wisconsin, Madison; the Purdue Research Park in West Lafayette, Indiana; and so on.

A technopark provides important advantages such as easier access to financial resources, strategic place with easy access to highways or airports, infrastructure support, fast growth for firms, etc. Thus many firms want to take place in technoparks, but because of either area constraints or specific innovative features required to firms, a selection process is necessary to choose the best firms for the technopark. A multi-criteria decision analysis (MCDA) process can support such a selection in which multiple conflicting criteria that embrace a wide range of disciplines and commonly defined under uncertainty are evaluated.

The interdisciplinary of this complex process not only implies the necessity of a heterogenous context of definition [11, 16, 24] in which different types of information can model the knowledge and the related uncertainties, but also makes that involved experts provide vague and imprecise information and hesitate about their assessments. The use of the fuzzy linguistic approach has provided good results [9, 12, 15, 21, 25] for dealing with uncertainty related to the imprecision and vagueness of meaning in assessments of qualitative criteria. However to cope with hesitancy, it is necessary a more flexible approach, different models have been proposed [1, 7, 20, 32, 35] but their expressions are far from common human beings expressions in decision situations. Recently, Rodríguez et al. introduced the concept of Hesitant Fuzzy Linguistic Term Set (HFLTS) [30] that facilitates, when experts hesitate, the elicitation of *comparative linguistic expressions* close to the expressions used by human beings in decision making and provides a fuzzy modelling for computing with them.

Initially, different Multicriteria Decicion Making (MCDM) models presented in the literature such as AHP [13], TOPSIS [3, 15, 36], VIKOR [23]; and outranking methods, ELECTRE, PROMETHEE [28], etc., could be used to solve the technopark selection process, but as far as we know none of them can directly deal with the type of information and uncertainties pointed out previously. Therefore, this paper proposes a technopark selection process based on a fuzzy TOPSIS MCDM method that will be able to deal with problems defined in a heterogeneous context conformed by numerical values, linguistic terms and comparative linguistic expressions based on

HFLTS. Additionally, this process will be integrated in FLINTSTONES [1] (Fuzzy LINguisTic DeciSion TOols eNhacemEnt Suite) [8] in order to support the whole selection process from the elicitation of information to the solution process including sensitive analysis.

The rest of this paper is organized as follows: Section 2 provides a short revision of technoparks and describes its objectives. Section 3 revises the modeling of hesitant information by means of the use of context-free grammars and HFLTS. Section 4 proposes a selection process based on a fuzzy TOPSIS MCDM method able to deal with comparative linguistic expressions, linguistic terms and numerical values to select the most suitable firms to take part in a technopark. Section 5 shows the implementation of the selection process into FLINTSTONES decision suite to support the resolution of a case study and a sensitive analysis of the solutions. Finally, some conclusions and suggestions for further research are pointed out in Section 6.

2. Technopark Problem

This section makes a brief review about technopark problem indicating some studies and results. It describes the goals of a technopark and explains how important the criteria selection is.

2.1. Related works

The aim of technoparks, and similarly named institutions, is to enhance knowledge agglomeration, networking among individual firms, and resource sharing [26, 29]. One of the best practices of technoparks is the Silicon Valley in USA which motivates other countries to build similar institutions [29]. The basic purpose behind establishing technoparks is the expectation that physical closeness of firms create additional value for technology-based firms located on the park. As a result of physical closeness, daily interaction between firms, universities and innovation services, an added value for firms is expected. The outcome of this close interaction can be in form of faster establishment, dealing with teething problems, infrastructural support, and easier access to financial sources, which lead to faster growth for firms.

The theoretical models and cause/effect relationships about technoparks have been in the focus of various studies in the literature. Lofsten and Lindelof [19] make an empirical research on Swedish technoparks and compare

[1] http://serezade.ujaen.es/flinstones/

163 new technology firms in 9 target science parks with 100 firms from off-park locations. On-park firms are detected to show higher sales-growth and employment growth, but the relationship between location and profitability cannot be proved in this study. In another study, Westhead and Storey [37] show that being in relationship with a higher education institution has a positive effect on survival of the on-park firms, but the study also show that the most of the relationships are informal. Phillimore [27] focuses on a technology park in Australia and shows that significant interactions take place within on-park firms and between firms and the university for knowledge transfer. On the other hand, Radosevic and Myrzakhmet [29] investigate the technoparks in Kazakhstan and find that technopark firms are no more innovative than other firms. The authors also propose that the key motivations of firms to take place in a technopark are lower rents and the possibility of accessing finance. Yang et al. [38] conduct a research using panel data for firms located within and outside the Hsinchu Science Industrial Park in Taiwan. The results show that the elasticity of R&D with respect to outputs of on-park firms is significantly higher than other firms and these firms invest more efficiently.

The expectations about technoparks positive effects on economic growth led to the establishment of such parks in developed countries starting from 1960s. Later, the trend also affected emerging economies and technoparks started to spread in early 2000s [29]. The first European country that tried to build technoparks was United Kingdom [22]. In early 1980s Italy and Germany established their first technoparks. The first Spanish initiative in this area was created in 1985. It has been reported [22] that in Spain by the year 2010 there were 80 technoparks with 5539 firms and more than 145000 employees. The turnover of these firms is reported to be 21475 million. The history of technoparks in Turkey is also in parallel with the world development. The first technoparks in Turkey were established in 2001 with the aim of integrating firms, universities and researchers to develop new products and production techniques. In July 2013, there were 52 technoparks geographically distributed in 40 different cities. There are 2247 firms operating in these active technoparks with a total of 19786 employers. By the end of June 2013 the total export amount had reached 897 million USD dollars [6].

Since on-park firms can benefit from various incentives, there is an immense interest on being tenant in a technopark. This leads to a new problem from the university or technopark's point of view; such as the evaluation and selection of suitable firms as tenants. This problem is critical since selecting right firms for technoparks may lead to economic growth, while a wrong choice may lead to waste of resources. The problem is a multicriteria decision making problem that contains various perspectives which will be discussed in the following section.

2.2. Criteria selection for technopark problem

The objectives of technoparks can be classified according to [2, 34, 37] as:

- Increasing interaction between industrial and academic research structures.
- Generating academic spin-offs.
- Replacing declining product technologies with new ones, enhancing founding process of startups.
- Enhancing technology transfer programs in the neighborhood.
- Enabling training programs to develop and manage emerging technologies.
- Providing management services to the firms located within the technopark.

These objectives are the main concern of technoparks and can be reached by means of the firms in the technoparks. Therefore, it is very important to select the most suitable firms which will take part in the technopark by using multiple criteria. Since, the right criteria selection should enable achieving technoparks objectives and leading to a superior technopark performance. The criteria selection used to evaluate the firms will depend on the technopark. In this paper, we utilize the criteria used by the technopark [2] of Istanbul Technical University (ITU), for the firms evaluation. In such a case, the set of criteria is organized in a hierarchical structure of main criteria and sub-criteria (see Fig. 1). *Appendix* **A** describes in further detail the criteria considered by the technopark of ITU.

[2] http://www.ariteknokent.com.tr

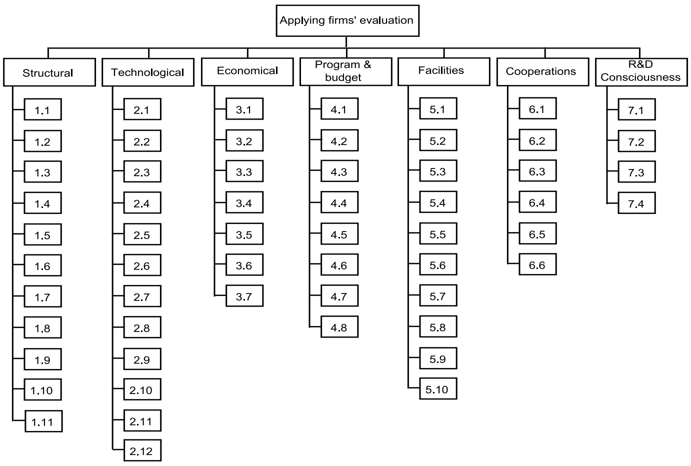

Figure 1: Hierarchical structure of the criteria considered by the ITU technopark

3. Managing Hesitant Situations in Decision Making

The complexity of assessing multiple criteria in a MCDM problem due to the lack of knowledge, time pressure, or lack of information, implies that experts might hesitate to express their assessments and they cannot provide only one value, because it is not enough to accurately reflect their knowledge. Recently, Torra has introduced the concept of HFS [33] to manage this type of uncertainty provoked by hesitancy that allows defining the membership of an element to a fuzzy set by several values.

Definition 1. *[33]:Let X be a reference set, a HFS on X is a function h that returns a subset of values in [0,1]:*

$$h : X \to \wp([0, 1]) \tag{1}$$

Similarly, in qualitative contexts it may happen that experts hesitate among several linguistic terms to assess a linguistic variable. To deal with these situations, Rodríguez et al. presented the concept of Hesitant Fuzzy Linguistic Term Sets (HFLTS).

Definition 2. *[30]: Let $S = \{s_0, \ldots, s_g\}$ be a linguistic term set, a HFLTS H_S, is defined as an ordered finite subset of consecutive linguistic terms of S:*

$$H_S = \{s_i, s_{i+1}, \ldots s_j\} \text{ such that } s_k{\in}S, k{\in}\{i, \ldots, j\} \tag{2}$$

Nevertheless, in real world decision making problems, experts do not provide their opinions or assessments by means of multiple linguistic terms, but by linguistic expressions. Therefore, Rodríguez et al. [30] improved the elicitation of hesitant linguistic information, generating *comparative linguistic expressions* close to human beings' expressions when they hesitate in decision situations by using context-free grammars. The context-free grammar can be adapted to generate different linguistic expressions according to the decision problem. A general context-free grammar G_H, was defined in [30] and extended in [31] to build *comparative linguistic expressions* similar to the expressions used by experts in real-world decision making problems.

Definition 3. *[31]: Let G_H be a context-free grammar and $S = \{s_0, \ldots, s_g\}$ be a linguistic term set. The elements of $G_H = (V_N, V_T, I, P)$ are defined as follows:*

$V_N = \{\langle primary\ term\rangle, \langle composite\ term\rangle, \langle unary\ relation\rangle, \langle binary\ relation\rangle, \langle conjunction\rangle\}$

$V_T = \{lower\ than, greater\ than, at\ least, at\ most, between, and, s_0, s_1, \ldots, s_g\}$

$I \in V_N$

$P = \{I ::= \langle primary\ term\rangle|\langle composite\ term\rangle$

$\langle composite\ term\rangle ::= \langle unary\ relation\rangle\langle primary\ term\rangle|\ \langle binary\ relation\rangle \langle primary\ term\rangle\langle conjunction\rangle\langle primary\ term\rangle$

$\langle primary\ term\rangle ::= s_0|s_1|\ldots|s_g$

$\langle unary\ relation\rangle ::= lower\ than|greater\ than|at\ least|at\ most$

$\langle binary\ relation\rangle ::= between$

$\langle conjunction\rangle ::= and\}$

Let's suppose the linguistic term set S={Very Low, Low, Medium, High, Very High}, two possible comparative linguistic expressions ll_1 and ll_2 generated by the context-free grammar G_H, could be the following ones:

$$ll_1 = at\ \ least\ \ Low, \quad ll_2 = between\ \ Medium\ \ and\ \ High$$

Such expressions are good enough for the goals chased in this paper for modelling experts' hesitancy. However it is still necessary to clarify how to compute with these expressions. Thus in [30] a transformation function E_{G_H}, was defined to transform the linguistic expressions into HFLTS and then operate on them.

Definition 4. *[30]: Let E_{G_H} be a function that transforms linguistic expressions ll, obtained from a context-free grammar G_H into HFTLS H_S, where S is the linguistic term set used by G_H, and S_{ll} is the set of linguistic expressions generated by G_H.*

$$E_{G_H} : S_{ll} \to H_S \tag{3}$$

E_{G_H} performance depends on the comparative linguistic expressions generated by the context-free grammar G_H. For the G_H in Def. 3 are:

- $E_{G_H}(s_i) = \{s_i | s_i \in S\}$
- $E_{G_H}(\text{at most } s_i) = \{s_j | s_j \in S \text{ and } s_j \leq s_i\}$
- $E_{G_H}(\text{lower than } s_i) = \{s_j | s_j \in S \text{ and } s_j < s_i\}$
- $E_{G_H}(\text{at least } s_i) = \{s_j | s_j \in S \text{ and } s_j \geq s_i\}$
- $E_{G_H}(\text{greater than } s_i) = \{s_j | s_j \in S \text{ and } s_j > s_i\}$
- $E_{G_H}(\text{between } s_i \text{ and } s_j) = \{s_k | s_k \in S \text{ and } s_i \leq s_k \leq s_j\}$

In order to facilitate the computations with HFLTS, a fuzzy envelope for HFLTS was proposed in [18] that represents the linguistic expressions by means of a fuzzy membership function obtained by the aggregation of the linguistic terms that form the HFLTS.

Definition 5. *[18]: Let $H_S = \{s_i, s_{i+1}, \ldots, s_j\}$ be a HFLTS, so that $s_k \in S = \{s_0, \ldots, s_g\}$, $k \in \{i, \ldots, j\}$.*

$$env_F\,(H_S) = T(a, b, c, d), \tag{4}$$

Being $T(\cdot)$ a trapezoidal or triangular fuzzy membership function. For further detail see [18].

Our proposal shall use this fuzzy envelope (see [18] for further detail) to facilitate the computing processes with comparative linguistic expressions in the fuzzy TOPSIS MCDM model proposed to solve the selection process of firms in the technopark problem.

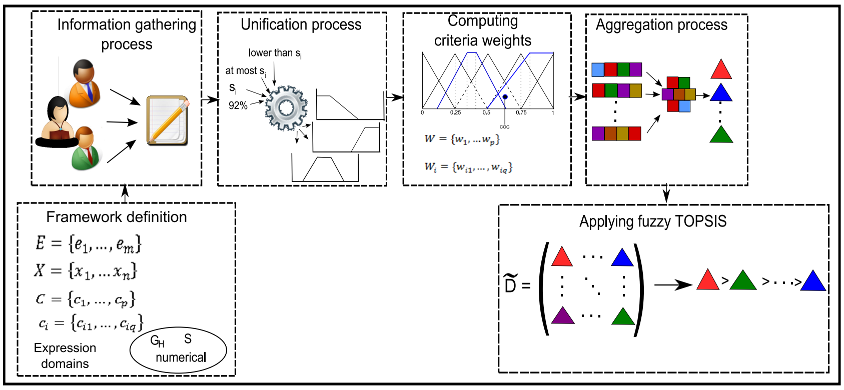

Figure 2: Solving process scheme for the technopark problem

4. A Selection Process based on Fuzzy TOPSIS for Technopark Firms Dealing with Hesitant Linguistic Information

This section presents a selection process based on a fuzzy TOPSIS MCDM method that ranks the set of firms that apply for a technopark position. It will be able to deal with a heterogeneous context in which either linguistic terms and numerical values or comparative linguistic expressions (based on HFLTS) are used to assess the criteria.

The selection process evaluates the involved criteria for the firms following a six-step process shown in Figure 2 and described below.

1. *Framework definition*: it defines the context and structure for the selection process of the technopark problem (experts, alternatives, main criteria and sub-criteria) and the expression domains in which assessments can be elicited.
2. *Information gathering process*: technopark experts express their opinions about the importance of the criteria to obtain the criteria weights and also their assessments about the alternatives criteria to obtain the ranking of firms.
3. *Unification process*: the assessments provided by using different type of information are unified into a fuzzy domain to facilitate the computing processes.
4. *Computing criteria weights*: it computes the criteria weights by using the technopark experts' opinions.

5. *Aggregation process*: the assessments about sub-criteria are aggregated to obtain a collective value for each alternative and main criterion.
6. *Applying fuzzy TOPSIS*: once the necessary input information has been obtained, a fuzzy TOPSIS method is applied to rank the set of firms applying for the technopark.

These phases are explained in further detail in the following sections.

4.1. Framework definition

In this phase, the framework for evaluating firms that apply for a technopark position, is established. It defines the main features and terminology of the selection process of the technopark problem in which technopark experts will provide their assessments.

A set of technopark experts $E = \{e_1, \ldots, e_m\}$, who express their assessments over a set of alternative firms $X = \{x_1, \ldots, x_n\}$, each alternative is defined by a set of main criteria $C = \{c_1, \ldots, c_p\}$, and each main criterion by a set of sub-criteria $c_i = \{c_{i1}, \ldots, c_{iq}\}$ $c_i \in C$. The assessments elicited by the technopark experts $e_k \in E$, over the alternatives $x_l \in X$, and the sub-criteria $c_{ij} \in C$, are represented by assessment vectors: $(r^{kl}_{ij}, \ldots, r^{kl}_{pq})$ with $i \in \{1, \ldots, p\}$ and $j \in \{1, \ldots, q\}$. The main criteria and sub-criteria weights are computed from technopark experts' opinions about the importance of the main criteria C, and sub-criteria c_i, and represented by the assessment vectors, $(w^k_1, \ldots, w^k_p)$ and $(w^k_{i1}, \ldots, w^k_{iq})$ respectively.

In our proposal, experts can express their assessments by using different expression domains (comparative linguistic expressions, linguistic terms and numerical values) fixed at this stage:

$$r^{kl}_{ij} \epsilon \left\{ \begin{array}{l} S_{ll} \\ S = \{s_0, \ldots, s_g\} \\ \vartheta \epsilon [0,1] \end{array} \right. \tag{5}$$

Experts will also provide their opinions about the criteria importance.

$$w^k_j, w^k_{ij} \epsilon \left\{ \begin{array}{l} S_{ll} \\ S \end{array} \right. \tag{6}$$

4.2. Information gathering process

Once the framework has been set up for the technopark problem, experts $e_k \in E$, provide their judgments over the alternative firms $x_l \in X$, and sub-criteria $c_{ij} \in C$, and express their opinions about the importance of the main criteria and sub-criteria by using heterogeneous information assessed in the expression domains defined in the framework (see Tables 1 and 2).

Table 1: Assessments over alternatives x_l and sub-criteria c_{ij}

e_1	$\{r_{ij}^{1l}, \ldots, r_{pq}^{1l}\}$
e_2	$\{r_{ij}^{2l}, \ldots, r_{pq}^{2l}\}$
$\ldots$	$\ldots$
e_m	$\{r_{ij}^{ml}, \ldots, r_{pq}^{ml}\}$

Table 2: Importance over main criteria w_j^k and sub-criteria w_{ij}^k

e_1	$\{w_1^1, \ldots, w_p^1\}$	$\{w_{i1}^1, \ldots, w_{iq}^1\}$
e_2	$\{w_1^2, \ldots, w_p^2\}$	$\{w_{i1}^2, \ldots, w_{iq}^2\}$
$\ldots$	$\ldots$	$\ldots$
e_m	$\{w_1^m, \ldots, w_p^m\}$	$\{w_{i1}^m, \ldots, w_{iq}^m\}$

4.3. Unification process

The heterogenous information elicited by technopark experts must be conducted into a unique expression domain to facilitate its management. In our case, a fuzzy domain to facilitate the treatment of the uncertainty involved in the problem and the computing processes is proposed. Such a unification process is accomplished in different ways according to the type of information:

1. The comparative linguistic expressions are transformed into HFLTS H_S by $E_{G_H}(\cdot)$ and then into its fuzzy representation by $env_F(\cdot)$.
$$env_F(E_{G_H}(r_{ij}^{kl})) = T(a, b, c, d)$$
being $T(a, b, c, d)$ a trapezoidal fuzzy membership function.
2. The linguistic terms $s_i \in S = \{s_0, \ldots, s_g\}$, are represented by trapezoidal fuzzy numbers. Therefore, a linguistic term s_i is represented by a trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$.
3. The numerical values are first normalized in the interval $[0, 1]$ and then transformed into trapezoidal fuzzy numbers by using the function R_N [14]:

Definition 6. *Let R_N be a function that transforms a numerical value into a trapezoidal fuzzy number:*

$$R_N : [0,1] \to \tilde{A} \\ R_N(\vartheta) = \tilde{A} = (\vartheta, \vartheta, \vartheta, \vartheta) \tag{7}$$

Being $\vartheta \in [0,1]$.

For the sake of clarity, the assessments and opinions about the criteria importance r_{ij}^{kl}, w_i^k and w_{ij}^k, transformed into fuzzy membership functions are represented by $\widetilde{r}_{ij}^{kl}$, $\widetilde{w}_i^k$ and $\widetilde{w}_{ij}^k$.

4.4. Computing criteria weights

In this phase the weights for the main criteria w_i, and sub-criteria w_{ij}, $i \in \{1, \ldots p\}$, $j \in \{1, \ldots q\}$, are computed by using the technopark experts' opinions. It consists of three steps:

1. Global fuzzy weights: the fuzzy weights obtained for the main criteria $\widetilde{w}_i^k$, and sub-criteria $\widetilde{w}_{ij}^k$, are aggregated by using Equation (8) obtaining a global fuzzy weight for each main criterion and sub-criterion $\widetilde{w}_i$, $\widetilde{w}_{ij}$ respectively.

$$\mu_{\widetilde{w}_{ij}}(z) = sup_{z=max(x_1,x_2,\ldots,x_k)} min\left(\mu_{\widetilde{w}_{ij}^1}(x_1), \mu_{\widetilde{w}_{ij}^2}(x_2), \ldots, \mu_{\widetilde{w}_{ij}^k(x_k)}\right), \quad x_t \in X, t \in \{1, \ldots, k\} \tag{8}$$

   being $k = \{1, \ldots, m\}$, $i = \{1, \ldots, p\}$ and $j = \{1, \ldots, q\}$, and X the universe of discourse.

2. Weighting values: the center of gravity method [4] is used to compute the weighting value of the global fuzzy weights, $\widetilde{w}_i$, $\widetilde{w}_{ij}$:

$$COG = \frac{\int \mu_{\widetilde{w}_{ij}}(x) * x dx}{\int \mu_{\widetilde{w}_{ij}}(x)}, x \in X \tag{9}$$

   Being X the universe of discourse.
   The result is a weighting vector for the main criteria $W = (w_1, \ldots, w_p)$, and another one for each set of sub-criteria $W_i = (w_{i1}, \ldots, w_{iq})$, with $i \in \{1, \ldots, p\}$.

3. Normalization: the weighting vectors are then normalized such that:

$$\sum_{i=1}^{p} w_i = 1 \qquad \sum_{j=1}^{q} w_{ij} = 1 \tag{10}$$

4.5. Aggregation process

Once the criteria weights are known, they are used to aggregate the technopark experts' assessments to obtain collective assessments for each main criterion, c_i and alternative, x_l by a two-step process:

1. Criteria aggregation: first, experts' assessments $\widetilde{r}_{ij}^{kl}$, over the sub-criteria c_{ij}, of each main criterion c_i, are aggregated by using the fuzzy weighted average operator (see Eq. (11)) to obtain a collective value $\widetilde{r}_i^{kl}$, for each main criterion c_i, alternative x_l and expert e_k.

$$\widetilde{r}_i^{kl} = \sum_{j=1}^{q} w_{ij} * \widetilde{r}_{ij}^{kl} \tag{11}$$

   where w_{ij} is the weight of the sub-criteria c_{ij}, obtained previously.
2. Experts aggregation: second, the collective values obtained $\widetilde{r}_i^{kl}$, are then aggregated by means of a fuzzy aggregation operator $f(\cdot)$, to obtain a collective value $\widetilde{r}_i^l$, for each main criterion c_i, and alternative x_l.

$$\widetilde{r}_i^l = f(\widetilde{r}_i^{kl}) \tag{12}$$

4.6. Applying fuzzy TOPSIS

Finally, the selection process aims at obtaining a ranking of alternatives to select the most suitable firms to take part in the technopark. Hence, from the collective values $\widetilde{r}_i^l$, it is applied the fuzzy TOPSIS MCDM method [3, 5, 36] that follows the below steps to obtain such a ranking:

1. Building a normalized decision matrix $\widetilde{R} = (\widetilde{r}_i^l)_{nxp}$ by using the collective values obtained in the previous phase.
2. Computing the weighting normalized fuzzy decision matrix $\widetilde{V} = (\widetilde{v}_i^l)_{nxp}$, where $\widetilde{v}_i^l = \widetilde{r}_i^l * w_i$.
3. Defining the fuzzy positive ideal solution $\widetilde{A}^+ = (\widetilde{v}_1^+, \ldots, \widetilde{v}_p^+)$ and the fuzzy negative ideal solution $\widetilde{A}^- = (\widetilde{v}_1^-, \ldots, \widetilde{v}_p^-)$, where $\widetilde{v}_i^+(1,1,1,1)$ and $\widetilde{v}_i^-(0,0,0,0)$.
4. Calculating the distances of each alternative from $\widetilde{A}^+$ and $\widetilde{A}^-$,

$$d^{l+} = \sum_{i=1}^{p} d(\widetilde{v}_i^l, \widetilde{v}_i^+) \qquad d^{l-} = \sum_{i=1}^{p} d(\widetilde{v}_i^l, \widetilde{v}_i^-) \tag{13}$$

   where $l = \{1, \ldots, n\}$ and $d(\cdot,\cdot)$ is the distance between two trapezoidal fuzzy numbers.

5. Calculating the closeness coefficient of each alternative,

$$CC^l = \frac{d^{l-}}{d^{l+} + d^{l-}} \tag{14}$$

6. Ranking the alternative firms according to CC^l.

5. A Decision Support FLINTSTONES Based System for Hesitant Linguistic Fuzzy TOPSIS Method: A Case Study in ITU Technopark

This section introduces the implementation of the method presented in previous section into a decision support system for the technopark problem based on the FLINTSTONES software [8] which facilitates and automates the technopark problem. A real case study of the ITU technopark problem is then presented that not only includes the ranking of alternatives, but also a sensitive analysis of the solutions.

5.1. Implementing the Hesitant Linguistic Fuzzy TOPSIS MCDM selection process in FLINTSTONES

FLINTSTONES is a novel decision tools suite introduced in [8] for solving decision making problems under uncertainty by using fuzzy and linguistic models. It is a componed-based application which has been developed by using Eclipse Rich Client Platform (Eclipse RCP)[3], a platform to build and deploy rich client applications. Eclipse RCP applications can easily be ported to Rich Internet Applications (RIA) by converting it to Eclipse Remote Application Platform (Eclipse RAP)[4].

Taking advantage of FLINTSTONES architecture, components, tools and Eclipse RAP features, we have implemented not only the selection process introduced in Section 4 for ranking alternatives, but also developed a Rich Internet Application (RIA) called Flintstones Gathering Cloud (FGC) (see Figure 5) which has been integrated in FLINTSTONES to facilitate the information gathering process because FGC allows users to import and distribute problems created by FLINTSTONES for gathering problem assessments in a remote, easy and distributed manner. Decision problems imported from

[3]http://www.eclipse.org/home/categories/rcp.php
[4]http://eclipse.org/rap/

FLINTSTONES into FGC are assigned to experts defined in the framework of the problem who will receive a notification by e-mail. Once all users taking part in the problem have submitted their assessments, the decision manager who distributed the problem will obtain a warning e-mail in order to download a file with all the assessments gathered for the problem which can be then loaded into the FLINTSTONES application for its resolution process. Figure 3 illustrates the scheme of operation and integration between FLINTSTONES and FGC that is used in the resolution of the ITU technopark case study shown in Section 5.2.

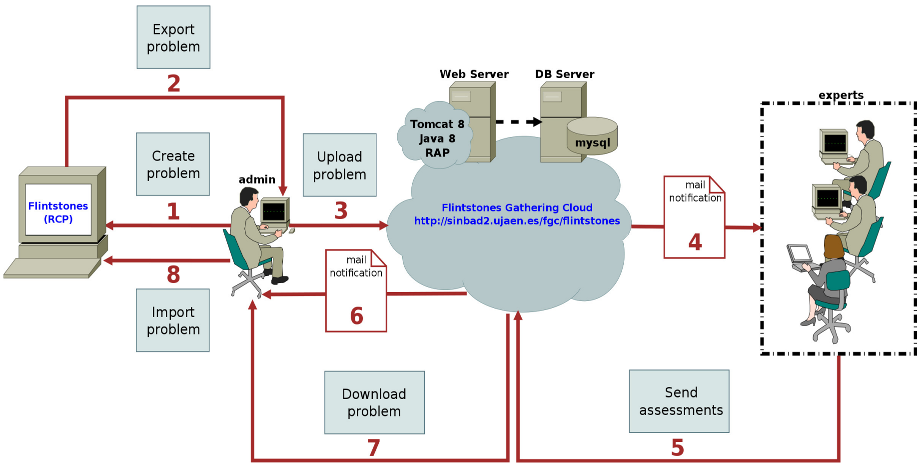

Figure 3: Integration between FLINTSTONES and FGC

5.2. Case study: Istanbul Technical University Technopark problem

In order to solve the ITU technopark problem, we have used FLINTSTONES software which implements the hesitant linguistic fuzzy TOPSIS MCDM selection process introduced in Section 4. The middle computations are shown by FLINTSTONES in a log format file, thus we will illustrate some results in Tables to see them in a better way.

1. *Framework definition*
   In this case study there are five industrial firms that apply to ITU technopark by means of their projects $\{p_1, p_2, p_3, p_4, p_5\}$ which are described below. These projects are evaluated by three experts $\{e_1, e_2, e_3\}$, considering the main criteria $C = \{c_1, \ldots, c_7\}$ and sub-criteria $c_i = \{c_{i1}, \ldots, c_{iq}\}\, c_i \in C$, described in *Appendix* **A**.

- p_1: project for hybrid engines for boats
- p_2: project for automobile sharts made of boron steels
- p_3: project for intelligent debt collection system
- p_4: project for integrated human resources software
- p_5: project for engine cylinder head design

The linguistic term set used by the context-free grammar G_H (see Def. 3) for the criteria importance is $S_1 = \{$*absolutely low importance, very low importance, low importance, medium importance, high importance, very high importance, absolutely high importance*$\}$ and for the alternatives evaluations is $S_2 = \{$*absolutely low aggrement, very low agreement, low agreement, medium agreement, high agreement, very high agreement, absolutely high agreement*$\}$.
Experts can elicitate their assessments about the criteria importance by using the comparative linguistic expressions generated by the context-free grammar G_H and the linguistic term set S_1. And they can assess the alternatives criteria and sub-criteria by using the comparative linguistic expressions generated by the context-free grammar G_H and the linguistic term set S_2 and numerical values depending on the nature of the criteria. *Appendix* **A** shows the expression domain defined for each criterion. Figure 4 illustrates the framework of this case study in FLINTSTONES.

2. *Information gathering process*
   We have implemented a Rich Internet Application (RIA) called Flintstones Gathering Cloud (FGC) which has been integrated in FLINTSTONES to facilitate the information gathering process. Figure 5 shows some criteria importance values provided by the expert e_2 using FGC.
3. *Unification process*
   All the assessments provided by technopark experts are unified into a fuzzy domain as it is shown in Section 4.3. The sub-criteria 1.11, 3.6, 3.7, 7.2, 7.3 and 7.4 whose expression domain used to assess them is boolean, are unified by using Eq. 7.
4. *Computing criteria weights*
   The criteria weights are computed using the opinions provided by technopark experts. The weights of the main criteria are shown in Table 3.

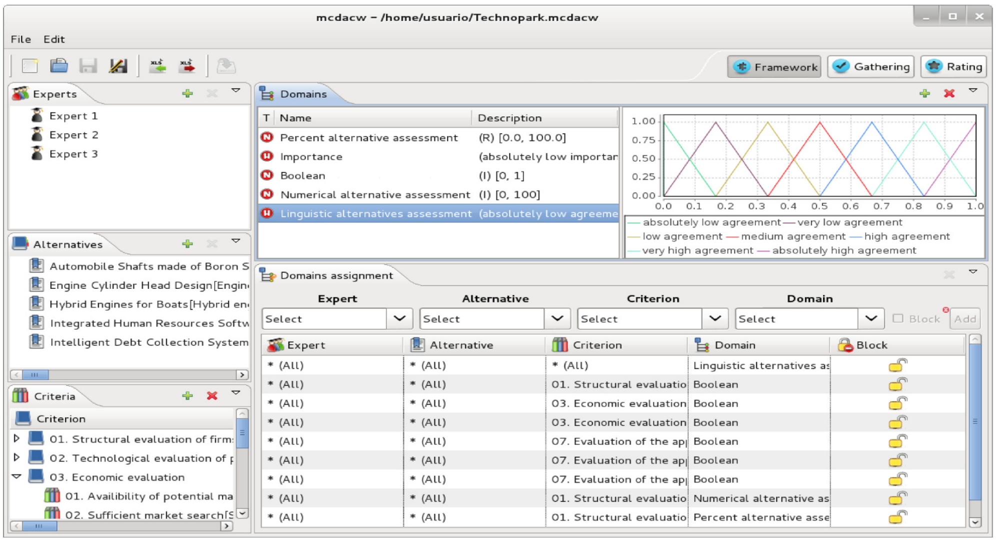

Figure 4: Framework

Table 3: Normalized weights of the main criteria

Main criteria	Weights
Structural evaluation of firms	0.038
Technological evaluation of project	0.208
Economic evaluation	0.232
Business plan and budget evaluation	0.149
Applying firm's facilities	0.132
Applying firm's plans for future cooperation	0.07
Evaluation of the applying firm's R and D consciousness	0.171

5. *Aggregation process*
   Once the criteria weights have been obtained, they are used to aggregate the technopark experts' assessments.
   (a) Criteria aggregation: the assessments over the sub-criteria are aggregated by means of the fuzzy weighted average operator.
   (b) Experts aggregation: in this case study all the technopark experts have the same importance, therefore we use the fuzzy arithmetic mean operator to aggregate the experts.

   Table 4 shows the collective values obtained for the alternative *Automobile Shafts made of Boron Steels*.
6. *Applying fuzzy TOPSIS*

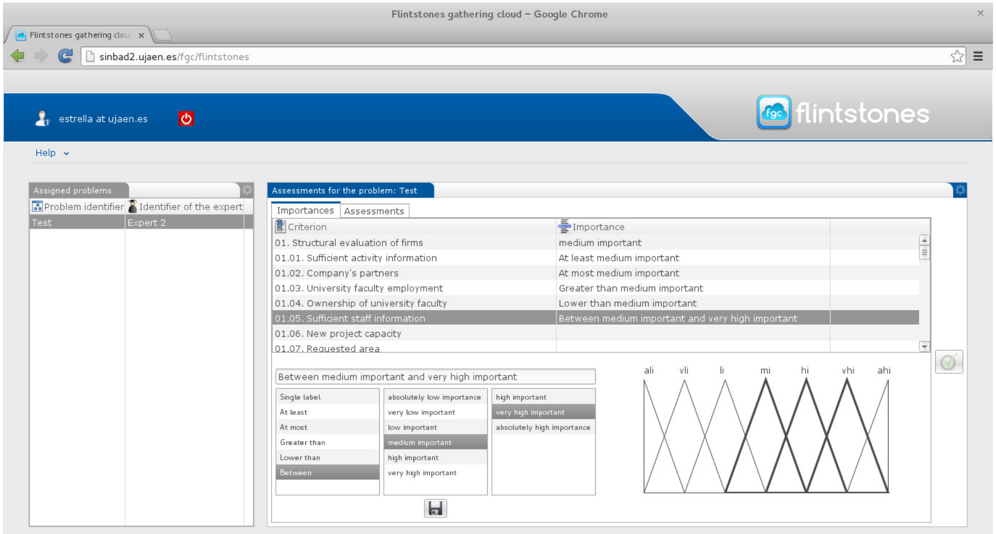

Figure 5: Flintstones Gathering Cloud

Table 4: Collective values for the alternative Automobile Shafts made of Boron Steels

Main criteria	**Collective values**
Structural evaluation of firms	(0.325, 0.468, 0.555, 0.637)
Technological evaluation of project	(0.604, 0.812, 0.888, 0.969)
Economic evaluation	(0.639, 0.816, 0.889, 0.967)
Business plan and budget evaluation	(0.231, 0.38, 0.548, 0.736)
Applying firm's facilities	(0.355, 0.556, 0.697, 0.847)
Applying firm's plans for future cooperation	(0.62, 0.872, 0.936, 0.989)
Evaluation of the applying firm's R&D consciousness	(0.54, 0.54, 0.54, 0.54)

(a) The normalized decision matrix is shown in Table 5.
(b) The weights obtained in the previous phase are used to compute the weighting decision matrix (see Table 6).
(c) The fuzzy positive and negative ideal solution are defined as $\tilde{A}^+ = (\tilde{v}_1^+, \ldots, \tilde{v}_7^+)$ and $\tilde{A}^- = (\tilde{v}_1^-, \ldots, \tilde{v}_7^-)$ where $\tilde{v}_i^+ = (1, 1, 1, 1)$ and $\tilde{v}_i^- = (0, 0, 0, 0)$, $i \in \{1, \ldots, 7\}$.
(d) The geometric distance [10] is used to compute the distance of each alternative from the fuzzy positive and negative ideal solutions.

$$d(A,B) = \begin{cases} \frac{1}{4}\left(|a_1 - a_2|^\lambda + |b_1 - b_2|^\lambda + |c_1 - c_2|^\lambda + |d_1 - d_2|^\lambda\right)^{\frac{1}{\lambda}}, & if\ 1 \leq \lambda < \infty \\ \max(|a_1 - a_2|, |b_1 - b_2|, |c_1 - c_2|, |d_1 - d_2|), & if\ \lambda = \infty \end{cases} \tag{15}$$

Table 5: Normalized decision matrix

Alternatives firms	Main Criteria 1	Main Criteria 2	Main Criteria 3
Automobile Shafts made of Boron Steels	(0.325, 0.468, 0.555, 0.637)	(0.604, 0.812, 0.888, 0.969)	(0.639, 0.816, 0.889, 0.967)
Engine Cylinder Head Design	(0.204, 0.322, 0.437, 0.547)	(0.554, 0.804, 0.88, 0.958)	(0.611, 0.772, 0.856, 0.945)
Hybrid Engines for Boats	(0.231, 0.358, 0.447, 0.57)	(0.372, 0.565, 0.693, 0.835)	(0.574, 0.725, 0.792, 0.896)
Integrated Human Resources Software	(0.2574, 0.387, 0.437, 0.54)	(0.037, 0.108, 0.242, 0.51)	(0.0398, 0.100, 0.192, 0.384)
Intelligent Debt Collection System	(0.459, 0.612, 0.653, 0.698)	(0.294, 0.484, 0.563, 0.742)	(0.483, 0.6199, 0.703, 0.839)
Main Criteria 4	**Main Criteria 5**	**Main Criteria 6**	**Main Criteria 7**
(0.231, 0.381, 0.548, 0.7362)	(0.355, 0.556, 0.697, 0.847)	(0.62, 0.872, 0.936, 0.989)	(0.54, 0.54, 0.54, 0.54)
(0.38, 0.574, 0.722, 0.882)	(0.421, 0.6578, 0.787, 0.919)	(0.612, 0.85, 0.917, 0.968)	(0.559, 0.559, 0.559, 0.559)
(0.249, 0.413, 0.532, 0.741)	(0.242, 0.397, 0.547, 0.743)	(0.452, 0.669, 0.761, 0.898)	(0.035, 0.035, 0.035, 0.035)
(0.423, 0.649, 0.764, 0.88)	(0.461, 0.657, 0.765, 0.905)	(0.275, 0.456, 0.533, 0.733)	(0.875, 0.875, 0.875, 0.875)
(0.346, 0.553, 0.664, 0.817)	(0.437, 0.636, 0.736, 0.888)	(0.334, 0.524, 0.618, 0.78)	(0.55, 0.55, 0.55, 0.55)

Table 6: Weighted decision matrix

Alternatives firms	Main Criteria 1	Main Criteria 2	Main Criteria 3
Automobile Shafts made of Boron Steels	(0.012, 0.018, 0.021, 0.024)	(0.125, 0.169, 0.185, 0.201)	(0.148, 0.1891, 0.206, 0.224)
Engine Cylinder Head Design	(0.008, 0.012, 0.016, 0.021)	(0.115, 0.167, 0.183, 0.199)	(0.142, 0.179, 0.198, 0.219)
Hybrid Engines for Boats	(0.0087, 0.0135, 0.017, 0.021)	(0.0774, 0.118, 0.144, 0.173)	(0.133, 0.168, 0.184, 0.208)
Integrated Human Resources Software	(0.01, 0.015, 0.016, 0.02)	(0.008, 0.0224, 0.0502, 0.106)	(0.009, 0.023, 0.045, 0.089)
Intelligent Debt Collection System	(0.0173, 0.0233, 0.025, 0.027)	(0.0611, 0.101, 0.117, 0.154)	(0.112, 0.144, 0.163, 0.195)
Main Criteria 4	**Main Criteria 5**	**Main Criteria 6**	**Main Criteria 7**
(0.0344, 0.057, 0.082, 0.11)	(0.047, 0.074, 0.092, 0.112)	(0.0434, 0.061, 0.066, 0.069)	(0.0854, 0.093, 0.093, 0.0996)
(0.057, 0.085, 0.108, 0.131)	(0.0558, 0.087, 0.104, 0.122)	(0.0428, 0.059, 0.064, 0.068)	(0.089, 0.096, 0.096, 0.103)
(0.0371, 0.0616, 0.079, 0.11)	(0.032, 0.053, 0.072, 0.098)	(0.032, 0.047, 0.053, 0.063)	(0.001, 0.006, 0.006, 0.013)
(0.063, 0.0979, 0.114, 0.131)	(0.061, 0.087, 0.101, 0.12)	(0.019, 0.0325, 0.038, 0.0513)	(0.143, 0.15, 0.15, 0.157)
(0.0516, 0.082, 0.099, 0.122)	(0.058, 0.085, 0.098, 0.118)	(0.023, 0.037, 0.043, 0.055)	(0.087, 0.094, 0.094, 0.101)

Tables 7 and 8 show the distances obtained by using Eq. 15 with $\lambda = 1$.

(e) The closeness coefficient of each alternative is calculated (see Figure 6).

7. Finally, the ranking of the industrial firms according to CC is:

$$p_2 > p_1 > p_5 > p_3 > p_4$$

5.3. Sensitivity Analysis

In order to see the robustness of our decision regarding the selected alternative, we analyze its sensitivity to the changes in the criteria weights. Figure 7 shows seven sensitivity graphs for the main criteria, showing each

Table 7: Positive ideal solution

Alternative firms	**Main criteria**						
Automobile Shafts made of Boron Steels	0.981	0.83	0.808	0.929	0.919	0.94	0.908
Engine Cylinder Head Design	0.986	0.834	0.815	0.905	0.908	0.941	0.904
Hybrid Engines for Boats	0.985	0.872	0.827	0.928	0.936	0.951	0.994
Integrated Human Resources Software	0.985	0.953	0.958	0.899	0.908	0.965	0.85
Intelligent Debt Collection System	0.977	0.892	0.847	0.911	0.911	0.961	0.906

Table 8: Negative ideal solution

Alternative firms	**Main criteria**						
Automobile Shafts made of Boron Steels	0.019	0.17	0.192	0.071	0.081	0.06	0.092
Engine Cylinder Head Design	0.014	0.166	0.185	0.095	0.092	0.059	0.096
Hybrid Engines for Boats	0.015	0.128	0.173	0.072	0.064	0.049	0.006
Integrated Human Resources Software	0.015	0.047	0.042	0.101	0.092	0.035	0.15
Intelligent Debt Collection System	0.023	0.108	0.153	0.089	0.089	0.039	0.094

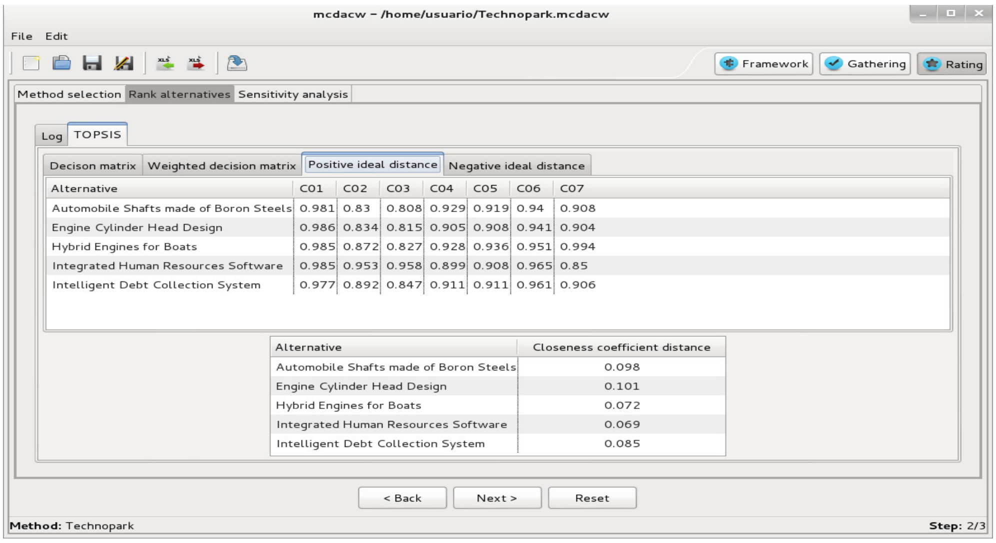

Figure 6: Selection process based on fuzzy TOPSIS MCDM method

one the closeness coefficient distances and corresponding ranking of the alternatives when we change a certain criterion weight. The cyan vertical lines represent the current weights of the criteria. For *Structural evaluation of firms*, a slight increase in the weight of this criterion changes our choice from *Engine Cylinder Head Design to Automobile Shafts made of Boron Steels*. For larger increases, *Intelligent Debt Collection System* will move toward the best alternative. For *Technological evaluation of project, Economic evalua-*

tion and *Applying firm's plans for future cooperation*, large increases in the weights of these criteria cause that *Engine Cylinder Head Design* leaves its first position in the ranking to *Automobile Shafts made of Boron Steels*. For evaluation of *The applying firm's R&D consciousness*, large increases in the weight of this criterion cause that *Integrated Human Resources Software* is selected. The alternative *Hybrid Engines for Boats* has no chance for selection in any case. Our decision is highly sensitive to the changes in the weight of the criterion, *Structural evaluation of firms*. Our decision is insensitive to the changes in the weights of *Applying firm's facilities* since *Engine Cylinder Head Design* has the highest rank along x axis.

6. Conclusions

Many firms want to take part in a technopark because it provides some benefits that lead to faster growth for companies. Since, it is necessary a selection process which implies multiple criteria that can be defined in a heterogeneous context. The complexity of assessing multiple criteria due to the lack of knowledge or lack of information implies that experts hesitate to express their opinions. The use of hesitant fuzzy linguistic term set allows modeling this hesitation and facilitates the elicitation of comparative linguistic expressions close to the expressions used by human beings in decision making.

In this paper, we have proposed a technopark selection process based on hesitant linguistic fuzzy TOPSIS MCDM method which is able to deal with problems defined in a heterogeneous context conformed by numerical values, linguistic terms and comparative linguistic expressions based on HFLTS. This process has been implemented and integrated in FLINTSTONES to support the complete selection process including the elicitation of information, information gathering process, solution process and a sensitive analysis to determine the most critical criteria for our decision. A real case study of the ITU tecnopark problem is presented to show the effectiveness of the proposed selection process.

For further research, we suggest another MCDM method to be used for comparison purposes, such as VIKOR. Alternatively, other extensions of fuzzy sets such as intuitionists fuzzy sets can be used.

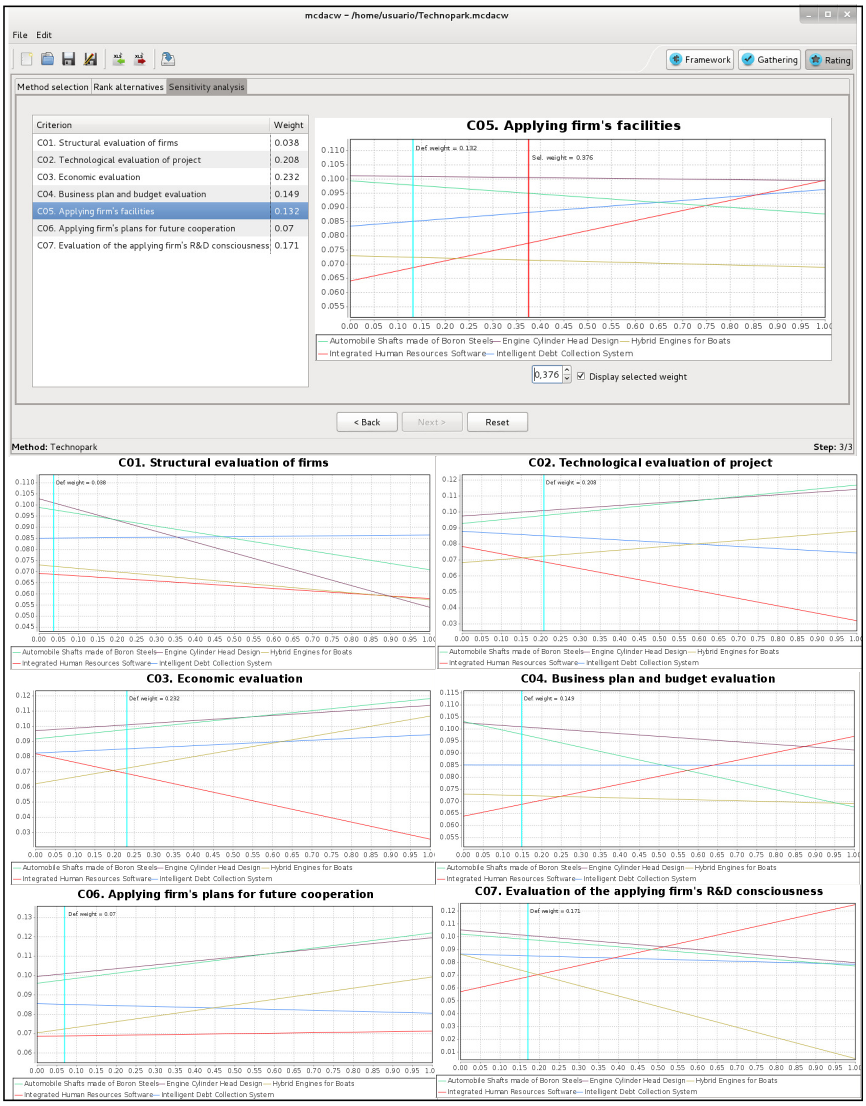

Figure 7: Technopark criteria sensitivity analysis

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Appendix A

The ITU technopark takes into account seven main criteria which contain several sub-criteria. (i) Structural evaluation of the criteria reveals how the firm is organized; (ii) the technological evaluation of project questions the innovativeness; (iii) economic attributes of the project shows the market potential [17] and its contribution to the national economy; (iv) business plan criterion shows the appropriateness of the business plans; (v) Firm's assets; (vi) applying firm's plans for future cooperation shows the resources and facilities used for the firm; and (vii) evaluation of the applying firm's R&D. The Tables 9, 10, 11, 12, 13, 14 and 15 show a description of the sub-criteria and the expression domain defined to assess them.

Table 9: Main criterion 1: Structural Evaluation of Firms

Sub-criteria	**Description and Expression domain**
Sufficient activity information	Sufficient information about the activities of the firm are given. (Linguistic)
Firm's partners	Firm partners' are capable of and willing to support the firm activities in Technopark. (Linguistic)
University Faculty Employment	The number of university faculties that are employed by the firm. (Numerical)
Ownership of University Faculty	The ownership percentage of the university faculties in the firm. (Numerical)
Sufficient staff information	Sufficient information is given on the quantity and qualifications of firm staff. (Linguistic)
New Project capacity	The firm has the capacity to produce new projects in the Technopark. (Linguistic)
Requested Area	The area that the firm requests is appropriate with the number of staff. (Linguistic)
Sufficient market information	Firm has sufficient information about current and target markets. (Linguistic)
Accomplished Projects	Firm has developed, managed R&D projects and fulfilled the desired targets. (Linguistic)
Capacity to Lead	Firm has the capacity to lead R&D projects. (Linguistic)
Tax Debt	Firm does not have tax debt. (Boolean)

Table 10: Main criterion 2: Technological Evaluation of Project

Sub-criteria	**Description and Expression domain**
Clear motivation	The motivation of the project is clearly explained. (Linguistic)
Clear main approach	The main approach (method, systematic etc.) of the project is clearly explained. (Linguistic)
Contribution to firm	The content and the output of the project contribute only to the firm's technological knowledge. (Linguistic)
Contribution to country	The content and the output of the project contribute to the country's technological knowledge. (Linguistic)
Contribution to world	The content and the output of the project contribute to the world's technological knowledge. (Linguistic)
Latest R&D usage	Latest R&D approaches and results are used in research and development. (Linguistic)
Uniqueness of the project output	Project output is unique in the targeted industry and/or market. (Linguistic)
Patent/Certificate of registry/Copyright potential	There is a potential for obtaining patent for new inventions, certificate of registry for new ideas and copyright for software. (Linguistic)
Patent/Certificate of registry/Copyright activities	Obtaining patent for new inventions, certificate of registry for new ideas and copyright for software is targeted and necessary activities are performed with this purpose. (Linguistic)
Benchmarking	Similar studies have been sufficiently investigated and benchmarked. (Linguistic)
Technological standards	The methodology, tools and scientific approach that will be used in the project is in line with technological standards. (Linguistic)
Explicit performance evaluation criteria	Project outputs and performance evaluation criteria is clearly explained. (Linguistic)

Table 11: Main criterion 3: Economic Evaluation

Sub-criteria	**Description and Expression domain**
Availability of Potential markets	A potential market exists or can be created, which is related to products and/or services of the finalized project. (Linguistic)
Sufficient market search	Sufficient search and assessment on the potential markets of products and/or services of the finalized project has been made. (Linguistic)
Sufficient cost/price analysis	Sufficient explanations on cost/price analyses related to the marketable products and/or services of the finalized project have been made. (Linguistic)
Contribution to National economy	Sufficient and realistic explanations for the expected contributions of the project to the firm and national economy are included. (Linguistic)
Possible changes in technology, economy, and policy	The effects of possible changes in technology, economy, and policy on the marketable products and /or services are sufficiently analyzed. (Linguistic)
Export of products or services	Export of products and/or services of the finalized project are aimed. (Boolean)
Substitution for the imported products or services	Substitution for the imported products and/or services with those from the finalized project is aimed. (Boolean)

Table 12: Main criterion 4: Business Plan and Budget Evaluation

Sub-criteria	**Description and Expression domain**
Explicit project flow	All the steps in project development (such as project processes) are sufficiently explained via detailed Gant diagrams. (Linguistic)
Milestone	Milestones are defined in time project schedule. (Linguistic)
Proper time schedule	Time schedule for the project and project steps is appropriate. (Linguistic)
Proper resource planning	Resources and the ways to gain resources are planned and explained sufficiently. (Linguistic)
Sufficient human resource	The total person/hour of the personnel that will be used for the project (when all the other activities of the personnel are considered) is sufficient. (Linguistic)
Proper resource usage	The planned machines, software and consultancy usage are appropriate and sufficient. (Linguistic)
Sufficient budget	Total project budget (human resource, tools, service, and other expenditures) is sufficient. (Linguistic)
Missing steps	All project steps which are necessary for project success are considered in the project planning. (Linguistic)

Table 13: Main criterion 5: Applying Firms Facilities

Sub-criteria	**Description and Expression domain**
Sufficient R&D Human Resource	The number of R&D staff is sufficient. (Linguistic)
Qualified R&D Human Resource	The qualifications and experience of R&D Staff is appropriate. (Linguistic)
R&D Certificates	R&D staff can demonstrate that they can fulfill their job with certificates and their documents. (Linguistic)
Turnover	The necessary actions have been taken for minimizing key staff's turnover ratio. (Linguistic)
Partners' Support	The managers and partners of the firm have the sufficient information about the Project. (Linguistic)
Similar Projects	Firm has managed similar projects in the past. (Linguistic)
Successful Projects	Firm is successfully managing other R&D projects. (Linguistic)
Cover Budget	The budget that is required for the Project (staff, hardware, software etc.) can be covered by the firm. (Linguistic)
Negative Factors	There are no factors that prevent successful completion of the project. (Linguistic)
Project Finalization	It is possible to manage and finalize the Project on time. (Linguistic)

Table 14: Main criterion 6: Applying Firms Plans for Future Cooperation

Sub-criteria	**Description and Expression domain**
Cooperation with the executive firm	The firm plans to cooperate with the executive firm during its activities at Technopark. (Linguistic)
Cooperation with other entrepreneurs	The firm plans to cooperate with other entrepreneurs at Technopark. (Linguistic)
Utilizing Universities human Resources	Firm plans to utilize the human resources of the university, e.g., consultancy, full time / part time work, and trainee. (Linguistic)
Utilizing University Facilities	Firm plans to utilize the facilities of University. (Linguistic)
Cooperation with University on other areas	Firm plans to cooperate with University on other areas, e.g., national/international publications, intellectual property rights, joint R&D projects. (Linguistic)
Cooperation with firms outside	Firm plans to cooperate with the national /international public or private institutes and foreign trade capital firms. (Linguistic)

Table 15: Main criterion 7: Evaluation of the Applying Firms R&D Consciousness

Sub-criteria	**Description and Expression domain**
Focusing on R&D projects	The ratio of R&D budget to the total budget. (Numerical)
Nationally-supported projects	The firm has R&D projects that were supported by national institutions. (Boolean)
EU-Supported projects	The firm has R&D projects that were supported by EU (European Union). (Boolean)
Other-supported projects	The firm has R&D projects that were supported by other investors, risk capital institutes. (Boolean)

4.4. Consensus under a fuzzy context: Taxonomy, analysis framework AFRYCA and experimental case of study

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Consensus under a fuzzy context: Taxonomy, analysis framework AFRYCA and experimental case of study

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Consensus reaching processes play an increasingly important role in the resolution of group decision making problems: a solution acceptable to all the experts participating in a problem is necessary in many real-life contexts. A large number of consensus approaches have been proposed to support groups in such processes, each one with its own characteristics, such as the methods utilized for the fusion of information regarding the preferences of experts. Given this variety of existing approaches in the literature to support consensus reaching processes, this paper considers two main objectives. Firstly, we propose a taxonomy that provides an overview and categorization of some existing consensus models for group decision making problems defined in a fuzzy context, taking into account the main features of each model. Secondly, the paper presents AFRYCA, a simulation-based analysis framework for the resolution of group decision making problems by means of different consensus models. The framework is aimed at facilitating a study of the performance of each consensus model, as well as determining the most suitable model/s for the resolution of a specific problem. An experimental study is carried out to show the usefulness of the framework.

1. Introduction

Decision making is a common process in daily life, characterized by the existence of several alternatives and the need to decide which one/s are the best or should be chosen as the solution to a problem. Group Decision Making (GDM) problems, in which several individuals or experts with different points of view take part in a decision problem with the aim of achieving a common solution, frequently occur in many organizations nowadays [1,2]. Although decision problems may take place in different environments (certainty, risk or uncertainty), most real-life GDM problems are often defined in uncertain environments. Due to the difficulty of dealing with uncertainty of a non-probabilistic nature, which is mainly caused by the imprecision and vagueness of information, experts must express their preferences over alternatives by means of information domains that allow them to deal with such uncertainty. To do so, fuzzy modeling and linguistic information has been utilized in such situations [3–5].

Traditionally, GDM problems have been solved by applying an alternative selection process [6], in which the preferences of each expert over the alternatives are gathered and the best alternative or subset of alternatives is chosen [7]. This resolution scheme does not take into account the existing level of agreement between experts, therefore some experts may not accept the decision made because they might consider that their individual preferences have not been taken into account sufficiently [8,9]. For this reason, Consensus Reaching Processes (CRPs) were introduced as an additional phase in the resolution of GDM problems [9]. In a CRP, experts discuss and modify their preferences, frequently coordinated by a human moderator, bringing their opinions closer to each other with the aim of increasing the level of agreement in the group.

Consensus has become a major research topic within the field of GDM. As a result, a large number of models and approaches to supporting CRPs have been proposed by several authors in the last few decades [10–17]. The earliest proposals of consensus approaches were developed with the objective of reaching a full degree of agreement in the group, i.e. unanimity [18], which is normally difficult to achieve in practice. Therefore, more flexible notions of consensus in which different partial degrees of agreement can be obtained, have since been proposed [2,19]. Consensus measures that are based on such flexible notions of agreement indicate

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how close experts' opinions are to unanimity. To do this, consensus degrees can be assessed in different ways, e.g. with numerical values in the unit interval [16,20,21], or linguistically [22–25].

A large number of consensus models have been proposed for dealing with GDM problems in fuzzy contexts, therefore they may present a high variety of features, such as: (i) the type of consensus measures utilized to determine the level of agreement, based on the fusion of information about experts' preferences [19,23,26], (ii) the use of different mechanisms to guide the discussion process [27], or (i) the type of preference structures (e.g. preference relations, preference orderings, utility vectors, etc. [28]) or information domains (e.g. numerical or linguistic information [22,29]) used by experts to express their preferences over alternatives, amongst others. Additionally, some models are focused on multiple criteria GDM problems (MCGDM) [29,30], in which information fusion approaches are often utilized to combine preferences evaluated according to several criteria, whilst other models have been defined to deal with a particular type of real-life decision problems [10,31].

Given this variety of existing consensus models, it would be desirable to have a clear characterization of them, with regard to the needs of each problem to be solved (type of preferences used by experts, necessity of giving the experts different importance weights, etc.), so that the most suitable models would be identified for solving such a problem. Moreover, some challenges are still present in the research topic of consensus, such as: (i) the large number of existing consensus models in the literature without a clear vision about which ones would be suitable for solving a specific type of GDM problem and (ii) the lack of a frame of reference for the practical study of consensus models, which makes the analysis of their main features, their advantages and weaknesses, as well as comparisons amongst them, more difficult. Such a comparison would be particularly useful for evaluating new proposals of consensus models, in order to determine their main contributions with respect to other existing ones.

As a result of a thorough literature review on consensus approaches in a fuzzy context, in this paper we tackle two objectives: (i) proposing a taxonomy of existing works and (ii) presenting an analytic framework called AFRYCA:

- We firstly present a taxonomy that provides an overview of a number of consensus models, with the main goal of providing a characterization of them, as well as pointing out the main characteristics of each proposal. The consensus models reviewed will be categorized into four groups, based on a double axis: (i) the use or not of feedback mechanisms to guide discussion, and (ii) the type of consensus measures applied (based on the method utilized for the fusion of information related to the preferences of the experts).
- Secondly, the paper introduces a prototype of simulation-based analysis framework called AFRYCA (A FRamework for the analYsis of Consensus Approaches). The framework has been developed to simulate the resolution of GDM problems by means of the different consensus models implemented in it. Therefore, its main purpose is to enable the analysis of the performance of each consensus model, as well as studying the results obtained by using different models for the resolution of a particular problem. AFRYCA has been implemented using Java and R technologies, and it incorporates several extendable modules and features, such as libraries that implement consensus models or patterns of expert behavior for its simulation, amongst others.

An experimental study is also presented to illustrate the usefulness of the analysis framework developed. For this, six consensus models of those reviewed in the taxonomy, have been implemented and used for the resolution of GDM problems.

The paper is organized as follows: in Section 2, some basic concepts regarding consensus in GDM are reviewed, together with some related works on consensus measures. Section 3 presents a taxonomy of consensus models. The analysis framework AFRYCA is presented in Section 4, followed by an experimental study that illustrates its usefulness in Section 5. Section 6 contains remarks on some of the lessons learnt and future directions in the use of AFRYCA. Finally, some conclusions are drawn in Section 7.

2. Background

In this section, we revise some basic concepts and approaches presented in the literature about GDM problems and consensus, in order to provide readers with a better understanding of the consensus models reviewed in the taxonomy presented in Section 3.

2.1. Group decision making problems

A GDM problem can be formally defined as a decision situation where [1]:

(i) There exists a group of m individuals or *experts*, $E = \{e_1, \ldots, e_m\}$, having each one their own knowledge and attitudes.
(ii) There is a decision problem consisting of n *alternatives* or possible solutions to the problem, $X = \{x_1, \ldots, x_n\}$.
(iii) The experts try to achieve a common solution.

In a GDM problem, each expert $e_i \in E, i \in \{1, \ldots, m\}$, expresses his/her preferences over alternatives in X, by means of a preference structure. One of the most common preference structures in GDM is the so-called preference relation [29]. A preference relation P_i associated to expert e_i can be represented, for X finite, as an $n \times n$ matrix as follows:

$$P_i = \begin{pmatrix} - & \cdots & p_i^{1n} \\ \vdots & \ddots & \vdots \\ p_i^{n1} & \cdots & - \end{pmatrix}$$

where each *assessment*, p_i^{lk}, represents the degree to which the alternative x_l is better than $x_k, l, k \in \{1, \ldots, n\}, l \neq k$, according to e_i. Other preference structures that have been considered in some GDM approaches are utility vectors [32] and preference orderings [33,34], amongst others.

Some problems are characterized by the existence of several attributes or criteria, $C = \{c_1, \ldots, c_q\}$ (e.g. location, neighborhood and size, in a problem about buying a new house). In such situations, experts must assess alternatives according to each of these criteria, $c_y \in C$, i.e. a Multi-Criteria Group Decision Making (MCGDM) problem is defined [1].

GDM problems are often defined in environments of uncertainty, characterized by the existence of vague and imprecise information. Such situations are also known as GDM problems in fuzzy contexts or *fuzzy GDM problems* in the literature [3]. In order to deal with such uncertainty, experts may utilize different information domains to provide their preferences out of the existing alternatives, depending on their knowledge area or level of expertise in the problem. Some information domains frequently utilized in GDM problems under uncertainty are [35,36]:

- *Numerical* [37]: Assessments are represented as numerical values belonging to a specific scale, e.g. values in the [0,1] interval or values in Saaty's 1–9 multiplicative scale [38].

- *Interval-valued* [39]: Assessments are represented as intervals, $I([0,1])$.
- *Linguistic* [40,41]: Assessments are represented as linguistic terms $s_u \in S, u \in \{0,\ldots,g\}$, being $S = \{s_0,\ldots,s_g\}$ a set of linguistic terms with granularity g.

The solution for a GDM problem can be derived by applying either a direct approach or an indirect approach [6]. In a *direct approach*, the solution is directly obtained from the individual preferences of experts, without constructing a social opinion first. In an *indirect approach*, however, a social opinion or *collective preference* (as it will be referred to in the rest of the paper) is determined a priori from individual opinions, and utilized to find a solution for the problem. Regardless of the approach considered, the classical alternative selection process for reaching a solution to GDM problems is composed of two phases [7], as shown in Fig. 1:

(i) *Aggregation phase*: the preferences of experts are combined, by using an aggregation operator.
(ii) *Exploitation phase*: This phase consists in obtaining an alternative or subset of alternatives as the solution to the problem, by means of a selection criterion.

2.2. Consensus in GDM: consensus measures and related works

The selection process for GDM problems described above does not guarantee the existence of agreement amongst experts before obtaining a solution to the problem. Therefore, it may be that such a solution is not accepted by some experts in the group, because they might consider that their individual opinions have not been taken into account sufficiently [8,9,42]. In many real-life GDM problems, obtaining a solution which is highly accepted by the whole group is crucial. In such cases, an additional phase called the consensus phase must be introduced into the resolution process for GDM problems [9]. This phase usually consists of a process of discussion and modification of preferences by experts, with the aim of reaching a high level of collective agreement (further detail regarding this process will be given in Section 2.3).

The concept of consensus has been interpreted from different points of view, from total agreement (unanimity), which is usually difficult to achieve in practice, to more flexible interpretations. In [9], Saint et al. defined consensus as "a state of mutual agreement among members of a group, where all legitimate concerns of individuals have been addressed to the satisfaction of the group". Kacprzyk et al. introduced the notion of *soft consensus*, based on the concept of fuzzy majority [2], which states that consensus exists when "most of the important individuals agree as to (their testimonies concerning) almost all of the relevant options" [19].

Flexible notions of consensus imply that it can be measured as different levels of partial agreement in the group, which indicate how far the opinions of experts are from unanimity. Therefore, the definition of appropriate *consensus measures*, which compute the current level of agreement in the group from the individual preferences of experts, has been an important subject of research within the field of consensus in GDM. A large number of consensus measures have been proposed by different authors in the literature

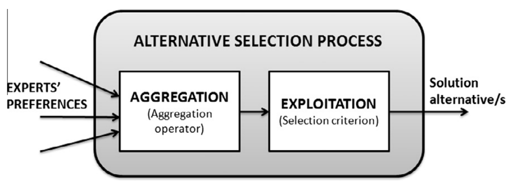

Fig. 1. Selection process for the resolution of GDM problems.

[19,24,43–45]. Based on a literature review of different consensus measures proposed by several authors, we have classified them into two categories, depending on the type of computations and information fusion procedures applied to measure consensus:

1. *Consensus measures based on distances to the collective preference*: A collective preference, denoted as P_c, that represents the global opinion of the group is computed by aggregating all individual preferences of experts, P_i, i.e. $P_c = \phi\{P_1,\ldots,P_m\}$, with ϕ being an aggregation operator. Consensus degrees are then obtained by computing the distances between each individual preference and the collective preference, $d(P_i,P_c)$ [24,43,44].
2. *Consensus measures based on distances between experts*: For each different pair of experts in the group, $(e_i,e_j), i<j$, the degrees of similarity between their opinions are computed, based on distance metrics. Similarity values $L(P_i,P_j)$ are then aggregated to obtain consensus degrees [19,22,25,45].

Fig. 2 shows a general scheme of the computations carried out in both types of consensus measures described above. In the following subsections, some consensus measures belonging to each of these two categories are briefly reviewed.

2.2.1. Consensus measures based on distances to the collective preference

Spillman et al. proposed in [43] one of the earliest consensus measures based on mathematical procedures taken from fuzzy set theory [4], thus complying with a notion of consensus which is more flexible and realistic in practice than the idea of consensus as unanimous agreement, as considered in other earlier works [18]. In their proposal, Spillman et al. measure the degree of consensus for each expert separately, as the distance between his/her reciprocal fuzzy preference relation and an "ideal" consensus matrix with maximum consensus degree, determined a priori by means of matrix calculus. Another complementary measure is the fuzziness degree, whose value is larger if the consensus degree is lower and vice versa, which is also introduced and utilized as a criterion to quantify the level of group agreement.

One of the first consensus measures for linguistic preferences was presented by Herrera et al. in [24], assuming that experts might sometimes have a vague knowledge about the problem and they would prefer to use linguistic assessments instead of numerical ones. Alternatives and experts have fuzzy importance degrees, inspired by Kacprzyk's *soft consensus* approach [2,19] (which will be revised in Section 2.2.2). Two different consensus measures are calculated: *consensus degrees*, which indicate the current level of agreement; and *linguistic distances*, used to evaluate the distance from each expert's linguistic preference relation to the collective opinion. Both measures are assessed linguistically, by means of linguistic terms s_u belonging to a finite term set $S = \{s_0,\ldots,s_g\}$ defined a priori, and they are calculated at three levels (using the LOWA operator [46] to aggregate information) by applying three steps sequentially: (i) a counting process, (ii) a coincidence process and (iii) a computing process [24].

In [23], Herrera et al. extended the consensus measures described above, by incorporating a process to control the consistency of preferences. The consistency control process is carried out before measuring consensus.

Ben-Arieh et al. studied in [47] the problem of aggregating linguistic preferences, expressed as fuzzy sets in a common linguistic term set by a group of experts who have associated linguistic importance weights. Firstly, they extended the Fuzzy-LOWA operator [44] to consider such importance weights in the aggregation of individual preferences into a collective preference. Then, they defined a consensus measure in which individual preference order-

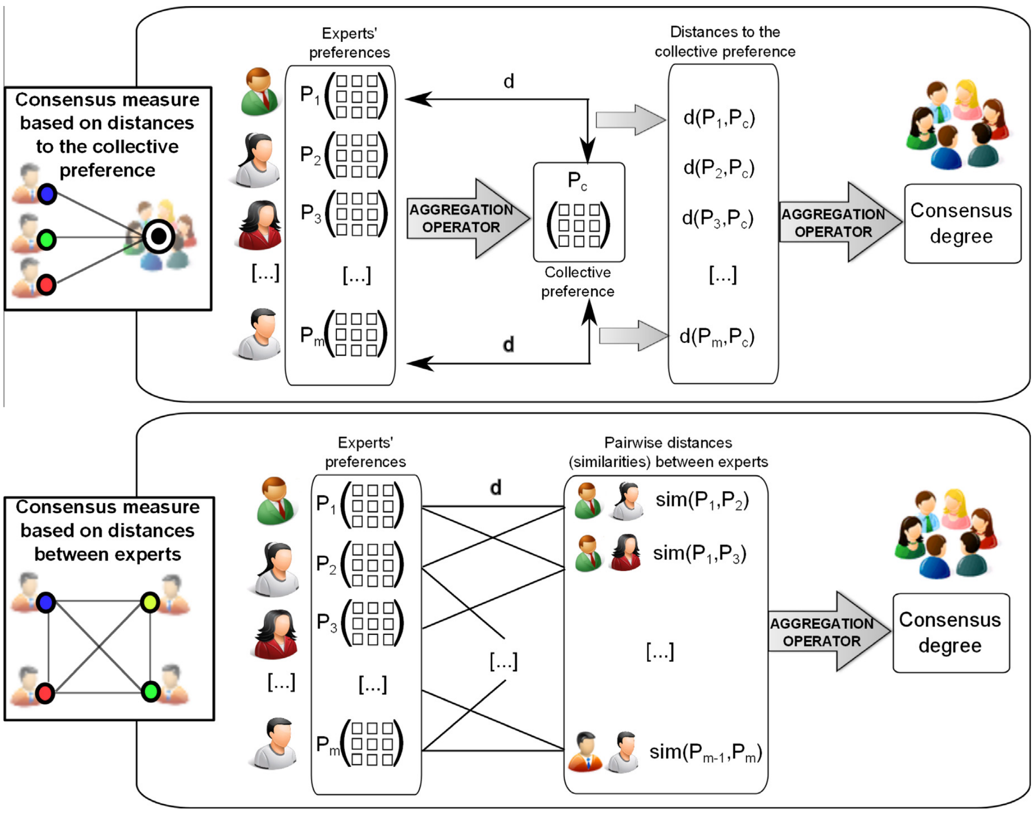

Fig. 2. Types of consensus measures.

ings and a collective preference ordering are compared. Such preference orderings are derived from their corresponding linguistic preferences. The degree of consensus C_l on an alternative x_l is computed as follows:

$$C_l = \sum_{i=1}^{m}\left[\left(1 - \frac{|O_i^l - O_c^l|}{n-1}\right) \times w_i\right] \tag{1}$$

with O_i^l and O_c^l being the ordered position of x_l, for expert e_i and the collective opinion respectively, and w_i the importance weight of e_i. The arithmetic mean operator is then used to compute the global consensus degree from all $C_l, l \in \{1 \dots n\}$.

2.2.2. Consensus measures based on distances between experts

Kacprzyk et al. conducted extensive research into human-consistent measures of consensus that reflect the human perception of consensus in practice in a better way than consensus as unanimous agreement. As a result, they proposed the notion of *soft consensus*, based on the concept of fuzzy majority [2]. One of the first consensus measures for fuzzy preference relations based on this notion was formalized in [19]. The consensus degree is hierarchically computed at multiple levels, starting by α-degrees of sufficient agreement (with $\alpha \in [0,1]$) between two experts (e_i, e_j) on a single assessment p_i^{lk}:

$$sim_{ij}^{lk} = \begin{cases} 1 & \text{if } |p_i^{lk} - p_j^{lk}| \leqslant 1 - \alpha \leqslant 1, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

The concept of fuzzy majority is reflected in the consensus measures by applying a fuzzy logic-based calculus of linguistically quantified propositions [2,48], taking into account the fuzzy importance weights assigned to experts and alternatives. The computation scheme of this "soft" consensus measure was slightly simplified in [45].

A different approach from *soft consensus* was taken into account by Szmidt and Kacprzyk in [49], where they extended the measures for fuzzy preference relations defined by Spillman et al. [43], and developed a consensus measure for reciprocal intuitionistic fuzzy preference relations. Consensus is computed as a scalar value in $[0,1]$, obtained from a consensus matrix of dimensions $m \times m$, in which each element cm_{ij} represents the degree of agreement between two experts e_i and e_j.

Herrera et al. proposed in [25] some consensus measures for linguistic GDM (linguistic consensus degrees and linguistic proximities, each one at three levels [24]), which pivot on determining degrees of fuzzy coincidence between pairs of experts, by means of a closeness measure between linguistic assessments. Different linguistic term sets can be used for the diverse elements of the GDM problem that are assessed linguistically, e.g. preferences, importance degrees of experts and alternatives, and consensus measures.

Another linguistic consensus measure was presented by Bordogna et al. in [22], being oriented towards MCGDM with linguistic preference matrices. This approach follows the concept of fuzzy majority, and it utilizes OWA operators [50] to aggregate preferences belonging to the different criteria. Such criteria are assessed linguistically by each expert. A linguistic consensus degree is computed for each alternative separately, based on degrees of agreement between pairs of experts.

Korshid et al. [51] presented a consensus measure based on coincidence between the positive and negative ideal degrees of agreement. Experts use linguistic terms to express their preferences by means of a vector of linguistic assessments. Such assessments are associated to triangular fuzzy numbers, and interval judgements are obtained by applying the α-cut operator [4] on fuzzy numbers, thus constructing an $m \times n$ fuzzy judgement matrix from the interval-valued assessments of all experts. Positive and negative agreement matrices are constructed taking into account similarities between pairs of experts, and then the relative closeness degrees to these two matrices are computed for each alternative.

Chen et al. defined in [52] a consensus measure for GDM problems with uncertain linguistic preference relations, with assessments given by uncertain linguistic terms expressed as $p_i^{lk} = [s_u, s_v], s_u, s_v \in S, u \leqslant v$ [53]. They determine the similarity between two experts' assessments upon a deviation measure, $d(p_i^{lk}, p_j^{lk})$, and an overlapping measure, $o(p_i^{lk}, p_j^{lk})$, as follows:

$$sim_{ij}^{lk} = \gamma(1 - d(p_i^{lk}, p_j^{lk})) + (1 - \gamma)o(p_i^{lk}, p_j^{lk}) \tag{3}$$

with $\gamma \in [0, 1]$ being the importance given to the deviation measure with respect to the overlapping measure, in the computation of similarity values. Consensus and proximity degrees are then computed at three levels. The Uncertain LOWA operator is utilized to aggregate uncertain linguistic preferences into a collective preference, which is necessary in order to calculate proximity degrees.

2.3. Consensus in GDM: Consensus Reaching Processes (CRPs)

As previously stated, reaching consensus normally implies that experts must modify their initial opinions over the course of a discussion process (i.e a CRP), bringing their positions closer to each other, towards a final collective opinion which satisfies the whole group [8,9,54].

Before initiating a CRP, it is important that some a priori assumptions are understood and accepted by the whole decision group [42]:

- Every member of the group *must* understand the process used to achieve an agreement, clarifying any possible doubts or questions before initiating it.
- Conducting a CRP implies that all experts *accept* the search for a common agreed solution, by means of collaboration.
- Experts *should move* from their initial positions, in order to make their preferences closer to each other.

A large number of consensus models have been proposed during recent decades [10–13,39,15–17]. Consensus models provide groups with the necessary guidelines to support them in CRPs carried out in different GDM frameworks.

The process to reach consensus is iterative and dynamic. Such a process is often coordinated by a human figure known as *moderator*, who is responsible for supervising and guiding the discussion between experts [42]. A general CRP scheme followed by all consensus models revised in the taxonomy (see Section 3), is shown in Fig. 3. Its main phases are described below:

1. *Consensus Measurement*: Preferences of all experts, $P_i, i \in \{1, \ldots, m\}$, are gathered to compute the current level of agreement in the group, by using consensus measures (see Section 2.2).
2. *Consensus Control*: The consensus degree is compared with a threshold level of agreement μ, defined a priori. If the level of consensus desired has been achieved, the group moves onto the selection process; otherwise, it is necessary to carry out another round of discussion. In order to prevent an excessive number of discussion rounds, a parameter indicating the maximum number of rounds allowed, $Maxround \in \mathbb{N}$, can also be taken into account.
3. *Consensus Progress*: A procedure is applied in order to increase the level of agreement in the following round of the CRP. Traditionally, such a procedure has consisted of applying a *feedback generation* process, in which the moderator identifies the assessments of experts which are farthest from consensus and advises them to modify such assessments [9,42]. Many existing consensus models incorporate feedback mechanisms based on this process [28,27,32,55]. However, some other proposed models do not incorporate such mechanisms, and instead they implement approaches that update information (e.g. assessments of experts) to increase consensus in the group automatically [44,56,57].

3. A taxonomy of consensus approaches in a fuzzy context

In this section, we propose a taxonomy that reviews different consensus models proposed by a variety of authors to support CRPs in GDM problems defined in a fuzzy environment. The main goal of the taxonomy is to categorize such models, so that those with similar characteristics are grouped in the same category.

Fig. 4 shows the structure of the taxonomy. In order to categorize the consensus models reviewed, we have considered two different kinds of criteria for constructing the taxonomy:

- *Feedback versus No Feedback*: Many consensus models define a feedback mechanism to support experts in the discussion and modification of their opinions. Such feedback mechanisms generate and provide experts with some advice, indicating to them how to modify their preferences in order to bring them closer to consensus, hence they must *supervise* this advice and decide whether to apply it or not [27,28,32,55]. Some other consensus models do not consider the use of feedback mechanisms, but instead implement other types of mechanisms that *automatically* update the preferences and/or importance weights of those experts whose opinions are not close enough to the rest of the group, thus making the human intervention of experts unnecessary in these models [44,56,57].
- *Type of consensus measure*: A key element in all consensus models is the consensus measure utilized to compute the level of agreement in the group. As previously reviewed in Section 2.2, such measures are normally either based on computing distances to the collective preference (see Section 2.2.1) or based on computing distances between experts (see Section 2.2.2).

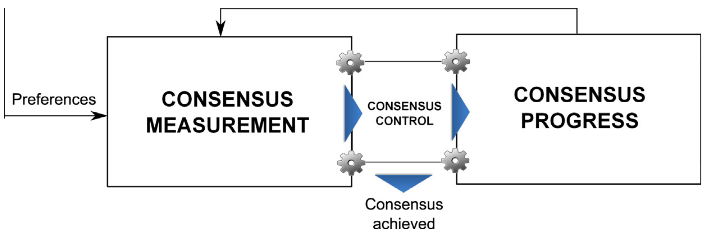

Fig. 3. General CRP scheme.

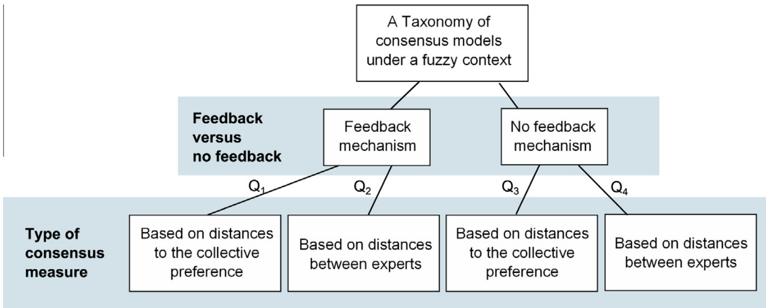

Fig. 4. A taxonomy of approaches for consensus reaching.

Table 1
Overview of consensus models reviewed in the taxonomy.

	Consensus measure based on distances to the collective preference	Consensus measure based on distances between experts
Feedback mechanism	(Q_1) Bryson [32,58] Herrera-Viedma et al. [28] Choudhury et al. [31] Dong et al. [62] Parreiras et al. [12,29] Jiang et al. [64]	(Q_2) Carlsson et al. [55] Eklund et al. [10,59] Herrera-Viedma et al. [60,61] Chiclana et al. [63] Mata et al. [27] Cabrerizo et al. [65] Pérez et al. [66] Alonso et al. [67] Kacprzyk et al. [13,68,69] Fu et al. [39,30,70,14]
No feedback mechanism	(Q_3) Lee [71] Ben-Arieh et al. [44] Chen et al. [74] Xia et al. [76], Xu et al. [77,78] Dong et al. [79], Zhang et al. [56] Gong et al. [15], Xu et al. [21] Wu and Xu [11,20,57,80,81]	(Q_4) Chen et al. [72] Zhang et al. [73] Palomares et al. [16,75]

Taking into account the two criteria described above, the classification of consensus models in the taxonomy is based on two axes, so that they are combined into four different quadrants that will categorize the consensus models revised in this paper (see Table 1):

- Q_1: Consensus models with feedback mechanism and a consensus measure based on computing distances to the collective preference, reviewed in Section 3.1.
- Q_2: Consensus models with feedback mechanism and a consensus measure based on computing pairwise similarities, reviewed in Section 3.2.
- Q_3: Consensus models without a feedback mechanism and with a consensus measure based on computing distances to the collective preference, reviewed in Section 3.3.
- Q_4: Consensus models without a feedback mechanism and with a consensus measure based on computing pairwise similarities, reviewed in Section 3.4.

Remark 1. For several consensus models reviewed throughout the following subsections, some figures with detailed schemes of their phases will be shown. The reason for showing the structure of these specific models in further detail rather than the other ones, is that they are already implemented in the initial version of the simulation-based analysis framework AFRYCA (see Section 4), and they will be utilized in the case study conducted in Section 5.

3.1. Q_1: feedback mechanism and consensus measure based on distances to the collective preference

In this section, we briefly review an assortment of consensus models characterized by: (i) the use of a feedback mechanism that provides some guidelines for experts on bringing their preferences closer to the rest of the group, and (ii) consensus measures based on the computation of distances between each expert's preference and the collective preference (see Fig. 5).

Bryson [32] proposed a model to assess the degree of group consensus and support group discussions under the Analytic Hierarchy Process (AHP) framework [38]. The model gathers, for each expert $e_i \in E$ a normalized numerical preference vector. Individual vectors are aggregated into a collective preference vector. Two thresholds and three consensus indicators are defined to decide whether the degree of consensus is sufficient or not, based on similarities between each individual vector and the collective vector. Bryson stated that the consensus preference vector should reflect an agreement that results from human interaction [32], hence the need for carrying out a negotiation process guided by a moderator [9], encouraging experts to interact with each other. Further guidelines and strategies to support such a negotiation (such as cooperation, communication and so on) by means of decision support tools in different scenarios, were later proposed by Bryson in [58], in which the use of qualitative assessments by experts, associated to numerical ranges (e.g. Poor: [0,40], Good: [60,80], etc.), was also introduced.

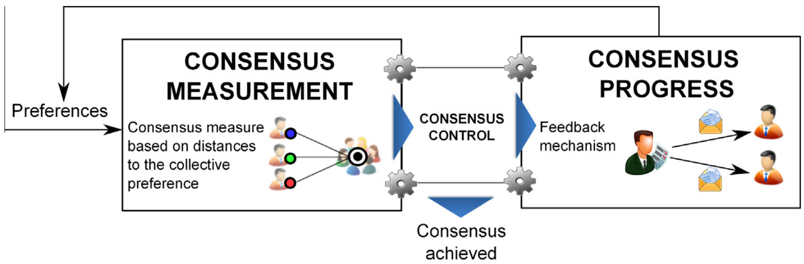

Fig. 5. General scheme of consensus models in Q_1.

The consensus model proposed by Herrera-Viedma et al. in [28] (represented in Fig. 6) allows experts to express their preferences by using different preference structures: (i) preference orderings O_i, (ii) utility functions U_i, (iii) fuzzy preference relations P_i and (iv) multiplicative preference relations A_i. Each expert chooses his/her most suitable preference structure according to the level of expertise he/she has in the problem. All preferences are conducted into fuzzy preference relations by means of several transformation functions. Furthermore, preference orderings of alternatives are obtained from individual fuzzy preference relations by computing the quantifier-guided dominance and non-dominance degrees for each alternative x_l (denoted in Fig. 6 as $QGDD_l$ and $QGNDD_l$, respectively). Such preference orderings are compared with a collective preference ordering to compute the consensus degrees. The model also introduces a feedback mechanism, based on proximity measures and a set of directions rules to suggest to experts how to increase/decrease some of their assessments.

Inspired by the consensus model with different preference structures proposed in [28], and considering its consensus measures, Choudhury et al. [31] proposed a consensus support system aimed at solving MCGDM problems in the context of advanced technology selection. Its main novelties with respect to previous models include the use of a multi-agent architecture [82] in which software agents with specific roles implement the different phases of the consensus model, as well as the aggregation of proximity degrees between experts and the collective preference, by means of the neat OWA operator, to obtain consensus degrees [83].

Dong et al. presented in [62] two consensus models for AHP-GDM with multiplicative preference relations [38]. The difference between the models is the nature of the consensus measure, which can be either ordinal or cardinal. Furthermore, unlike the above reviewed proposals, consensus measures are characterized by the application of a prioritization method that derives a prioritization vector of alternatives (instead of a preference ordering) from each preference relation. The collective preference is computed by

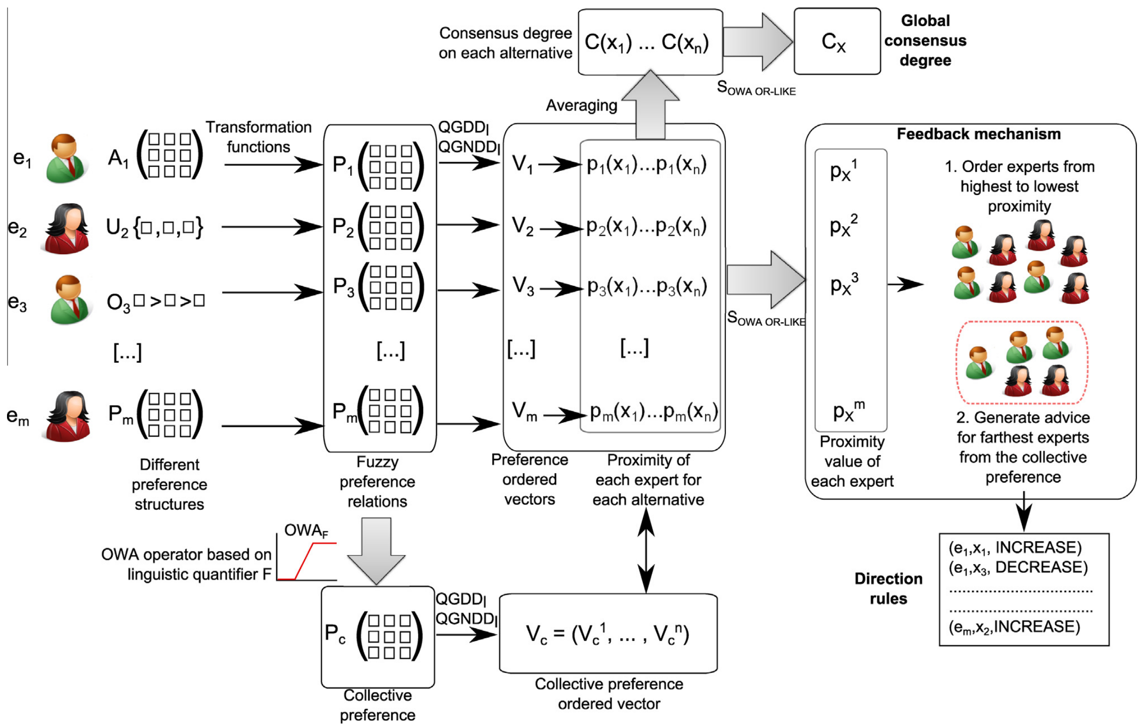

Fig. 6. Computation of consensus degrees and feedback mechanism in the model of Herrera-Viedma et al. [28].

means of the Weighted Geometric Mean operator. The proposed feedback mechanism identifies the expert farthest from consensus, determines some updated values for his/her preferences and shows the updated values to the human expert, who decides whether to accept or not the recommended changes.

Parreiras et al. proposed two consensus models for MCGDM problems. In their first model [12], experts utilize preference matrices with linguistic multi-granular assessments for each alternative and criterion, with the semantics of the linguistic terms given by trapezoidal membership functions. Since experts have importance weights according to their influence or position in the group, the authors suggested two methods to obtain them: either based on a discordance measure or by means of an optimization algorithm. The model presented in [29] introduces a measure of comparability to identify experts who experience difficulties in expressing their preferences (which are given by nonreciprocal fuzzy preference relations). In order to deal with such experts, other group members who are more sure of their opinion are invited to assist them. In both works, when the degree of consensus is insufficient, the moderator analyzes the concordance index of each expert with the collective preference, and suggests that the most discordant expert modifies his/her assessments.

More recently, Jiang et al. [64] defined a compatibility measure between intuitionistic multiplicative preference relations, and proposed two consensus models in which consensus degrees are measured for each expert separately, based on this compatibility measure. As occurred with [12,29,62], these models detect the farthest expert from the group opinion and invite him/her to modify his/her assessments. The second consensus model presented in [64] introduces identification rules in the feedback mechanism, in order to identify multiple discordant experts at the same discussion round and make the CRP more efficient.

3.2. Q_2: feedback mechanism and consensus measure based on distances between experts

A large number of consensus models in the literature calculate the closeness between all the different pairs of experts in the group for the measurement of consensus [19,22,23,60,84]. This section revises some consensus models that present this type of consensus measure and incorporate a feedback mechanism to guide experts across the CRP (Fig. 7).

Carlsson et al. [55] developed one of the first distributed consensus support systems to assist a group of experts connected to a local computer network. Its underlying consensus model follows an AHP framework for MCGDM problems in which experts provide preference matrices with assessments for each alternative and criterion, as well as the subjective importance weights they want to consider for each criterion. The consensus degree in the group is given by the maximum pairwise geometric distance between experts, i.e. $\max_{ij} d(P_i, P_j)$. The feedback mechanism finds the farthest expert from consensus and suggests to him/her how to bring his/her preferences towards a central point between the rest of the experts' preferences. Based on the consensus measure defined by Carlsson et al. in which consensus is given by the maximum distance between two experts, Eklund et al. developed some models for consensus reaching in committees [59] and dynamic political contexts with coalition formation [10]. Their works include a detailed comparison between their consensus model and several voting schemes and rules, e.g. majority vote, plurality vote and Borda rule [85].

Herrera-Viedma et al. presented the first model aimed at letting experts with diverse levels of expertise express their preferences by means of different linguistic term sets (multi-granular linguistic preference relations) [60]. In order to deal with multi-granular linguistic information, they introduced a unification phase to conduct preferences into fuzzy sets in a common linguistic term set. This consensus model adopted some features which have been later considered by the authors in several works, such as: (i) a scheme for the computation of consensus degree at three levels (assessment, alternative and preference relation) upon pairwise similarities of experts, and (ii) a feedback mechanism consisting of identification and direction rules for experts, based on the computation of proximity degrees with the collective preference.

Several works have since been proposed, based on the consensus measure and feedback mechanism defined in Herrera-Viedma et al.'s model [60]. Their work in [61] is characterized by dealing with incomplete fuzzy preference relations whose missing assessments are computed by applying an estimation procedure. The model of Chiclana et al. [63] (see Fig. 8), incorporates a consistency control process applied before beginning the CRP to ensure consistency in individual fuzzy preference relations, and proposes an adaptive feedback mechanism in which the direction rules generated for experts depend on the level of agreement achieved at each round, which is compared with several consensus thresholds, $\theta_1 < \theta_2 < \mu$. The adaptive consensus model proposed by Mata et al. [27] considers the use of multi-granular linguistic information [61], and implements the adaptive feedback mechanism proposed in [63]. The consensus model of Cabrerizo et al. [65] is capable of dealing with unbalanced fuzzy linguistic information, given by linguistic terms distributed in a non-symmetrical and non-uniform way around a central term. Computational processes on unbalanced linguistic information are carried out by means of the 2-tuple linguistic model [86,87]. A mobile consensus support system model for dynamic GDM, was presented by Pérez et al. in [66]. The system allows experts connected to their own mobile device to use different preference structures to provide their opinions [28], and it considers dynamic problems in which the set of alternatives X may vary over time. Finally Alonso et al. proposed in [67] a linguistic consensus model for Web 2.0 communities, in which the set of experts might vary during the CRP. A delegation scheme

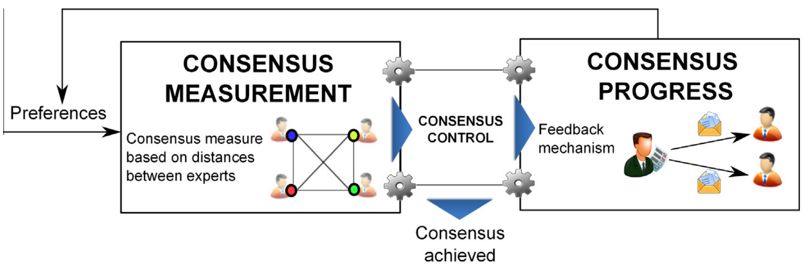

Fig. 7. General scheme of consensus models in Q_2.

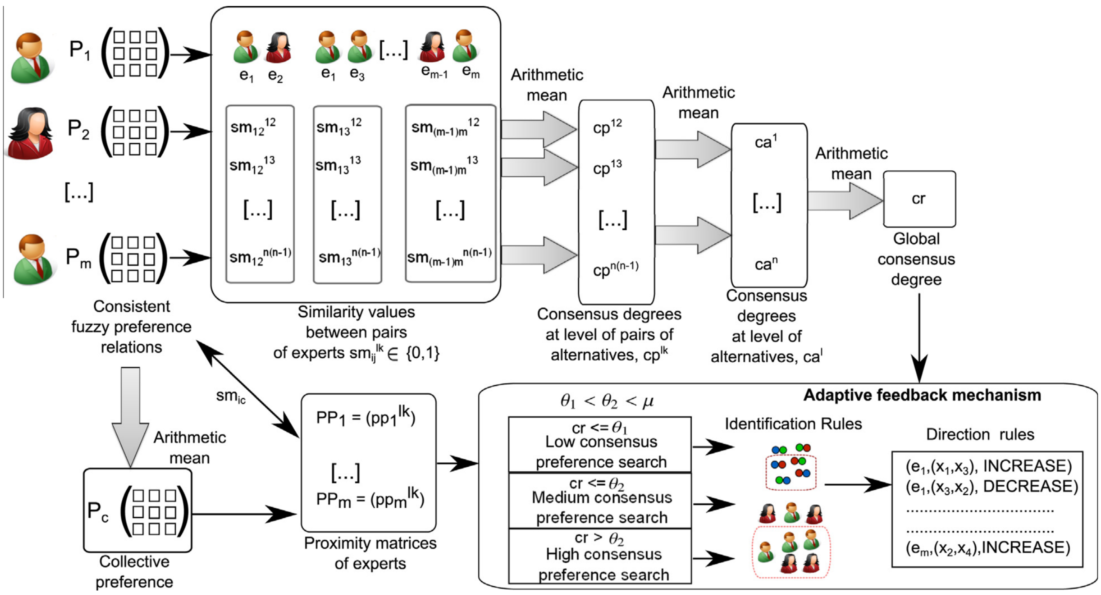

Fig. 8. Adaptive consensus model of Chiclana et al. [63].

based on trust weights between similar experts is defined to simplify GDM processes with large groups.

Kacprzyk et al. developed several consensus models based on their notion of *soft consensus* and fuzzy majority (see Section 2.2.2). In [68], they proposed a consensus model in which the moderator identifies experts and alternatives with difficulties in achieving a consensus by means of linguistic data summaries [88]. This proposal does not assign importance weights to experts and alternatives. Instead, two linguistic quantifiers F_1 and F_2 are utilized to capture the concept of fuzzy majority in the computation of consensus degrees at multiple levels [68], as illustrated in Fig. 9. The authors also proposed some models of consensus support systems that implement their previous ideas. For instance, in [13,69] a concept of Web-based consensus support system that not only implements previous models, but also includes a guidance system based on several approaches, such as rule generation and collaborative filtering, is shown. In [13], ontologies are utilized to formalize knowledge managed by the system with regard to the consensus reaching processes and each particular GDM problem. In addition, the system incorporates a feedback mechanism consisting of computing quantifier-guided degrees of agreement over pairs of alternatives, identifying the pairs of alternatives in which the experts

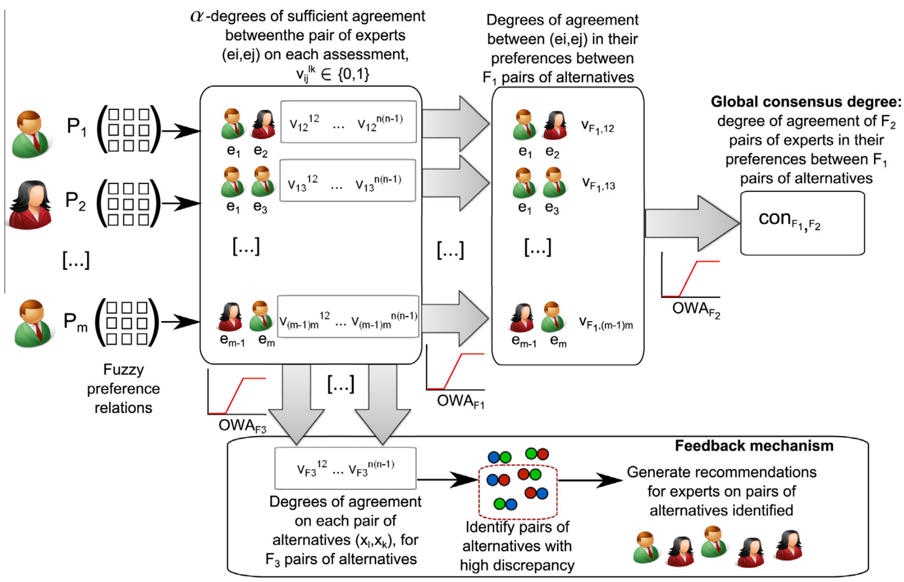

Fig. 9. Computation of consensus degree based on the concept of fuzzy majority, and feedback mechanism proposed by Kacprzyk et al. in [68,13], respectively.

present a higher degree of discrepancy, and providing recommendations to experts, based on several rules (see feedback mechanism in Fig. 9).

In [39,30,70,14], Fu et al. developed four consensus models for MCGDM problems in evidential reasoning contexts, where assessments of alternatives according to different criteria are given by distributed vectors of belief degrees, based on Dempster–Shafer evidence theory [89]. Such belief degrees can be either numerical [30,70] or interval-valued [39]. Assessments of pairs of experts are compared by means of a compatibility measure. Consensus degrees are then computed at three levels, similarly to [60]. In [39,70], they introduce a feedback mechanism consisting of identification rules and direction rules for experts, taking into account assessments related to criteria with the highest importance weights only. In [14], they extend the feedback mechanism, so that if consensus is not reached after some consecutive rounds of generating feedback, weights of experts are adjusted based on an optimization algorithm to ensure convergence to consensus.

3.3. Q_3: no feedback mechanism and consensus measure based on distances to the collective preference

Some consensus models do not incorporate a feedback mechanism and are designed to carry out the whole CRP automatically, so that the preferences and/or importance weights of experts are adjusted in order to reach a high level of agreement without the need for human intervention. This section revises several consensus models characterized by: (i) not incorporating any feedback mechanism and (ii) defining consensus measures based on the computation of distances to the collective preference (see Fig. 10).

In [71], Lee developed an iterative algorithmic approach to finding an optimal level of group consensus by adjusting the importance weights of experts and computing a collective preference based on them, so that the weighted sum of distances to the collective preference becomes minimal. The collective preference is given by the weighted average of individual preferences, which are expressed as trapezoidal fuzzy numbers. The consensus reaching algorithm is applied for each alternative separately.

Ben-Arieh et al. presented a consensus model for autocratic GDM [44] in a linguistic framework. Experts use linguistic preference relations, from which preference orderings are obtained to compute distances to the collective preference. Then consensus degrees are computed at the alternative and global level. If consensus is not enough, the degree of contribution of each expert towards consensus is determined, and weights of the least cooperating experts are penalized. More recently, Chen et al. defined in [74] an aggregation operator called ILLOWA (Interval Linguistic Labels Ordered Weighted Averaging) to facilitate the management of preferences expressed as interval linguistic labels, together with a consensus model that extends the one presented in [44] to manage this type of information.

Xu [77] considered the problem of consensus reaching in MCGDM, and developed a model that automatically updates all experts' assessments at the end of each consensus round if the level of agreement is not sufficient. To do so, an update coefficient $\eta \in (0,1)$, which partially takes into account values of the collective preference to update experts' assessments, is defined and utilized. A convergent iterative algorithm that automates the whole CRP is proposed. Unlike previous automatic consensus approaches, the importance weights of the experts remain fixed across the CRP. They are utilized to compute the collective preference. Consensus is only achieved when *all* distances between experts and the collective preference fall below a threshold, i.e. $d(P_i,P_c) \leqslant \mu, \forall e_i \in E$. An extension of this work was proposed by Xia et al. in [76], in which an automatic consistency improvement algorithm on reciprocal fuzzy preference relations is also defined.

The work of Xu et al. [78] proposes a number of goal and quadratic programming models oriented towards the maximization of consensus in groups of experts whose preferences are given in the form of fuzzy and multiplicative preference relations. Such programming models aim to find the optimal weights of experts that minimize their deviation with respect to the collective preference.

Wu and Xu have proposed several automatic consensus models in the last few years [11,20,57,80,81], in which the process used to compute and control individual consensus degrees similar to [77] in all of them. The model in [80] is aimed at the resolution of MCGDM problems with cost/benefit criteria, hence a normalization of assessments in the unit interval is applied before proceeding to measure consensus. Its mechanism to bring preferences closer to each other consists of obtaining at each CRP round a weighted distance matrix *DM*. Then its maximum element is identified, and the corresponding assessment is updated by assigning the value of the collective assessment to the preferences of those experts with the largest distance from the group preference. Their subsequent works [11,20,57,81] utilize a simpler mechanism that updates the preferences of all experts whose distance to consensus exceeds a specified threshold. The updating of assessments is based on the updating coefficient, η [77]. Each of these proposals is characterized by the use of a different preference structure: linguistic preference relations [11], multiplicative preference relations [57], uncertain linguistic preference relations [81], and reciprocal fuzzy preference relations [20]. Fig. 11 shows the procedure used to compute consensus degrees and update preferences, corresponding to the consensus model based on reciprocal fuzzy preference relations.

The work of Dong et al. [79] focuses on the use of two different representational models to deal with linguistic preferences (continuous linguistic model [90] and 2-tuple fuzzy linguistic model [86,87]). They define a consensus measure based on an aggregation operator called Extended-OWA, to obtain the collective preference from continuous linguistic information. As stated in [77], all the experts must be close enough to the collective preference in order to reach a consensus, otherwise a quadratic programming algorithm

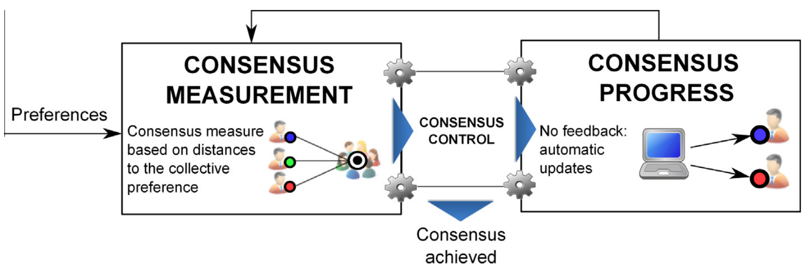

Fig. 10. General scheme of consensus models in Q_3.

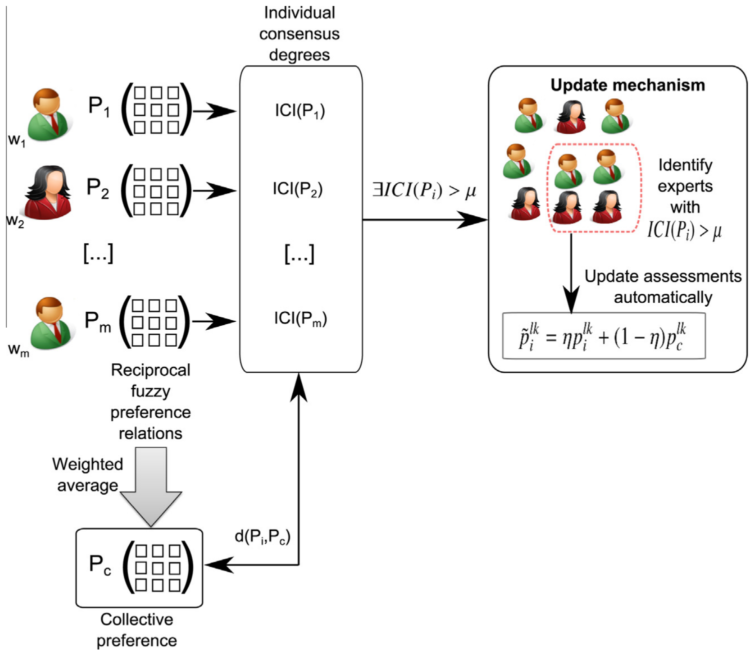

Fig. 11. Consensus model of Wu et al. [20].

that seeks the minimum required changes to individual preferences to find an agreement, is applied. Such an algorithm has since been considered by Zhang et al. in [56], in which a more generic consensus model under numerical preferences and the use of OWA operators is proposed.

Gong et al. formulated in [15] an optimization algorithm that, given a set of experts with associated weights and preferences expressed as 2-tuple linguistic preference relations, minimizes the deviation between all individual preferences and the collective preference. The optimization technique is applied to the values of experts' weights only, and no consensus thresholds are defined to decide on the existence of sufficient agreement, therefore the process ends when optimal weights are found. The additive consistency of preferences is also controlled.

The work of Xu et al. in [21] (see Fig. 12) proposes two distance-based consensus models for fuzzy and multiplicative preference relations, respectively. Two consensus measures are used in both models: Individual Consensus Indices $ICI(P_i) = d(P_i, P_c)$ for each $e_i \in E$, and a Group Consensus Index GCI for the whole group. The feedback mechanism to update preferences must be applied if $ICI(P_i) > \mu$ for at least one $e_i \in E$, or $GCI > \lambda$, with μ and λ being the individual and group consensus threshold, respectively, with $\lambda \leqslant \mu$. In such a case, the assessments of discordant experts with the greatest differences among them are updated by assigning the corresponding value of the collective preference to them. This procedure is similar to the one previously shown in [80].

3.4. Q_4: no feedback mechanism and consensus measure based on distances between experts

Most automatic consensus models compute consensus degrees based on distances to the collective preference (see Section 3.3), but a small number of them carry out computations of similarities between pairs of experts to measure consensus. Some automatic and semi-automatic models based on computing distances between experts (Fig. 13) are reviewed in this section, corresponding to the fourth quadrant of the taxonomy presented in this paper.

An adaptive consensus support system model inspired by the ideas of [27] was proposed by Chen et al. in [72]. Its main novelties with respect to the work of Mata et al. are: (i) preferences are given by intervals of linguistic 2-tuples, (ii) the system modifies preferences of experts by adjusting interval-valued assessments, and (iii) despite the underlying consensus model being automatic, the human expert can optionally decide to revise the changes applied to the preferences and accept them or not.

Zhang et al. extended in [73] the consistency-driven consensus model of Chiclana et al. [63], by introducing a linear optimization model to update preferences that ensures a minimum cost of modifying preferences, expressed as fuzzy preference relations. The main advantage of applying a linear optimization model is its low computational cost. Therefore, such a technique is utilized not only to conduct the CRP, but also to reach a high level of consistency for each individual preference relation.

In [16], Palomares et al. developed and presented a consensus support system based on a multi-agent architecture [82]. The main novelty of such a system is its capacity to automate the CRP completely, not only for the human moderator, but also for experts. To do this, experts provide their initial preferences (expressed as fuzzy preference relations) and delegate to autonomous software agents the revision of the advice received and the application of changes to preferences throughout the overall CRP. The underlying consensus model (see Fig. 14) follows some of the guidelines proposed in [27,60], such as: (i) the computation of pairwise similarities between experts by using the euclidean distance, (ii) the computation of consensus degrees at three levels, and (iii) although there is no real feedback for human experts, an agent-oriented feedback scheme consisting of identification and direction rules is implemented. Software agents are responsible for checking and applying direction rules on experts' preferences automatically. Moreover, two ontologies are defined and integrated in the model to facilitate communication and exchange of information amongst

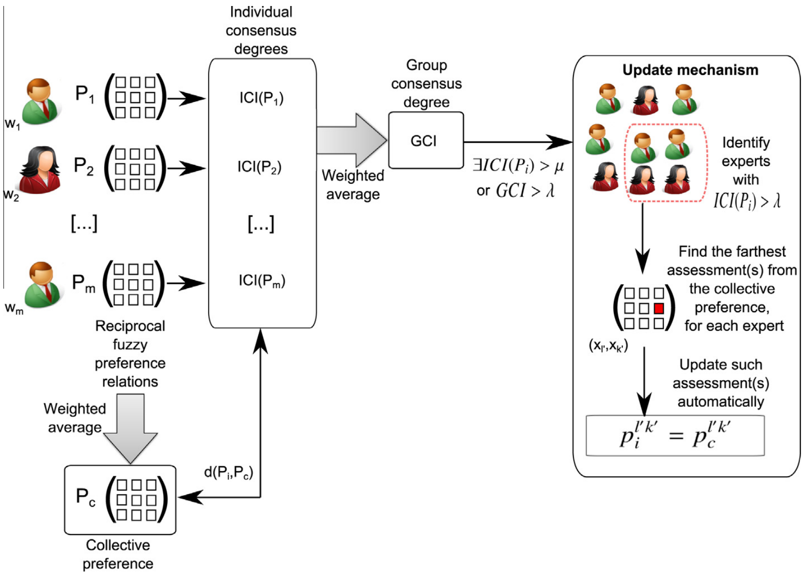

Fig. 12. Consensus model of Xu et al. [21].

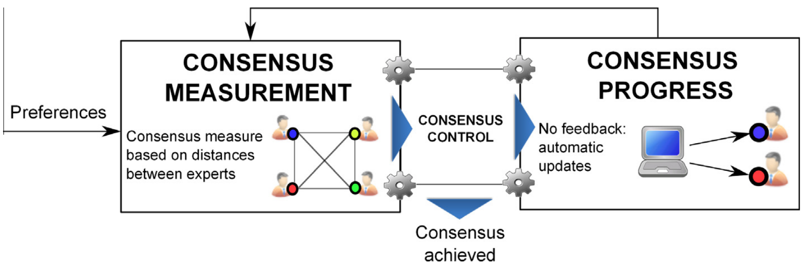

Fig. 13. General scheme of consensus models in Q_4.

agents, based on the ideas propounded by Kacprzyk and Zadrozny in [13]. Palomares et al. suggested the implementation and flexible use of different aggregation operators to measure consensus.

The system presented by Palomares et al. allows a full automation of human experts, regarding the process of supervising and modifying preferences. However, in [75], they argued that in some specific situations, it might be desirable that the human expert supervises the advice generated on an assessment p_i^{lk}, e.g. if such advice implies an important change to his/her preference. Based on this idea, they propose an agent-based semi-supervised approach that allows software agents to carry out most revisions of preferences by themselves, so that they only request human intervention when critical changes must be applied. Such an approach is based on the definition of several behavioral profiles that define how agents apply changes autonomously, as well as a rule-based mechanism to indicate the situations in which the human expert must revise his/her opinions. Its main advantage is the capacity of automating the CRP for human experts to a high degree, while preserving their sovereignty.

4. AFRYCA: A FRamework for the analYsis of Consensus Approaches

This section introduces a novel software framework called AFRYCA to simulate the resolution of GDM problems by using different consensus models proposed in the literature, many of which have been categorized and reviewed in the taxonomy previously presented. AFRYCA is mainly oriented towards a practical study of consensus models, for discovering the advantages and weaknesses of each model, analyzing the performance of a model under different settings, etc. The framework also aims at: (i) providing a better understanding of which models would be the most suitable to solve a specific type of GDM problem, and (ii) enabling comparisons between different consensus models, which could be useful to find out the main contributions of new proposals with respect to other existing works, for instance.

Firstly, we present the architecture and technologies of the framework (Section 4.1). A methodology for the use of the framework is then briefly described. Finally, we undertake a case study to show the performance of several consensus models implemented in the framework, for the resolution of several GDM problems (Section 5).

4.1. Architecture of AFRYCA

Here, the architecture of AFRYCA and the technologies that have been utilized in the analysis framework are presented.

AFRYCA has been developed under Java language, by means of the set of plugins *Rich Client Platform* (RCP), which enables the development of client desktop applications with rich functionality. One of the main advantages of RCP is its appropriateness for building component-based software applications based on high quality

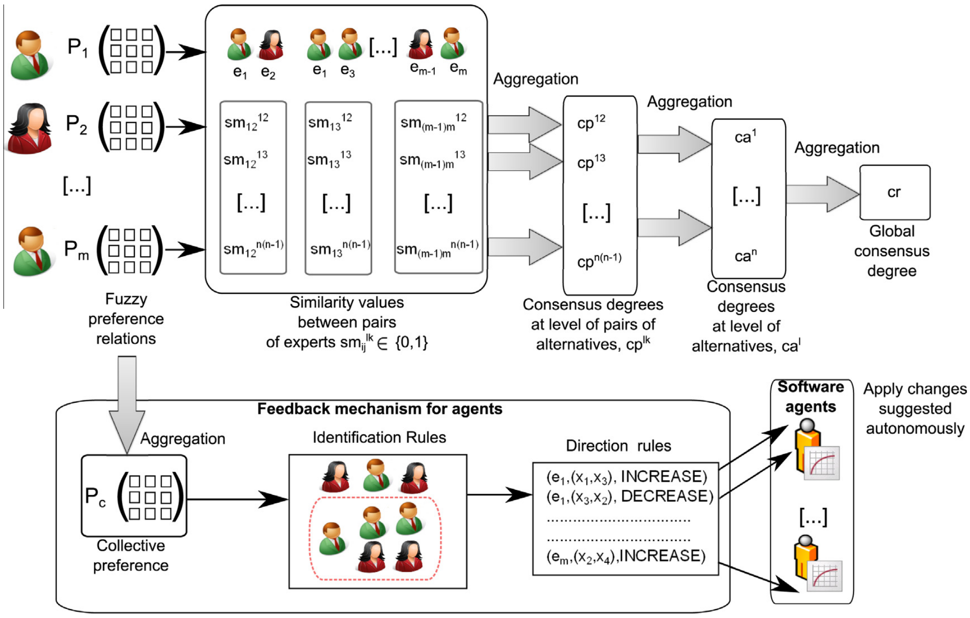

Fig. 14. Agent-based consensus model of Palomares et al. [16].

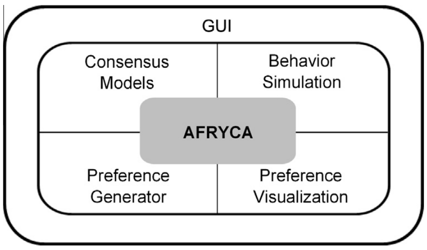

Fig. 15. Architecture of AFRYCA.

components that are easy to maintain and extend, due to the high cohesion degree within each component and the low coupling between different components. Additionally, the software suite R[1] for statistical computing and graphics has been utilized to develop some components of the framework.

The framework is divided into five modules, as shown in Fig. 15. Such modules implement the functionalities and tools included in AFRYCA for the simulation and analysis of GDM problems based on consensus models, and they are described below:

- *Consensus Models*: Libraries that develop several existing consensus models. Each library corresponding to an existing consensus model is implemented in Java, and it includes the different phases (e.g. computation of consensus degrees, advice generation, etc.), operators (e.g. OWA, weighted mean, etc.) and parameters (e.g. consensus thresholds, linguistic quantifiers, etc.) necessary to apply such a model in practice. The flexible, loosely coupled architecture of AFRYCA facilitates the introduction of new libraries that implement additional consensus models easily. The current version of the framework incorporates the necessary libraries for using six consensus models based on the use of fuzzy preference relations:
  - Three consensus models with feedback mechanism: Herrera-Viedma et al. [28] (see Fig. 6), Chiclana et al. [63] (see Fig. 8), and Kacprzyk et al. [68,13] (see Fig. 9).
  - Three consensus models without feedback mechanism: Wu et al. [20] (see Fig. 11), Xu et al. [21] (see Fig. 12), and Palomares et al. [16] (see Fig. 14).

Remark 2. In AFRYCA, the current implementation of Herrera-Viedma et al.'s consensus model [28] omits the initial phase of unifying different preference structures, because the model deals with fuzzy preference relations only. Besides, in the model of Kacprzyk et al. in [68], the feedback mechanism based on linguistic summaries has been replaced by a feedback mechanism based on the criterion of "lack of arguments" suggested in [13].

- *Behavior Simulation*: This module has been designed to choose and simulate different patterns of behavior adopted by experts when accepting/ignoring feedback and modifying their assessments across the CRP. Such behavior patterns are utilized by the consensus models that have a feedback mechanism (see Sections 3.1 and 3.2). Two key aspects must be taken into account to define a behavioral pattern in AFRYCA. These two aspects are modeled by generating values belonging to different probability distributions, as follows:
  - The amount of recommendations on assessments that an expert e_i may accept or ignore. This feature can be modeled by means of a generator of discrete random values (e.g. 1 for *accept* or 0 for *ignore*) belonging to a probability distribution (e.g. binomial), whose parameter values (e.g. probability of success p in binomial distribution) can be fixed by the developer.
  - The degree of change that e_i may apply to the assessment p_i^{lk}, the modification of which he/she has accepted. This feature can be modeled with either a discrete or continuous probability distribution (e.g. Normal or Negative Binomial), so that values generated with R under this distribution represent the degree of change applied to the assessment.

[1] http://www.r-project.org.

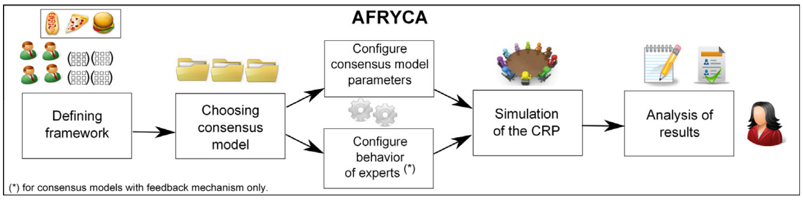

Fig. 16. Methodology for the analysis of consensus models based on simulation of GDM problems.

A number of built-in *R* functions for the generation of random values under different probability distributions are utilized. R functions are invoked from Java code, by means of a third-party Java-R interface library. As occurred with consensus model libraries, this component can also be extended in the future. Moreover, such patterns can be used by different consensus models flexibly, in the sense that the user of AFRYCA may configure which behavioral pattern may be utilized with a specific consensus model at a given moment.

- *Preference Generator*: A Java implementation of the method proposed in [91] to construct consistent reciprocal fuzzy preference relations P_i from a set of $n-1$ values of assessments $p_i^{l(l+1)}, l \in \{1,\ldots,n-1\}$. Although such $n-1$ assessments are initialized randomly, the rest of the assessments are constructed taking into account the method mentioned above, thus ensuring consistency in preferences. This module allows the generation of data sets of experts' preferences. Each data set contains a specified number m of preference relations, as well as the formulation of a GDM problem, alternatives, etc. Such information is specified a priori, through the AFRYCA user interface. Data sets can be stored on a disk for future use.
- *Preference Visualization*: This module, inspired by the graphical monitoring tool of preferences presented in [92], provides a graphical 2-D representation of experts' preferences and the group preference, P_c, obtained after having conducted a CRP during the resolution of a GDM problem. Such a visualization is shown to the user of AFRYCA, together with the results of the GDM problem resolution. Some built-in R multi-dimensional scaling functions have been considered for the implementation of this module.
- *Graphical User Interface* (GUI): This allows users to interact with the rest of the modules in the framework. The GUI of AFRYCA has been implemented with the SWT (*Standard Widget Toolkit*) library, and it includes the necessary interfaces to: (i) choose the GDM problem and consensus model to utilize, (ii) configure the consensus model and select the behavioral pattern to simulate experts' behavior, (iii) visualize a summary of results after having applied the consensus model. It is also possible to generate a log file with more detailed results of the CRP conducted.

The architecture of AFRYCA offers several advantages, some of which are:

- Since it has been developed as a Java-based RCP, the framework can be used on any platform provided with a Java Virtual Machine, regardless of the operating system.
- The structure of AFRYCA, which is divided into separated modules, makes it possible to upgrade or extend some of its components (e.g. consensus model libraries and behavioral patterns, as mentioned above) without having to carry out changes that affect the whole framework.

A downloadable version of AFRYCA, as well as further details and documentation about the framework and its modules, can be found on the AFRYCA website.[2]

4.2. Methodology for using AFRYCA to simulate the resolution of GDM problems

Here, we describe the methodology for using AFRYCA to simulate the resolution of a GDM problem by using a consensus model implemented in the framework, and analyze different aspects of such a model, e.g. determining the strong points, weaknesses and types of GDM problems that can be solved with such a model, studying its performance with respect to other models, etc. The methodology is divided into the following steps, as depicted in Fig. 16:

1. *Defining Framework*: An instance of a GDM problem is chosen, to be solved by applying the consensus model previously chosen. To do so, the user can either select a data set file with an already existing GDM problem, or he/she can use the *Preference Generator* module to create a data set for a new GDM problem with m experts.
2. *Choosing consensus model*: A consensus model is chosen from amongst those included in the framework. The GUI of the framework provides a description and the main features of each model, as shown in Fig. 17.
3. *Configure parameters of the consensus model and behavior of experts*: Before proceeding to carry out the CRP, it is necessary to configure the values of parameters in the consensus model chosen (e.g. consensus thresholds, aggregation operators, etc.). For consensus models with a feedback mechanism, it is also necessary to specify the pattern of behavior adopted by experts when they receive recommendations and apply changes to their preferences (see *behavior simulation* module, Section 4.1).
4. *Simulation of the CRP*: Once the consensus model settings are fixed, the CRP is carried out.
5. *Analysis of results*: When consensus is achieved, an alternative selection process based on fuzzy non-dominance degrees of alternatives is applied [37], and the results of the GDM problem resolution are shown, in order to allow the user to analyze them. Results shown in the GUI include: (i) the initial consensus degree in the group and the final consensus degree achieved, (ii) the number of discussion rounds required, (iii) the ranking of alternatives and alternative/s chosen as the solution, and (iv) a visualization of experts' preferences and the group preference at the end of the CRP (see Fig. 18). AFRYCA also offers the possibility of storing a log file with more detailed results of the CRP performance.

5. Experimental study

In order to illustrate the purpose of AFRYCA, in this section we show an experimental study conducted to study the performance

[2] The AFRYCA website can be found at: http://sinbad2.ujaen.es/afryca.

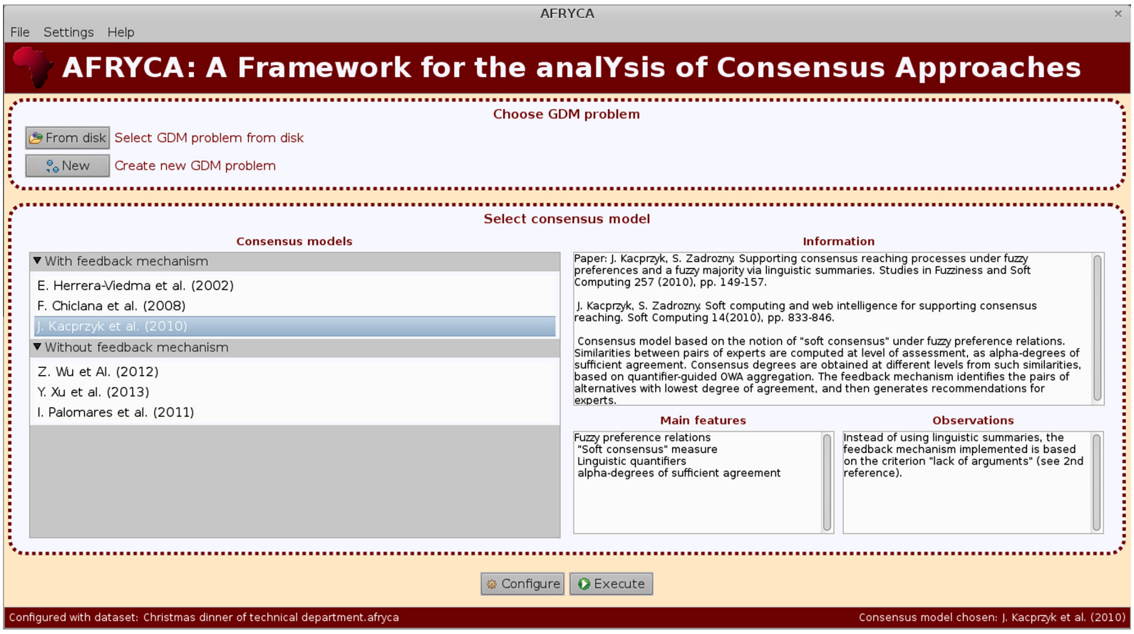

Fig. 17. Main interface of AFRYCA for the selection of a GDM problem and consensus model.

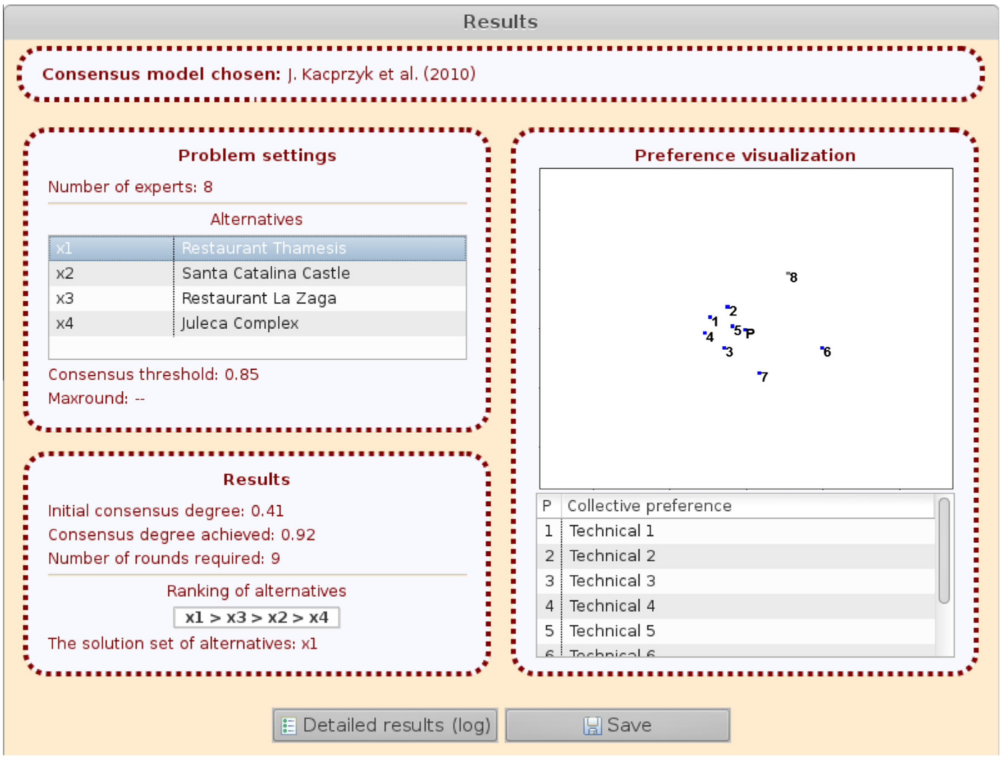

Fig. 18. Interface of results in AFRYCA.

of the consensus models integrated in the analysis framework [13,16,20,21,28,63,68], during the resolution of GDM problems with four different groups of experts.

Let us suppose a company composed of 32 employees, divided into four departments of equal size: Technical Department, $E_T = \{e_{T1}, \ldots, e_{T8}\}$, Human Resources Department,

$E_H = \{e_{H1}, \ldots, e_{H8}\}$, Marketing Department, $E_M = \{e_{M1}, \ldots, e_{M8}\}$ and Sales Department, $E_S = \{e_{S1}, \ldots, e_{S8}\}$. Each department plans to celebrate a Christmas dinner separately, hence each group must make a common decision about choosing a restaurant to celebrate their dinner, from amongst four possible alternatives for all of them: $X = \{x_1 : \textit{Restaurant Thamesis}$, $x_2 : \textit{St. Catalina Castle}, x_3 : \textit{Restaurant La Zaga}, x_4 : \textit{Juleca Complex}\}$.

All experts express their preferences as fuzzy preference relations. These preferences are included in four data sets that will be used in this case study: one for each department. The minimum level of agreement required is $\mu = 0.85$ for all groups, and the maximum number of discussion rounds, *Maxround*, will not be taken into account in this case study, therefore all simulations will be carried out without the CRPs ending due to having exceeded the number of discussion rounds permitted.

The case study is divided into three parts: (i) simulation of consensus models with a feedback mechanism, (ii) simulation of consensus models without a feedback mechanism, and (iii) discussion of results. At each stage, the four GDM problems defined above are solved by means of three different consensus models. Then, the results obtained are analyzed and compared.

5.1. Consensus models with a feedback mechanism

The phases of the methodology shown in Section 4.2 to simulate CRPs and analyze the performance of consensus models are carried out for each GDM problem and consensus model separately:

(1) *Defining Framework.*
(2) *Choosing consensus model.*
(3) *Configure parameters of the consensus model and behavior of experts:* Table 2 summarizes the values chosen for parameters that need to be configured by the user of AFRYCA for each consensus model. Further information about such parameters, as well as the rules of the feedback mechanism and operations carried out during the different phases of the CRP, can be found in the reference associated to each model. Regarding the pattern utilized to simulate the behavior of experts in this case study, the degree of acceptance or rejection of recommendations to modify preferences is modeled by means of a Binomial Distribution, and the degree of change applied to accepted recommendations is modeled by means of Negative Binomial Distribution.
(4) *Simulation of the CRP.*
(5) *Analysis of Results:* The results of the performance of the CRP and the solution set of alternatives obtained with each consensus model, are summarized in Table 3. They will be discussed in Section 5.3.

5.2. Consensus models without a feedback mechanism

The previous methodology is applied again to solve the four GDM problems by means of each of the three consensus models without a feedback mechanism, with the only difference being that no experts' behavior needs to be configured for its simulation in the third phase.

(1) *Defining Framework.*

Table 2
Parameters of consensus models with a feedback mechanism.

	Herrera-Viedma et al. [28]	Chiclana et al. [63]	Kacprzyk et al. [68]
Consensus threshold	$\mu = 0.85$	$\mu = 0.85, \theta_1 = 0.75, \theta_2 = 0.8$	$\mu = 0.85$
Quantifier for aggregating information	F_{most}	–	$F_1 = F_2 = F_3 = F_{most}$
Quantifier for $QGNDD_l$	$F_{as\,many\,as\,possible}$	–	–
S_{OWA} OR-LIKE behavior	$\beta = 0.8$	–	–
S_{OWA} OR-LIKE behavior	$\beta = 0.8$	–	–
Recommendation rule in feedback mechanism	–	–	Lack of arguments [13]

Table 3
Results of the GDM problem resolution for consensus models with feedback mechanism.

	Herrera-Viedma et al. [28]	Chiclana et al. [63]	Kacprzyk et al. [68]
Technical Dept. (E_T)			
Initial consensus degree	0.79	0.77	0.41
Number of rounds	2	15	9
Final consensus degree	0.85	0.85	0.92
Ranking	$x_1 \succ x_3 \succ x_2 \succ x_4$	$x_1 \succ x_3 \succ x_2 \succ x_4$	$x_1 \succ x_3 \succ x_2 \succ x_4$
Alternative/s chosen	x_1	x_1	x_1
Human Res. Dept. (E_H)			
Initial consensus degree	0.76	0.69	0.1
Number of rounds	4	15	20
Final consensus degree	0.88	0.86	0.92
Ranking	$x_1 \succ x_3 \succ x_2 \succ x_4$	$x_1 \succ x_2 \sim x_3 \succ x_4$	$x_1 \succ x_2 \succ x_3 \succ x_4$
Alternative/s chosen	x_1	x_1	x_1
Marketing Dept. (E_M)			
Initial consensus degree	0.78	0.63	0.11
Number of rounds	7	24	26
Final consensus degree	0.86	0.85	0.86
Ranking	$x_3 \succ x_1 \succ x_2 \succ x_4$	$x_1 \sim x_3 \succ x_2 \succ x_4$	$x_1 \succ x_3 \succ x_2 \succ x_4$
Alternative/s chosen	x_3	x_1, x_3	x_1
Sales Dept. (E_S)			
Initial consensus degree	0.71	0.61	0.09
Number of rounds	7	26	25
Final consensus degree	0.88	0.85	0.89
Ranking	$x_3 \succ x_1 \succ x_2 \succ x_4$	$x_1 \sim x_3 \succ x_2 \succ x_4$	$x_3 \succ x_2 \succ x_1 \succ x_4$
Alternative/s chosen	x_3	x_1, x_3	x_3

Table 4
Parameters of consensus models without a feedback mechanism.

	Wu et al. [20]	Xu et al. [21]	Palomares et al. [16]
Consensus threshold	$\mu = 0.15$	$\mu = 0.2, \lambda = 0.15$	$\mu = 0.85$
Normalized weights of experts	$w_i = 1/8, i = 1, \ldots, 8$	$w_i = 1/8, i = 1, \ldots, 8$	–
Updating coefficient	$\eta = 0.8$	–	–
Choice of aggregation operator	–	–	Arithmetic mean
Degree of change on assessments	–	–	0.05

(2) *Choosing consensus model.*
(3) *Configure parameters of the consensus model:* The values chosen for parameters that require configuration in AFRYCA for each consensus model are shown in Table 4. Notice that the consensus thresholds in [20,21] are distance-based thresholds, i.e. in this case consensus indices *below* these thresholds represent a satisfactory level of agreement, hence the values assigned to them are equal to $1 - \mu = 0.15$.
(4) *Simulation of the CRP.*
(5) *Analysis of Results:* Table 5 shows the results obtained from conducting the CRP with each consensus model and applying an alternatives selection process. In order to facilitate the comparison of consensus models, the consensus degrees shown in the table for the models of Wu et al. and Xu et al. are given by $1 - GCI$, because these models utilize group and individual distance-based consensus indices (denoted as GCI and ICI respectively, as shown in Section 3.3). The consensus degrees depicted in the table for the model of Wu et al. correspond to the ICI of the most distant expert in the group, i.e. $1 - \max_i ICI(P_i)$. The results are described in Section 5.3.

5.3. Discussion of the experimental study

Once the results of the experimental study have been set out, they are briefly discussed and analyzed, regarding their convergence towards agreement and the solution achieved.

From results of simulation with the consensus models with feedback mechanism (Section 5.1, Table 3), it can be observed that:

1. **Convergence**
   (a) The consensus model of Herrera-Viedma et al. presents a significantly higher convergence towards consensus for all the GDM problems, i.e. a lower number of consensus rounds are necessary to achieve the required level of agreement, $\mu = 0.85$.
   (b) The consensus model of Chiclana requires a large number of rounds to reach consensus, due to the values chosen for intermediate consensus thresholds θ_1 and θ_2, and the nature of its adaptive feedback mechanism, which generates a much lower amount of advice when the consensus degree exceeds θ_1.
   (c) Consensus degrees are much lower in the model of Kacprzyk et al., due to its similarity measure being based on α-degrees of sufficient agreement (see Eq. (2)), which is a rather strict measure.
2. **Solution**: The ranking of alternatives is very similar in the groups of experts belonging to the Technical and Human Resources Departments, with x_1 being the alternative chosen in both of them, regardless of the consensus model utilized. In the Marketing and Sales departments, either x_1 or x_3, or both of them, can be chosen as the solution to the GDM problem, depending on the model used.

Regarding the results of simulation with the consensus models without feedback mechanism (Section 5.2, Table 5), it can be observed that:

Table 5
Results of the GDM problem resolution for consensus models without feedback mechanism.

	Wu et al. [20]	Xu et al. [21]	Palomares et al. [16]
Technical Dept. (E_T)			
Initial consensus degree	0.7	0.84	0.77
Number of rounds	10	3	6
Final consensus degree	0.86	0.9	0.85
Ranking	$x_1 \succ x_3 \succ x_2 \succ x_4$	$x_1 \succ x_3 \succ x_2 \succ x_4$	$x_1 \succ x_3 \succ x_2 \succ x_4$
Alternative/s chosen	x_1	x_1	x_1
Human Res. Dept. (E_H)			
Initial consensus degree	0.67	0.79	0.69
Number of rounds	16	3	10
Final consensus degree	0.85	0.87	0.85
Ranking	$x_1 \succ x_3 \succ x_2 \succ x_4$	$x_1 \succ x_3 \succ x_2 \succ x_4$	$x_1 \succ x_2 \succ x_3 \succ x_4$
Alternative/s chosen	x_1	x_1	x_1
Marketing Dept. (E_M)			
Initial consensus degree	0.41	0.75	0.63
Number of rounds	19	4	14
Final consensus degree	0.86	0.89	0.86
Ranking	$x_1 \succ x_3 \succ x_2 \succ x_4$	$x_3 \succ x_1 \succ x_2 \succ x_4$	$x_3 \succ x_1 \succ x_2 \succ x_4$
Alternative/s chosen	x_1	x_3	x_3
Sales Dept. (E_S)			
Initial consensus degree	0.46	0.73	0.60
Number of rounds	20	4	12
Final consensus degree	0.85	0.87	0.86
Ranking	$x_3 \succ x_1 \succ x_2 \succ x_4$	$x_3 \sim x_2 \succ x_1 \succ x_4$	$x_3 \succ x_1 \succ x_2 \succ x_4$
Alternative/s chosen	x_3	x_3	x_3

1. **Convergence**
   (a) The convergence towards consensus is higher in the model of Xu et al., due to the fact that the identified assessments are directly updated with the value of the collective preference (see Fig. 12), therefore experts' preferences may experience significant changes in a single round.
   (b) The consensus model of Wu et al. applies small changes to preferences at each round (since $\eta = 0.8$ and the closer η is to 1, the smaller the changes applied [20]), hence its lower convergence.
   (c) The model of Palomares et al. also presents a lower convergence, because it has been applied with a low degree of autonomous change (increase/decrease) to assessments, 0.05.
2. **Solution**: x_1 is the best alternative at the Technical and Human Resources Departments, x_3 is the best alternative at the Sales Department, and either x_1 or x_3 could be the chosen alternative at the Marketing Department, depending on the consensus model.

We draw the following conclusions from the experimental case of study conducted:

- A similar solution is obtained at each group, regardless of the consensus model used for simulation: similar consensus degrees have been achieved, with slight differences in the alternative/s chosen as solution to the GDM problem.
- The main distinguishing element amongst the performances of consensus models, is the convergence that each one presents. Such a convergence is evaluated as the number of iterations or discussion rounds carried out before reaching a sufficient consensus degree. This could be an important factor for groups of experts, when they have to choose the most suitable consensus model in terms of usability.

6. Lessons learnt and future directions

The simulation of CRPs with AFRYCA provides multiples advantages and possibilities, some of which are:

- The framework makes it possible to simulate the resolution of a GDM problem under different consensus models, provided that they are suitable for dealing with such types of problems (e.g. consensus models for GDM problems with fuzzy preference relations). Thus, a decision maker, i.e. a person responsible for making the group decision, is able to study the performance and results obtained with each model.
- For a specific problem and consensus model, AFRYCA offers the possibility of investigating the different settings of such a model, based on the parameters or operators defined in it. Moreover, for those models with a feedback mechanism, the problem might be simulated under different patterns of expert behavior, in order to observe the effect of considering different types of behavior in the simulation.
- Although the decision group may prefer to conduct a real CRP, AFRYCA could provide them with a rough idea a priori about the performance of results that would be obtained, taking into account the initial preferences of experts and defining the appropriate problem settings that would reflect the real context of the problem.
- The experimental study presented has not focused on the use of different representational formats (e.g. linguistic preferences) to assess alternatives, but it is possible to implement and utilize any other existing types of preferences or representational formats in AFRYCA, for simulation purposes.

Six consensus models have been implemented in AFRYCA so far. Nevertheless, we note again that the architecture of the framework is designed to allow the inclusion of new consensus models (based on other types of preferences, information domains or even focused on MCGDM problems), as well as the further comparison between new models introduced and the existing ones.

Multiple proposals of consensus models have been presented in the specialized literature without showing a comparison with other existing models, hence their usefulness and main contributions are not justified properly. AFRYCA enables the implementation and analysis of these new proposals to find out their main contributions, with respect to the already existing ones.

Future work on extending the functionalities of AFRYCA, will mainly be oriented towards the definition of new metrics to measure the performance of a CRP. Such metrics would evaluate not only the discussion process itself, but also the quality of the collective solution achieved (in terms of its degree of acceptance by each member of the group, for instance), with the aim of facilitating a more comprehensive comparative study amongst different consensus models. This is currently one of the most important challenges in consensus: defining good performance measures would make it possible to evaluate the real usefulness of both new and existing proposals in the future.

7. Concluding remarks

Consensus has become a prominent research area in the field of group decision making. A large number of approaches to support consensus reaching have been proposed – and continue to be proposed – by a variety of authors.

In this paper, we have presented a taxonomy of existing consensus models for group decision making problems defined in a fuzzy context, which categorizes a number of consensus models based on their main characteristics, e.g. the type of information fusion techniques utilized to measure consensus in the group, or the procedures applied to increase the level of agreement throughout the discussion process. Besides characterizing a large number of existing consensus models, the taxonomy would also be useful to determine which could be considered for comparison with a new proposal, based on its characteristics and taking into account the taxonomy structure. Comparative studies are necessary to analyze the real capabilities of new proposals, instead of undertaking straightforward consensus exercises with them directly.

We have also presented a prototype of simulation-based analysis framework called AFRYCA, for the simulation of group decision making problems under consensus, by means of implementations of different existing consensus models in the literature. An experimental study has been shown to illustrate the usefulness of AFRYCA. To do this, six consensus models have been implemented and utilized in the study, based on the use of fuzzy preference relations to represent and manage preferences. As a result of the study conducted with AFRYCA, we suggest some future directions in the research topic of consensus: (i) the importance of comparing new proposals with existing ones, in order to show their contributions and (ii) the definition of new performance measures for consensus reaching processes, as a major challenge in the topic.

Finally, some recent approximations for consensus reaching consider different perspectives, e.g. agent-based consensus support systems [75], consensus models for large-scale group decision making problems [**?**,94], etc. These works could also be considered for their simulation in the framework.

Acknowledgments

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Capítulo 5

Conclusiones y Trabajos Futuros

Este capítulo cierra la memoria de investigación revisando las diferentes conclusiones obtenidas de las propuestas que se han realizado en la misma y exponiendo las líneas de investigación sobre trabajos futuros que podrían realizarse partiendo de los resultados presentados en ella. Finalmente, se indican el conjunto de publicaciones derivadas de la investigación realizada y el software registrado.

5.1. Conclusiones

A diario, los individuos se enfrentan a múltiples problemas de decisión en los que deben elegir una alternativa o conjunto de alternativas de entre un conjunto dado. La TD se centra en el estudio de estos problemas con objeto de proporcionar diferentes métodos y herramientas que ayuden a tomar decisiones difíciles y complejas.

A menudo, los problemas de TD están definidos en contextos de incertidumbre de carácter no probabilístico que hacen que sea apropiado emplear términos lingüísticos para valorar las alternativas del problema. La lógica difusa y el enfoque lingüístico difuso brindan un conjunto de herramientas que permiten modelar y gestionar la incertidumbre por medio de variables lingüísticas, proporcionando una mejora en la flexibilidad y fiabilidad de los modelos de decisión.

El uso de variables lingüísticas en los procesos de TD o TDL, ha proporcionado buenos resultados para manejar la incertidumbre envuelta en el problema. Sin embargo, en la actualidad no existe ningún software para llevar a cabo la resolución de problemas de TDL de un modo simple y automatizado lo cual, frena la aplicación de los modelos de TDL en los problemas del mundo real.

Para cubrir la carencia de herramientas software en TDL mencionada, en esta memoria de investigación se ha realizado un estudio de diferentes modelos de TDL para tratar con problemas de TD bajo incertidumbre en marcos de decisión lingüísticos y complejos, con objeto de proporcionar una herramienta genérica y extensible que pueda ser utilizada en diferentes ámbitos de aplicación. Fruto de ese estudio, se ha propuesto una herramienta software denominada FLINTSTONES, la cual integra el modelo lingüístico 2-tupla y sus extensiones para tratar con problemas de TD bajo incertidumbre en marcos de decisión lingüísticos y complejos.

La elección de dicho modelo viene motivada por tratarse de un modelo capaz de realizar los procesos de computación con palabras necesarios para resolver los problemas de TDL de forma precisa. Además, han sido implementadas sus extensiones, ya que permiten resolver problemas de TD en marcos de decisión complejos como son los marcos multigranulares, heterogéneos o no balanceados. Para conseguir que la herramienta sea fácilmente extensible, ha sido diseñada en base a un esquema flexible unificado de resolución que permite adaptar los modelos de TDL existentes en la literatura y acoplar nuevos modelos.

Así, la propuesta en esta memoria de investigación de la herramienta FLINTSTONES, proporciona una completa suite para la resolución de problemas de TD en marcos de decisión lingüísticos y complejos, diseñada para ser genérica, fácil de utilizar y sencilla de ampliar. Adicionalmente a la propuesta de la herramienta, se ha realizado su registro como software en el Registro de la Propiedad Intelectual y desarrollado un sitio web[1] para darla a conocer y fomentar su uso. Desde dicho

[1] http://sinbad2.ujaen.es/flintstones

sitio web es posible descargar la herramienta así como acceder a una multitud de recursos relacionada con la misma.

Para demostrar el gran valor de la herramienta FLINTSTONES para aplicar los modelos de TDL en diferentes ámbitos, en esta memoria se ha propuesto realizar el desarrollo de dos herramientas software que permiten simplificar y automatizar la resolución de problemas del mundo real empleando la base de componentes de FLINTSTONES, la primera de ellas destinada a la evaluación sensorial del aceite de oliva virgen y la segunda a la selección de empresas para formar parte de un parque tecnológico.

Para la primera aplicación, centrada en la evaluación sensorial del aceite de oliva virgen, se ha propuesto un nuevo modelo de evaluación sensorial con un nuevo conjunto de términos lingüísticos no balanceado, el cual permite recoger de forma adecuada las intensidades de los atributos sensoriales por los expertos, para así clasificar correctamente las muestras según las categorías comerciales establecidas por el Consejo Oleícola Internacional. Basándonos en FLINTSTONES, se ha implementado una herramienta software que implementa el modelo de evaluación sensorial propuesto y que permite, a partir de las valoraciones expresadas por los expertos, obtener la clasificación de una muestra. Cabe mencionar que la herramienta es capaz de emplear diferentes conjuntos de términos lingüísticos no balanceados, no quedando vinculada a un conjunto concreto.

Para la segunda aplicación, centrada en la selección de empresas para formar parte de un parque tecnológico, se ha propuesto la definición de un proceso de selección basado en un nuevo modelo TOPSIS difuso, capaz de tratar con problemas de TD definidos en marcos heterogéneos en los que pueden emplearse valores numéricos, términos lingüísticos y expresiones lingüísticas comparativas basadas en conjuntos de términos lingüísticos difusos dudosos. Empleando FLINTSTONES, se ha desarrollado una herramienta software que implementa el nuevo modelo y que permite llevar a cabo todas las fases del proceso de selección, incluyendo además un análisis sensitivo de los resultados obtenidos. Además, para facilitar la recogida de las valoraciones, se ha desarrollado *Flintstones Gathering Cloud*, una aplicación rica de internet que permite que los expertos expresen sus valoraciones de forma remota y distribuida.

Adicionalmente a estas propuestas, nuestra investigación también se ha centrado en los problemas de TDG en los que se lleva a cabo un proceso de consenso con objeto de encontrar una solución colectiva que satisfaga a todos los individuos que participan en el problema. De modo similar a la situación existente en TDL, nos encontramos con que el creciente número de modelos de procesos de consenso para TDG bajo incertidumbre propuesto, unido a la ausencia de herramientas que permitan analizar su comportamiento, hace que sea complicado encontrar el modelo más adecuado para un problema determinado.

En vista de esta problemática, en esta memoria de investigación se ha propuesto un framework de simulación denominado AFRYCA[2], el cual facilita el análisis de los procesos de consenso y la realización de estudios comparativos entre modelos. Al igual que FLINTSTONES, AFRYCA ha sido diseñado de modo que la integración de nuevos modelos de procesos de consenso resulte un proceso sencillo.

En esta memoria hemos proporcionado un conjunto de herramientas software para TDL y procesos de consenso de gran valor para el análisis, la investigación y la aplicación de los resultados teóricos obtenidos. Como podemos observar, todos los objetivos que perseguíamos al inicio de esta investigación han sido alcanzados a través de las propuestas presentadas en esta memoria.

5.2. Trabajos Futuros

En base a la investigación realizada, proponemos a continuación las líneas de los trabajos futuros en los que estamos trabajando a partir de los resultados de este trabajo de investigación.

[2]http://sinbad2.ujaen.es/afryca

Los trabajos futuros que proponemos están enfocados a la ampliación de las propuestas presentadas, en aras de incrementar su funcionalidad, su difusión y su promoción:

- Incorporar en la suite FLINTSTONES nuevos modelos de TD y operadores de agregación así como funcionalidades adicionales con objeto de dar soporte a una mayor variedad de problemas de TD y casuísticas diferentes.
- Aumentar la funcionalidad del componente de análisis sensitivo desarrollado en la propuesta presentada en la Sección 4.3 para la selección de empresas en un parque tecnológico, a fin de analizar con FLINTSTONES la robustez de los resultados obtenidos en cualquier problema de TD.
- Desarrollar nuevas aplicaciones basadas en FLINTSTONES para problemas de decisión existentes en diversos ámbitos como calidad del servicio en red [39] o políticas energéticas [35] para los que, actualmente, no existe ninguna solución software disponible.
- Definir diferentes métricas que posibiliten el análisis de los procesos de consenso y el estudio comparativo entre ellos.
- Aumentar la funcionalidad del framework AFRYCA, permitiendo llevar a cabo desde el mismo la simulación de un mayor número de modelos de procesos de consenso, así como el análisis de su comportamiento.
- Continuar con el proceso de registro del framework AFRYCA para que sea reconocida su autoría.

5.3. Publicaciones Derivadas y Software Registrado

Para finalizar, presentamos la lista de publicaciones que han derivado de los resultados presentados en esta memoria así como el software que ha sido registrado:

- *Publicaciones:*

- En Revistas Internacionales Indexadas:
  - F. J. Estrella, M. Espinilla, F. Herrera, L. Martínez, FLINTSTONES: A fuzzy linguistic decision tools enhancement suite based on the 2-tuple linguistic model and extensions. Information Sciences, vol. 280, pp. 152-170, 2014.
  - F. J. Estrella, M. Espinilla, L. Martínez, Fuzzy Linguistic Olive Oil Sensory Evaluation Model based on Unbalanced Linguistic Scales. Journal of Multiple-valued Logic and Soft Computing, vol. 22, pp. 501-520, 2014.
  - F. J. Estrella, S. Çevik, R. Rodríguez, B. Öztayşi, L. Martínez, C. Kahraman, Selecting firms for University Technoparks: A Hesitant Linguistic Fuzzy TOPSIS model, Computers & Industrial Engineering, sometido a revisión, 2015.
  - I. Palomares, F. J. Estrella, L. Martínez, F. Herrera, Consensus under a Fuzzy Context: Taxonomy, Analysis Framework AFRYCA and Experimental Case of Study. Information Fusion, vol. 20, issue November 2014, pp. 252-271, 2014.
- En Congresos Internacionales:
  - F.J. Estrella, I. Palomares, L. Martínez, A Novel Distance-based Metric to Evaluate the Solution for Group Decision Making Problems under Consensus, 16th World Congress of the International Fuzzy Systems Association (IFSA) and 9th Conference of the European Society for Fuzzy Logic and Technology (EUSFLAT) (IFSA-EUSFLAT 2015). Aceptado.
  - F.J. Estrella, R. Rodríguez, L. Martínez, A Hesitant Linguistic Fuzzy TOPSIS Approach Integrated into FLINTSTONES, 16th World Congress of the International Fuzzy Systems Association (IFSA) and 9th Conference of the European Society for Fuzzy Logic and Technology (EUSFLAT) (IFSA-EUSFLAT 2015). Aceptado.

        - F.J. Estrella, M. Espinilla, L. Martínez, Fuzzy Linguistic Decision Tools Enhancement Suite to Solve Linguistic Decision Making Problems, 11th International FLINS Conference on Decision Making and Soft Computing (FLINS2014), João Pessoa (Paraíba), Brazil. August 17-20th, 2014.
        - M. Espinilla, F.J. Martínez, F.J. Estrella, Integration of dependent features on sensory evaluation processes, 2014 Joint Rough Set Symposium, Granada-Madrid (Spain). July 9-13th, 2014.
        - F.J. Estrella, R.M. Rodríguez, M. Espinilla, L. Martínez, On the use of Hesitant Fuzzy Linguistic Term Set in FLINTSTONES, 2014 IEEE World Congress on Computational Intelligence, Beijing (China). July 6-11th, 2014.
        - M. Espinilla, F.J. Estrella, L. Martínez, A New Unbalanced Linguistic Scale for the Classification of Olive Oil based on the Fuzzy Linguistic Approach, 7th International Conference on Intelligent Systems and Knowledge Engineering (ISKE 2012). Advances in Intelligent and Soft Computing, Beijing (China). December 15-17th, 2012.
    - En Congresos Nacionales:
        - F.J. Estrella, M. Espinilla, L. Martínez, FLINTSTONES: Una suite para la toma de decisiones lingüísticas basada en 2-tuplas lingüísticas y extensiones, XVII Congreso Español sobre Tecnologías y Lógica Fuzzy (ESTYLF 2014), Zaragoza (Spain). February 5-7th, 2014.

- *Software registrado:*
    - Suite software FLINTSTONES:
        - Registro general de la propiedad intelectual en la comunidad autónoma de Andalucía. Número de asiento registral 04/2014/10539.

Apéndice A

English Summary

This appendix covers an English summary of the thesis entitled: *Herramientas y Utilidades Software de Apoyo a la Toma de Decisión Lingüística y a Procesos de Consenso Bajo Incertidumbre (Software tools to support linguistic decision making and consensus processes under uncertainty)*, which is written in English language, as partial fulfillment for obtaining the International Ph.D.

Firstly, a brief introduction to the research topic and a motivation for the research conducted is shown. The objectives established in such research are then exposed and the structure of chapters that compose this research memory is described. After that, a summary of the research proposals of this memory is presented. Finally, some conclusions, future works and publications related to this research are pointed out.

A.1. Motivation

In our daily life, humans are continually exposed to different situations in which we must choose an option or option set to solve a given problem. Ideally, the selection of the optimal option set is carried out after assessing each option based on all related criteria. However, it is not always possible to conduct a rational and objective process, since there are external and subjective factors which affect the decision [4, 60]. For this reason, decision theory [31, 120], which is dedicated to the study of decision processes, has become a prominent research topic in recent decades.

In general, *Decision Making* (DM) can be defined as the process by which we select the best alternative or alternative set of a given set [20, 107]. DM problems can take place in different contexts or decision environments [31, 120]:

1. *Certainty environment*: All the elements involved in the problem are accurately known.
2. *Risk environment*: At least one of the elements involved in the problem depends on chance.
3. *Uncertainty environment*: At least one of the elements involved in the problem is not exactly known, having an incomplete, vague or imprecise information about it.

In DM problems defined in uncertainty environments, since the uncertainty has non-probabilistic nature because it is related to the vagueness of the information involved in the problem, it is common that the information are modeled by means of linguistic terms [90]. Fuzzy logic and fuzzy linguistic approach provide tools to model and manage such uncertainty by means of linguistic variables [141], which improving the flexibility and reliability of the decision models. The use of linguistic variables in DM processes or *Linguistic Decision Making* (LDM) has provided good results in different areas such as sensory evaluation [83, 85, 86, 112], clinical diagnostic [24] or systems theory [13, 101] among others [3, 6, 8–10, 35, 39, 40, 50, 51, 54–59, 81, 84, 89, 118, 119, 134, 139].

Currently there are software tools that can carry out DM processes in certainty environments and risk environments [62, 138], however, there is no software tool to solve problems defined in uncertainty environments using LDM in a simple and automated way. This fact provokes that the resolution of this type of problems is addressed by traditional methods such as spreadsheets, forms or text documents, which makes the process a slow and error-prone task, limiting the dissemination of the theoretical results and their application in real-world problems.

Another issue to consider in this memory occurs in *Group Decision Making* (GDM) [5] problems. Usually, for reaching a solution to DM problem is carried out a selection process in which, experts' assessments are aggregated and, based on the obtained values, an alternative or alternative set is selected as the solution to the problem.

Although an alternative selection process is enough in many cases, in a GDM problem, in which should be considered the preferences of several experts, it may be necessary to conduct an additional process to reach an agreement between experts in order to ensure that the obtained solution is satisfactory for all of them. To achieve the agreement between experts, a possible option is to carry out a *consensus reaching process* or *consensus process*. A consensus process is an iterative process which seeks to make experts' preferences closer to each other before obtaining a solution to the DM problem [14, 88, 113].

The study of consensus processes in GDM problems under uncertainty environments has gained great research interest in recent years, having been proposed in scientific literature a large number of consensus process models [11, 45, 52, 53, 56, 72, 91, 99, 100, 113, 128, 131]. However, the large number of available models, the absence of a framework that characterizes them, and the lack of software tools to analyze a consensus process imply a difficult process to select the most appropriate model for a particular problem.

As in LDM, even with numerous theoretical results that allow dealing with multiple DM problems under uncertainty in the real world, the lack of software tools based on these models makes difficult the use of them in the resolution of real-world problems. The need of having software tools to apply LDM in real-world problems and to analyze consensus processes, motivates the research conducted in this memory, which has focused on overcoming these challenges:

- The lack of software tools for LDM hinders its application in real-world problems. Therefore it is necessary to provide software tools to solve LDM problems applying the theoretical results.

- The large number of consensus process models provokes a complex selection of the most appropriate model to use in a specific GDM problem. Therefore, it is necessary to offer software tools to analyze consensus processes in order to facilitate the achievement of better solutions.

A.2. Objectives

Based on the motivation and considerations raised in the previous section, the purpose of this research is focused on the proposal of software tools for LDM and consensus processes under uncertainty to be useful for analysis, research and application of theoretical results obtained in real-world DM problems.

Based on this purpose the following objectives are considered:

1. To study the LDM models presented in the literature, their application in real-world problems and to analyze the concepts of DM processes that allow modeling the resolution of LDM problems regardless of the context in which they are defined.

2. To develop a software tool for LDM based on an unified flexible resolution approach to deal with DM problems under uncertainty defined in linguistic and complex contexts. The tool should allow to adapt and to implement easily LDM models in order to integrate new models and hence allows to apply these models in real-world problems.

3. To apply the proposed software in the resolution of real-world DM based problems to speed up and to automate the processes involved in such problems, in which information modeling is a tedious and error-prone task.

4. To develop a software tool to simulate consensus processes in GDM problems defined in uncertainty environments to analyze the behavior of these processes. The tool should facilitate carry out comparative studies between different consensus process models by its characteristics. Additionally, the software tool must be able to be extended with new consensus process models.

A.3. Structure

In order to achieve the objectives previously formulated, and in accordance with the article 23, point 3, of the current regulations for Ph.D. studies in the University of Jaén, corresponding to the program established in the RD. 99/2011, this research memory is presented as compendium of articles published by the Ph.D. candidate.

Such publications constitute the nucleus of the thesis, and they correspond to three scientific articles published in International Journals indexed by the *JCR* (Journal Citation Reports) database, produced by *ISI* (Institute for Scientific Information). In addition, another article submitted to revision in an International Journal indexed by JCR at the time of finishing this memory has also been included.

Therefore, the memory is composed of four publications, three of them published in prestigious journals. It is divided into five chapters, but the English summary just includes the discussion of main research findings related to them as well as some concluding remarks and future works. Therefore, the research memory is structured as follows:

- **Chapter 1**: It presents a general introduction to the research problem addressed in this research memory and the objectives to be achieved, followed by the structure of chapters that compose this research memory.
- **Chapter 2**: It reviews the theoretical concepts that have been used in our proposals. First, the basics of LDM are reviewed to then conduct a review of the used models in the resolution of such problems herein, the 2-tuple linguistic

model and its extensions for decision problems defined in complex decision frameworks. Finally, the basic concepts related to consensus processes in GDM are introduced, together as a classification for such processes.

- **Chapter 3**: It presents a summary of the research proposals that form this memory showing, for each of them, a brief discussion of the results obtained.

- **Chapter 4**: It constitutes the nucleus of the thesis, and it contains the four obtained publications from this research.

- **Chapter 5**: It discusses the conclusions extracted from this research and some future works.

- **Appendix A**: It presents a English summary of this research memory.

A.4. Discussion of Results

This section presents a summary of the proposals considered in this research memory, presenting for each of them a brief discussion of the obtained research results.

The section is structured around four proposals:

1. *Linguistic Decision Making Software Tool Based on 2-tuple Linguistic Model and its Extensions: FLINTSTONES.*

2. *FLINTSTONES Applications Based on Linguistic Decision Making.* This proposal is divided into two proposals:

   a) *Virgin Olive Oil Sensory Evaluation.*

   b) *Selecting firms for University Technoparks.*

3. *Software Tool for Consensus Processes: AFRYCA.*

A.4.1. Linguistic Decision Making Software Tool Based on 2-tuple Linguistic Model and its Extensions: FLINTSTONES

The 2-tuple linguistic model and its extensions have been successfully applied in different real-world DM problems [35,39,87]. However, there are no software tools to carry out the resolution of LDM problems using these models. Given the absence of software solutions of this type, the development of a software suite for LDM based on 2-tuple linguistic model and its extensions called *FLINTSTONES* (Fuzzy LINguisTic deciSion TOols eNhancEment Suite)[1] has been proposed. FLINTSTONES integrates the 2-tuple linguistic model and its extensions in order to solve decision problems defined in linguistic and complex frameworks, offering linguistic results that facilitate their understandability. Furthermore, the suite is based on an unified flexible resolution approach which allows to adapt general LDM models and add new models. To publicize FLINTSTONES and encourage to its use has been deployed a website[2] that includes documentation of interest and the link to download the software.

The article associated to this proposal is (see Section 4.1):

F. J. Estrella, M. Espinilla, F. Herrera, L. Martínez, FLINTSTONES: A fuzzy linguistic decision tools enhancement suite based on the 2-tuple linguistic model and extensions. Information Sciences, vol. 280, pp. 152-170, 2014.

A.4.2. FLINTSTONES Applications Based on Linguistic Decision Making

In this section, two software tools that have been developed using FLINTSTONES and focused, one of them in the area of sensory evaluation and the other in the selection of firms, are proposed. The aim of these proposals is to demonstrate

[1]Registration of Intellectual Property: Registro general de la propiedad intelectual en la comunidad autónoma de Andalucía. Número de asiento registral 04/2014/10539

[2]http://sinbad2.ujaen.es/flintstones

the suitability of the suite for the creation of derivate applications based on LDM models, as well as to obtain useful software tools in real-world DM problems for which there is no currently available software.

A.4.2.1. Virgin Olive Oil Sensory Evaluation

Sensory evaluation processes [2,27,83,112,115] imply uncertainty and vagueness and, generally, are defined in unbalanced contexts. This proposal focuses on a sensory evaluation process of virgin olive oil defined in an unbalanced context. The quality of a virgin olive oil sample is established by its sensory profile in which each sensory attribute is measured by a trained expert panel in order to classify the virgin olive oil in four categories.

A fuzzy linguistic sensory evaluation model was proposed in [85] to establish the quality category of a virgin olive oil sample through the definition of an unbalanced linguistic term set [42]. However, in the validation of this model was revealed that the unbalanced linguistic term set used does not classify correctly samples that are doubtful between two categories.

Therefore, we propose a new sensory evaluation model for virgin olive oil based on the model presented in [85]. Our model proposes the use of a new unbalanced linguistic term set [42] that can measure adequately experts' perceptions in order to correctly classify doubtful virgin olive oil samples between two categories.

To simplify and automate the virgin olive oil sensory evaluation using the new model, we propose a software tool based on FLINTSTONES that implements the new sensory evaluation model for virgin olive oil and classify a virgin olive oil sample based on the experts' perceptions. Furthermore, in this software tool can be used different unbalanced linguistic term set to carry out the sensory evaluation process, so the software is not limited to a specific unbalanced linguistic term set.

The article associated to this proposal is (see Section 4.2):

F. J. Estrella, M. Espinilla, L. Martínez, Fuzzy Linguistic Olive Oil Sensory Evaluation Model based on Unbalanced Linguistic Scales. Journal of Multiple-valued Logic and Soft Computing, vol. 22, pp. 501-520, 2014.

A.4.2.2. Selecting firms for University Technoparks

A technopark is an innovation center established at an university campus to enhance the collaboration between the university, industry and government [105]. A technopark provides important advantages such as easier access to financial resources, strategic place with easy access to highways or airports, infrastructure support and fast growth for firms. For this reason, a selection process is necessary to choose the best firms for the technopark.

A multicriteria decision analysis process can support such a selection in which multiple conflicting criteria that embrace a wide range of disciplines and commonly defined under uncertainty are evaluated. The interdisciplinary of this complex process does not only implies the necessity of a heterogeneous context of definition [49] in which different types of information can model the knowledge and the related uncertainties, but also makes that involved experts provide vague and imprecise information and hesitate about their assessments. Recently, Rodríguez et al. introduced the concept of Hesitant Fuzzy Linguistic Term Set (HFLTS) [109] that facilitates, when experts hesitate, the elicitation of comparative linguistic expressions close to the expressions used by human beings in decision making and provides a fuzzy modeling for computing with them.

Therefore, we propose a technopark selection process based on a fuzzy TOPSIS [78, 94, 123] method that will be able to deal with problems defined in a heterogeneous context conformed by numerical values, linguistic terms and comparative linguistic expressions based on HFLTS.

To speed up the selection process, we propose a software tool based on FLINTSTONES that integrated this process in order to support the whole selection process from the elicitation of information to the solution process, including sensitive analysis [121]. This software tool is complemented by a *Rich Internet Application* (RIA) proposal called *Flintstones Gathering Cloud* (FGC), which allows users to import and distribute problems created by FLINTSTONES for gathering assessments in a remote, easy and distributed way.

The article associated to this proposal is (see Section 4.3):

F. J. Estrella, S. Çevik, R. Rodríguez, B. Öztayşi, L. Martínez, C. Kahraman, Selecting firms for University Technoparks: A Hesitant Linguistic Fuzzy TOPSIS model, Computers & Industrial Engineering, submitted, 2015.

A.4.3. Software Tool for Consensus Processes: AFRYCA

In a GDM problem it may be necessary to reach agreement among the experts before the selection of alternatives to ensure that the obtained solution is considered valid by all of them. The consensus processes emerge as an additional iterative phase in the processes of GDM focused on finding a satisfactory solution for all experts involved in the problem [14, 88, 113]. In recent years, consensus processes have come to play a prominent role in GDM problems defined in uncertainty environments. So a broad variety of models to carry out the consensus processes in such contexts have been proposed [11, 45, 52, 53, 56, 72, 91, 99, 100, 113, 128, 131].

Due to the large number of consensus process models for GDM problems defined in uncertainty environments, we propose a simulation framework which we have called *AFRYCA* (A FRamework for the analYsis of Consensus Approaches) and which allows to analyze the behavior of consensus process models on GDM problems and to carry out comparative studies between different consensus process models by its characteristics. As FLINTSTONES (see Section A.4.1), the framework AFRYCA has been designed to allow the development of new consensus process models without modifying the existing functionality, and to promote its use, a website for the tool has been deployed[3].

The article associated to this proposal is (see Section 4.1):

I. Palomares, F. J. Estrella, L. Martínez, F. Herrera, Consensus under a Fuzzy Context: Taxonomy, Analysis Framework AFRYCA and Experimental Case of Study. Information Fusion, vol. 20, issue November 2014, pp. 252-271, 2014.

A.5. Conclusions and Future Works

Finally, this section concludes this research memory, reviewing the main proposals and results obtained, and pointing out some future works.

A.5.1. Conclusions

Every day, individuals face multiple decision problems in which they must choose an alternative or alternatives set from a given set. DM focuses on the study of these problems in order to provide methods and tools to make difficult and complex decisions in a rational way.

Often, DM problems are defined in uncertainty environments, being appropriate the use of linguistic terms to assess the alternatives of the problem. Fuzzy logic and fuzzy linguistic approach provide a set of tools to model and manage uncertainty by means of linguistic variables, providing improved flexibility and reliability of decision models.

The use of linguistic variables in the process of DM or LDM, has provided good results to handle the uncertainty involved in the problem. However, currently there is no software tools for resolution of LDM problems in a simple and automated manner, which hampers application of LDM models in real-world problems.

[3]http://sinbad2.ujaen.es/afryca

In order to overcome the lack of LDM software tools mentioned, in this research memory a study of different LDM models to allow dealing with DM problems under uncertainty, defined in linguistic and complex frameworks, has been carried out. As result of this study, we have proposed a software tool based on an unified flexible resolution approach called FLINTSTONES[4], which integrates the 2-tuple linguistic model and its extensions to deal with DM problems under uncertainty defined in linguistic and complex contexts, and which allows adapt existing LDM models and add new models easily, to facilitate the creation of derivate applications for the resolution of real-world DM based problems. In addition to the proposal of the suite, a website[5] has been developed to promote its use. From this website, it is possible to download the tool as well as access to a multitude of resources related to it.

To demonstrate the great value of FLINTSTONES in the application of LDM models in various areas, in this memory has been proposed the development of two software tools that simplify and automate the resolution of two real-world problems, one of them focused on the sensory evaluation of virgin olive oil and the second on the selection of firms to be part of a technology park.

For the first application, which is focused on sensory evaluation of virgin olive oil, a new sensory evaluation model with a new unbalanced linguistic term set have been proposed. The model provide a way to measure accurately experts' perceptions in order to classify virgin olive oil samples. Based on FLINTSTONES, a software tool that implements the sensory evaluation model proposed and allows to obtain the classification of a sample from the assessments provided by experts has been developed. It is noteworthy that the software tool is able to use different unbalanced linguistic term sets, not being tied to a specific set.

For the second application, which is focused on the selection of firms to be part of a technology park, a selection process based on a fuzzy TOPSIS method, able

[4]Registration of Intellectual Property: Registro general de la propiedad intelectual en la comunidad autónoma de Andalucía. Número de asiento registral 04/2014/10539

[5]http://sinbad2.ujaen.es/flintstones

to deal with problems defined in heterogeneous contexts conformed by numerical values, linguistic terms and comparative linguistic expressions based on HFLTS has been proposed. Using FLINTSTONES, a software tool that implements the selection process and allows to perform all the phases of the process, including a sensitivity analysis of the obtained results has been developed. Furthermore, to facilitate the information gathering process, a RIA called *Flintstones Gathering Cloud*, that allows experts to express their opinions in a remote and distributed manner has been developed.

In addition to these proposals, our research has also been focused on GDM problems in which it is conducted a consensus process in order to find a collective solution that satisfies all individuals involved in the problem. Similarly to what happens with LDM models, the increasing number of consensus process models for GDM under uncertainty proposed, and the lack of tools to analyze their behavior, makes it difficult to find the most appropriate model for a given problem. In view of this problem, in this research memory a simulation framework called AFRYCA[6] that facilitates the analysis of consensus processes operation and conduct comparative studies between different consensus process models by its characteristics has been presented. As FLINTSTONES, AFRYCA has been designed in order to its expansion can be performed easily.

In this research memory, we have provided a collection of software tools to LDM and consensus processes which are of great value for analysis, research and application of theoretical results. As we can see, all the objectives indicated at the beginning of this research have been achieved with the proposals presented in this research memory.

[6]http://sinbad2.ujaen.es/afryca

A.5.2. Future Works

Taking into account these research results, our future works are focused on the extension of the proposals presented, in order to increase their functionality, their dissemination and their promotion:

- Incorporate in the suite FLINTSTONES new DM models and new aggregation operators as well as additional features in order to support a wider variety of DM problems and different cases.
- Increase the functionality of sensitivity analysis component developed in the proposal presented in Section 4.3 for selecting firms in a technology park, in order to analyze with FLINTSTONES the robustness of the results of any DM problem.
- Develop new FLINTSTONES based applications for existing decision problems in various areas such as quality of network service [39] or energy policies [35] for which there is currently no software solution available.
- Define different metrics that enable to analyze consensus processes and make comparative studies between them.
- Increase the functionality of the framework AFRYCA, allowing the simulation of more consensus process models and analyzing their behavior.
- Continue the process of framework AFRYCA registering to be recognized its authorship.

A.5.3. Publications and Registered Software

To conclude, we present the list of publications that have derived from the results presented in this research memory as well as the registered software:

- *Publications:*

- In International Journals Indexed:

  - F. J. Estrella, M. Espinilla, F. Herrera, L. Martínez, FLINTSTONES: A fuzzy linguistic decision tools enhancement suite based on the 2-tuple linguistic model and extensions. Information Sciences, vol. 280, pp. 152-170, 2014.
  - F. J. Estrella, M. Espinilla, L. Martínez, Fuzzy Linguistic Olive Oil Sensory Evaluation Model based on Unbalanced Linguistic Scales. Journal of Multiple-valued Logic and Soft Computing, vol. 22, pp. 501-520, 2014.
  - F. J. Estrella, S. Çevik, R. Rodríguez, B. Öztayşi, L. Martínez, C. Kahraman, Selecting firms for University Technoparks: A Hesitant Linguistic Fuzzy TOPSIS model, Computers & Industrial Engineering, submitted, 2015.
  - I. Palomares, F. J. Estrella, L. Martínez, F. Herrera, Consensus under a Fuzzy Context: Taxonomy, Analysis Framework AFRYCA and Experimental Case of Study. Information Fusion, vol. 20, issue November 2014, pp. 252-271, 2014.

- In International Conferences:

  - F.J. Estrella, I. Palomares, L. Martínez, A Novel Distance-based Metric to Evaluate the Solution for Group Decision Making Problems under Consensus, 16th World Congress of the International Fuzzy Systems Association (IFSA) and 9th Conference of the European Society for Fuzzy Logic and Technology (EUSFLAT) (IFSA-EUSFLAT 2015). Accepted.
  - F.J. Estrella, R. Rodríguez, L. Martínez, A Hesitant Linguistic Fuzzy TOPSIS Approach Integrated into FLINTSTONES, 16th World Congress of the International Fuzzy Systems Association (IFSA) and 9th Conference of the European Society for Fuzzy Logic and Technology (EUSFLAT) (IFSA-EUSFLAT 2015). Accepted.

- F.J. Estrella, M. Espinilla, L. Martínez, Fuzzy Linguistic Decision Tools Enhancement Suite to Solve Linguistic Decision Making Problems, 11th International FLINS Conference on Decision Making and Soft Computing (FLINS2014), João Pessoa (Paraíba), Brazil. August 17-20th, 2014.
- M. Espinilla, F.J. Martínez, F.J. Estrella, Integration of dependent features on sensory evaluation processes, 2014 Joint Rough Set Symposium, Granada-Madrid (Spain). July 9-13th, 2014.
- F.J. Estrella, R.M. Rodríguez, M. Espinilla, L. Martínez, On the use of Hesitant Fuzzy Linguistic Term Set in FLINTSTONES, 2014 IEEE World Congress on Computational Intelligence, Beijing (China). July 6-11th, 2014.
- M. Espinilla, F.J. Estrella, L. Martínez, A New Unbalanced Linguistic Scale for the Classification of Olive Oil based on the Fuzzy Linguistic Approach, 7th International Conference on Intelligent Systems and Knowledge Engineering (ISKE 2012). Advances in Intelligent and Soft Computing, Beijing (China). December 15-17th, 2012.

- In National Conferences:
  - F.J. Estrella, M. Espinilla, L. Martínez, FLINTSTONES: Una suite para la toma de decisiones lingüísticas basada en 2-tuplas lingüísticas y extensiones, XVII Congreso Español sobre Tecnologías y Lógica Fuzzy (ESTYLF 2014), Zaragoza (Spain). February 5-7th, 2014.

- *Registered Software:*
  - FLINTSTONES software suite:
    - Registro general de la propiedad intelectual en la comunidad autónoma de Andalucía. Número de asiento registral 04/2014/10539.

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